

# Four-Dimensional Mathematics in the Structure of the Super Universe

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## Abstract

In the previous publication “Four-Dimensional Mathematics Creates the Super Universe”, the model of the Universe is the surface volume of the four-dimensional sphere. The surface of the four-dimensional sphere is a good model for our Universe. In the previous publication, the method to compute the surface volume of the four-dimensional sphere was right, but one thing went astray. The corrected calculus method of this study has been proved to be correct, and it yields the same formulae as the mathematicians have derived. In the present study, a new mathematical method is to construct the four-dimensional cube by using four different construction blocks. In the publication “The Solution to the Dark Energy Mystery in the Universe of Four Distance Dimensions”, the redshift measurements have shown that the structure of our three-dimensional Universe is the surface volume of the four-dimensional sphere. In this study, there was an error in the redshift theory which in the present paper has been corrected. However, the red shift calculations are the same, and also the result is about the same, and the Universe has been calculated to have a very slow expansion. The previous publication: “Theory to the Mystery of Super Massive Black Holes” continues in the present study. In the previous publication, it has been proved that the mysterious dark matter which rotates the spiral galaxies is the super massive Black Holes in the fourth dimension beneath our three-dimensional Universe. This four-dimensional Universe is here called the Super Universe. In the present study, the locations of the Black Holes in the fourth dimension have been calculated, and it reveals the structure of the Super Universe.

## Keywords

Mathematics of the Four-Dimensional Space, Dark Matter, Dark Energy, Expansion of Universe, Big Bang, Four-Dimensional Sphere, Four-Dimensional Matter, Atom Theory

## 1. Introduction

In publication [1], it is calculated that 69% of the total mass energy of the Universe is dark energy that accelerates the Universe. The common knowledge is that the ordinary mass is only about 5%, dark matter that rotates the galaxies comprises 27%, and dark energy that expands the Universe at an accelerating speed is 68%. This evaluation of the dark energy is due to the measurements of the wavelengths of light emitted from distant galaxies. The measurements have indicated that the wavelengths have become longer. The wavelength of light has shifted towards red, and it is inferred that the redshift results from the speed of increasing distances of the far-away galaxies. Therefore, it seems to be that the whole Universe is expanding at an increased rate of acceleration, and the age of the Universe would be  $13.8 \cdot 10^9$  light years. Publication [1] has been presented problems of this model of the Universe: “These twin tensions between expectations and observations, between the early and late universe may reflect some deep flaw in the Standard Model of cosmology.”

In publication [2] “Galaxy Rotation in the Space of Four Distance Dimensions” the dark matter mystery has been solved in the Universe of four distance dimensions. Publication [2] presented a new idea that dark matter is located at the fourth distance dimension in the center of the galaxy. In the same manner as a structure of three dimensions can be drawn in the cross-sections of two dimensions, a structure of four dimensions can also be drawn in the cross-sections of two or three dimensions. Therefore, the determination of the location of dark matter in the fourth dimension is no problem. The study of publication [2] contains the solution to the dark matter mystery of spiral galaxies by using the space of four distance dimensions  $x, y, z, x'$ , in which  $x'$  is the fourth distance dimension. The four-dimensional mass  $M$ , which generates the main gravitation field of the galaxy, is located in the fourth dimension at the distance  $x' = X'$  and the other dimensions are zero  $x = 0, y = 0, z = 0$ . Publication [3] has presented the rotational speed distribution curves of the galaxy NGC 3198. The speed distribution curve of the galactic halo in publication [3] corresponds to the speed distribution curve of the four-dimensional mass in publication [2]. In order to find out how well this four-dimensional model functions, the speed distribution curve of the four-dimensional mass was calculated, and it was fitted into the speed distribution curve of the galactic halo in publication [3]. The conclusion was that the calculated distribution curve of the four-dimensional mass is a good match for the halo curve in publication [3]. The accuracy of the fitting was not very good, but good enough for this conclusion. Furthermore, four rotational speed distribution curves were calculated using different values of the distance  $X'$ , which yielded different values for the maximum radius of the galaxy. In this manner, the different galaxy models of the publication [4] were obtained, and therefore, this solution to the dark matter mystery was proved.

Publication [5] “The Solution to the Dark Energy Mystery in the Universe of Four Distance Dimensions”, solves the mystery of the dark energy by using the

structure of the four-dimensional Universe. In the common theory of the Universe, the redshift of the light wavelength from distant stars indicates the speed of the star. In publication [5], the model of the Universe is the surface volume of the four-dimensional sphere, and this type of structure of the Universe creates the same kind of an accelerating redshift increase which has been measured. The cause of the redshift in this model of the Universe is its structure, and therefore, there is no dark energy accelerating the Universe. The surface of the four-dimensional sphere is a volume, and this volume is a good model for the Universe. In publication [5], the surface volume of the four-dimensional sphere has been explained by a model of four-dimensional cube, within which the forming of surface volume can be easily shown. The four-dimensional cube has a similar surface volume of eight three-dimensional cubes. Therefore, the four-dimensional cube is a model for understanding the structure of the surface volume of the four-dimensional sphere. In publication [5], the construction of the surface volume of four-dimensional cube was shown by the construction of its six side cubes. As model of four-dimensional cube that was good, simple and easy to understand. But it had one shortcoming: This model had only six side volumes, while in reality there are eight side volumes. Therefore, a more complicated model was needed, and that model is in the present study. In publication [5], the equation for the redshift which is caused by the theoretical Universe was solved, and this equation was fitted into the measured redshift in the real Universe. The measured redshift in the real Universe was obtained from the model of the expanding Universe in the publication [6]. A similar model of the Universe has been constructed by NASA, publication [7]. The derivation of the redshift wavelength of light was not correct in publication [5], and it is corrected in the present study. However, the result of the fitting, the redshift wavelength into the theoretical wavelength, was about the same, and so also in the present study, the four-dimensional model of the Universe yielded the Universe that was found to have a slow expansion and a Big Bang which is not very big. Publication [8] "Four-Dimensional Mathematics Creates the Super Universe" continues the study of the dark energy mystery in publication [5]. Publication [8] presents our three-dimensional Universe as the surface volume of the four-dimensional sphere Universe which here is called the Super Universe. The volume of our three-dimensional Universe is finite, and beneath it there is the infinite volume four-dimensional Super Universe.

Two important basic formulae in publications [5] and [8] have been corrected in publication [9]. In publications [5] and [8], the formula for the surface volume of the four-dimensional sphere was derived, and the principle of the derivation was correct, but there was one error which in calculation which has been corrected, and furthermore, the formula for the fourth power volume of the four-dimensional sphere has been also corrected. The surface volume of the four-dimensional sphere is  $2\pi^2 R^3$  in which  $R$  is the radius of the sphere, and the fourth-power volume of the four-dimensional sphere is  $1/2\pi^2 R^4$ . The volume of the Universe has been calculated  $2\pi^2 R^3 = 39.5 \cdot 10^{30} \text{ ly}^3$ .

Publication [10] “Theory to the Mystery of Super Massive Black Holes” continues the dark matter research of publication [2]. Vera Rubin measured rotational speeds of galaxies, publication [4] in 1983, and she found that the masses of galaxies were not enough to produce the measured speeds of rotation. Therefore, it was inferred that there must be unknown matter which mass is many times the masses of known visible and dark matter. The common theory to this dark matter mystery is that it is the result of the gravitational force of unknown fundamental particles, and that these unknown fundamental particles would be located in the halo structures round about the spiral galaxies. Despite of thorough searching the unknown fundamental particles have not been found, and it seems to be that halo theory is very improbable. In any case, the galaxies have ordinary fundamental particles which have an effect on galaxies’ motion dynamics, and these particles may create halo structures, but in any case they have too little mass to be the solution to the dark matter mystery. In publications [2] and [9] the solution to the dark matter mystery of spiral galaxies is the four-dimensional mass in the space of four dimensions  $x,y,z,x'$ , at the distance  $x' = X$ . It is inferred that the four-dimensional mass is a black hole, which generates the main gravitation field of the galaxy. The rotational speed distributions curves of the galaxy NGC 3198 have been presented in publication [3]. The speed distribution curve of the galactic halo in that publication corresponds to the speed distribution curve of the four-dimensional black hole in publication [10]. The speed distribution curve of the black hole is a good match to the halo curve of the publication [3]. Furthermore, in publication [10], the rotational speed distribution curves of the four-dimensional black holes were calculated by using different values of the reduced distance  $X'$ , which yielded at the distance  $X' = 0$ , a black hole of radius  $R = 7.7 \times 10^{17}$  m. By using the relativistic Lorentz transformation, it was shown in this study that a star falling into the four-dimensional black hole remains rotating it at near speed of light, and cannot fall into the actual black hole. This function of the Lorentz transformation was invented by Albert Einstein. It is also shown by calculations that three-dimensional black holes are not real black holes, but they leak out radiation. The real black holes are four-dimensional.

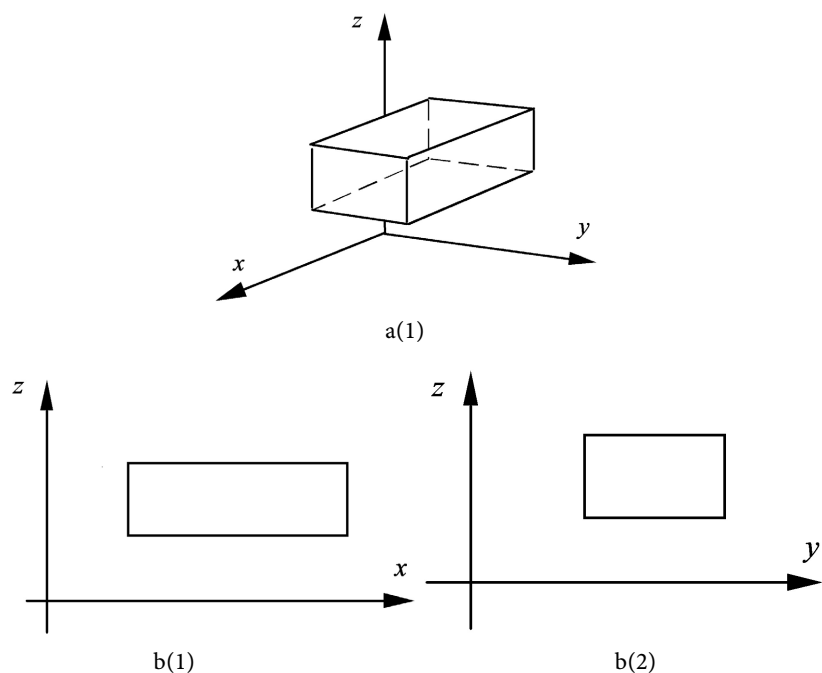
In publication [11], Lior Shamir has studied the effect of the galaxy’s rotational speed on the measured redshifts. The current understanding of the Universe is that the redshift of distant galaxies results from the speed of galaxies. Because the speeds of galaxies computed from the redshift are very high, and the rotational speeds of galaxies compared to that about zero, it can be concluded that the galaxy’s rotational speed has a negligible effect on redshift. However, the measurements of publication [11] have shown consistent redshift effect caused from the galaxy rotation. Therefore, it has been concluded that redshift cannot be caused by galaxy’s speed but that Zwicky’s TL theory may be right, publication [12], and light photon loses its energy in the travel of great distances. But this cannot be the solution of redshift mystery, because in that case the loss of energy (redshift distance curve) should have a constant increase, but it has not. However, the redshift

measurements of publication [11] are a direct observation that the energy of light photon loses its energy in its travel from distant galaxies, and this is also a proof for the energy loss theory of this study in Equation (7) and **Figure 8**.

The aim of the present study is to solve the dark energy mystery, the major problem of the prevailing model of the Universe. According to this model, the dark energy is the energy that expands the Universe at an accelerated speed of expansion. In this model, the redshift of the wavelength of the light photon is caused from the expansion speed of the Universe. In the present study, the cause of the redshift is the loss of the energy, as the light photon travels through the four-dimensional Universe.

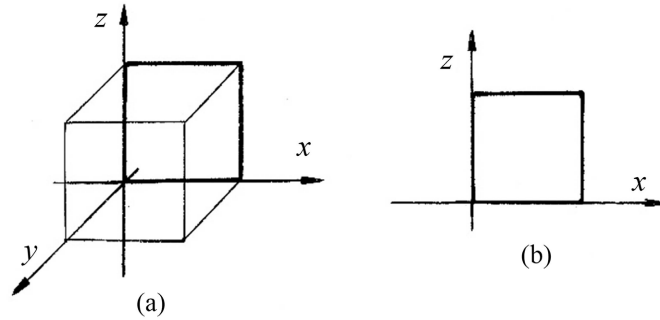
## 2. The Universe in the Four Distance Dimensions

In the following, a simple method has been presented to see the space of four distance dimensions, and to perform calculations in it. In **Figure 1(a)**, a box is drawn in the three-dimensional space  $x,y,z$ . In **Figure 2(b)** left, the same box is drawn in the two-dimensional space  $z,x$  and on **Figure 1(b)** right it is drawn in the two-dimensional space  $y,z$ . There is still a third possible set of coordinates  $x,y$ , but it is not needed to determine the shape of the box. Here, it is seen that a structure of three dimensions can be drawn in a two-dimensional coordination system. In the same manner, a structure of four dimensions  $x,y,z,x'$  can be drawn in the coordination with the three dimensions  $(x,y,z)$ ,  $(y,z,x')$ ,  $(x,z,x')$ ,  $(x,y,x')$ . If the structure is simple, it is possible that only one two-dimensional coordination system is needed to determine the form of the structure.

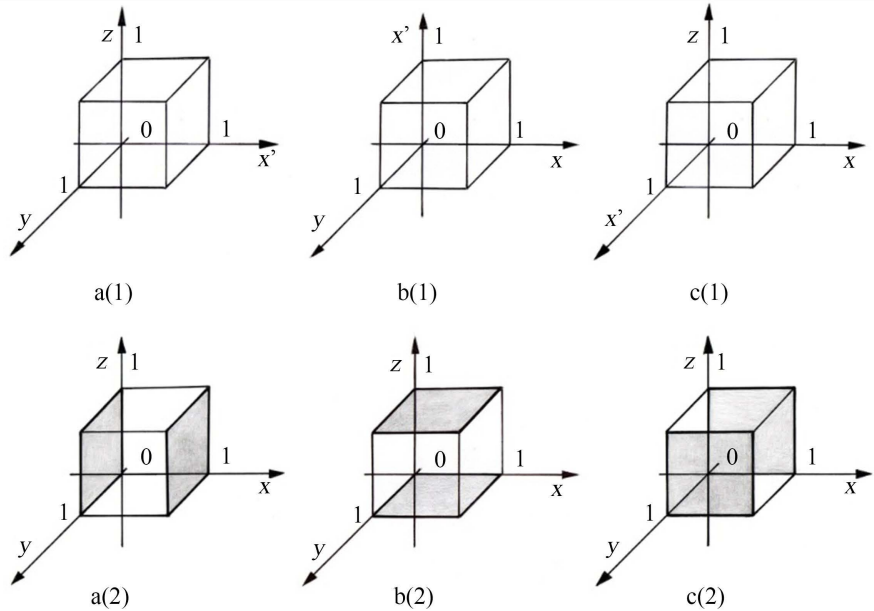


**Figure 1.** A box is drawn in a coordination system of the three-dimensional space  $x,y,z$ ; b(1) the box is drawn in a coordination plane of the two-dimensional space  $x,z$  and; b(2) the box is drawn in the coordination plane  $y,z$ .

The technique which is used in the mathematics of the four-dimensional space is presented in **Figure 2(a)** and **Figure 2(b)**. In **Figure 2(b)** the three-dimensional cube in **Figure 2(a)** is presented so that the volume of the cube in **Figure 2(a)** is the area of the square in **Figure 2(b)**, and the area of the four sides of the cube in **Figure 2(a)** is the four lines in **Figure 2(b)**. The thick lines indicate the area.



**Figure 2.** In (a), there is a cube drawn in a coordination system of the three-dimensional space  $x,y,z$  and, in (b), there is its cross-section on the  $x,z$  plane. The thick-lined square in the cube is the location of the cross-section. The cube in (a) is located in **Figure 3(b)** in the square so that the volume of the cube is the area of the square, and the four sides of the cube, which are at a right angle to the cross-section square, are the four thick lines of the square in (b). This technique is used in the following mathematics of the four-dimensional space.

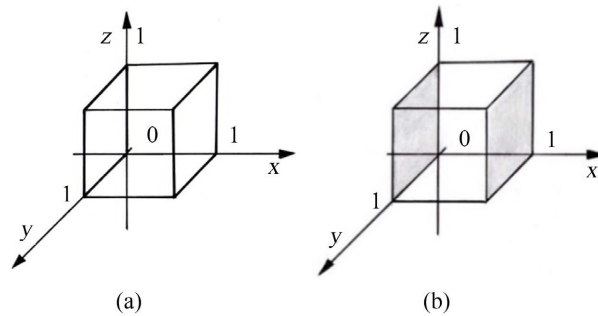


**Figure 3.** The four-dimensional cube can be constructed using the four three-dimensional cubes: a(1), b(1), c(1). The three-dimensional cube a(1) is located at the dark sides of the four-dimensional cube a(2), and the three-dimensional cube b(1) is located at the dark sides of the four-dimensional cube b(2), and the three-dimensional cube c(1) is located at the dark sides of the four-dimensional cube c(2). In this manner, all six sides of the four-dimensional cube have been constructed. The three-dimensional human sense can see this structure as a cube with volume surface sides, but actually it is one end of the four-dimensional cube.

**Figure 3** shows the construction of one end of the four-dimensional cube. The three-dimensional cubes a(1), b(1) and c(1) are the construction blocks of the four-dimensional cube. The three-dimensional cube a(1) is located into the four-dimensional cube a(2) so that  $x$  coordinates are  $x = 0$  and  $x = 1$ . In **Figure 3**, they can be seen as two dark sides of the cube a(2) in which the area of the dark sides indicates the volume and the thick lines round about the dark sides indicate the area. In **Figure 3**, all six side volume cubes of the four-dimensional cube have been constructed in the same manner, and one end of the four-dimensional cube has been completed. The three-dimensional human sense sees this structure as a cube with volume surface sides, and this is an important ability of understanding four-dimensional space. Therefore, it can also be seen that the surface of the four-dimensional sphere is a volume, and in this study, the surface volume of four-dimensional sphere is the model of the Universe. **Figure 4** shows the construction of the whole four-dimensional cube. Except the six side cubes which are constructed in **Figure 3**, the four-dimensional  $x, y, z, x'$  cube has also two other side cubes. In **Figure 4**, the whole four-dimensional cube has been constructed so that the three-dimensional  $x, y, z$  cube in **Figure 4(a)** is located in **Figure 4(b)** into the four-dimensional  $x, y, z, x'$  cube so that  $x' = 0$  and  $x' = 1$ , and they are seen as the two dark sides of the cube in **Figure 4(b)**. The four other sides of the four-dimensional cube in **Figure 4(b)** are the same as on **Figure 4(a)**, so that the four-dimensional cube has eight three-dimensional side cubes. The construction of the whole four-dimensional cube is done so that the three-dimensional  $x, y, z$  cube on **Figure 4(a)** is determined to be  $0 \leq x' \leq 1$  in **Figure 4(b)**.

In the same manner, the  $x, y, z$  section of the four-dimensional  $x, y, z, x'$  sphere has a surface which is the three-dimensional volume. In this study, this type of surface volume is a model of the Universe, and it produces a good fitting with the redshift measurements, proving it to be correct. In **Figure 5**, a four-dimensional sphere has been constructed from an ordinary three-dimensional sphere. The ordinary dimensions are  $x, y, z$  and the fourth-dimension  $x'$ . **Figure 5(a)** is a sphere of ordinary  $x, y, z$  dimensions, and **Figure 5(b)** is a sphere of four dimensions  $x, y, z, x'$  which has been shown as its  $y, z, x'$  section. The cross-sections of the both spheres at the  $y, z$  plane are the same. The fourth-dimension  $x'$  is zero ( $x' = 0$ ) within the whole three-dimensional sphere on **Figure 5(a)**. Therefore, the whole three-dimensional sphere, **Figure 5(a)**, is in the four-dimensional sphere, **Figure 5(b)**, at the cross-section of the  $y, z$  plane, in which the fourth dimension  $x'$  is zero ( $x' = 0$ ). The surface area of the three-dimensional sphere, **Figure 5(a)**, is at the distance  $R$  from the center at all angle values of coordination system (two angles and distance from center  $R$ ). Therefore, the surface area of the three-dimensional sphere, **Figure 5(a)**, is at the circumference line of the  $y, z$  cross-section of the four-dimensional sphere, **Figure 5(a)**. The increased thickness of the circumference line indicates the location of the area of the three-dimensional sphere. In the same manner the volume of the three-dimensional sphere, **Figure 5(a)**, is located at the area of  $y, z$  cross-section of the four-dimensional sphere, **Figure 5(b)**. The For-

mation of the four-dimensional sphere is done by rotation of the circumference line of the  $y,z$  cross-section about the A-B axis in **Figure 5(b)**.



**Figure 4.** The construction of the four-dimensional cube continues. The three-dimensional cube in (a) is located at the dark sides of the four-dimensional cube in (b). The real four-dimensional cube is between dark sides, and its four-dimensional volume is of fourth power, and its three-dimensional volume is infinite. The location of the end of the four-dimensional cube which is constructed in **Figure 3** is so that the left dark side of the cube in (b) is the location of the three-dimensional  $x,y,z$  cube in **Figure 3**. The four sides round the cube are the four sides of the six sides constructed in **Figure 3**. The four-dimensional cube has eight side cubes, six side cubes in **Figure 3** and two side cubes in **Figure 4**.

The calculation method of the surface volume of the four-dimensional sphere in publications [5] and [9] was right, but one important thing went astray. Here is the corrected calculation of the surface volume of the four-dimensional sphere:

1) **Figure 3** and **Figure 4**, the four-dimensional cube has been presented in the three-dimensional space. In the same manner as in **Figure 3** and **Figure 4**, the four-dimensional  $x,y,z,x'$  cube has the surface volume of eight side cubes, the four-dimensional sphere has a similar surface volume.

2) The  $y-z$  cross-sections are the same in **Figure 5(a)** and **Figure 5(b)**.

3) The whole surface area of the three-dimensional sphere in **Figure 5(a)** is located at the circumference of the  $y-z$  cross-section of the four-dimensional sphere in **Figure 5(b)**. This is using the coordination system of two angles and the distance from the center  $R$ , and the technique of **Figure 2**. The thick circumference line indicated area.

4) The area within the length of the circumference line  $Rd\varphi$ , **Figure 6(a)**

$$dA = 2\pi R \cos \varphi \cdot R d\varphi = 2\pi R^2 \cos \varphi d\varphi \tag{1}$$

5) Rotation of the infinitesimal area  $dA$  round about the axis A-B is the increment of the volume, **Figure 6(b)**

$$dV = 2\pi R \sin \varphi \cdot 2\pi R \cos \varphi \cdot R d\varphi = 4\pi^2 R^3 \sin \varphi \cos \varphi d\varphi \tag{2}$$

6) Integration of the infinitesimal area  $dA$  within angles  $0 - \pi/2$  is the total surface volume of the four-dimensional sphere, **Figure 6(b)**

$$\int dV = \int_0^{\pi/2} 4\pi^2 R^3 \sin \varphi \cos \varphi d\varphi = 4\pi^2 R^3 \frac{1}{2} (\sin^2(\pi/2) - \sin^2(0)) = 2\pi^2 R^3 \tag{3}$$

7) The total surface volume of the four-dimensional sphere is equal to the volume of the Universe

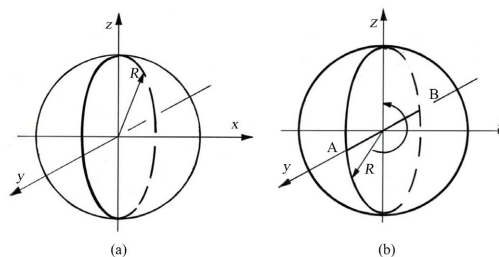
$$V = 2\pi^2 R^3 \tag{4}$$

The surface volume of the four-dimensional sphere  $2\pi^2 R^3$  divided by the center sphere of the four-dimensional sphere  $4/3\pi R^3$  is 4.71. This result can be compared with the corresponding value of the four-dimensional cube. In **Table 1**, corresponding values of Circle - Sphere and Square - Cube divisions are compared.

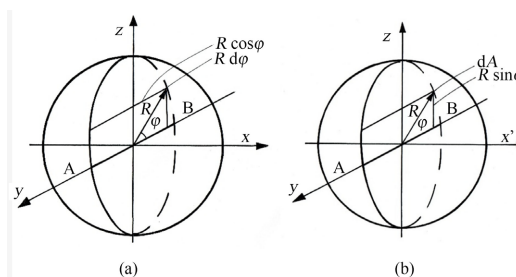
**Table 1.** Comparison of Circle - Sphere and Square - Cube divisions.

Circle - Sphere radius $R$		Square - Cube length of the side $a$	
$2\pi R/2R$	3.14	$4a/a$	4
$4\pi R^2/\pi R^2$	4	$6a^2/a^2$	6
$2\pi^2 R^3/4/3\pi R^3$	4.71	$8a^3/a^3$	8

The value 4.71 seems to be correct in comparison with the progress of Square - Cube divisions, and the formula of the surface volume of four-dimensional sphere seems to be correct. In Internet a similar formula is in publication [13].



**Figure 5.** In (a) is an ordinary three-dimensional sphere and in (b) is a four-dimensional sphere. The coordinate axes on the left side are  $x, y, z$  and on the right side are  $y, z, x'$  and the whole  $x$  coordinate axis is located in (b) at the zero point of coordinate axes. The thick circumference line indicates the surface area of the sphere in (a). The  $y$ - $z$  cross-sections of both spheres are the same. The four-dimensional sphere is formed by rotating the  $y$ - $z$  cross-section round about A-B axis, and it is the same manner than the ordinary three-dimensional sphere can be formed. In (b), the formation of the four-dimensional surface volume is formed by the rotation of the  $x'$  axis' direction.



**Figure 6.** The figure presents two three-dimensional sections  $y, z, x$  and  $y, z, x'$  of the four-dimensional  $x, y, z, x'$  sphere. The surface layer of the four-dimensional sphere is the model of the Universe. In (a), the whole surface area of the sphere of (a) is transformed into the thick circle line at the radius  $R$  on the  $y$ - $z$  plane. In (b) the surface volume of the sphere is calculated by rotating the minimal area  $dA$ , Equation (1), round about the A-B axis, the rotation in the direction of  $x'$  axis.

The calculation of the four-power volume of the four-dimensional sphere is performed as follows: The three-dimensional volume of the surface of the four-dimensional sphere is in Equation (4)

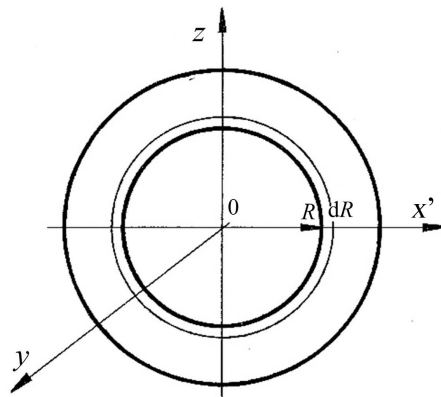
$$V = 2\pi^2 R^3 \tag{4}$$

In **Figure 7**, the surface volume is shown as a thick circle line at the radius  $R$ , and  $dR$  is the increment of the radius. The increment of the four-power volume of the four-dimensional sphere is

$$dW = 2\pi^2 R^3 dR \tag{5}$$

and the whole four-power volume is the integral

$$W = \int_0^R 2\pi^2 R^3 dR = \frac{1}{2} \pi^2 R^4 \tag{6}$$



**Figure 7.** The calculation of the four-power volume of the four-dimensional sphere is presented in the cross-section of the sphere. The thick circle line indicates the surface volume of the four-dimensional sphere  $V = 2\pi^2 R^3$ .

### 3. Calculation of the Redshift

In publication [5], the redshift theory was correct, but there was an error which was due to the confusion of the determination of the redshift. However, the result of the calculations is about the same, and the Universe has been calculated to have a very slow expansion. The determination of the redshift within the wavelength is

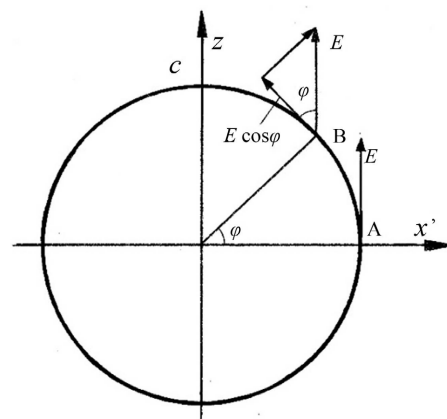
$$Z = \frac{\Delta\lambda}{\lambda(0)} \tag{7}$$

in which  $Z$  is the relative redshift,  $\Delta\lambda$  is the redshift within the wavelength of light,  $\lambda(0)$  is the initial wavelength of light. In publication [6], the redshift is resulted from the speeds of distant galaxies, and in that case, the increase of distance multiplied by the distance of the galaxy is equal to the redshift.

$$\frac{\Delta\lambda}{\lambda(0)} = \frac{\Delta R}{R(0)} \tag{8}$$

in which  $\Delta R$  is the increase of the galaxy’s distance, and  $R(0)$  is the galaxy’s distance. Therefore, it can be concluded that the redshift curve is the same form than

the expansion speed curve of the Universe, and therefore the curves of the expansion of the Universe in publications [6] and [7] are also redshift curves. In publication [1] it is presented the current theory that there is an unknown dark energy that expands the whole Universe at an accelerating speed. This dark energy is due to the measurements of the wavelengths of light from distant galaxies. The measurements have indicated that the wavelengths have become longer, and have shifted towards red, and it is inferred that this redshift results from the speed of increasing distances of the far-away galaxies. However, this theory cannot be correct because according to this theory, the age of the Universe is  $13.8 \cdot 10^9$  light years, and the James Webb Space Telescope has shown that there are galaxies very near to that time. Also many redshift publications have come to this conclusion, for instance publication [11].



**Figure 8.** This figure is a two-dimensional  $x,z$  cross-section of the four-dimensional  $x,y,z,x'$  sphere. The model of the Universe is the three-dimensional volume surface of the four-dimensional sphere. The three-dimensional volume is presented as the circumference of the radius  $R$ . As the light photon travels from point A to point B, its energy decreases from  $E$  to  $E \cos \varphi$ .

In the present study, it is shown that there is no dark energy expanding the Universe, but the redshift of light emitted from distant galaxies is the result of the structure of the Universe of four distance dimensions. **Figure 8** presents the four-dimensional  $x,y,z,x'$  sphere as its two-dimensional cross-section  $x,z$ . The model of the Universe is the surface volume of the fourth-dimensional sphere at the distance  $R$  from the center. The photon of light has been emitted at point A, and it has been observed at point B. The angle between the points A and B is  $\varphi$  radians. At point A, the photon's energy is  $E$ , and as the photon travels to point B, its Energy is  $E \cos \varphi$ . This is because all the three surface volume dimensions must be at a right angle to the radius vector  $R$  of the four-dimensional sphere. Therefore, it can be concluded that the photon, which has been emitted at point A, as it travels to point C, will completely disappear. This is a theory, but the next part of this chapter will demonstrate that the theory functions correct in the calculations of the redshift of light in the Universe.

The increase of the wavelength in the Universe was calculated from the decrease

of the energy of the light photon. Basic equations of light are speed of light and energy of the light photon.

$$c = \lambda_0 \cdot f$$

$$E = h \cdot f$$

$$\lambda_0 = hc/E$$

The equation of the redshift wavelength of light in **Figure 8**

$$\lambda = \frac{hc}{E \cos \varphi} \tag{9}$$

in which

$f$  Frequency of light,

$h$  Planck's constant,

$c$  Speed of light,

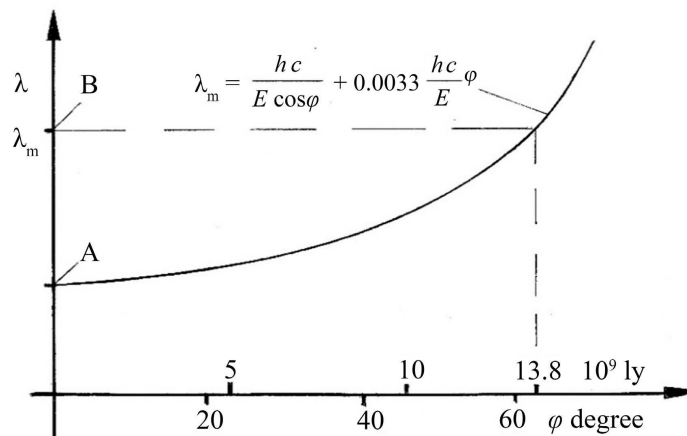
$E$  Photon's energy at point A in **Figure 8**,

$E \cos \varphi$  Photon's energy at point B,

$\lambda_0$  Original wavelength,

$\lambda$  Redshift wavelength.

The wavelength of Equation (9) was fitted into the model of the Universe which was constructed according to the redshift measurements. The measurements were published in scientific journals as well as in newspapers around the world. The model of the Universe in this study is in the publication [6]. This model of the Universe presents the increase of the theoretical radius of the Universe which is directly proportional to the increase of the wavelength of the measured light from distance stars in Equation (8). The equation of the redshift wavelength  $\lambda$ , Equation (9), fitted the model of the Universe,  $\lambda_m$  in **Figure 9**, but was not good enough and therefore, a component of the expansion of the Universe was added to the equation, and then it did match, Equation (10).



**Figure 9.** The vertical axis  $\lambda_m$  is the measured wavelength of the light photon emitted from distant star, beneath the horizontal axis is the angle of the Universe  $\varphi$  which is the angle in **Figure 8**, and above the horizontal axis is the corresponding distance in light years. The curve  $\lambda_m$  is the fitting of Equation (9) into the measured wavelength of light. The fitting matches with an additional term  $0.0033(hc/E)\varphi$ .

In **Figure 9**, the wavelength at the zero point is  $A = \lambda_0 = hc/E$ , and in **Figure 8** the corresponding zero point is A. Then the light travels 13.8 years, and in **Figure 9**, the wavelength is B, which in **Figure 8** corresponds to the point B. If the wavelength of light at the zero point is for example 500 nm, then according to the scale of the wavelength axis, the wavelength  $A = 500$  nm corresponds to the wavelength  $B = 1200$  nm. The redshift in the Universe is the increase of wavelength according to the equation  $(\lambda - \lambda_0)/\lambda_0$ . The redshift has two components: the first component is the effect of Equation (9), and the second component is the effect of the approximate expansion speed of the Universe  $0.0033 (hc/E)\varphi$ . In **Figure 9**, the equation of the approximate measured wavelength in the Universe is

$$\lambda_m = \frac{hc}{E \cos \varphi} + 0.0033 \frac{hc}{E} \varphi \quad (10)$$

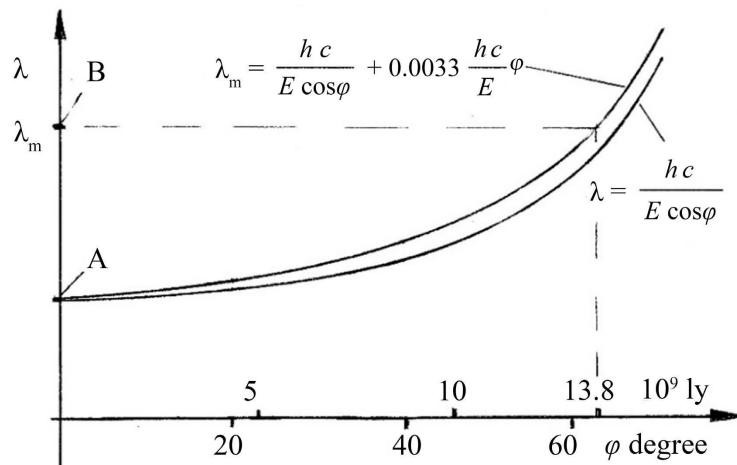
In **Figure 10**, the measured wavelength is  $\lambda_m$  Equation (10), and the wavelength according to Equation (9) is  $\lambda$ , which is the effect of the turning of the Universe's dimensions. The first part of Equation (10) is the effect of the shape of the Universe on the red shift, and the most of redshift is the result of this turning effect of the Universe's dimensions. The second part of the Equation (10) is the effect of the Universe's expansion and this expansion is much less than that which is predicted by current theory of the Universe's expansion, and it has no acceleration at all. The matching of Equation (10) to the measured wavelength curve of publication [6] means that the theory of the Universe appears to be correct. The range of the wavelength measurements is so large that the fitting cannot be coincidental. The model of the Universe in publication [6] by The Swedish Academy of Science indicates that our time is  $13.8 \cdot 10^9$  years after Big Bang, at which point of time the expansion of the Universe is at a high rate of acceleration. The Universe of the four-dimensional sphere is different. Our galaxy and the galaxies of Big Bang are both at the surface area of the sphere of the Universe, and they are both moving in opposite directions. In **Figure 8**, our time is at point B which in **Figure 9** and **Figure 10** corresponds to the time  $13.8 \cdot 10^9$  light years. The intersection of the measured and redshift wavelength curves,  $\lambda_m$  and  $\lambda$ , in **Figure 9** and **Figure 10** indicates that the time of Big Bang must be more than  $13.8 \cdot 10^9$  light years. At the point of Big Bang, the both curves should become the same curve, and they should not intersect.

The radius of the Universe  $R$  in **Figure 8** can be calculated based on **Figure 9**. The angle  $\varphi$  between points A and B in **Figure 8** is in **Figure 9** the angle  $\varphi$  beneath the horizontal axis. The maximum distance of measurement  $13.8 \cdot 10^9$  ly corresponds to the angle  $\varphi = 62.7^\circ$  in **Figure 9**, and in **Figure 8** the angle  $\varphi = 360^\circ$  corresponds to the distance  $(360^\circ/62.7^\circ) \cdot 13.8 \cdot 10^9$  ly =  $79.2 \cdot 10^9$  ly which is the circumference of the four-dimensional sphere, and the radius of the Universe is  $R = 79.2 \cdot 10^9$  ly/ $2\pi = 12.6 \cdot 10^9$  ly. Calculation of the surface area of the Universe using Equation (4)

$$V = 2\pi^2 R^3 = 2\pi^2 \cdot 12.63 \cdot 10^{27} \text{ ly}^3 = 39.5 \cdot 10^{30} \text{ ly}^3 \quad (11)$$

The volume of the center sphere of the Universe is

$$\frac{4}{3} \pi R^3 = \frac{4}{3} \pi \cdot 12.6^3 \cdot 10^{27} \text{ ly}^3 = 8.38 \cdot 10^{30} \text{ ly}^3 \quad (12)$$

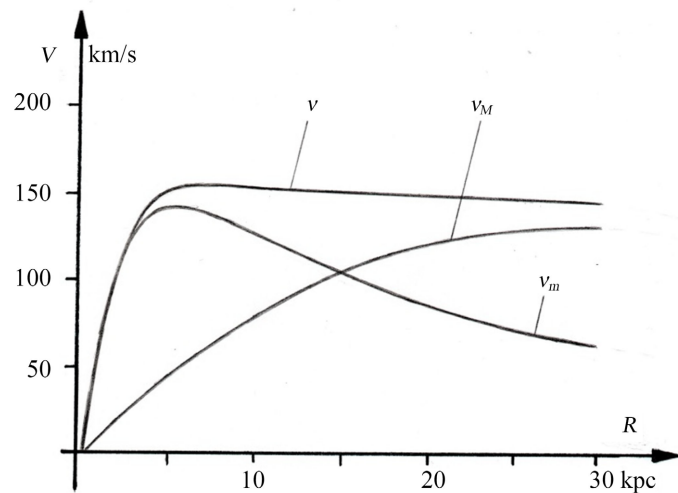


**Figure 10.** The measured wavelength is  $\lambda_m$ , Equation (10), and the redshift wavelength of light Equation (9) is  $\lambda$ , which is the effect of the turning of the Universe’s dimensions. The difference of these wavelengths is directly proportional to the expansion speed of the Universe. The difference of the wavelengths  $\lambda_m - \lambda$ , the expansion speed, indicates that there has been at some point something like Big Bang. The expansion speed has no acceleration, and it is much less than the expansion speed concluded by the current theories.

#### 4. Calculations of the Super Universe

In publication [10], it was inferred that the massive Black Holes in the fourth dimension create and rotate the spiral galaxies in our three-dimensional Universe. The generation process of the galactic system can be easily explained in the Universe of four dimensions. The gravitational force of the four-dimensional mass  $M$  acts in the rotational plane of the galaxy. It has no component at the direction of the fourth dimension because the three-dimensional mass of galaxy’s stars and other matter has no gravitational force of the fourth dimension. The result is that the four-dimensional mass  $M$  generates a gravitational field which has a great hole at the center of the galaxy. The gravitational field of the four-dimensional mass  $M$  accelerates the three-dimensional mass of stars and other matter into the speed of rotation, at which speed it rotates round about the center of galaxy. In this manner the hole in the gravitational field of the four-dimensional mass fills up, and the typical constant speed outer boundary regions of the galaxies have been generated. The total gravitational field of the four-dimensional mass  $M$  and the galaxy’s ordinary mass accelerates a star to the maximum rotational speed, in **Figure 11** approximately 150 km/s, in which case it retains rotating the galaxy at the border region, and if the star loses kinetic energy and speed, it begins to rotate the galaxy at the center region. In publication [10], Equation (11), it has been calculated that the galaxy’s maximum rotational speed curve is determined by the Black Hole’s mass and its four-dimensional distance from the center of the galaxy. The major

proof for this model of galaxy is the fact that the theoretical curve of rotational speed of the Black Hole  $v_M$  is the same form that the corresponding measured curve of the real galaxy in publication [3].



**Figure 11.** The rotational speeds of the theoretical model of the galaxy’s NGC 3198. The rotational speeds have been presented as the function of the radius  $R$  from the center of the galaxy. The rotational speed of the whole galaxy  $v$  is approximately 150 km/s within the level section. The rotational speed of the four-dimensional mass  $v_M$  has been calculated from the Black Hole of mass  $M = 80 \cdot 10^{61}$  kg and distance in the fourth dimension  $X' = 8.8 \cdot 10^{20}$  m, publication [10]. The rotational speed of the visible mass and ordinary dark mass  $v_m$  has been calculated with Equation (5) in publication [10].

**Table 2.** The table presents Black Holes’ masses  $M$  and four-dimensional distances  $X'$  in the Super Universe. Black Holes are at the four-dimensional distance  $X'$  from our three-dimensional Universe which is the surface volume of the Super Universe. **Table 2** is a revised version of **Table 1** in publication [10] with added galaxies Milky Way and Andromeda.

Galaxy	$X'$ kpc	$X' \cdot 10^{20}$ m	$v_{Mx} \cdot 10^5$ m/s	$M \cdot 10^{61}$ kg
UGC 2885	100	30.8	2.6	3800
Andromeda	78	24.0	1.95	1300
NGC 801	50	15.4	1.9	510
NGC 2998	35	10.8	1.75	210
Milky Way	27	8.25	2.2	200
NGC 3198	30	8.8	1.3	80
NGC 2742	12	3.7	1.6	20

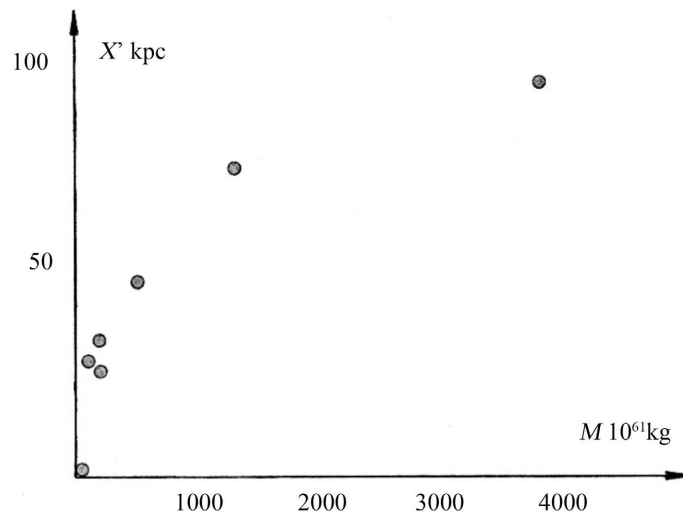
In **Figure 11**, the rotational speed curve  $v$  is the speed of the star rotating in the galaxy. The rotational speed curve of the Black Hole  $v_M$  is the speed of rotation in the Black hole’s gravitational field and  $v_m$  is the rotational speed in the gravitational field of galaxy’s ordinary mass. The rotational speed curve  $v_M$  is calculated in publication [10] by using the values of the mass  $M = 80 \cdot 10^{61}$  kg and its four-dimensional distance from the center of galaxy  $X' = 8.8 \cdot 10^{20}$  m, and it is the best fitting to the mysterious dark matter’s rotational speed curve of the real galaxy

NGC 3198 in publication [3]. The maximum value of the rotational speed curve is  $v_{Mx} = 131$  km/s at the radius  $R_x = 8.8 \cdot 10^{20}$  m, and it is approximately the same as the maximum radius of the real galaxy NGC 3198 in the publication [3],  $R = 9.0 \cdot 10^{20}$  m. According to Equation (22) in publication [10], the radius  $R_x$  of the maximum value of the rotational speed curve is equal to the four-dimensional distance of the Black Hole  $X' = R_x$ . As the radius  $R_x$  is also equal to the maximum radius of the galaxy, it is possible to determine the distances of the Black Holes in the fourth dimension beneath our three-dimensional Universe, and this reveals the structure of the Super Universe in **Table 2** and **Figure 12**.

In **Table 2**, the Black Holes' distances from our three-dimensional Universe have been presented, and it seems to be that at the longer distances there are more massive Black Holes. In **Figure 11**, at the radius  $R_x$ , the maximum value of the speed curve  $v_{Mx}$  is 131 km/s, and the maximum speed of the whole galaxy is 150 km/s, and the ratio  $130/150 = 0.87$ . In publication [4], the rotation curves of spiral galaxies have been presented, and the radius  $R_x$  is equal to the length of the rotational curve, and therefore it is also equal to the distance  $X'$ . In publication [4], it can be seen the maximum rotational speeds of the galaxies, and the maximum value of the rotational speed curve  $v_{Mx}$  can be computed by the multiplier  $130/150 = 0.87$ . In publication [10], the formula for the Black hole's mass is Equation (28)

$$M = \frac{4 R_x^2 (v_{Mx})^2}{\gamma} \tag{13}$$

By this means, the more accurate Black Hole masses have been calculated for this study



**Figure 12.** The figure presents Black Hole's mass  $M$  and distances  $X'$  in the Super Universe. Our three-dimensional Universe is the surface volume of the Super Universe, and the distance  $X'$  is Black Hole's four-dimensional distance from our three-dimensional Universe.

### 5. Conclusions

In the present study, the model of the Universe is the surface volume of the four-

dimensional sphere. In order to understand this four-dimensional Universe, a simple computation and construction system of the four-dimensional space has been developed. The function of this new 4D system is verified by computing the surface volume of the four-dimensional sphere  $2\pi^2 R^3$ , and another important formula, the fourth power volume of the four-dimensional sphere  $1/2\pi^2 R^4$ . Because these results are the same as the mathematicians have calculated by a much more complex method, publication [13], it is verified that this 4D computation system functions correct.

Within the redshift phenomena, the wavelength of the light photon increases, so it is possible to think that the wavelength of the light of the most distant galaxies has become so long that it has become the cosmic microwave background.

The decrease of the energy of the photon as it travels through the four-dimensional Universe is caused by the turning of the energy vector at the direction of the movement. This is the theory, and to verify this theory, the theoretical equation, Equation (9), is fitted into the measured wavelength of the light, in **Figure 10**. The fitting matches the theoretical equation, into which a slow expansion of the Universe has been added, Equation (10).

Black holes have a boundary beneath which, to escape the speed, must be higher than the speed of light, and therefore light cannot escape it. Within the three-dimensional black holes, this boundary for the vertical radiation has a more different location than for the horizontal radiation. The calculations are in publication ([10], p. 121). Einstein's relativity theory has no effect on the movement because the moving masses are eliminated from the equations. The calculations are about masses, but the spot of the boundary can be inferred.

In the study of the Universe, the measurements of the rotational speeds of the galaxies, and also the measurements of the wavelength of the light from distant galaxies, have shown that besides our three-dimensional Universe there must be also the four-dimensional Super Universe, and to understand this super Universe we must study the mathematics of the four-dimensional space. In the present study, the method to construct the four-dimensional cube by using four pairs of three-dimensional construction blocks has been developed. The construction of the four-dimensional cube with the four pairs of cubes in the four different directions functions in the same manner as the construction of the three-dimensional cube with the three pairs of side squares in the three different directions. But as we have the understanding of the three-dimensional space, we know that actually we can have six similar squares which can be turned into the right direction. How is then the four-dimensional space, is there also eight three-dimensional cubes which can be turned into right direction. This is the conclusion because the spaces of different number of dimensions have a similar function in this respect. The studies of the Super Universe in publication [5] and [8] were bad luck studies. There were three errors in these papers. The calculation method of the surface volume of the four-dimensional sphere in publications [5] and [8] was right, but there was an error in the calculation of infinitesimal area of the Super Universe.

In this study, the corrected calculation of the surface volume of the four-dimensional sphere yields the same equation which has been derived by the mathematicians. Furthermore, the method to check this formula has been renewed, which also proves that the formula is correct. The derivation of the surface volume of the four-dimensional sphere had one more thing that must be checked, and I used the map ball of the earth to see that the intersections of the longitudinal and latitudinal lines are always at right angle. This was needed for the calculation of area and volume. Mathematicians have sophisticated theory and calculus for the four-dimensional space, but it has been too hard science for astronomers and astrophysicists. The need for the mathematics of the four-dimensional space has been perceived, but the thinking has been that as we have understanding of the three-dimensional space, and we do not have the understanding of the four-dimensional space, so it is impossible to understand it. In this study, the calculation of the four-dimensional space is easy to comprehend, and therefore it is applicable to the study of the Universe

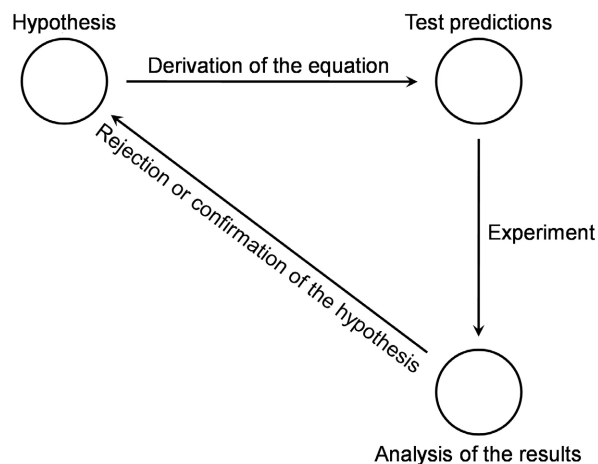
The second major error in publication [5] was that there was not a certain definition for the red shift, and it went wrong. This was corrected in the publication [8], and in the present study the definition of the red shift is in Equation (7), and its application to the expansion model of the Universe is in Equation (8). The third error is in publication [8], in which the equations in **Figure 8** and **Figure 9** are defective. The corrections are in the present study in **Figure 9** and **Figure 10**. The red shift theory was right in publication [8]. This is the redshift theory: the red shift is due to the light photon travelling in the four-dimensional Universe, and because of the turning of the dimension in the direction of the travel the energy of the light photon decreases. This theory has been proved with the procedure of **Figure 13**, in which the progress of research and testing the hypothesis is presented in the same manner as in Karl Popper's book publication [14]. The progress of testing is as follows: The first step, the hypothesis, is that the redshift measurements from distant galaxies are due to the shape of the four-dimensional Universe, and also in some degree due to the expansion of the Universe. The second step is derivation of the equations. This phase of proceeding involves the derivation of mathematics of the four-dimensional space. At this stage, the equation of the light photon travelling in the spherical surface volume of the four-dimensional Universe has been derived, Equation (9). The third step is the test prediction which is that the equation of the light photon travelling in the surface volume of the four-dimensional Universe is the same as the measured redshift values in the Universe. The fourth step is the measurements, which can be calculated based on the model of the Universe in publication [6]. In the present theory of the Universe, it is presumed that the wavelength of light is directly proportional to the expansion speed of the Universe, and because the present theory of the Universe has no model of the Universe, therefore it must be that the largeness of the Universe is shown in the measured wavelength curve. Therefore, the measured wavelength of light is the model of the Universe in publication [6]. A perfect fitting of the measured

wavelength of the model of the Universe in publication [6] to the theoretical wavelength of the present study in **Figure 9** was obtained. The fifth step is the analysis of the result. In the analysis, the measured wavelength with real measured red shift is compared to the wavelength with theoretical red shift which is due to the shape of the Universe, **Figure 10**. The fitting yielded the Universe of a slow expansion. Therefore, it can be concluded that there may be Big Bang which is not very big. In this model of the Universe, the theoretical dark energy, which has been calculated to be 69% of the total mass energy of the Universe, publication [1], does not exist. In this manner, the theory of this study has been proved to be correct.

Measurements of the rotational speeds of the spiral galaxies have shown that most of the matter that generates the galaxies' rotational speed is missing, publication [3] and [4]. Despite of thorough search for appropriate fundamental particles, none has been found. Furthermore, the halo structure which has been the solution to this problem is quite unrealistic. In this study, the missing matter is four-dimensional Black Holes in the four distance dimension beneath our three-dimensional Universe. The fact that the rotations of the spiral galaxies have been generated by the Black Holes in the fourth dimension has been justified by many proofs. The most important proof is that the theoretical rotational speed distribution curve of spiral galaxies generated by the Black Hole is approximately the same as the real galaxy rotational speed distribution curve of the red shift measurements. **Figure 11**, the theoretical rotational speed curve of the total galaxy  $v$ , the Black Hole  $vM$  and the visible and ordinary dark matter  $vM$  are about the same than the corresponding rotational speed curves obtained in red shift measurements, publication [3]. The more precise presentation of the justification of this theory is in publication [10], in which the progress of the research and testing the hypothesis has been presented in the same manner as in Karl Popper's book, **Figure 13**. The Black Hole study continues in the present study, and the locations and masses of the Black Holes in the fourth-dimensional space have been determined from the measurements of publication [4]. The distribution of the Black Holes' locations and the masses in the fourth dimension reveals the structure of the Super Universe.

The first round of the progress triangle is publication [2] "Galaxy Rotation in the Space of Four Dimensions", the second round is publication [5] "The Solution to the Dark Energy Mystery in the Universe of Four Distance Dimensions" the third round is publication [10] "Theory to the Mystery of Super Massive Black Holes", the fourth round is publication [8] "Four-Dimensional Mathematics Creates the Super Universe", the fifth round is "Erratum to "Four-Dimensional Mathematics Creates the Super Universe" and the present study is the sixth round of the study of the four-dimensional space. This method of progress was also used for the solution to the Hill's equation. The famous British Nobel laureate A. V. Hill invented this equation in 1938. The major problem of this equation was that it was derived from the experiments of muscle contractions with frog muscles. Within the motion of frogs and fleas this equation functions right, but no one was

interested in that. Instead of that, it has been applied to human movement, and within that kind of motion it does not function right. This is because the muscle force is proportional to the area of the cross section of the muscle and the inertial forces of the accelerated masses are proportional to volume. Therefore, in human movement there is so much mass that the muscle force cannot accelerate it to the speed of Hill's equation. This was the major problem of the Hill's equation, and within five rounds that problem was solved, publications [15]-[19].



**Figure 13.** The progress of testing the hypotheses of the fourth-dimensional mass is presented by using the method of Karl Popper, publication [14].

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

## References

- [1] Ananthaswamy, A. (2022) Cosmic Conflict. *Scientific American*, **327**, 60-65.
- [2] Rahikainen, A. (2020) Galaxy Rotation in the Space of Four Distance Dimensions. *World Journal of Mechanics*, **10**, 83-94. <https://doi.org/10.4236/wjm.2020.107007>
- [3] van Albada, T.S., Bahcall, J.N., Begeman, K. and Sancisi, R. (1985) Distribution of Dark Matter in the Spiral Galaxy NGC 3198. *The Astrophysical Journal*, **295**, 305-313. <https://doi.org/10.1086/163375>
- [4] Rubin, V.C. (1983) The Rotation of Spiral Galaxies. *Science*, **220**, 1339-1344. <https://doi.org/10.1126/science.220.4604.1339>
- [5] Rahikainen, A. (2021) The Solution to the Dark Energy Mystery in the Universe of Four Distance Dimensions. *World Journal of Mechanics*, **11**, 95-110. <https://doi.org/10.4236/wjm.2021.115008>
- [6] Jarnestad, J. (2019) Graphics, the Royal Swedish Academy of Sciences, ML/HS. Helsingin Sanomat. Fysiikan Nobelistit Oivalsivat. Tiede. <https://www.hs.fi/tiede/art-2000006265660.html>
- [7] Wollacl, E.J. (2012) Timeline of the Universe Image. NASA/WMAP Science Team. <https://map.gsfc.nasa.gov/media/060915/index.html>
- [8] Rahikainen, A. (2023) Four-Dimensional Mathematics Creates the Super Universe. *World Journal of Mechanics*, **13**, 135-148. <https://doi.org/10.4236/wjm.2023.137008>

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- [9] Rahikainen, A. (2024) Erratum to “Four-Dimensional Mathematics Creates the Super Universe” [World Journal of Mechanics, 2023, 13, 135-148]. *World Journal of Mechanics*, **14**, 205-208. <https://doi.org/10.4236/wjm.2024.1411011>
- [10] Rahikainen, A. (2023) Theory to the Mystery of the Super Massive Black Holes. *World Journal of Mechanics*, **13**, 107-126. <https://doi.org/10.4236/wjm.2023.135006>
- [11] Shamir, L. (2024) An Empirical Consistent Redshift Bias: A Possible Direct Observation of Zwicky’s TL Theory. *Particles*, **7**, 703-716. <https://doi.org/10.3390/particles7030041>
- [12] Zwicky, F. (1929) On the Redshift of Spectral Lines Through Interstellar Space. *Proceedings of the National Academy of Sciences*, **15**, 773-779. <https://doi.org/10.1073/pnas.15.10.773>
- [13] Weisstein, E.W. (2025) Four-Dimensional Geometry. From MathWorld-A Wolfram Web Resource. <https://mathworld.wolfram.com/Four-DimensionalGeometry.html>
- [14] Popper, K. (2002) *Conjectures and Refutations: The Growth of Scientific Knowledge*. (Routledge Classics). Routledge. <https://routledge.com/conjectures-and-Refutations-The-Growth-of-Scientific-knowledge>
- [15] Rahikainen, A. (2019) Constant Power Solution of Hill’s Equation. Book Publisher International (a part of SCIENCEDOMAIN International). <https://doi.org/10.9734/bpi/mono/978-81-940613-2-8>
- [16] Rahikainen, A. and Virmavirta, M. (2019) Hill’s Equation in Arm Push of Shot Put and in Braking of Arm Rotation. In: Rusu, T., Ed., *Advances in Applied Science and Technology*, Vol. 6, Book Publisher International, 49-75.
- [17] Rahikainen, A. and Virmavirta, M. (2014) Constant Power Model in Arm Rotation—A New Approach to Hill’s Equation. *World Journal of Mechanics*, **4**, 157-169. <https://doi.org/10.4236/wjm.2014.46018>
- [18] Rahikainen, A., Avela, J. and Virmavirta, M. (2012) Modeling the Force-Velocity Relationship in Arm. *World Journal of Mechanics*, **2**, 90-97. <https://doi.org/10.4236/wjm.2012.22011>
- [19] Rahikainen, A., Virmavirta, M. and Ranta, M. (2016) Hill’s Equation in the Arm Push of Shot Put. *British Journal of Applied Science & Technology*, **18**, 1-12. <https://doi.org/10.9734/bjast/2016/30766>

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## List of Variables

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Dimensions of ordinary space	$x, y, z$
Fourth distance dimension	$x'$
Four-dimensional mass	$M$
Distance of the four-dimensional mass from center of galaxy	$X'$
Speed distribution component of the four-dimensional mass	$v_M$
Radius of the four-dimensional spherical Universe	$R$
Coordination angle of the four-dimensional spherical Universe	$\varphi$
Distance, Light years	ly
Time, Years	$t$
Energy of light photon	$E$
Planck's constant	$h$
Speed of light	$c$
Volume of the four-dimensional spherical Universe	$V$
Area of the surface of the sphere Universe	$A$
fourth power volume of the sphere Universe	$W$
Original wavelength	$\lambda_0$
Redshift wavelength	$\lambda$
Measured wavelength	$\lambda_m$

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