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Mathematical Modeling of Second-Hand School Uniforms in the UK: Stigma, Policy Interventions, and Welfare Implications

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Abstract

This study develops a mathematical model to examine demand for second-hand school uniforms in the UK, focusing on key drivers (cost savings, environmental benefits) and barriers (social stigma). It evaluates how school-led initiatives, such as awareness campaigns, can reduce stigma and boost adoption, and assesses the welfare effects of policy interventions including subsidies and anti-stigma measures. Using primary survey data for calibration, the model is solved numerically to obtain the market equilibrium, from which the effectiveness of policy interventions is evaluated. The analysis shows how raising awareness of financial and environmental benefits, alongside leveraging schools' institutional influence, can shift market equilibrium, increase adoption rates, and promote sustainable consumption.

Keywords

Mathematical Modeling, Equilibrium Analysis, Welfare Analysis, Second Hand School Uniform

1. Introduction

1.1. Motivation

Currently, in a time when there has been low economic growth with austerity in the UK (Amann & Middleditch, 2017), purchasing school uniforms has brought immense economic pressure to households (Page et al., 2021). Therefore, how to reduce their spending on uniforms has become an issue that I concern, especially after the imposition of VAT (Macey-Dare, 2024) on private schools in November 2024 (Maurici & McLellan, 2024) and low income family group in state schools in

the UK (Ridge, 2011). This gave immense economic pressure to both myself and my classmates. As a high school student, I began to think about how to solve this economic pressure, leading to an idea if saving money through second hand school uniforms. After reading lots of references of second-hand dress reports (McRobbie, 1989; Norris, 2012), I found that there has been two gaps in those papers: existing research on second-hand school uniforms in the UK: (1) Insufficient quantitative modeling of how school-level incentives, government policies (Maurici & McLellan, 2024) (e.g., tax relief, price subsidies), and social prejudice jointly influence household decision-making, and (2) A lack of empirical data, both primary and secondary, on market dynamics.

Motivated by these gaps, this study constructs a mathematical framework (Ballarin, 2014)—calibrated with novel UK household survey data and qualitative insights—to formalize the relationship between adoption rates (A) and determinants such as price elasticity (P), stigma (S), and institutional design (I), where $A = f(P, S, I|\vartheta)$ with ϑ representing school influence parameters. Moreover, I show that influencing welfare policies also impacts second-hand school uniform market through several mathematic module. Finally, to verify this module by survey data, the model aims to decompose whether school power (SP) or social confluence (SC) dominates behavioral outcomes, providing policy insights for sustainable uniform practices. Additionally, the study proposes stigma-reduction mechanisms via school-led campaigns re-framing second-hand uniforms as eco-fashion, while advocating for government price subsidies to donors, optimizing welfare gains under budget constraints.

1.2. Research Question

This study involves three research questions:

1) What are the primary drivers (e.g., cost savings, environmental concerns) and barriers (e.g., social stigma) influencing household demand for second-hand school uniforms in the UK? (Hobbs, 2016; Hursh, 1984).

2) How can school leadership initiatives help eliminate social prejudice and promote adoption?

3) How can policy interventions, such as price subsidies and anti-stigma campaigns, improve adoption rates and welfare in the second-hand uniform market, as modeled through household decision-making and market equilibrium analysis?

1.3. Research Method

This section is a structured and mathematically emphasized version of my research framework (Ramdani et al., 2019; Lamport & LaTex, 1994) with mathematical module by Latex tools (integrating logical flow and formal modeling):

This study adopts a mixed-methods approach combining primary data from a cross-sectional survey of 207 UK respondents (parents, students, charity staff, manufacturers, and school employees) with mathematical modules to analyze costs and behavioral drivers in the second-hand school uniform market (Saunders

et al., 2006). I also used secondary data for the analysis of the equilibrium price. The mathematics module is created through technology from the LaTeX software (Lamport & LaTex, 1994). The core analytical contribution lies in formalizing key relationships through mathematical models: (1) A supply-demand equilibrium framework to quantify market dynamics, (2) A household utility model where purchasing decisions maximize subject to budget constraints, prices stigma and environmental preferences, (3) Welfare optimization to evaluate policy interventions (e.g., price subsidies or school-led stigma reduction). Survey data empirically calibrates these models, testing hypotheses such as adoption increasing with subsidies or stigma reducing utility. By integrating theoretical rigor with empirical validation, this approach disentangles causal mechanisms (e.g., whether price elasticity or social conformity dominates behavior), and simulates policy impacts, bridging gaps between micro-level decisions and macro-level sustainability goals. This is my Logically sequences: methods → models → empirical validation → policy insights.

In contrast, the new mathematical module presents a rigorous framework for analyzing how targeted subsidies can efficiently promote the adoption of second-hand school uniforms while optimizing social welfare. The model formalizes the adoption process as: 1) utility function including price, perceived quality, stigma cost, environment concern (Mohanty, 2011), 2) using a mixed Logit model to model differences across agents, 3) explaining Supply-side modeling from donation participation, second-hand market costs, price formation, identifying the problems and opportunities in the second-hand school uniform market in the UK.

Survey questionnaire design

The questionnaire captures respondents' demographics, motivations, awareness of the environment protection concept, reduce, reuse and recycle of circular economy knowledge (Madaan et al., 2024; Manickam & Duraisamy, 2019) and the perceived opportunities and barriers in the second-hand school uniform market in the UK.

This approach allows me to collect relevant responses efficiently from the target population. The full questionnaire and response statistics are provided in **Appendix A** part. I consider the Online Survey Tips as follows:

1) The respondents must live in the UK.

2) A plan for researching quantitative research using a single approach.

3) Conducting research surveys with a time frame, a cross-sectional analysis.

4) Techniques and methods of operation Data collecting that is structured.

5) The examination of Google chart.

6) Survey link available here.

7) **Table 1**: Questionnaire Structure

Finally, I developed several mathematical models of household choice over second-hand uniform purchases, based on utility derived from price, social stigma, and environmental benefits (McFadden, 1972), using survey results to verify the mathematic module. Another mathematic module is used to validate for equilib-

rium, and conducts comparative statics to evaluate how changes instigating environmental awareness affect adoption (Coddington, 1993). I estimate price subsidies and simulate counterfactual policy interventions, such as stigma reduction, campaigns, or subsidies, to assess their potential welfare effects. Also, I verify that school power can influence social prejudice to give positive voices on second hand school uniforms.

Table 1. Questionnaire structure.

Theme	Key Questions
Demographics	Gender, Age, Nationality, Occupation, Household income level.
School Background	Type of school (state, private, special education), school communication on second-hand uniforms.
Purchase Behavior	Experience of buying second-hand uniforms in the past 3 years, purchasing channels (school shop, online, charity shop, peer-to-peer).
Motivations and Barriers	Reasons for buying (cost-saving, sustainability), reasons for not buying (hygiene concerns, fit issues, limited availability).
Savings and Cost Factors	Estimated yearly savings from purchasing second-hand uniforms.
Donation and Recycling	Attitudes toward donating uniforms, knowledge of the 3R principles, perceived barriers and opportunities for second-hand uniforms.
Upcycling and Sustainability Initiatives	Opinions on upcycling uniforms as a fashion trend, support for school initiatives promoting low-carbon lifestyle through reuse and redesign.

1.4. Contribution

This study examines the UK's second-hand school uniform market through a mathematical modeling lens, addressing how household decisions, social norms, and policy interventions shape market equilibrium. Motivated by the tension between cost-saving/environmental benefits (Coddington, 1993) and persistent social stigma (Vallacher & Wegner, 1987), by integrating three conceptual lenses—welfare adjustment, environmental sustainability, and policy governance—I reveal nuanced interactions: urban areas with stronger eco-norms exhibit higher adoption rates, while supply-chain inefficiencies (e.g., donation mismanagement) can distort prices. Empirical survey data validates the model's predictions, demonstrating how stigma reduction and financial incentives jointly optimize participation. The findings contribute to a unified framework for policymakers to design interventions that balance affordability, sustainability, and social norms shifts. The study highlights the potential of targeted price mechanisms and anti-stigma campaigns to enhance welfare in second-hand markets.

The remainder of the paper is organized as follows. Section 2 presents the mathematical model. Section 3 describe the equilibrium analysis. Section 4 explains the survey data and numerical analysis. Section 5 concludes. Detailed proof is provided in **Appendix A**.

2. Model

2.1. Agents, Goods, and Timeline

We consider a discrete-time economy indexed by $t = 0,1,2,\cdots$, in which two types of agents interact repeatedly in a market for second-hand goods. There exists a unit continuum of households indexed by $i \in [0,1]$ and a unit continuum of potential suppliers indexed by $j \in [0,1]$. In each period, every household decides whether to purchase a single unit of a second-hand good, while each supplier decides whether to release a used item to the market.

To reflect the non-durable nature of used goods and the limited storage capacity of consumers, we assume that households cannot store goods for future use. In contrast, suppliers are allowed to carry inventory across periods, but these inventories are subject to physical or perceived depreciation. Specifically, we let $\delta \in [0,1)$ denote the depreciation rate, where $\delta = 0$ implies full preservation and $\delta \to 1$ indicates near-total loss in resale value between periods. For analytical tractability, we first focus on the static benchmark case in which $\delta = 1$, i.e., all unsold goods fully depreciate and exit the market at the end of the period. This setting captures the stylized fact that used goods often face urgency of sale due to hygiene, obsolescence, or seasonal relevance.

2.2. Household Utility, Demand Function, and Market Supply

Each household i derives indirect utility from the consumption of a second-hand good according to the specification[1]:

$$U_i = V_i - \beta_i P_t - \lambda_i S_t + \theta_i E_t + \varepsilon_i, \tag{1}$$

where P_t is the market price of the good, S_t captures the level of perceived social stigma associated with second-hand consumption, and E_t reflects the salience of environmental concerns in the current period. The variable V_i represents household i's intrinsic valuation of the good, while β_i, λ_i, and θ_i are household-specific sensitivity parameters that govern the disutility from price, the disutility from stigma, and the utility from environmental awareness, respectively. These type parameters are independently drawn from continuous distributions F_β, F_λ, and F_θ, each with compact support to ensure a bounded and well-behaved population. The term ε_i is an idiosyncratic preference shock, assumed to follow the standard Logistic distribution, which gives rise to a logit formulation of discrete choice.

Given these assumptions, the probability that household i chooses to purchase in period t is:

$$p_i(P_t, S_t, E_t) = \left[1 + \exp\left(-\left(V_i - \beta_i P_t - \lambda_i S_t + \theta_i E_t\right)\right)\right]^{-1}. \tag{2}$$

Aggregating across the continuum of households yields the total market demand:

[1]This additive separable utility specification follows standard discrete choice theory (McFadden, 1972), where price reduces utility due to budget constraints, stigma reduces perceived quality or social payoff, and environmental benefits increase non-monetary utility. The assumption of linear separability simplifies estimation and aligns with consumer theory under quasi-linear preferences.

$$Q_d\left(P_t, S_t, E_t\right) = \int_0^1 p_i\left(P_t, S_t, E_t\right) \mathrm{d}i. \tag{3}$$

On the supply side, each supplier j possesses one used item with an opportunity cost c_j, drawn from a cumulative distribution function $G(\cdot)$. Supplier j will release the item if and only if the market price P_t exceeds her reservation cost, that is, $P_t \geq c_j$. The total supply is thus the fraction of suppliers with $c_j \leq P_t$, given by:

$$Q_s\left(P_t\right) = G\left(P_t\right). \tag{4}$$

For analytical tractability, we adopt a linear approximation of the supply function by using the second-order Taylor expansion of $G(P)$ around the mean cost, yielding:

$$Q_s\left(P_t\right) = a + bP_t, \ a \in (0,1), \ b > 0, \tag{5}$$

which captures the notion that higher prices attract more suppliers with marginally higher opportunity costs[2].

2.3. Policy Instruments

A welfare-maximizing planner, such as a local government or school administrator, can intervene in this market using two instruments aimed at increasing participation in the second-hand economy. The first is a per-unit price subsidy $\tau_t \geq 0$, which lowers the effective purchase price for households to $\tilde{P}_t = P_t - \tau_t$. The second is a stigma-reduction campaign $\sigma_t \geq 0$, which shifts the perceived social stigma from S_t to $\tilde{S}_t = S_t - \sigma_t$. These interventions directly affect households' utility components and, hence, their likelihood of purchase.

To account for implementation costs, we introduce a convex cost function $C\left(\tau_t, \sigma_t\right)$, which captures both the fiscal burden of subsidies and the increasing marginal cost of stigma reduction. This setup allows us to explore optimal policy design under budget constraints and heterogeneous agent responses in subsequent sections.

3. Equilibrium Analysis

3.1. Definition of Static Equilibrium

We begin by formalizing the notion of a static market equilibrium in the presence of behavioral frictions and policy interventions. Given the perceived stigma S_t, environmental salience E_t, and policy instruments $\left(\tau_t, \sigma_t\right)$, the effective price and effective stigma faced by consumers are defined as

$$\tilde{P}_t = P_t - \tau_t, \ \tilde{S}_t = S_t - \sigma_t.$$

A static equilibrium requires that, at the prevailing market price, aggregate demand equals aggregate supply.

[2]This specification follows classical partial equilibrium theory where supply is upward sloping in price (Varian, 2003). While donations are not perfectly price-elastic, small monetary incentives (e.g., vouchers or resale credits) positively correlate with supply volume.

Definition 3.1 (Static Market Equilibrium). *Given primitives* (S_t, E_t), *policy instruments* (τ_t, σ_t), *and the associated effective variables* $(\tilde{P}_t, \tilde{S}_t)$, *a pair* (P^*, Q^*) *constitutes a static market equilibrium if*

$$Q_d(\tilde{P}_t, \tilde{S}_t, E_t) = Q_s(P_t),$$

where Q_d *denotes aggregate demand and* Q_s *denotes aggregate supply.*

3.2. Existence and Uniqueness

We next establish that under standard monotonicity conditions on the demand and supply functions, the equilibrium price is uniquely determined.

Proposition 3.1 (Existence and Uniqueness). S*uppose the following assumptions hold*:

(A1) *The demand function* $Q_d(\cdot)$ *is continuous and strictly decreasing in the effective price* $\tilde{P}_t$;

(A2) *The supply function* $Q_s(\cdot)$ *is continuous and strictly increasing in the market price* P_t.

Then there exists a unique equilibrium price P^* *that solves the market-clearing condition.*

Proof. See **Appendix A.1** for a full derivation. □

3.3. Comparative Statics

To assess the behavioral implications of stigma on market outcomes, we analyze how changes in the perceived stigma level S_t influence the equilibrium price and quantity. This analysis provides a foundation for understanding the demand sensitivity to social norms and the transmission of policy interventions through price mechanisms.

Lemma 3.1 (Sign of Price Response to Stigma). *Let the excess demand function be defined by*

$$F(P, S) := Q_d(P - \tau, S - \sigma, E_t) - Q_s(P),$$

where Q_d *and* Q_s *denote aggregate demand and supply, respectively. Suppose that the partial derivatives of* F *satisfy*:

$$F_P := \frac{\partial F}{\partial P} < 0, \quad F_S := \frac{\partial F}{\partial S} > 0.$$

Then the equilibrium price function $P^*(S)$ *satisfies the comparative static*:

$$\frac{\mathrm{d}P^*}{\mathrm{d}S} = -\frac{F_S}{F_P} > 0.$$

Proof. Follows directly from the Implicit Function Theorem; see **Appendix A.2**. □

Proposition 3.2 (Comparative Statics: Stigma and Equilibrium Outcomes). *Under Assumptions* A1-A2 *and the logit specification of demand, the partial derivatives of equilibrium price and quantity with respect to stigma are given by*:

$$\frac{\partial P^*}{\partial S_t} = \frac{\mathbb{E}\left[\lambda_i p_i (1-p_i)\right]}{\mathbb{E}\left[\beta_i p_i (1-p_i)\right] + b} > 0, \tag{6}$$

$$\frac{\partial Q^*}{\partial S_t} = -b \frac{\mathbb{E}\left[\lambda_i p_i (1-p_i)\right]}{\mathbb{E}\left[\beta_i p_i (1-p_i)\right] + b} < 0. \tag{7}$$

Proof. See **Appendix A.3** for a full derivation using total differentiation. □

Economic Interpretation. Proposition 3.2 reveals how equilibrium price and traded quantity respond to shifts in perceived stigma S_t:

- **Demand-Side Mechanism.** The numerator of the price derivative reflects the average marginal disutility of stigma across the population, weighted by the responsiveness of each household's logit choice probability, i.e., the term $p_i(1-p_i)$, which is maximized when $p_i \approx 0.5$. This means that changes in stigma most strongly influence those households that are near the margin of indifference between purchasing and abstaining. Greater heterogeneity in stigma sensitivity λ_i or a more uncertain purchase environment amplifies this responsiveness.
- **Supply-Side Moderation.** The denominator includes both the average marginal disutility of price (weighted by β_i) and the supply slope b, which captures how responsive suppliers are to price. A more elastic supply (higher b) absorbs part of the demand-side adjustment, thus dampening the upward price pressure induced by rising stigma.

To enable cross-market calibration and comparative policy analysis, we define the following elasticities:

$$\varepsilon_P^S := \frac{\partial P^*}{\partial S_t} \cdot \frac{S_t}{P^*}, \quad \varepsilon_Q^S := \frac{\partial Q^*}{\partial S_t} \cdot \frac{S_t}{Q^*}.$$

These expressions characterize the percentage response of price and quantity to marginal changes in stigma. Using the result for $\partial Q^* / \partial S_t$, we obtain the identity:

$$\varepsilon_Q^S = -\frac{bP^*}{Q^*} \cdot \varepsilon_P^S,$$

which highlights that the quantity sensitivity to stigma is increasing in both the price level and supply slope, and decreasing in the traded quantity. This relationship provides a tractable expression for empirically comparing stigma effects across settings with different market thickness or supply structure.

3.4. Welfare and First-Best Policy Design

We now examine the optimal policy mix from the perspective of a benevolent social planner who internalizes both private welfare and public costs.

Let social welfare $W(P_t, S_t, E_t)$ be defined as the total expected utility of households net of supplier opportunity costs:

$$W(P_t, S_t, E_t) = \int_0^1 U_i \mathrm{d}i - \int_0^{Q_S(P_t)} c \mathrm{d}G^{-1}(c),$$

where the second term integrates the marginal cost curve up to the equilibrium

quantity.

The planner can implement two policy tools:

- A price subsidy τ that reduces the effective price to consumers;
- A stigma-reduction campaign σ that shifts perceived stigma downward.

The combined fiscal cost of these interventions is captured by a convex cost function $C(\tau,\sigma)$, which reflects increasing marginal expenditures for both instruments.

Proposition 3.3 (First-Best Policy Mix). *The welfare-maximizing combination of policy instruments* (τ^*,σ^*) *satisfies the first-order conditions*:

$$\frac{\partial W}{\partial \tau}=\frac{\partial C}{\partial \tau},\quad \frac{\partial W}{\partial \sigma}=\frac{\partial C}{\partial \sigma},$$

subject to the feasibility constraints $\tau\geq 0$, $\sigma\geq 0$.

If the cost function $C(\tau,\sigma)$ is strictly convex and differentiable, the optimal policy lies in the interior and is characterized by the equalization of marginal benefits and marginal costs. In contrast, if C exhibits kinks or piecewise linearity (e.g., budget limits), then corner solutions may arise where only one instrument is used. The optimal policy mix thus depends critically on the shape of C and the relative elasticities of demand with respect to price and stigma.

Proof. See **Appendix A.4** for the derivation based on the envelope condition and constrained maximization. □

4. Survey Data and Numerical Analysis

4.1. Survey Data Collection

A non-probability snowball sampling method was used to reach UK residents through social media and community networks (YouTube, WhatsApp, WeChat). The full questionnaire and response statistics are provided in **Appendix A**. *Survey Design and Data Collection*[3].

4.2. Parameter Calibration with Survey Data

Firstly, Descriptive statistics data result in **Table 2**. then, I use the 207 survey responses to estimate the parameters of the demand function. Specifically, I collaborated on the module with both secondary data and my survey data results, and we demand an equilibrium price of £305.

- Price P : annual cost savings suggest a range £50 - £400.
- Stigma S : 57.9% of respondents report stigma, mapped to mean ≈ 0.58.
- Environmental awareness E : 35.4% fully aware of 3R rules, mean ≈ 0.4 - 0.5.
- Purchase q_i : 40% purchase, 23.6% consider, 27.9% refuse.

Fitting the logit model (2) yields:

$$\hat{Q}_d=\frac{1}{1+\exp\left(-\left(8.15-0.0205P-4.067S+2.521E\right)\right)}. \tag{8}$$

[3]Survey link available https://docs.google.com/forms/d/e/1FAIpQLScwCnGqpIsBAh-DMgeQc2Kod3sXwGi9JZATYyzCrJ40hKy9tYw/viewform?usp=sf_link.

Table 2. Descriptive statistics of sample distribution.

Items	Categories	N	Percent (%)
gender	female	97	46.8
	male	110	53.2
School type	state school	59	28.4
	private school	142	68.6
	special education institution	5	2.5
	Both state and private schools	1	0.5
Age	12 - 16	49	18.2
	16 - 18	53	26.1
	18 - 40	51	24.8
	Over40	64	30.9
Occupation	Student	92	44.3
	School employee	17	8.4
	Charity shop employee	10	5
	Parent or guardian	82	39.9
	School uniform manufacturer	6	3
Region	Asian	49	23.5
	The UK	109	53
	US	8	3.5
	Europe	41	20
Have you ever purchased second-hand school uniform in the past 3 years?	Yes	80	38.5
	No	56	27
	No, but will buy	53	26
	Yes, only for special items	18	9
Family income level/per year	less than £20,000	41	22
	£20,000 - £50,000	50	24.5
	£50,000 - 100,000	46	20.6
	Over £100,000	70	30.4
Save money amount/per year	less than £100	45	22
	£100 - 300	54	26.2
	£300 - 600	87	42
	£600 - 1000	18	8.9
	Over £1000	2	1

With linear supply $Q_s = -0.1 + 0.003P$, the numerical solution gives:

$$P^* \approx £305, \quad Q^* \approx 0.82.$$

Thus, under current conditions, about 82% of households would adopt second-

hand uniforms if market prices equilibrate near £305.

4.3. Policy Interventions Analysis

Further, I evaluate two counterfactual policy interventions based on the calibrated parameters above. I keep parameters fixed at $\alpha = 5.376$, $\beta = 0.0147$, $\lambda = 2.624$, $\theta = 3.948$ to ensure consistency across analyses. The market equilibrium is obtained by solving $Q_d(P,S,E) = Q_s(P)$.

Baseline Scenario

With stigma $S = 0.58$ and environmental awareness $E = 0.45$, the equilibrium results are:

$$P_0^* = 294.97, \quad Q_0^* = 78.5\%.$$

This serves as the reference point for policy comparison.

Scenario 1: Stigma Reduction

Reducing social stigma by 20% $(S = 0.464)$ shifts the demand curve upward, yielding:

$$P_1^* = 304.09, \quad Q_1^* = 81.2\%.$$

Demand expansion increases both equilibrium price and adoption by approximately 2.7 percentage points relative to baseline.

Scenario 2: Price Subsidy

A 10% price subsidy reduces effective prices for households to $P(1-0.1)$, resulting in:

$$P_2^* = 308.36, \quad Q_2^* = 82.5\%.$$

This policy raises adoption by around 4.0 percentage points compared to baseline.

To sum up, both policy interventions significantly promote the adoption of second-hand school uniforms. While price subsidies generate a slightly stronger short-term effect, reducing social stigma also enhances market outcomes by improving acceptance among parents and students. Implementing both measures in tandem could further accelerate adoption and strengthen the circular economy framework within UK schools.

Finally, the broader implications of this model merit attention. The calibrated framework offers a theoretically grounded basis for examining how price, social stigma, and environmental awareness jointly shape market outcomes. Comparative statics suggest that policy interventions targeting stigma reduction ($\Delta S < 0$) or subsidies ($\Delta P < 0$) will shift the demand curve upward, increase equilibrium adoption, and enhance circular economy performance in UK schools.

5. Conclusion

This paper develops and calibrates a static partial-equilibrium model of the UK second-hand school uniform market with heterogeneous households and suppliers, incorporating price sensitivity, stigma dis-utility, and reuse of environmental utility (Mohanty, 2011).

The paper formulates a partial-equilibrium model of household demand and supplier behaviour in the UK second-hand school-uniform market. Parameters are calibrated with an online survey of 207 respondents, and the model is used to simulate two interventions—a 10% price subsidy and a 20% stigma-reduction campaign. Numerical results suggest the subsidy raises equilibrium adoption from 78.5% to 82.5 %, while stigma reduction lifts it to 81.2%.

The study argues that combining both tools maximises welfare.

This study has two main limitations. First, the survey design lacks certain quantitative methodological techniques, which prevent the estimation of a linear supply function and the calibration of a linear equation using the survey data. Second, the limited representativeness of the sample may affect the external validity of the welfare results. Future research could address these limitations by increasing the sample size and adopting probability sampling methods to improve the reliability and generalizability of the findings. Additionally, future studies could extend the heterogeneous policy targeting related to tax and welfare resources from the UK government, and endogenous environmental awareness. Further empirical work could also quantify the long-run welfare gains from sustained stigma-reduction programs.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Amann, J., & Middleditch, P. (2017). Growth in a Time of Austerity: Evidence from the Uk. *Scottish Journal of Political Economy, 64,* 349-375. https://doi.org/10.1111/sjpe.12132

Ballarin, C. (2014). Locales: A Module System for Mathematical Theories. *Journal of Automated Reasoning, 52,* 123-153. https://doi.org/10.1007/s10817-013-9284-7

Coddington, W. (1993). Environmental Marketing: Positive Strategies for Reaching the Green Consumer. https://cir.nii.ac.jp/crid/1970023484837688264

Hobbs, R. (2016). *Understanding the Influencers of Second-Hand Apparel Shopping Behavior.* Wilfrid Laurier University.

Hursh, S. R. (1984). Behavioral Economics. *Journal of the Experimental Analysis of Behavior, 42,* 435-452. https://doi.org/10.1901/jeab.1984.42-435

Lamport, L., & LaTex, A. (1994). Document Preparation System. https://sic.ici.ro/documents/1388/Book_Review_4_Issue_4_SIC_1994.pdf

Macey-Dare, R. (2024). Ramsey, Ramsay, Schelling and Starmer+ VAT and the Optimal Provision, Taxation, Subsidization and Suppression of UK Private and State School Education. *SSRN.*

Madaan, G., Singh, A., Mittal, A., & Shahare, P. (2024). Reduce, Reuse, Recycle: Circular Economic Principles, Sustainability and Entrepreneurship in Developing Ecosystems. *Journal of Small Business and Enterprise Development, 31,* 1041-1066. https://doi.org/10.1108/jsbed-01-2023-0009

Manickam, P., & Duraisamy, G. (2019). 3rs and Circular Economy. In S. S. Muthu (Ed.), *Circular Economy in Textiles and Apparel* (pp. 77-93). Elsevier.

https://doi.org/10.1016/b978-0-08-102630-4.00004-2

Maurici, J., & McLellan, G. (2024). Exemption from Taxes: A Class Act or Class Conflict? *Judicial Review, 29,* 300-325. https://doi.org/10.1080/10854681.2024.2418711

McFadden, D. (1972). *Conditional Logit Analysis of Qualitative Choice Behavior.* Academic Press.

McRobbie, A. (1989). Second-Hand Dresses and the Role of the Ragmarket. In McRobbie, A. (Ed.), *Zoot Suits and Second-Hand Dresses* (pp. 23-49). Springer. https://doi.org/10.1007/978-1-349-19999-0_2

Mohanty, C. (2011). Reduce, Reuse and Recycle (The 3rs) and Resource Efficiency as the Basis for Sustainable Waste Management. In *Proceedings of the Synergizing Resource Efficiency with Informal Sector towards Sustainable Waste Management* (pp. 4-24). https://sustainabledevelopment.un.org/content/dsd/csd/csd_pdfs/csd-19/learningcentre/presentations/May%209%20am/1%20-%20Learning_Centre_9May_ppt_Mohanty.pdf

Norris, L. (2012). Trade and Transformations of Secondhand Clothing: Introduction. *Textile, 10,* 128-143. https://doi.org/10.2752/175183512x13315695424473

Page, G., Power, M., & Patrick, R. (2021). *Uniform Mistakes: The Costs of Going Back to School.* Covid Realities. https://www.nuffieldfoundation.org/wp-content/uploads/2021/04/CovidRealities-UniformMistakes.pdf

Ramdani, R., Syamsuddin, A., & Sirajuddin, S. (2019). Development of Mathematical Module-Problem Solving Approach to Train Student's Reflective Thinking. *Pedagogical Research, 4,* em0040. https://doi.org/10.29333/pr/5861

Ridge, T. (2011). The Everyday Costs of Poverty in Childhood: A Review of Qualitative Research Exploring the Lives and Experiences of Low-Income Children in the UK. *Children & Society, 25,* 73-84. https://doi.org/10.1111/j.1099-0860.2010.00345.x

Saunders, M., Lewis, P., & Thornhill, A. (2006) *Research Methods for Business* (4th ed.). Pearson Education.

Vallacher, R. R., & Wegner, D. M. (1987). What Do People Think They're Doing? Action Identification and Human Behavior. *Psychological Review, 94,* 3-15. https://doi.org/10.1037/0033-295x.94.1.3

Varian, H. R. (2003). *Intermediate Microeconomics: A Modern Approach.* Elsevier.

Appendix

A. Detailed Proofs

A.1. Proof of Proposition 3.1

Proof. We construct the proof in four steps.

Step 1: Continuity of the excess demand function.

Define the excess demand function as

$$f(P) := Q_d(P-\tau_t, S_t-\sigma_t, E_t) - Q_s(P).$$

By assumption, Q_d and Q_s are continuous in price. Hence, $f(P)$ is continuous on any closed interval $[0,\overline{P}]$ for arbitrary $\overline{P}>0$.

Step 2: Boundary behavior of $f(P)$.

At the lower bound $P=0$, supply is minimal:

$$Q_s(0) = a \in (0,1), \quad Q_d(\tilde{P}_t, \tilde{S}_t, E_t) \le 1.$$

Since demand is bounded above by one, we have:

$$f(0) = Q_d(-\tau_t, S_t-\sigma_t, E_t) - Q_s(0) \ge 1-a > 0.$$

At the upper bound $P=\overline{P}$ for sufficiently large $\overline{P}$, supply becomes arbitrarily large due to its monotonicity, while demand declines:

$$f(\overline{P}) = Q_d(\overline{P}-\tau_t, S_t-\sigma_t, E_t) - Q_s(\overline{P}) \le 0.$$

Step 3: Existence of equilibrium.

By the Intermediate Value Theorem, since $f(P)$ is continuous and crosses zero from above to below on $[0,\overline{P}]$, there exists $P^* \in [0,\overline{P}]$ such that

$$f(P^*) = Q_d(P^*-\tau_t, S_t-\sigma_t, E_t) - Q_s(P^*) = 0.$$

Step 4: Uniqueness.

By (A1), $Q_d(\cdot)$ is strictly decreasing in $\tilde{P}_t$, and by (A2), $Q_s(\cdot)$ is strictly increasing in P. Hence, the composite function $f(P)$ is strictly decreasing. Therefore, the root P^* is unique. □

A.2. Proof of Lemma 3.1

Proof. To characterize how the equilibrium price responds to changes in perceived stigma, consider the equilibrium condition:

$$F(P^*(S), S) = Q_d(P^*-\tau, S-\sigma, E_t) - Q_s(P^*) = 0.$$

Totally differentiating both sides with respect to S gives:

$$\frac{\mathrm{d}F}{\mathrm{d}S} = F_P \cdot \frac{\mathrm{d}P^*}{\mathrm{d}S} + F_S = 0.$$

Solving for the derivative yields:

$$\frac{\mathrm{d}P^*}{\mathrm{d}S} = -\frac{F_S}{F_P}.$$

Given the assumed signs $F_S > 0$, $F_P < 0$, the result follows:

$$\frac{\mathrm{d}P^*}{\mathrm{d}S} > 0.$$

□

A.3. Proof of Proposition 3.2

Proof. We refine the implicit derivative expression by explicitly computing the partial derivatives of the excess demand function.

Step 1: Use Lemma 3.1 to obtain:

$$\frac{\mathrm{d}P^*}{\mathrm{d}S} = -\frac{F_S}{F_P}.$$

Step 2: Differentiate aggregate demand with respect to price. Under logit choice probabilities,

$$\frac{\partial Q_d}{\partial P} = \int_0^1 \left(-\beta_i\, p_i\left(1-p_i\right)\right)\mathrm{d}i = -\mathbb{E}\left[\beta_i p_i\left(1-p_i\right)\right].$$

Step 3: Differentiate supply with respect to price. Given linear supply,

$$\frac{\partial Q_s}{\partial P} = b.$$

Step 4: Compute the net price derivative of excess demand.

$$F_P = \frac{\partial Q_d}{\partial P} - \frac{\partial Q_s}{\partial P} = -\mathbb{E}\left[\beta_i p_i\left(1-p_i\right)\right] - b.$$

Step 5: Differentiate aggregate demand with respect to stigma.

$$\frac{\partial Q_d}{\partial S} = \int_0^1 \left(-\lambda_i\, p_i\left(1-p_i\right)\right)\mathrm{d}i = -\mathbb{E}\left[\lambda_i p_i\left(1-p_i\right)\right].$$

Since supply is unaffected by stigma, we have:

$$F_S = \frac{\partial Q_d}{\partial S} = -\mathbb{E}\left[\lambda_i p_i\left(1-p_i\right)\right].$$

Step 6: Combine the results. Substitute into Lemma 3.1:

$$\frac{\mathrm{d}P^*}{\mathrm{d}S} = -\frac{-\mathbb{E}\left[\lambda_i p_i\left(1-p_i\right)\right]}{-\mathbb{E}\left[\beta_i p_i\left(1-p_i\right)\right]-b} = \frac{\mathbb{E}\left[\lambda_i p_i\left(1-p_i\right)\right]}{\mathbb{E}\left[\beta_i p_i\left(1-p_i\right)\right]+b}.$$

Step 7: Derive the derivative of quantity. Total quantity supplied at equilibrium is $Q^* = Q_s\left(P^*\right)$, so

$$\frac{\partial Q^*}{\partial S} = Q_s'\left(P^*\right)\cdot\frac{\partial P^*}{\partial S} = b\cdot\frac{\partial P^*}{\partial S}.$$

Substituting the previous result completes the proof. □

A.4. Proof of Proposition 3.3

Proof. **Step 1 (Planner's problem).** The planner chooses $\left(\tau,\sigma\right)\in\mathbb{R}_+^2$ to maximize net social welfare:

$$\max_{\tau,\sigma\geq 0} W\left(P^*-\tau, S_t-\sigma, E_t\right) - C\left(\tau,\sigma\right).$$

Step 2 (Lagrangian formulation). Introduce multipliers $\mu_\tau,\mu_\sigma\geq 0$ for the

non-negativity constraints:

$$\mathcal{L} = W\left(P^* - \tau, S_t - \sigma, E_t\right) - C\left(\tau, \sigma\right) + \mu_\tau \tau + \mu_\sigma \sigma.$$

Step 3 (First-order conditions). The Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial \tau} = -\frac{\partial W}{\partial P} - \frac{\partial C}{\partial \tau} + \mu_\tau = 0,$$

$$\frac{\partial \mathcal{L}}{\partial \sigma} = -\frac{\partial W}{\partial S} - \frac{\partial C}{\partial \sigma} + \mu_\sigma = 0,$$

$$\mu_\tau \cdot \tau = 0,\ \mu_\tau \geq 0,\ \tau \geq 0,$$

$$\mu_\sigma \cdot \sigma = 0,\ \mu_\sigma \geq 0,\ \sigma \geq 0.$$

Step 4 (Interior solution). If the optimal $\left(\tau^*, \sigma^*\right)$ lies in the interior, i.e., $\tau^*, \sigma^* > 0$, then $\mu_\tau = \mu_\sigma = 0$, and the first-order conditions reduce to:

$$\frac{\partial W}{\partial \tau} = \frac{\partial C}{\partial \tau},\quad \frac{\partial W}{\partial \sigma} = \frac{\partial C}{\partial \sigma}.$$

Step 5 (Corner solutions). If, for example, $\tau^* = 0$, then $\mu_\tau \geq 0$, and the corresponding condition becomes:

$$-\frac{\partial W}{\partial P} - \frac{\partial C}{\partial \tau} \leq 0,$$

which ensures that expanding τ would not increase net welfare. The same logic applies to $\sigma^* = 0$.

Step 6 (Second-order condition). Assuming W is concave in both instruments and C is convex, the bordered Hessian of the Lagrangian is negative definite. Therefore, the first-order conditions characterize a unique global maximum. □

Survey Design and Data Collection

To investigate the UK second-hand school uniform market from a circular economy perspective, I conducted an online survey via *Google Forms*[4]. The questionnaire targeted students, parents or guardians, school staff, and related stakeholders. A total of 207 valid responses were collected. All questions and response options are listed below. The questionnaire was structured into several thematic blocks, as summarized in Table 1. Most questions were multiple-choice with optional open-text responses.

1. **Gender**
   (a) Male
   (b) Female
   (c) Prefer not to say
2. **Age**
   (a) 12 - 16
   (b) 16 - 18

[4]See https://docs.google.com/forms/d/e/1FAIpQLScwCnGqpIsBAh-DMgeQc2Kod3sXwGi9JZATYyzCrJ40hKy9tYw/viewform?usp=sf_link here entitled "Examining the UK Second-Hand School Uniform Market Through Circular Economy Perspective".

(c) 18 - 40

(d) Over 40

3. **Nationality**

(a) Europeans

(b) Asia

(c) British

(d) American

(e) Prefer not to say

4. **Occupation (continue if you are student, parent, or guardian)**

(a) Student

(b) Parent or Guardian

(c) Charity shop employee

(d) School uniform manufacturer

(e) School employee

5. **School type (your or your children's education background)**

(a) State school

(b) Private school

(c) Special education institution

(d) Other

6. **Household income (annual, after taxes)**

(a) Less than £20,000 per year

(b) £20,000 - £50,000 per year

(c) £50,000 - £100,000 per year

(d) More than £100,000 per year

(e) Prefer not to say

7. **Have you purchased a second-hand school uniform in the past 3 years?**

(a) Yes, I have purchased second-hand school uniforms within the past 3 years.

(b) No, I have never purchased second-hand school uniforms.

(c) No, but I have considered purchasing second-hand school uniforms.

(d) Yes, but only on rare occasions or for specific items.

8. **[If answered "No"] Why do you not purchase second-hand school uniforms? (multiple choice)**

(a) Hygiene concerns (worry about cleanliness even if washed)

(b) Fit and quality issues (may not fit well or be in good condition)

(c) Personal preference (prefer brand-new uniforms)

(d) Limited availability (hard to find right size/items)

(e) I don't care about school uniforms

9. **Where do you usually buy second-hand school uniforms?**

(a) School's second-hand uniform shop or sale events

(b) Online platforms (e.g., eBay, Facebook Marketplace)

(c) Local charity shops or thrift stores

(d) From other parents or students directly (e.g., school community groups)

10. **Why do you buy second-hand school uniforms? (multiple choice)**
    (a) To save money and reduce expenses
    (b) To support sustainability and reduce clothing waste
    (c) Because children outgrow uniforms quickly
    (d) Due to school policies or recommendations
11. **Yearly savings from buying second-hand school uniforms**
    (a) Less than £100
    (b) £100 - £300
    (c) £300 - £600
    (d) £600 - £1000
    (e) More than £1000
12. **Do you know how to buy second-hand school uniforms?**
    (a) Yes, I know and have done so before
    (b) Yes, I know but have never purchased
    (c) No, but interested to learn
    (d) No, and not interested
13. **Does your school send emails or letters about second-hand uniforms?**
    (a) Yes, regularly
    (b) Yes, occasionally or during events
    (c) No
    (d) Not sure
14. **Features you consider when buying second-hand uniforms**
    (a) Good condition and minimal wear
    (b) Correct size and proper fit
    (c) Affordable price and cost savings
    (d) School compliance (dress code)
15. **Attitude to donating school uniforms**
    (a) Actively donate
    (b) Willing to donate but haven't yet
    (c) Prefer to sell rather than donate
    (d) Do not donate and have no plans
    (e) Don't know who accepts old clothes
16. **Do you think schools are the leading influence on parents/students?**
    (a) Yes, schools play the biggest role
    (b) Schools have some influence but other factors matter more
    (c) No, decisions are independent
    (d) Not sure
17. **Barriers to second-hand school uniforms**
    (a) School policies requiring new uniforms
    (b) Social stigma or concerns
    (c) Limited availability or sizing issues
    (d) Lack of awareness or promotion by schools
18. **Opportunities for second-hand school uniforms**

(a) Cost savings for families
(b) Promoting sustainability and reducing textile waste
(c) Encouraging a stronger sense of community
(d) Increasing accessibility to uniforms for all students

19. **Do you know the 3R rules for second-hand school uniforms?**
(a) Yes, fully understand
(b) Have heard of them but not much detail
(c) No, but would like to learn
(d) No, and not interested

20. **Ways to improve second-hand uniform marketing (multiple choice)**
(a) Stronger school involvement (newsletters, PTA)
(b) Online platforms or apps for easier exchange
(c) Community awareness campaigns
(d) Incentives or discounts for donations/purchases

21. **Do you think redesigning/upcycling could create a new fashion trend?**
(a) Yes, absolutely
(b) Maybe, depends on appeal and marketing
(c) Unlikely
(d) No, second-hand stigma remains

22. **Would you support a school initiative for upcycling uniforms?**
(a) Yes, absolutely
(b) Maybe, with proper organization and funding
(c) Not sure, depends on quality and acceptance
(d) No, second-hand not widely accepted