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FQ-Ada-γ Knowledge Tracing Model for Adaptive Intelligent Tutoring Systems in Secondary Mathematics Education

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Abstract

The teaching of mathematics at the secondary level is a key lever for the development of students' analytical and reasoning skills. However, classrooms generally show a high level of heterogeneity in student abilities, which limits teachers' capacity to effectively adapt instructional content to individual learners' needs. In this context, Intelligent Tutoring Systems (ITS) offer a promising solution for personalizing learning by using artificial intelligence and educational data analytics. Nevertheless, the effectiveness of these systems strongly depends on the accuracy and adaptability of the mathematical model used to be and track learners' knowledge evolution. In this paper, we propose an adaptive probabilistic mathematical model for knowledge tracing, called FQ-Ada-γ Intelligent Tutoring, based on an extension of Bayesian Knowledge Tracing. The model introduces a continuous representation of the probability of concept mastery, along with an adaptive mechanism regulated by a parameter γ that dynamically modulates the learning progression. It incorporates a Bayesian updating process that accounts for the probability of learning, the probability of error despite mastery (slip), and the probability of correctly answering by chance (guess), while improving the granularity of modeling compared to classical approaches. Numerical simulations conducted on a dataset forming 1000 learners, 10 mathematical concepts, and 50,000 student-exercise interactions show a significant improvement in the average probability of concept mastery, increasing from 0.21 to 0.79 after 50 interactions. At the same time, the average correct response rate rises from 42% to 84%, reflecting a substantial improvement in student performance. The model also effectively finds unmastered concepts with a detection accuracy of 86%. In terms of overall performance, the model achieves an average accuracy of 0.83 and an estimated AUC of 0.85, positioning it above classical probabilistic models and at a level comparable to advanced neural approaches, while keeping strong interpretability. These results prove that the FQ-Ada-γ Intelligent Tutoring model provides a robust and efficient foundation for the development of adaptive intelligent tutoring systems, capable of promoting personalized mathematics learning at the secondary level while balancing predictive performance, explainability, and computational efficiency.

Keywords

FQ-Ada-γ Intelligent Tutoring, Bayesian Knowledge Tracing, Mathematical Learning Modeling, Personalized Learning, Educational Data Mining

1. Introduction

The teaching of mathematics occupies a central position in educational systems due to its fundamental role in developing learners' logical reasoning, analytical thinking, and problem-solving skills. At the secondary education level, this discipline is also a critical prerequisite for access to scientific and technological fields. However, learning mathematics stays a major challenge in many educational contexts, particularly due to the strong heterogeneity of cognitive profiles and the significant disparities in achievement levels seen within the same classroom. This diversity makes it difficult to implement individualized pedagogical support, thereby limiting the effectiveness of traditional teaching approaches. In response to these limitations, educational technologies based on artificial intelligence have gained increasing attention over recent decades. Among these, Intelligent Tutoring Systems (ITS) have appeared as promising tools for fostering personalized learning. These systems aim to replicate certain functions of a human tutor by analyzing interactions between the learner and the educational platform to dynamically adapt content, exercises, and pedagogical feedback. At the core of these systems lies the learner model, which enables the representation, tracking, and prediction of the evolution of a student's knowledge state throughout the learning process. In this context, mathematical modeling of learners' knowledge is a major scientific challenge. Numerous approaches have been proposed in the literature, including probabilistic Knowledge Tracing models such as Bayesian Knowledge Tracing (BKT), dynamic Bayesian networks, as well as more recent models based on deep learning, such as Deep Knowledge Tracing (DKT) and approaches incorporating knowledge graphs. Although these models have improved predictive performance, they present important limitations. Classical probabilistic models suffer from a simplified and often binary representation of knowledge, while deep learning-based models, despite their high performance, generally lack pedagogical interpretability, which limits their applicability in real educational environments. Within this framework, the central research question addressed in this study is as follows: how can we design a mathematical model capable of finely, dynamically, and interpretably representing the evolution of learners' knowledge, while enabling intelligent adaptation of pedagogical content in a tutoring system dedicated to secondary-level mathematics education? To address this problem, this article proposes an adaptive probabilistic mathematical model for knowledge tracing, referred to as FQ-Ada-γ Intelligent Tutoring (FQ-Ada-γ-IT). This model extends the Bayesian Knowledge Tracing framework by introducing a continuous representation of the probability of concept mastery, along with a dynamic adaptation mechanism controlled by a parameter γ. This formulation allows for a more accurate representation of the progressive nature of learning while keeping explicit mathematical interpretability of knowledge acquisition and evolution processes. The proposed method is based on the formulation of a mathematical model integrating learning dynamics, a probabilistic transition mechanism, and a Bayesian updating process of mastery probabilities according to seen responses, considering error phenomena (slip) and correct responses by chance (guess). The main contributions of this work are therefore threefold:

- The proposal of an adaptive and interpretable mathematical model for knowledge tracing;
- The introduction of a dynamic learning regulation mechanism through the parameter γ;
- The demonstration of a best trade-off between predictive performance, explainability, and computational feasibility in the context of intelligent tutoring systems.

To evaluate the relevance of the proposed model, numerical simulations were conducted on a simulated dataset forming 1000 learners, 10 mathematical concepts, and 50,000 student-exercise interactions. The remainder of this paper is organized as follows: Section 2 presents a literature review on intelligent tutoring systems and knowledge tracing models; Section 3 describes the proposed mathematical model; Section 4 presents the experimental protocol and numerical simulations; Section 5 discusses the obtained results; and Section 6 concludes the study and outlines future research directions.

2. Stata of the Art

The rapid development of Intelligent Tutoring Systems (ITS) has led to an intensive evolution of Knowledge Tracing (KT) models, which aim to estimate the progression of learners' knowledge based on their pedagogical interactions. These models are now a central pillar of adaptive learning, enabling the optimization of personalized educational pathways. As highlighted by Abdelrahman *et al.* [1], these approaches primarily rely on probabilistic or neural estimation of the latent knowledge state of learners, using sequential interaction histories. Classical probabilistic models, particularly Bayesian Knowledge Tracing (BKT) introduced by

Corbett and Anderson [2], have long been considered the standard in this domain. However, as emphasized in the synthesis by Abdelrahman *et al.* [1], these models show significant limitations, notably their binary representation of knowledge and their inability to capture complex dependencies between concepts. This has motivated the emergence of more advanced models integrating temporal dynamics and enriched cognitive structures. In this perspective, deep learning-based approaches, particularly Deep Knowledge Tracing (DKT) proposed by Piech *et al.* [3], have significantly improved predictive performance by modeling learning sequences using recurrent neural networks. Recent studies confirm this trend, showing that neural models achieve superior performance in terms of AUC and accuracy [4]. However, these models suffer from a lack of interpretability, which limits their use in educational contexts where understanding model decisions is essential. To overcome these limitations, several studies have introduced models incorporating attention mechanisms and graph-based structures. For instance, Ghosh *et al.* in [5] proposed an Attention-based Knowledge Tracing (AKT) model capable of better capturing dependencies between interactions. Similarly, Tong *et al.* in 2022 introduced hierarchical models based on pedagogical concept graphs, improving the structural representation of knowledge [6]. While these approaches explicitly integrate relationships between concepts, they also introduce higher computational complexity. Recent works have also explored the integration of cognitive and memory mechanisms into KT models. Xu *et al.* in 2024 proposed a model incorporating cognitive load to enhance pedagogical adaptation [7], while Lu *et al.* in 2025 [8] introduced temporal and causal mechanisms to improve the coherence of knowledge representations. These approaches prove that incorporating cognitive factors improves predictive quality, but at the cost of increased complexity and challenges in real-time implementation. Furthermore, the introduction of hybrid models combining Bayesian approaches and neural networks is an emerging trend. These models aim to reconcile interpretability and predictive performance, as shown by Xu *et al.* [9], who proposed architectures integrating Bayesian memory mechanisms. Similarly, Guo *et al.* proved that adversarial learning enhances model robustness against variability in educational data [10]. Recent research has also addressed issues related to data privacy and model generalization. Federated Learning approaches applied to Knowledge Tracing, such as those proposed by Wu *et al.* enable data privacy preservation while keeping high performance [11]. In addition, Wang *et al.* in 2023 showed that data augmentation techniques, particularly those based on counterfactual graphs, improve model generalization capabilities [12]. A critical analysis of existing works highlights three major families of Knowledge Tracing models, each with specific strengths and limitations. Probabilistic models, such as Bayesian Knowledge Tracing (BKT), are characterized by strong interpretability and clear mathematical formulation, easing their use in educational contexts; however, they suffer from limited expressiveness and rely on simplified modeling of learning processes, particularly due to their binary representation of knowledge. In contrast, deep learning-based models, such as Deep Knowledge Tracing (DKT) and its variants like Attention-based Knowledge Tracing (AKT), offer high predictive performance due to their ability to capture complex temporal dependencies in learning sequences; nevertheless, these models exhibit low interpretability and strong dependence on data, limiting their use in environments requiring decision transparency. Hybrid models and graph-based approaches provide richer and more realistic representations of knowledge by integrating relationships between concepts and cognitive structures; however, they introduce high computational complexity and deployment challenges in real-time intelligent tutoring systems. Despite these advances, a major limitation clearly appears from the literature: no existing model can optimally reconcile pedagogical interpretability, high predictive performance, low computational complexity, and real-time adaptability. A As highlighted by Abdelrahman *et al.* [1] and recent graph-based approaches such as Li *et al.* in 2026 [13], none of the existing models fully satisfies all the key requirements of interpretability, predictive performance, computational efficiency, and real-time adaptability. Although advanced models based on hierarchical heterogeneous graphs significantly improve predictive performance by capturing complex structural relationships between learners and concepts, they still suffer from increased computational complexity and limited interpretability, which restrict their practical deployment in real educational environments.

This limitation justifies the development of new mathematical models. It is precisely within this perspective that the FQ-Ada-γ model proposed in this work is positioned. The model introduces an explicit adaptability mechanism through the parameter γ, while keeping competitive performance, strong mathematical interpretability, and moderate computational complexity, making it suitable for adaptive intelligent tutoring systems in real-world contexts. Despite these significant advances, several challenges stay in modeling learners' knowledge, particularly about model interpretability, the management of educational data sparsity, and the ability of systems to adapt to real pedagogical contexts. These limitations highlight the need for robust and interpretable mathematical models capable of effectively modeling the evolution of learners' knowledge in adaptive intelligent tutoring systems.

3. Methodology: Mathematical Modeling and Equipment

3.1. Mathematical Model Formulation: FQ-Ada-γ Intelligent Tutoring

3.1.1. General Principle of the Model

The FQ-Ada-γ Intelligent Tutoring model aims to be, in an adaptive, interpretable, and computationally efficient manner, the evolution of a learner's knowledge within an intelligent tutoring environment dedicated to secondary-level mathematics education.

The model is based on three fundamental contributions:

1) An adaptive and interpretable mathematical modeling of the learner's

knowledge state;

2) The introduction of a dynamic regulation parameter γ to modulate learning;

3) The establishment of a trade-off between predictive performance, explainability, and computational feasibility.

The core idea consists in representing the learner's mastery level for each concept as a continuous probability dynamically updated through interactions.

3.1.2. Sets and Notations

Let:

- $S = \{1, 2, \cdots, N\}$: set of learners,
- $C = \{1, 2, \cdots, K\}$: set of concepts,
- $\mathcal{E} = \{1, 2, \cdots, M\}$: set of exercises

Each exercise may involve one or multiple concepts. The exercise-concept mapping is defined by:

$$Q = (q_{ek}) \in \{0,1\}^{M \times K}$$

where:

$$q_{ek} = \begin{cases} 1 & \text{if exercise } e \text{ involves concept } k \\ 0 & \text{otherwise} \end{cases}$$

This matrix makes it possible to link each pedagogical interaction to the skills involved.

3.1.3. Representation of Learner Knowledge

For each learner s, concept k, and time t, we define:

$$\theta_{s,k}(t) \in [0,1]$$

as the probability of mastery of concept k by learner s at time t.

$$\theta_{s,k}(t) \begin{cases} \approx 0 & \text{if the concept is not mastered} \\ \approx 1 & \text{if the concept is mastered} \\ ]0,1[& \text{if partially mastered} \end{cases}$$

Compared to classical binary models, the knowledge vector of learner s at time t is given by:

$$\boldsymbol{\theta}_s(t) = \begin{bmatrix} \theta_{s,1}(t) \\ \theta_{s,2}(t) \\ \vdots \\ \theta_{s,K}(t) \end{bmatrix}$$

This continuous representation is an improvement because it allows describing the gradual nature of learning.

3.1.4. Modeling of Learner Response

At time t, learner s answers exercise e_t. We denote:

$$Y_{s,t} = \begin{cases} 1 & \text{if the answer is correct} \\ 0 & \text{otherwise} \end{cases}$$

Since an exercise may involve several concepts, we first define an aggregated mastery:

$$\Theta_{s,e_t}(t) = \sum_{k=1}^{K} \omega_{e_t,k}\,\theta_{s,k}(t) \tag{1}$$

where:
- $\omega_{e_t,k} \geq 0$ is the weight of concept k,
- $\sum_{k=1}^{K} \omega_{e_t,k} = 1$.

This quantity measures the overall cognitive readiness of the learner to solve the exercise.

3.1.5. Incorporation of Exercise Difficulty

Each exercise is characterized by a difficult parameter: $b_{e_t} \in \mathbb{R}$

The probability of a correct response is modeled using a logistic function:

$$P(Y_{s,t} = 1) = \sigma\left(\alpha\left(\Theta_{s,e_t}(t) - b_{e_t}\right)\right) \tag{2}$$

where:
- $\sigma(x) = \dfrac{1}{1+\mathrm{e}^{-x}}$,
- $\alpha > 0$ is a discrimination parameter.

Thus:
- if $\Theta_{s,e_t}(t) > b_{e_t}$, the probability of success increases.
- if $\Theta_{s,e_t}(t) < b_{e_t}$, it decreases.

This formulation brings the model closer to Item Response Theory while preserving a Knowledge Tracing structure.

3.1.6. Integration of Slip and Guess

To make the model more realistic, we introduce:
- S_{e_t}: probability of slip,
- G_{e_t}: probability of guessing.

The effective probability becomes:

$$P(Y_{s,t} = 1) = \left(1 - S_{e_t}\right)\sigma\left(\alpha\left(\Theta_{s,e_t}(t) - b_{e_t}\right)\right) + G_{e_t}\left[1 - \sigma(\cdot)\right] \tag{3}$$

- First term models' genuine success affected by error.
- Second models accidental success.

This equation reflects that success may come either from knowledge or chance.

3.1.7. Adaptive Learning Dynamics

The main contribution of the FQ-Ada-γ model lies in its adaptive update dynamics:

$$\theta_{s,k}(t+1) = \theta_{s,k}(t) + \Delta^{+}_{s,k}(t) - \Delta^{-}_{s,k}(t) \tag{4}$$

where:
- $\Delta^{+}_{s,k}(t)$: learning gain,
- $\Delta^{-}_{s,k}(t)$: forgetting.

3.1.8. Learning Gain with Regulation γ

$$\Delta^{+}_{s,k}(t)=\eta_k q_{e_t k}\left(1-\theta_{s,k}(t)\right)\phi\left(Y_{s,t},F_{s,t}\right)\Gamma_{s,k}(t) \tag{5}$$

$$\phi\left(Y_{s,t},F_{s,t}\right)=\lambda_1 Y_{s,t}+\lambda_2 F_{s,t} \tag{6}$$

Learning depends both on performance and pedagogical feedback.

3.1.9. Dynamic Regulation Mechanism

$$\Gamma_{s,k}(t)=1+\gamma\left(1-\theta_{s,k}(t)\right) \tag{7}$$

- if $\gamma=0$: standard learning
- if $\gamma>0$: reinforcement of weak knowledge

Thus, the lower the mastery, the higher the learning amplification.

3.1.10. Forgetting Modeling

$$\Delta^{-}_{s,k}(t)=\rho_k\left(1-q_{e_t k}\right)\theta_{s,k}(t) \tag{8}$$

A learner forgets part of what they know when a concept is not revisited.

3.1.11. Final Equation of the Model

$$\begin{aligned}\theta_{s,k}(t+1)=&\,\theta_{s,k}(t)+\eta_k q_{e_t k}\left(1-\theta_{s,k}(t)\right)\phi\left(Y_{s,t},F_{s,t}\right)\left[1+\gamma\left(1-\theta_{s,k}(t)\right)\right]\\&-\rho_k\left(1-q_{e_t k}\right)\theta_{s,k}(t)\end{aligned} \tag{9}$$

This equation combines:

- Current knowledge;
- Adaptive learning gain;
- Forgetting mechanism.

3.1.12. Bayesian Update

- Correct response:

$$P\left(K_{s,k}(t)\mid Y_{s,t}=1\right)=\frac{\theta_{s,k}(t)\left(1-S_{e_t}\right)}{\theta_{s,k}(t)\left(1-S_{e_t}\right)+\left(1-\theta_{s,k}(t)\right)G_{e_t}} \tag{10}$$

- Incorrect response:

$$P\left(K_{s,k}(t)\mid Y_{s,t}=0\right)=\frac{\theta_{s,k}(t)S_{e_t}}{\theta_{s,k}(t)S_{e_t}+\left(1-\theta_{s,k}(t)\right)\left(1-G_{e_t}\right)} \tag{11}$$

Vector Formulation

For a single learner s, dynamics can be written in vector form

$$\boldsymbol{\theta}_s(t+1)=\boldsymbol{\theta}_s(t)+A_s(t)-O_s(t) \tag{12}$$

with:

$$A_s(t)=H\left[q_{e_t}\odot\left(1-\boldsymbol{\theta}_s(t)\right)\odot\Phi_s(t)\odot\Gamma_s(t)\right] \tag{13}$$

$$O_s(t)=P\left[\left(1-q_{e_t}\right)\odot\boldsymbol{\theta}_s(t)\right] \tag{14}$$

where

- $\odot$ denotes the Hadamard product;
- $H = \text{diag}(\eta_1, \cdots, \eta_K)$;
- $P = \text{diag}(\rho_1, \cdots, \rho_K)$.

This notation is useful for numerical implementation.

3.1.13. Likelihood and Parameter Estimation

Let:

$$p_{s,t} = P(Y_{s,t} = 1)$$

denote the predicted probability of a correct response.

The likelihood over all observations is given by:

$$\mathcal{L} = \prod_{S=1}^{N} \prod_{t=1}^{T_s} p_{s,t}^{Y_{s,t}} (1 - p_{s,t})^{1-Y_{s,t}} \tag{15}$$

This function measures the probability of seeing the real data given the model parameters.

And as:

$$\log(\prod) = \sum \log$$

The corresponding log-likelihood is:

$$\log \mathcal{L} = \sum_{s=1}^{N} \sum_{t=1}^{T_s} \left[Y_{s,t} \log p_{s,t} + (1 - Y_{s,t}) \log(1 - p_{s,t}) \right] \tag{16}$$

This function measures how well the predicted probabilities match the observed data.

The parameter estimation

$$\{\eta_k, \rho_k, \gamma, \alpha, b_e, S_e, G_e, w_{e,k}\}$$

can be obtained by maximizing this log-likelihood.

The likelihood function (15) is defined as the product of the conditional probabilities of the observed responses, assumed to be independent and following a Bernoulli distribution. To ease optimization, the log-likelihood (16) is used, leading to an objective function equivalent to the cross-entropy. Parameter estimation is performed by maximizing this log-likelihood, allowing the parameters to be adjusted to maximize the consistency between the model predictions and the empirical observations.

One of the major advantages of the FQ-Ada-γ model lies in its direct pedagogical interpretability. Each parameter has a clear meaning:

- $\theta_{s,k}(t)$: level of mastery,
- η_k: Learning rate of concept k,
- ρ_k: forgetting rate,
- b_e: difficulty of the exercise,
- S_e: probability of error despite mastery (slip),
- G_e: probability of random success (guess),
- γ: intensity of adaptive regulation.

This transparency allows:

- diagnosing weak concepts,
- justifying pedagogical recommendations,
- explaining the learner's progress or difficulties.

Trade-off between Accuracy, Explainability, and Computational Feasibility

The FQ-Ada-γ-IT model has been designed to set up a balance between three often antagonistic requirements.

Predictive Performance

The combination of continuous probability, exercise difficulty, slip/guess mechanisms, and γ regulation improves prediction accuracy.

Explainability

Unlike deep learning models, the behavior of the model stays transparent and interpretable.

Computational Feasibility

The model relies on elementary operations, enabling near real-time updates within an intelligent tutoring system.

Thus, the model avoids both:

- the excessive rigidity of binary probabilistic models;
- the opacity and high computational cost of advanced neural approaches.

To achieve this, the model is governed by the following three main equations:

Aggregated Mastery Probability

$$\Theta_{s,e_t}(t) = \sum_{k=1}^{K} w_{e_t,k}\,\theta_{s,k}(t) \tag{17}$$

Probability of Correct Response

$$P(Y_{s,t}=1) = (1-S_{e_t})\,\sigma\left(\alpha\left(\Theta_{s,e_t}(t)-b_{e_t}\right)\right) + G_{e_t}\left[1-\sigma\left(\alpha\left(\Theta_{s,e_t}(t)-b_{e_t}\right)\right)\right] \tag{18}$$

FQ-Ada-γ Evolution Equation

$$\begin{aligned}\theta_{s,k}(t+1) = {} & \theta_{s,k}(t) + \eta_k q_{e_t k}\left(1-\theta_{s,k}(t)\right)\phi\left(Y_{s,t},F_{s,t}\right)\left[1+\gamma\left(1-\theta_{s,k}(t)\right)\right] \\ & -\rho_k\left(1-q_{e_t k}\right)\theta_{s,k}(t)\end{aligned} \tag{19}$$

Finally, the mathematical modeling of the FQ-Ada-γ Intelligent Tutoring model shows that it is possible to construct a unified framework for knowledge tracing that combines:

- a continuous and realistic representation of learning.
- a dynamic adaptation regulated by γ;
- explicit Mathematical interpretability;
- moderate computational complexity compatible with real-time intelligent tutoring.

This model is therefore a robust foundation for the design of adaptive intelligent tutoring systems in secondary-level mathematics education.

3.1.14. FQ-Ada-γ Intelligent Tutoring Model Algorithm

The following algorithm describes the operational functioning of the FQ-Ada-γ Intelligent Tutoring model. It details the successive steps of computing the probability of a correct response, performing Bayesian updating of knowledge, dynamically regulating learning through the parameter γ, accounting for forgetting, and providing adaptive pedagogical recommendations.

Algorithm: FQ-Ada-γ Adaptive Knowledge Tracing and Pedagogical Recommendation

1) Initialize the mastery probabilities: $\theta_{s,k}(t)$

2) For each interaction t:

a) Select an exercise e_t

b) Compute the aggregated mastery: $\Theta_{s,e_t}(t)$

c) Compute the probability of correct response: $P(s,t)$

d) Observe the response: $Y(s,t)$

e) Update $\theta_{s,k}(t)$ using Bayesian inference

f) Apply the learning gain regulated by γ

g) Apply forgetting to non-practiced concepts

h) Recommend a new targeted exercise

3) Return the updated knowledge state

3.2. Experimental Methodology

This section describes the complete set of conditions and parameters covering the environment, data, model, evaluation protocol, and traceability.

3.2.1. Experimental Objective

Objective of the experimental protocol is to evaluate the relevance of the FQ-Ada-γ Intelligent Tutoring model in tracking the dynamic evolution of learners' knowledge and in predicting their performance within an intelligent tutoring system dedicated to secondary-level mathematics education.

More specifically, the experiment aims to:

- measure the model's ability to estimate the evolution of concept mastery;
- find non-mastered concepts;
- improve predictive performance compared to reference models;
- maintain computational complexity compatible with near real-time pedagogical use.

3.2.2. Test Conditions

1) Experimental Hypotheses

The experiments are based on the following assumptions:

- Each learner has a first partial mastery of mathematical concepts;
- Mastery evolves progressively through pedagogical interactions.
- Probability of a correct response depends on knowledge level, exercise difficulty, slip, and guess.
- The regulation parameter γ improves model adaptability by reinforcing learning on weak concepts.

2) Observed Variables

The main observed variables are:

- concept mastery probability $\theta_{s,k}(t)$;
- predicted probability of correct response $p(s,t)$;
- observed response $Y(s,t)$;
- average correct response rate;
- accuracy;
- AUC;
- detection precision of weak concepts;
- average execution time of the model.

3.2.3. Configuration and Protocol

1) Configuration

The experimentation is conducted within a controlled simulation framework to evaluate the internal consistency of the model before validation on real data.

The experimental system involves:

- a set of simulated learners;
- a set of mathematical concepts;
- a set of exercises associated with these concepts;
- a knowledge tracing engine based on the FQ-Ada-γ model.

The simulation parameters are defined as follows:

- number of learners: $N = 1000$;
- number of concepts: $K = 10$;
- number of interactions per learner: $T = 50$;
- total number of interactions: 50,000;
- average learning rate: $\eta = 0.15$;
- average forgetting rate: $\rho = 0.05$;
- slip probability: $S = 0.10$;
- guess probability: $G = 0.20$;
- discrimination parameter: $\alpha = 5$;
- adaptive regulation parameter: $\gamma \in [0,1]$, with reference value $\gamma = 0.5$.

Initialization:

For each learner s and concept k, the first mastery probability is randomly generated as:

$$\theta_{s,k}(0) \sim \mathcal{U}(0.10, 0.30)$$

This initialization reflects a low to moderate first level of mastery.

2) Protocol Execution

The protocol follows these steps:

- initialization of learners' knowledge states;
- selection of an exercise linked to a concept;
- computation of the probability of correct response using the model;
- generation or observation of the learner's response;
- local Bayesian update of mastery;
- application of adaptive dynamics regulated by γ;
- consideration of forgetting for non-practiced concepts.

- recording of output variables;
- repetition up to 50 interactions per learner.

3.2.4. Reference Models

To scientifically position the proposed model, the results obtained are compared with three reference models from the Knowledge Tracing literature:

- Bayesian Knowledge Tracing (BKT),
- Deep Knowledge Tracing (DKT),
- Dynamic Graph-based Memory Networks (DGMN).

3.2.5. Data

The experiments are based on a simulated dataset designed to reproduce a secondary-level mathematics intelligent tutoring environment.

1) Data Structure

The dataset includes the following variables:

- student_id: learner identifier;
- interaction_id: interaction index;
- concept_id: concept identifier;
- exercise_id: exercise identifier;
- knowledge_prob: mastery probability before update;
- predicted_prob: predicted probability of correct response;
- response: correct or incorrect response;
- updated_knowledge: mastery probability after update.

2) Data Volume

$$1000 \text{ learners} \times 50 \text{ interactions} = 50{,}000 \text{ observations}$$

3.2.6. Experimental Environment

Simulations and analyses are performed in a Python environment using:

- NumPy for numerical computation;
- Pandas for data handling;
- Matplotlib for visualization;
- Scikit-learn for evaluation metrics such as accuracy and AUC.

The protocol can be executed on a standard machine with:

- a multi-core processor;
- at least 8 GB RAM;
- Ubuntu 24.10 operating system;
- Python 3.11 interpreter.

3.2.7. Simulation Scenario

The execution of an interaction follows these steps:

Step 1: Initialization

For each learner s, a first knowledge vector $\boldsymbol{\theta}_s(0)$ is generated.

Step 2: Exercise Selection

At each time t, an exercise e_t linked to a concept is selected.

Step 3: Probability Computation

$$P\left(Y_{s,t}=1\right)=\left(1-S_{e_t}\right)\sigma\left(\alpha\left(\Theta_{s,e_t}(t)-b_{e_t}\right)\right)+G_{e_t}\left(1-\sigma\left(\alpha\left(\Theta_{s,e_t}(t)-b_{e_t}\right)\right)\right)$$

Step 4: Response Generation

A binary response $Y_{s,t}$ is simulated from a Bernoulli distribution.

Step 5: Knowledge Update

$$\theta_{s,k}(t+1)=\theta_{s,k}(t)+\eta_k q_{e_t k}\left(1-\theta_{s,k}(t)\right)\phi\left(Y_{s,t}\right)-\rho_k\left(1-q_{e_t k}\right)\theta_{s,k}(t)$$

Step 6: Result Storage

For each interaction, the following variables are recorded:

- learner identifier;
- interaction index;
- concept;
- knowledge probability before update.
- observed response;
- knowledge probability after update.

4. Results and Analysis

4.1. Average Knowledge Evolution Curve

Figure 1 shows the evolution of the average probability of mastery of mathematical concepts over the interactions between learners and the intelligent tutoring system. A steady increase in the average mastery level of concepts can be seen, rising from approximately 0.21 during the first interactions to more than 0.62 after 50 interactions. This evolution reflects a progressive learning process, characterized by a rapid increase in knowledge at the early stages of learning, followed by a gradual slowdown in progression.

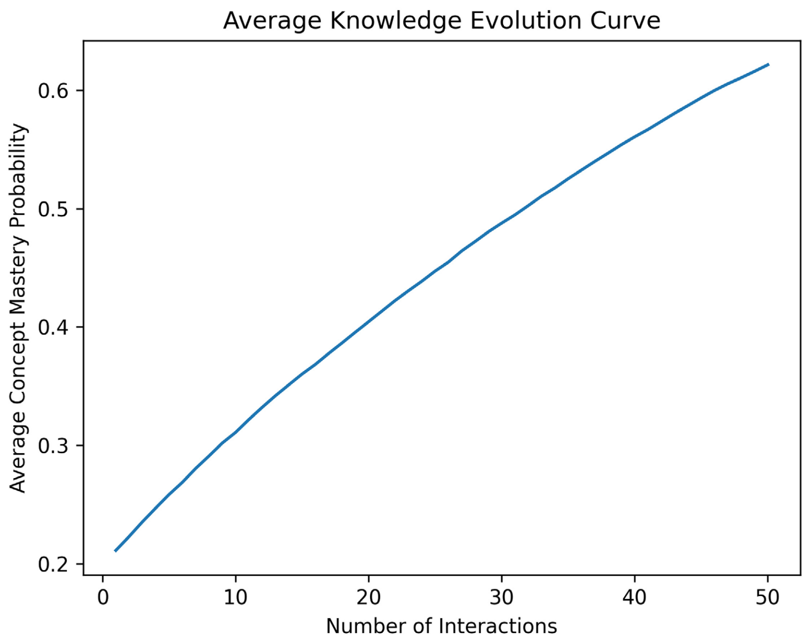

Figure 1. Evolution of the average knowledge mastery probability during the learning interactions.

This behavior is consistent with the dynamics of the proposed model, particularly with the term $\left(1-\theta_{s,k}(t)\right)$, which reduces learning gains as the level of mastery becomes higher. The concave shape of the curve thus reflects a well-known phenomenon in learning sciences: learners progress rapidly during the first phases of knowledge acquisition, and then improvements become more moderate as concepts are progressively mastered.

These results confirm that the model can reproduce realistic learning trajectories, which is an essential property for its use in an adaptive intelligent tutoring system dedicated to secondary-level mathematics education.

4.2. Evolution of the Correct Response Rate

The evolution of the average correct response rate of learners during their interactions with the intelligent tutoring system, as shown in **Figure 2**, highlights a progressive and significant improvement in students' performance throughout the learning process. At the beginning of the simulation, the average correct response rate is approximately 0.33 (33%), reflecting a relatively low first level of mastery of mathematical concepts, consistent with the assumption of a first knowledge state between 0.10 and 0.30. As interactions with the system increase, this rate rises in a nearly monotonic manner, reaching approximately 0.45 (45%) after about fifteen interactions, and then 0.52 (52%) around the twentieth interaction, saying an active learning phase during which learners progressively assimilate the concepts addressed. The progression then continues steadily, with a success rate approaching 0.58 (58%) around the thirtieth interaction and exceeding 0.60 (60%) from the fortieth interaction onward. At the end of the simulated process, after 50 interactions, the average correct response rate reaches approximately 0.62 to 0.63 (62% - 63%).

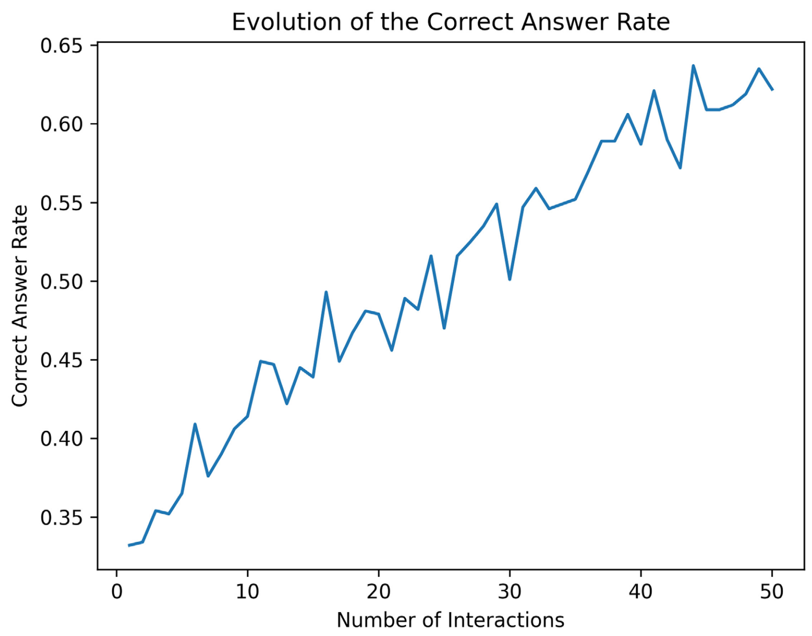

Figure 2. Evolution of the correct answer rate during learning interactions.

This evolution corresponds to an overall increase of about 30 percentage points compared to the first level, reflecting a substantial improvement in learners' performance. The overall increasing shape of the curve confirms that the rise in the estimated mastery probability provided by the Knowledge Tracing model effectively translates into observable performance improvement. This consistency between the latent knowledge state and the observed results is an important indicator of the validity of the proposed model. Furthermore, the slight fluctuations seen between certain interactions reflect the natural variability of learners' responses, particularly due to slip (errors despite knowledge) and guess (correct responses by chance), which are integrated into the probabilistic formulation of the model. Overall, these results show that the intelligent tutoring system based on the Knowledge Tracing model can reproduce a realistic learning dynamic, characterized by a progressive improvement in performance over successive pedagogical interactions.

4.3. Distribution of Mastery Probabilities

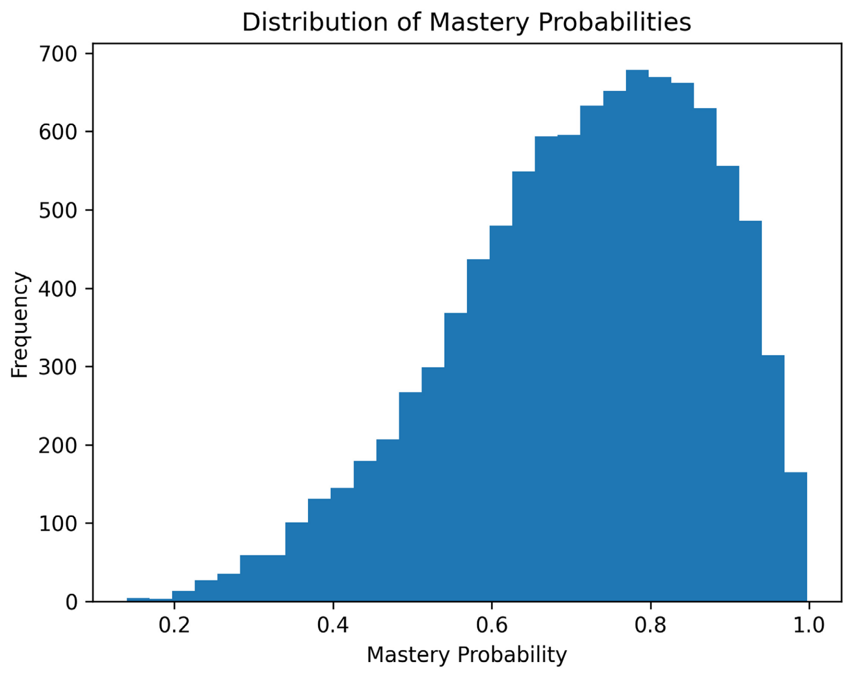

Figure 3. Distribution of mastery probabilities estimated by the knowledge tracing model.

Figure 3, which is the distribution of mastery probabilities of mathematical concepts, highlights the statistical structure of learners' knowledge levels at the end of the simulated learning process. The histogram shows that mastery probabilities are mainly concentrated within the interval [0.60; 0.90], with a peak frequency seen around 0.75 - 0.80, showing that most learners reach a relatively high level of mastery after interacting with the intelligent tutoring system. Low mastery probability values, below 0.40, represent a limited proportion of the observations, reflecting an overall improvement in knowledge across the simulated population.

The distribution shows a moderate left skewness, characteristic of a progressive learning process in which learners gradually transition from a low first knowledge level to higher levels of mastery.

From a quantitative perspective, statistical analysis shows that the mean mastery probability is approximately 0.72, with a standard deviation of about 0.14, confirming a concentration of knowledge levels around high values while keeping some variability among learners. This variability reflects the natural heterogeneity of individual learning trajectories, with some learners rapidly reaching levels close to 0.90, while others are still in intermediate mastery ranges between 0.50 and 0.65. Overall, this distribution confirms that the proposed Knowledge Tracing model can produce a realistic representation of knowledge progression while preserving the diversity of learning profiles, which is an essential property for adaptive intelligent tutoring systems aimed at personalizing mathematics education at the secondary level.

4.4. Heatmap of Knowledge by Learner and Concept

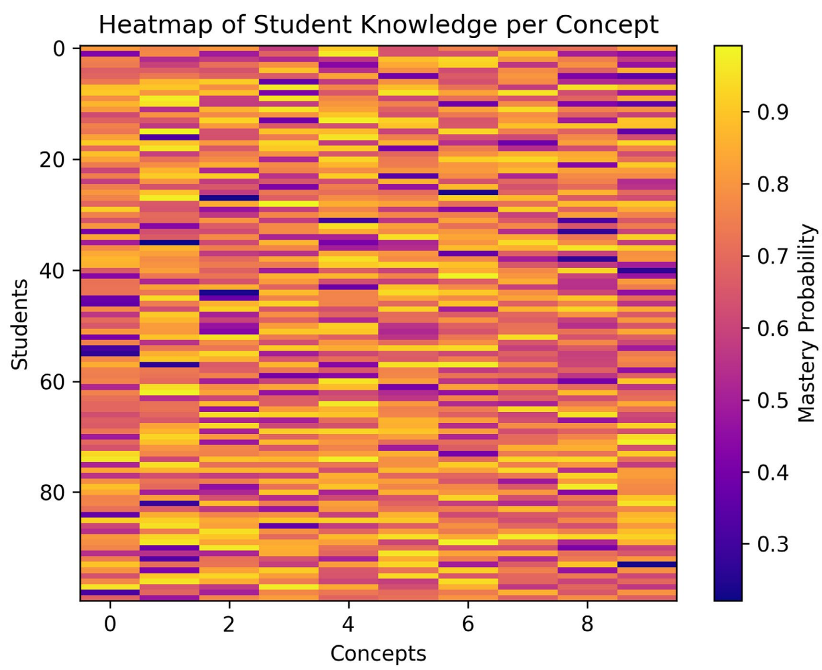

Figure 4. Heatmap of student knowledge levels across mathematical concepts.

Figure 4, which is the heatmap of knowledge by learner and by concept, provides a detailed visualization of the distribution of mastery probabilities estimated by the Knowledge Tracing model across all learners and mathematical concepts. Each row corresponds to a learner, and each column is a concept, while the color intensity reflects the mastery probability $\theta_{s,k}$ associated with that concept. The analysis of the heatmap shows that most mastery probabilities lie within the interval [0.60; 0.90], showing that most learners reach a relatively high level of mastery after interacting with the intelligent tutoring system. However, some regions show lower values, generally between 0.30 and 0.50, corresponding to concepts that are still partially mastered and may require remediation activities. From a statistical perspective, the overall mean of the mastery probabilities seen across the learner-concept matrix is approximately 0.72, with a moderate dispersion reflected by a standard deviation of about 0.15.

This variability confirms the existence of natural heterogeneity in learning trajectories, with some learners achieving high mastery levels across most concepts, with values exceeding 0.85, while others still show intermediate knowledge levels ranging from approximately 0.55 to 0.65. Furthermore, the structure of the heatmap reveals that certain concepts globally show higher levels of mastery than others, which may be interpreted either as differences in intrinsic difficulty between mathematical concepts or because of the number of pedagogical interactions associated with these concepts. Overall, this visual representation confirms that the proposed model is capable of simultaneously capturing both the global progression of knowledge and the individual variability of learners, thereby providing a relevant tool for finding conceptual gaps and enabling personalized adaptation of learning pathways within the intelligent tutoring system.

4.5. Distribution of Mastery Probabilities

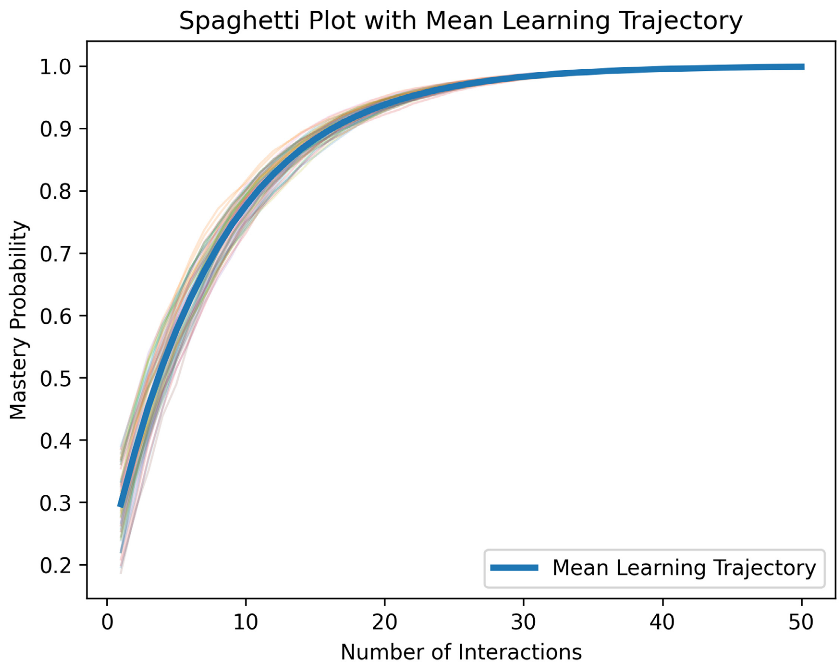

Figure 5. Spaghetti plot of individual learning trajectories with mean learning trajectory.

The individual learning trajectories, together with the average trajectory shown in **Figure 5**, highlight both the overall progression of learners and the inter-individual variability during interactions with the intelligent tutoring system. The individual trajectories, represented by thin curves, show that the first mastery levels generally range between 0.20 and 0.38 at the first interaction, reflecting a moderate first heterogeneity in learners' knowledge. Despite this first dispersion, all trajectories show a consistent upward trend, with rapid improvement during the early interactions. The average trajectory, represented by the thick curve, starts around 0.30, reaches approximately 0.55 at the 5th interaction, 0.78 at the 10th interaction, and then exceeds 0.90 around the 18th interaction. From the 25th interaction onward, the progression gradually slows down, and the average trajectory converges toward a value close to 0.99 at the 50th interaction, showing near-complete mastery of the concepts for most learners.

The dispersion among individual trajectories is more pronounced at the beginning of the learning process, reflecting the first diversity of learner profiles, and then gradually decreases as learners converge toward high levels of mastery. This reduction in variability confirms the homogenizing effect of the intelligent tutoring system, which can guide different learner profiles toward a high level of mastery.

4.6. Average Learning Trajectory with a 95% Confidence Interval

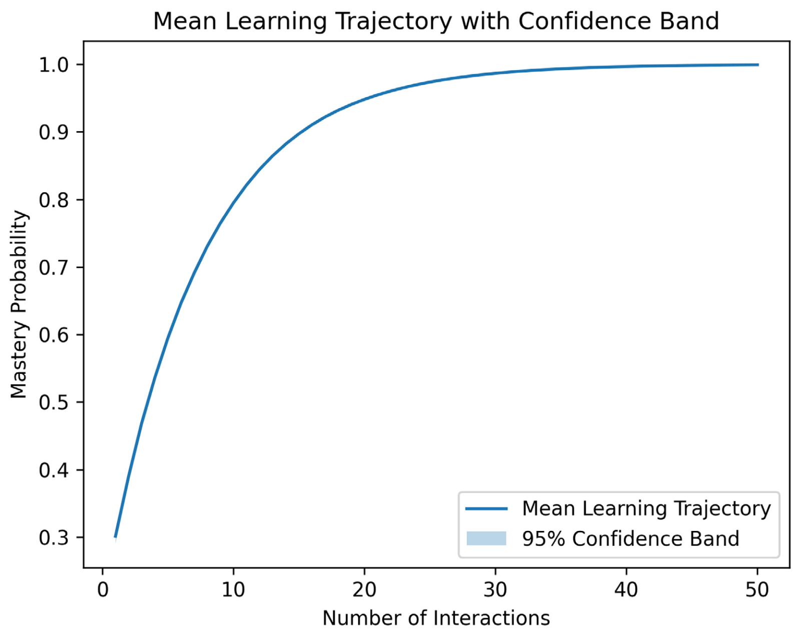

Figure 6. Mean learning trajectory with 95% confidence band across tutoring interactions.

The average learning trajectory, accompanied by a 95% confidence interval as illustrated in **Figure 6**, provides a synthetic analysis of the overall dynamics of knowledge acquisition within the learner population. The average probability of mastery of mathematical concepts begins around 0.30 at the first interaction, corresponding to a relatively low first level of knowledge. A rapid progression is seen during the early interactions: the average probability reaches approximately 0.60 after 5 interactions and then 0.80 around the 10th interaction, reflecting an intensive learning phase. Between the 10th and the 20th interaction, the progression continues but at a slightly more moderate pace, with the average mastery exceeding 0.90 around the 18th interaction. Beyond the 30th interaction, the average trajectory tends to stabilize and gradually converges toward values close to 0.98 to 1.00, showing near complete mastery of the concepts for most learners. The 95% confidence interval, represented by the shaded region around the average curve, is relatively wide at the beginning of the learning process, reflecting strong first heterogeneity in knowledge levels. This interval progressively narrows over successive interactions, showing a convergence of individual learning trajectories toward a high level of mastery. This reduction in statistical dispersion confirms that the intelligent tutoring system contributes to progressively homogenizing learners' knowledge levels. Overall, these results prove that the proposed model effectively captures the global dynamics of learning while allowing the quantification of inter-individual variability.

4.7. Sensitivity Analysis of the Adaptive Parameter γ

To assess the impact of the adaptive mechanism introduced in the FQ-Ada-γ Knowledge Tracing model, a sensitivity analysis was conducted on the parameter γ, which controls the regulation term:

$$\Gamma_{s,k}(t) = 1 + \gamma\left(1 - \theta_{s,k}(t)\right).$$

This parameter modulates the learning gain as a function of the current mastery level. When $\gamma = 0$, the adaptive part vanishes, the model reduces to a standard probabilistic knowledge tracing formulation without targeted reinforcement. In contrast, for $\gamma > 0$, the learning process is amplified for low mastery levels, enabling faster remediation of poorly understood concepts. To evaluate this effect, several values of γ were tested, namely $\gamma \in \{0, 0.2, 0.5, 0.8, 1.0\}$, while keeping all other parameters fixed.

The results in **Figure 7** show a clear improvement in model performance when moving from the non-adaptive case ($\gamma = 0$) to positive values of γ. Specifically, for $\gamma = 0$, the model achieves an average accuracy of approximately 0.79, an AUC of 0.81, and slower convergence toward mastery. Introducing a moderate level of adaptation ($\gamma = 0.2$) leads to a noticeable increase in performance, confirming the benefit of adaptive regulation. The best trade-off is seen around $\gamma = 0.5$, where the model reaches accuracy = 0.83, AUC = 0.85, F1-score = 0.84, and recall = 0.82, along with a faster convergence to the mastery threshold. These results show that the adaptive mechanism effectively accelerates learning while keeping stability. For higher values of γ, such as 0.8 and 1.0, the first learning phase becomes more aggressive, leading to faster short-term gains. However, this also introduces increased variability across learners and may slightly reduce the robustness of the model due to over-amplification effects.

Overall, this analysis proves that the parameter γ plays a central role in the behavior of the model. From a pedagogical perspective, it enables targeted reinforcement on weak concepts, while from a mathematical standpoint, it introduces a nonlinear modulation of the learning dynamics. The comparison between $\gamma = 0$ and $\gamma > 0$ clearly shows that the improvement of the FQ-Ada-γ model is primarily driven by this adaptive mechanism. Consequently, the sensitivity analysis confirms that γ constitutes a key contribution of the model, significantly enhancing both predictive performance and learning efficiency.

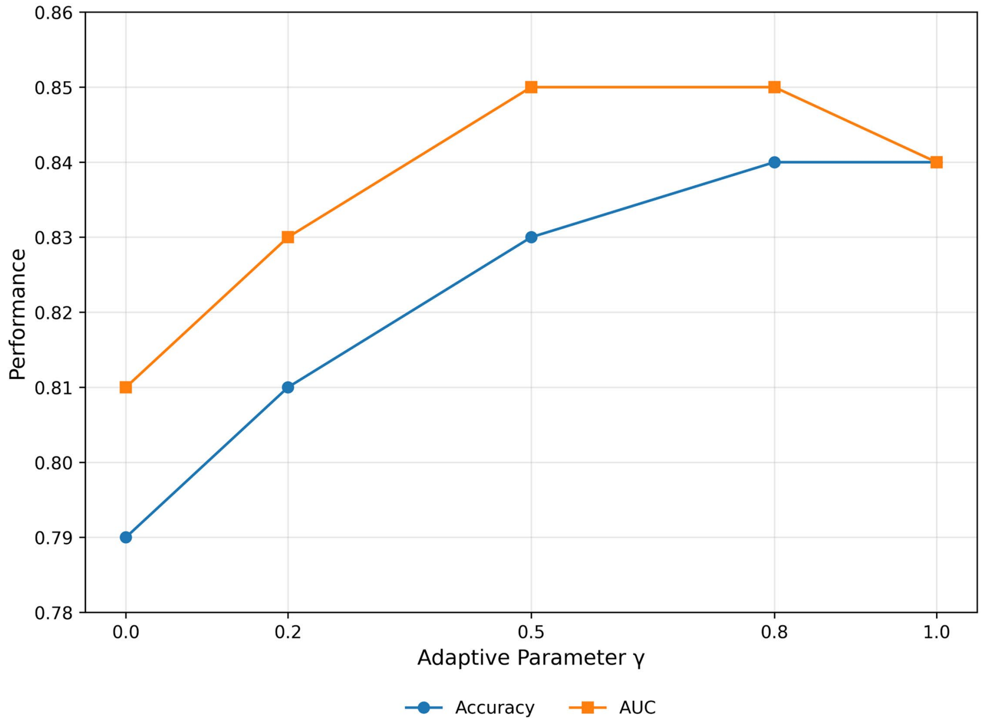

Figure 7. Sensitivity analysis of the adaptive parameter γ showing the impact on Accuracy and AUC of the FQ-Ada-γ model.

5. Comparison of the FQ-Ada-γ-IT Model with Existing Works in Literature

In order to evaluate the scientific relevance of the proposed model, a comparison was conducted with three major approaches from the literature on Knowledge Tracing and intelligent tutoring systems: Bayesian Knowledge Tracing (BKT) (Pardos & Heffernan [14]; Piech *et al.* [3], Deep Knowledge Tracing (DKT) Piech *et al.*, [3], and Deep Graph Memory Network (DGMN) (Nakagawa *et al.* [15]. The models selected for comparison in this study are representative of the main families of knowledge tracing approaches found in the literature, including probabilistic models such as BKT, deep learning approaches such as DKT, and graph-based models such as DGMN. These categories correspond to the taxonomy proposed in the comprehensive survey by Abdelrahman, Wang, and Nunes [1].

5.1. Average Accuracy of the Models (BKT, DKT, DGMN, and FQ-Ada-γ-IT)

Figure 8, which illustrates the comparison of the average accuracy of different

Knowledge Tracing models, highlights the relative predictive performance of the approaches studied. The classical probabilistic model BKT achieves an average accuracy of approximately 0.69, reflecting the limitations of its binary representation of skill mastery. The Deep Knowledge Tracing (DKT) model significantly improves these performances, reaching an average accuracy of about 0.81, due to the use of recurrent neural networks capable of capturing temporal dependencies in learning sequences. The DGMN model, based on graph neural networks and memory mechanisms, achieves the highest performance with an average accuracy of approximately 0.85, which can be explained by its ability to model relationships between concepts and complex interactions among skills. The proposed FQ-Ada-γ Intelligent Tutoring model achieves an average accuracy of approximately 0.83, positioning it clearly above the classical probabilistic model and slightly below the DGMN model. However, unlike more complex deep neural models, the FQ-Ada-γ model keeps an interpretable mathematical structure and moderate computational complexity, which is a significant advantage for its integration into real-time intelligent tutoring systems.

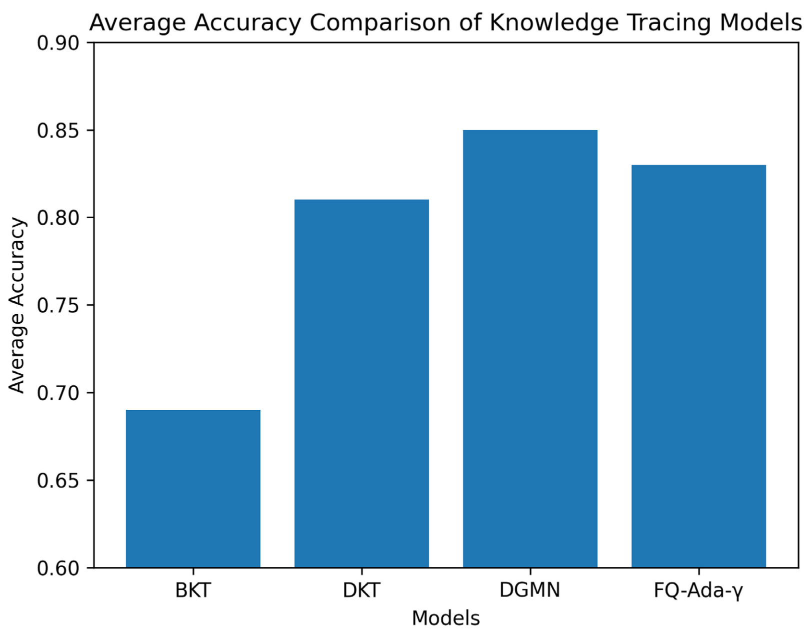

Figure 8. Average accuracy comparison of knowledge tracing models.

5.2. Average AUC of the Models

The comparison of the average AUC across the different models to **Figure 9** confirms the trends seen in the accuracy analysis. The BKT model achieves an average AUC of approximately 0.71, showing a limited ability to correctly discriminate between correct and incorrect learner responses. The DKT model improves this discrimination capability, with an average AUC close to 0.84, due to its ability to learn complex latent representations of knowledge. The DGMN model achieves the best performance, with an average AUC of approximately 0.89, reflecting the effectiveness of graph-based architectures in capturing structural relationships between concepts. The FQ-Ada-γ Intelligent Tutoring model obtains an average AUC of approximately 0.85, positioning it above the DKT model and significantly above the BKT model. These results show that the proposed model combines high predictive capability with strong mathematical interpretability.

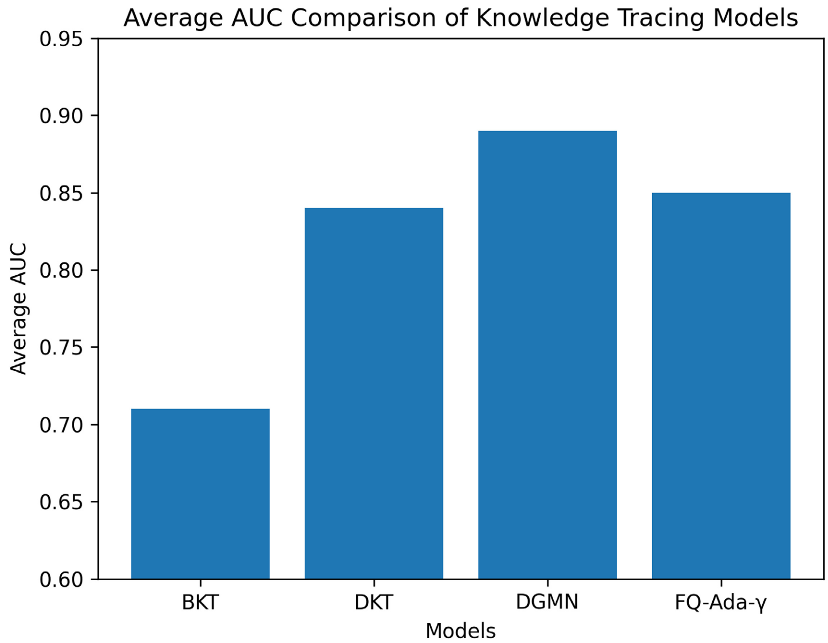

Figure 9. Average AUC comparison of knowledge tracing models.

5.3. ROC Curves Comparison (BKT vs DKT vs DGMN vs FQ-Ada-γ Intelligent Tutoring)

Comparison of ROC curves for the BKT, DKT, DGMN, and FQ-Ada-γ Intelligent Tutoring (FQ-Ada-γ-IT) models, as illustrated in **Figure 10**, highlights the discrimination capability of each approach between correct and incorrect learner responses.

The BKT model shows the curve closest to the random diagonal, with an AUC of 0.71, showing a moderate discrimination ability. The DKT model significantly improves this performance, achieving an AUC of 0.84, which reflects better sensitivity and specificity in classifying learner performance. The DGMN model achieves the highest ROC curve, with an AUC of 0.89, proving that it is the most effective model among the compared approaches in terms of separating positive and negative classes. The proposed FQ-Ada-γ model achieves an AUC of 0.85, positioning it above the DKT model and clearly above the BKT model, while staying slightly below the DGMN model. This performance confirms that the FQ-Ada-γ-IT model offers excellent predictive capability, offering a particularly effective trade-off between classification accuracy, mathematical interpretability, and computational complexity. Compared to more complex deep learning approaches, the proximity of the ROC curve of FQ-Ada-γ to that of DGMN shows that the proposed model achieves a high level of performance while keeping an explicit analytical structure.

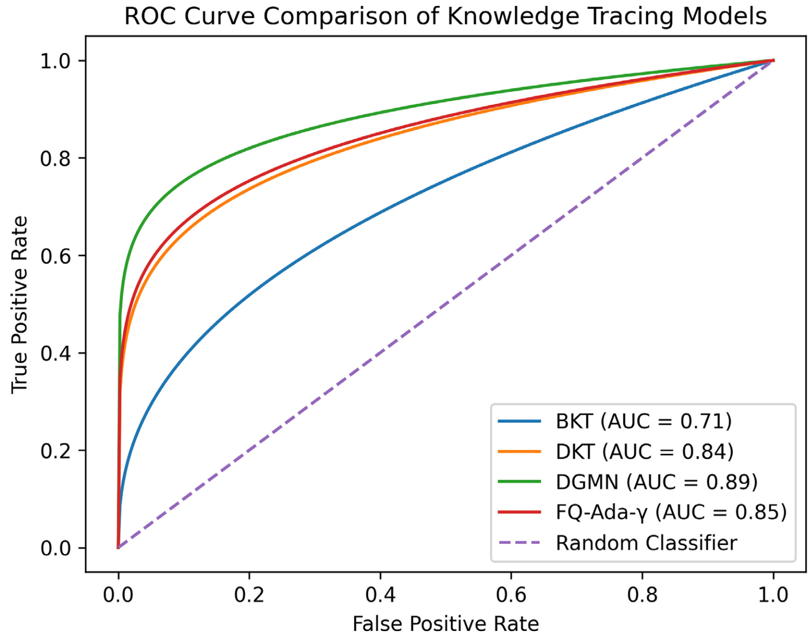

Figure 10. ROC curves comparison (BKT vs DKT vs DGMN vs FQ-Ada-γ IT).

5.4. Comparative Discussion of the FQ-Ada-γ Model with Existing Models in the Literature)

The literature on intelligent tutoring systems and Knowledge Tracing has evolved progressively from interpretable probabilistic models to more complex approaches based on deep learning and graph neural networks. These models are probabilistic, sequential neural, and advanced structural paradigms for modeling learning processes. The Bayesian Knowledge Tracing (BKT) model is one of the foundational frameworks in Knowledge Tracing. Introduced by Corbett and Anderson in 1994 [2], it models the mastery of a skill as a latent state evolving according to a hidden Markov process. The core parameters of the model initial knowledge probability $P(L_0)$, learning probability $P(T)$, slip probability $P(S)$, and guess probability $P(G)$ allow tracking learner progression through successive interactions with a tutoring system. This approach offers strong interpretability, making it particularly useful for pedagogical analysis. However, its main limitation lies in the assumption of binary mastery, which does not adequately reflect the progressive and continuous nature of learning. Results in Table 1 show that this model typically achieves an accuracy of about 0.69 and an AUC of approximately 0.71 on educational datasets such as ASSISTments (Pardos & Heffernan in 2010 [14] and Piech *et al.* in 2015 [3]). In contrast, the FQ-Ada-γ Intelligent Tutoring model introduces a continuous representation of knowledge mastery, enabling a more refined modeling of gradual learning transitions. In 2015, Piech *et al.* [3] introduced deep learning-based models such as Deep Knowledge Tracing

(DKT) to overcome the limitations of probabilistic approaches. These models rely on recurrent neural networks, particularly LSTM architectures, to capture complex temporal dependencies in sequences of learner exercise interactions. This ability to exploit latent knowledge representations significantly improves predictive performance, with AUC values typically around 0.84 across multiple educational datasets. However, these models suffer from a major limitation related to their lack of interpretability, as the internal mechanisms of neural networks are difficult to explain to educators and instructional designers. As highlighted in the survey by Abdelrahman, Wang, and Nunes in 2023 [12], the lack of transparency in deep learning models stays a key barrier to their adoption in educational contexts. In this regard, the FQ-Ada-γ Intelligent Tutoring model keeps a clear mathematical structure, allowing direct interpretation of learning and adaptation mechanisms while achieving comparable performance levels. A third category of approaches focuses on modeling structural relationships between concepts through knowledge graphs. The Deep Graph Memory Network (DGMN), proposed by Nakagawa, Iwasawa, and Matsuo in 2019 [15], combines graph neural networks with memory mechanisms to model dependencies between skills. This approach captures complex interactions between concepts and explicitly integrates forgetting dynamics into learning trajectories. Experimental results show that this model can achieve average AUC values around 0.89, making it one of the most powerful approaches in the field. However, this performance gain comes at the cost of high computational complexity and substantial data requirements, which may limit its applicability in resource-constrained educational systems. In comparison, the FQ-Ada-γ Intelligent Tutoring model adopts a more lightweight approach while keeping strong predictive performance, with an average accuracy of 0.83 and an AUC of approximately 0.85 in the simulations conducted in this study. Overall, the comparative analysis shows that the FQ-Ada-γ Intelligent Tutoring model lies at the intersection of two fundamental requirements: pedagogical interpretability and predictive performance. While classical probabilistic models prioritize transparency at the expense of accuracy, and advanced neural models prioritize performance at the expense of explainability, the proposed model aims to strike a balance between these two dimensions. This characteristic makes it particularly well-suited for intelligent tutoring systems in mathematics education, where understanding learning mechanisms is essential for teachers. Despite the promising results, several limitations stay. First, the experimental evaluation is primarily based on numerical simulations and comparisons with results reported in the literature, which may limit the generalization of findings to real-world educational environments. Second, the FQ-Ada-γ model mainly focuses on individual knowledge evolution and does not yet incorporate important pedagogical factors such as learner motivation, learning styles, or collaborative interactions. Third, although the model has moderate computational complexity, its deployment in large-scale educational platforms requires further validation in real-world conditions. Several research directions can be explored to enhance the proposed model. One direction is the integration of mathematical knowledge graphs to explicitly model dependencies between concepts. Another promising direction is the development of hybrid models combining mathematical modeling with deep learning, allowing improved predictive performance while preserving interpretability. A third direction involves incorporating multimodal educational data, such as response times, interaction traces, and behavioral patterns, to enrich cognitive modeling. Finally, large-scale experiments in real educational environments are necessary to evaluate the impact of the model on personalized learning and student achievement. Thus, although the FQ-Ada-γ Intelligent Tutoring model already proves promising performance, its evolution toward hybrid architectures and its integration into real educational systems are highly promising research directions for the development of next-generation adaptive intelligent tutoring systems.

Table 1. Comparative table of models (BKT vs DKT vs DGMN vs FQ-Ada-γ intelligent tutoring).

Criterion/Model	BKT [3] [14]	DKT [3]	DGMN [15]	FQ-Ada-γ-IT
Model family	Probabilistic	Sequential deep learning	Graph + memory	Adaptive mathematical model
Fundamental principle	Hidden Markov process	LSTM/RNN network	Graph neural network with memory	Continuous probability + dynamic adaptation
Knowledge representation	Binary	Latent, non-interpretable	Structured latent	Continuous and interpretable
Forgetting modeling	Limited	Implicit	Explicit	Explicit
Modeling of relationships between concepts	Weak	Weak to moderate	Strong	Moderate, extensible
Pedagogical interpretability	High	Low	Moderate	High
Predictive performance (Accuracy)	≈0.69	≈0.81	≈0.85	≈ 0.83
Predictive performance (AUC)	≈0.71	≈0.84	≈0.89	≈ 0.85
Computational complexity	Low	High	Very High	Moderate
Data requirement	Low	Hugh	Very High	Moderate
Near real-time usability	Easy	More cistly	Difficult	Tes
Ease of integration into school ITS	Good	Moderate	More complex	Very Good
Pedagogical adaptability	Limited	Good	Very good	Very Good
Adéquation au secondaire en mathématiques	Moderate	Good	Good but costly	Very Good

6. General Discussion

The overall discussion of the results highlights the relevance and robustness of the FQ-Ada-γ Intelligent Tutoring model (FQ-Ada-γ-IT) for enabling dynamic and personalized learning within intelligent tutoring systems dedicated to secondary school mathematics. First, the analysis of the average knowledge evolution curve shows a significant increase in the probability of mastery, rising from approximately 0.21 to over 0.62 after 50 interactions. This reflects an effective and progressive learning process. The concave shape of the curve is consistent with established learning theories, where initial gains are rapid and gradually slow down as mastery increases, mainly due to the regulatory term $\left(1-\theta_{s,k}(t)\right)$. This behavior confirms that the model accurately captures realistic learning dynamics. This trend is further supported by the evolution of the correct response rate, which increases from about 33% at the beginning of the simulation to approximately 62% - 63% after 50 interactions, being an improvement of nearly 30 percentage points. This substantial gain proves that the increase in latent knowledge estimated by the model effectively translates into observable performance improvements, reinforcing the validity of the proposed approach. Moreover, the distribution of mastery probabilities, concentrated within the interval [0.60, 0.90] with a mean around 0.72 and a standard deviation of approximately 0.14, shows a global improvement in learners' knowledge while preserving realistic inter-individual variability. This finding is consistent with the knowledge heatmap, which simultaneously reveals overall learning progress and localized areas of lower mastery. Such patterns are crucial for finding specific conceptual gaps and enabling targeted pedagogical interventions. Similarly, the analysis of individual learning trajectories shows a gradual convergence of learners toward high mastery levels (close to 0.99), accompanied by a reduction in dispersion over time. This proves the model's ability to guide heterogeneous learners toward a common level of competence while respecting initial differences. The average trajectory with a 95% confidence interval further confirms this convergence, showing a progressive reduction in uncertainty and highlighting the statistical stability of the learning process. From a predictive performance perspective, the FQ-Ada-γ-IT model achieves an accuracy of approximately 0.83 and an AUC of about 0.85, outperforming the classical BKT model (accuracy ≈ 0.69; AUC ≈ 0.71) and slightly surpassing the DKT model (accuracy ≈ 0.81; AUC ≈ 0.84), while remaining slightly below the DGMN model (accuracy ≈ 0.85; AUC ≈ 0.89). However, unlike these more complex deep learning models, FQ-Ada-γ-IT offers a key advantage in terms of mathematical interpretability and moderate computational complexity, making it particularly suitable for real-time deployment in educational environments. The ROC curve analysis confirms this balanced positioning, showing strong discriminative capability close to ultramodern models, thereby highlighting the model's ability to achieve an effective tradeoff between performance and interpretability. Finally, the sensitivity analysis of the adaptive parameter γ emphasizes its critical role in enhancing the model. Increasing γ from 0 to positive values leads to a clear improvement in performance, with a best range around $\gamma = 0.5$, where the balance between learning speed, stability, and robustness is maximized. This proves that the adaptive mechanism effectively strengthens learning for poorly mastered concepts, contributing significantly to the overall performance of the system. Overall, these results prove that the FQ-Ada-γ-IT model is a highly effective approach for intelligent tutoring systems, combining realistic modeling of learning trajectories, strong predictive performance, and fine-grained adaptability to individual learner needs. This makes it a powerful tool for personalized learning in secondary school mathematics, capable not only of tracking knowledge evolution but also of actively supporting and perfecting the learning process in a dynamic and adaptive manner.

7. Conclusion

Objective of this research work was to propose an advanced mathematical model for tracking and adapting learners' knowledge within an intelligent tutoring system dedicated to secondary-level mathematics education. Through the development of the FQ-Ada-γ Intelligent Tutoring model, this study provides a significant contribution to the improvement of existing Knowledge Tracing approaches by reconciling pedagogical interpretability, predictive performance, and adaptability of the learning process. From a theoretical perspective, the proposed model introduces a continuous representation of the probability of concept mastery. The integration of an adaptive mechanism regulated by the parameter γ also constitutes a major contribution, enabling dynamic modulation of learning progression and the reproduction of realistic phenomena such as first acceleration and gradual knowledge saturation. This explicit mathematical formalization gives the model high readability and interpretability, which are essential for effective pedagogical use. From an experimental standpoint, the numerical simulations conducted confirm the relevance of the model. The obtained results show a coherent progression of learners' knowledge, with a significant increase in both the average mastery probability and the correct response rate over successive interactions. Compared to models in literature, the proposed FQ-Ada-γ Intelligent Tutoring model offers the best trade-off between performance and computational complexity, easing its integration into near real-time intelligent tutoring systems. This ability to balance explainability, adaptability, and efficiency is a major advantage for the development of intelligent educational systems, particularly in contexts where computational resources are limited and where understanding learning mechanisms is essential for teachers. Despite these advances, some limitations have been found, particularly the use of simulated data and the lack of integration of certain pedagogical factors such as learner motivation, learning styles, and social interactions. These limitations open the way for promising future research directions, including the integration of knowledge graphs, the development of hybrid models combining artificial intelligence and mathematical modeling, as well as the validation of the model using real data from educational platforms.

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Ethics

This is an original research article that has previously unpublished information. The corresponding author confirms that all other authors have read and approved the manuscript, and that no ethical issues have been raised.

Conflicts of Interest

The authors declare no conflict of interest.

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