Open Access Library Journal
2026, Volume 13, e15055
ISSN Online: 2333-9721
ISSN Print: 2333-9705

Mass Energy and Space-Time: On the Mass Increase of Moving Matter and Negative Energy Occurrence

Chana Pongpothakul*, Nawapol Bourin

Metropolitan Waterworks Authority, Bangkok, Thailand
Email: *chana_mwa@yahoo.com

How to cite this paper: Pongpothakul, C. and Bourin, N. (2026) Mass Energy and Space-Time: On the Mass Increase of Moving Matter and Negative Energy Occurrence. *Open Access Library Journal*, **13**: e15055.
https://doi.org/10.4236/oalib.1115055

Received: February 23, 2026
Accepted: March 28, 2026
Published: March 31, 2026

Abstract

The equation describing the relationship between mass and energy has long been a subject of debate among physicists. One perspective holds that the mass of an object changes with velocity, referred to as *relativistic mass*, while another maintains that mass remains constant regardless of velocity, referred to as *invariant mass*. However, many experiments have shown that the concept of relativistic mass is required to explain experimental results. Moreover, when analyzed using Newtonian or classical mechanics under the assumption that mass depends on velocity, the result coincides with Einstein's mass-energy relation. This makes it difficult to deny the validity of relativistic mass. A question arises, however: if the increase of mass with velocity is indeed real, how can an object's velocity continue to increase when all the work or energy applied to it seemingly contributes solely to increasing its mass? To resolve this while remaining consistent with the law of conservation of energy, the authors propose a theory derived from classical mechanics, under the assumption that mass varies with velocity, together with the consideration that space-time itself imposes constraint on motion. This space-time constraint introduces a resistive constraint force against motion. The result shows that the mass of an object increasing with velocity as relativistic mass, given by $m_v = \frac{m_o}{\sqrt{1-(v/c)^2}}$, is possible and that this increase in mass is accompanied by the occurrence of *negative energy*, with a value of $-\frac{1}{2}m_v v^2$.

Subject Areas

Modern Physics

Keywords

Mass, Energy, Relativistic Mass, Space-Time Constraint, Negative Energy

1. Introduction

In 1905, Albert Einstein [1] demonstrated the relationship between mass and energy in his article *On the Electrodynamics of Moving Bodies*, specifically in section 10, *Dynamics of the Slowly Accelerated Electron*. He stated that an electron with mass *m* (as long as its motion is slow), when accelerated from rest in an electromagnetic field until it reaches velocity *v*, has kinetic energy given by Equation (1):

$$E = mc^2(\gamma - 1) \tag{1}$$

where γ is Lorentz's factor which is equal to $\frac{1}{\sqrt{1-(v/c)^2}}$ and c is the speed of light in a vacuum (empty space).

In that article, Einstein discussed the mass of the electron as it appeared in the transformed equations of motion, from a moving reference frame (with the same velocity as the electron) to a stationary reference frame. He defined the mass in the component of the equation parallel to the direction of motion as *longitudinal mass*, and the mass in the component perpendicular to the motion as *transverse mass*. The longitudinal mass equals $m\gamma^3$, while the transverse mass equals $m\gamma^2$.

From the kinetic energy equation of the electron, it seems that a moving electron has a mass of $m\gamma$. This led to debates among physicists as to whether mass changes with velocity. One side argued that an object has a rest mass m_o when stationary, and a moving mass $m_v = m_o\gamma$, referred to as *relativistic mass*. The other side held that mass does not change with velocity, but instead is an *invariant mass* (represented simply as m), equal to the rest mass, and uses term mass only without distinction between *rest mass* and *relativistic mass*.

The lack of a clear explanation, free from doubt, regarding whether mass actually increases with velocity has caused confusion among physicists when referring to mass. Even Einstein himself, in 1948 (as cited in Okun, 1989) [2], expressed a view different from what he wrote in 1905, stating: It is not good to introduce the concept of the mass $M = \frac{m}{\sqrt{1-(v/c)^2}}$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the rest mass m. Instead of introducing M, it is better to mention the expression for the momentum and energy of a body in motion. Nowadays, in modern physics, the term mass will refer to invariant mass; m, with a value given by:

$$m = \frac{\sqrt{E^2 - (pc)^2}}{c^2}$$

This article presents a theory aimed at resolving the question of whether mass truly increases with velocity (relativistic mass), by applying Newtonian or classical mechanics under the assumption that mass depends on velocity, and considering that the motion of matter is subject to the *space-time constraint*. From these two assumptions, it is found that the increase of mass with velocity occurs simultaneously with the emergence of *negative energy* which may also be interpreted as negative mass. This effect explains why matter cannot reach the speed of light and how the negative energy or negative mass can happen.

2. The Increase of Mass with Velocity: Experimental Evidence

Many physicists have conducted experiments to verify whether the mass of an object changes with velocity. For example:

- Bucherer's experiment (1908) (cited in Petkov, 2023) [3]
- Neumann's experiment (1914) (cited in Kaufmann-Bucherer-Neumann experiments, 2024: Wikipedia) [4]
- Guye and Lavanchy's experiment (1915) (cited in Kaufmann-Bucherer-Neumann experiments, 2024: Wikipedia) [4]
- Rogers et al.'s experiment (1940) [5]
- Bertozzi's experiments (1962 and 1964) (cited in Tests of relativistic energy and momentum, 2023: Wikipedia) [6]

In addition, there are experiments designed for students at educational institutions, such as the one carried out by the Department of Physics and Astronomy at Union College, Schenectady, New York, published by Marvel and Vineyard (2011) [7].

All of these experiments consistently demonstrate that relativistic mass must be applied in order to explain the results.

3. The Relationship between Mass and Energy Using Newtonian or Classical Mechanics

From classical mechanics, when a force is applied to a rigid body that is free to move and exists in the absence of resistance or friction, that force causes the rate of change of the body's momentum with respect to time (by generalizing Newton's second law of motion), as shown in Equation (2):

$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \tag{2}$$

where $\boldsymbol{p}$ is the momentum of the body, equal to mass multiplied by velocity.

If, at any velocity $\boldsymbol{v}$, the mass of the body is expressed as m_v equal to $\frac{m_o}{\sqrt{1-(v/c)^2}}$, then:

$$\boldsymbol{p} = m_v \boldsymbol{v} \tag{3}$$

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \frac{\mathrm{d}(m_v \boldsymbol{v})}{\mathrm{d}t} \tag{4}$$

When the body moves over a distance $\mathrm{d}\boldsymbol{s}$ during a time interval $\mathrm{d}t$, the work performed by the force is equal to the force multiplied by the distance, as in Equation (5):

$$\mathrm{d}W = \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s} \tag{5}$$

Work, therefore, causes a change in the energy of the body; that is, $\mathrm{d}E$. For linear motion in one dimension or rectilinear motion in three-dimensional space (with the reference axis set parallel to the direction of motion), the energy change of the body can be written as in Equations (6)-(9):

$$\mathrm{d}E = \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s} \tag{6}$$

$$= \frac{\mathrm{d}\left(m_v \boldsymbol{v}\right)}{\mathrm{d}t} \cdot \mathrm{d}\boldsymbol{s} \tag{7}$$

$$= \left(m_v \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} + \boldsymbol{v} \frac{\mathrm{d}m_v}{\mathrm{d}t} \right) \cdot \mathrm{d}\boldsymbol{s} \tag{8}$$

$$= m_v v \mathrm{d}v + v^2 \mathrm{d}m_v \tag{9}$$

If the work done by the applied force causes the change in kinetic energy of the body from rest to any velocity v, then the kinetic energy of the body at that instant is expressed by Equations (10)-(12):

$$E_K = \int_{v=0}^{v} m_v v \mathrm{d}v + \int_{v=0}^{v} v^2 \mathrm{d}m_v \tag{10}$$

$$= \left(-\Delta mc^2 + m_v v^2\right) + \left(2\Delta mc^2 - m_v v^2\right) \tag{11}$$

$$= \Delta mc^2 \tag{12}$$

which is essentially the same as Einstein's equation published in 1905.

4. Questions Arising from the Mass-Energy Relationship Equation

From the derivation above, the relationship between mass and energy was obtained using Newtonian or classical mechanics, under the assumption that the mass of a body depends on its velocity. Although this differs from Einstein's 1905 method, the result is the same. Since considering mass as increasing with velocity is the only condition that leads to this consistent result, it is difficult to deny that the mass of a body changes with velocity, i.e., relativistic mass:

$$m_v = \frac{m_o}{\sqrt{1-\left(v/c\right)^2}}$$

However, one significant question arises: if the increase of mass with velocity is real, then **how can the velocity of a body continue to increase?** This is because all the work or energy imparted to the body would already have been consumed entirely by the increase of its mass.

When we consider the energy imparted to the body, its kinetic energy, and the portion of energy converted into mass, it becomes apparent that the equation does not comply with the **law of conservation of energy**, as follows:

According to the law of conservation of energy, the total energy of a system is constant, though it may transform between different forms. In a closed system, the sum of all changes in energy must equal zero, as expressed in Equation (13):

$$\sum_{i=1}^{N} \Delta E_i = 0 \tag{13}$$

where E_i is the energy in i form, and $\Delta E_i = E_{i,final} - E_{i,initial}$ represents the change in the amount of energy in i form.

In mechanics, the **mechanical energy** of a body exists in the form of potential energy (E_P) and kinetic energy (E_K). If the force acting on the body is solely due to a conservative potential field (which depends on position but not on time), then the work done results in a conversion between potential and kinetic energy, equal in magnitude to the potential difference between the two positions. This type of work does not change the total energy of the system. Thus, if the kinetic energy of the body increases, its potential energy must decrease by the same amount. Such work is called *work by a conservative force*.

In contrast, if the body does not change its potential energy, then the work done by the applied force is entirely concerted into kinetic energy, thereby changing the total energy of the system. Such work is called *work by a non-conservative force*.

Therefore, in general, work can be regarded as the transformation of the form and/or magnitude of energy. The relationship between work and the energy of a body can be written as Equations (14) and (15):

$$W = W_{con} + W_{non\text{-}con} \tag{14}$$

$$W - W_{con} = W_{non\text{-}con} = \Delta E_T = \Delta E_P + \Delta E_K \tag{15}$$

where:

- W is the total work done by the applied force
- W_{con} is the work done by conservative forces, equal to $-\Delta E_P = \Delta E_K$
- $W_{non\text{-}con}$ is the work done by non-conservative forces
- ΔE_T is the total change in energy of the body
- ΔE_P is the change in potential energy
- ΔE_K is the change in kinetic energy

In Section 3, when deriving the energy of a moving body, we assumed that its mass changes with velocity. Therefore, in addition to potential energy and kinetic energy, the energy in the form of **mass** should also be considered. In this case, the relationship between work and energy can be expressed as Equation (16):

$$W - W_{con} = W_{non\text{-}con} = \Delta E_T = \Delta E_P + \Delta E_K + \Delta E_m \tag{16}$$

where ΔE_m is the change in energy stored in the mass of the body.

Considering the results obtained in Section 3, two scenarios arise:

1) Case 1: Work due only to conservative forces

$$W - W_{con} = \Delta E_T = 0 \tag{17}$$

Since work by conservative forces equals $-\Delta E_P$, then:

$$\Delta mc^2 = -\Delta E_P = \Delta E_K + \Delta E_m \tag{18}$$

But, by assumption, the mass increases with velocity, so $\Delta E_m = \Delta mc^2$. Substituting into (18):

$$\Delta mc^2 = \Delta E_K + \Delta mc^2 \tag{19}$$

$$\Delta E_K = 0 \tag{20}$$

This result means the body's velocity does not change -i.e., the body remains stationary. This demonstrates that the solution from Equation (12) would violate the conservation of energy if mass actually increases with velocity.

2) Case 2: Work due only to non-conservative forces, in this case we get

$$W_{non-con} = \Delta E_T = \Delta E_P + \Delta E_K + \Delta E_m \tag{21}$$

If the body's potential energy does not change ($\Delta E_P = 0$), so

$$\Delta mc^2 = \Delta E_K + \Delta E_m \tag{22}$$

$$\Delta mc^2 = \Delta E_K + \Delta mc^2 \tag{23}$$

$$\Delta E_K = 0 \tag{24}$$

This result is identical in meaning to the first case.

Nevertheless, experimental results, along with the derivation of the mass-energy relationship using Newtonian or classical mechanics (under the assumption that mass increases with velocity), yield results consistent with Einstein's equation. Therefore, the increase of mass with velocity (relativistic mass) should still be considered as a valid assumption, but must be incorporated with additional concepts in the development of a theory that also complies with the law of conservation of energy.

5. The Governing Factors of Moving Matter: The Concept of Space-Time Constraint, Equations of Motion, and Energy Equations of Motion

To establish an equation that describes the relationship between mass and energy, which reflects the validity of relativistic mass without contradicting the law of conservation of energy, the proposed theory is based on two assumptions:

1) The mass of a body, which considered as a point mass, changes with velocity according to the equation:

$$m_v = \frac{m_o}{\sqrt{1-\left(v/c\right)^2}}$$

2) The motion of a body is subjected to the **space-time constraint**, which introduces a resistive constraint force; $\boldsymbol{F}_C$ opposite to the applied force; $\boldsymbol{F}_A$. This arises due to changes in mass with velocity. The resultant of the applied force and the constraint force is the inertial force; $\boldsymbol{F}_i$ which causes the momentum change (rate of change with respect to time) of the body. At any instant, the change in the constraint force; $\mathrm{d}\boldsymbol{F}_C$, is equal to the product of the change in mass and acceleration, expressed as:

$$\mathrm{d}\boldsymbol{F}_c = \frac{\mathrm{d}m_v \mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \tag{25}$$

where:

- m_v is the relativistic mass of the body at time t
- m_o is the rest mass of the body
- v is velocity of the body at time t
- c is the speed of light in a vacuum
- $\mathrm{d}m_v$ is change in mass between t and $t+\mathrm{d}t$
- $\mathrm{d}v$ is change in velocity between t and $t+\mathrm{d}t$
- $\mathrm{d}t$ is a small time interval

Thus, the governing force equation is:

$$\boldsymbol{F}_A - \boldsymbol{F}_c = \boldsymbol{F}_i \tag{26}$$

or equivalently:

$$\boldsymbol{F}_A = \boldsymbol{F}_i + \boldsymbol{F}_c \tag{27}$$

where:

$$\boldsymbol{F}_i = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \tag{28}$$

and

$$\boldsymbol{F}_c = \int \mathrm{d}\boldsymbol{F}_c = \int_{v_1}^{v_2} \frac{\mathrm{d}m_v \mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \tag{29}$$

Therefore, the equation of motion of the body can be expressed as:

$$\boldsymbol{F}_A = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} + \int_{v_1}^{v_2} \frac{\mathrm{d}m_v \mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \tag{30}$$

$$= \frac{\mathrm{d}(m_v \boldsymbol{v})}{\mathrm{d}t} + \int_{v_1}^{v_2} \frac{\mathrm{d}m_v \mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \tag{31}$$

For rectilinear motion in one dimension, or linear motion in three-dimensional space with the reference axis aligned along the body's direction of motion, if the body moves a distance $\mathrm{d}\boldsymbol{s}$ in time $\mathrm{d}t$, then the infinitesimal energy change of the body is given by:

$$\mathrm{d}E = (\boldsymbol{F}_i + \boldsymbol{F}_c) \cdot \mathrm{d}\boldsymbol{s} \tag{32}$$

$$= \frac{\mathrm{d}(m_v \boldsymbol{v})}{\mathrm{d}t} \cdot \mathrm{d}\boldsymbol{s} + \int_{v_1}^{v_2} \frac{\mathrm{d}m_v \mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \cdot \mathrm{d}\boldsymbol{s} \tag{33}$$

$$= m_v \frac{\mathrm{d}v}{\mathrm{d}t} \mathrm{d}s + v \frac{\mathrm{d}m_v}{\mathrm{d}t} \mathrm{d}s + \int_{v_1}^{v_2} \frac{\mathrm{d}m_v \mathrm{d}v}{\mathrm{d}t} \mathrm{d}s \tag{34}$$

$$= m_v v \mathrm{d}v + v^2 \mathrm{d}m_v + \int_{v_1}^{v_2} v \mathrm{d}v \mathrm{d}m_v \tag{35}$$

The energy change of the body can be shown another way as follows:

From Equation (25), infinitesimal work done from the infinitesimal constraint force shall be:

$$\mathrm{d}\boldsymbol{F}_c \cdot \mathrm{d}\boldsymbol{s} = \frac{\mathrm{d}m_v \mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \cdot \mathrm{d}\boldsymbol{s} \tag{36}$$

$$= v\mathrm{d}v\mathrm{d}m_v \tag{37}$$

so energy change due to the constraint force can be expressed as:

$$\int\int \mathrm{d}\boldsymbol{F}_c \cdot \mathrm{d}\boldsymbol{s} = \int_{v_1}^{v_2}\int_{v_1}^{v_2} v\mathrm{d}v\mathrm{d}m_v \tag{38}$$

The energy change due to the inertial force work done is:

$$\int \boldsymbol{F}_i \cdot \mathrm{d}\boldsymbol{s} = \int_{v_1}^{v_2} m_v v\mathrm{d}v + \int_{v_1}^{v_2} v^2\mathrm{d}m_v \tag{39}$$

So the total energy change of the body shall be expressed as:

$$\int_{E_1}^{E_2} \mathrm{d}E = \int_{v_1}^{v_2} m_v v\mathrm{d}v + \int_{v_1}^{v_2} v^2\mathrm{d}m_v + \int_{v_1}^{v_2}\int_{v_1}^{v_2} v\mathrm{d}v\mathrm{d}m_v \tag{40}$$

If the work done by the applied force changes the energy of the body from rest to velocity v, the total energy change at that instant is:

$$E - E_o = \int_{v=0}^{v} m_v v\mathrm{d}v + \int_{v=0}^{v} v^2\mathrm{d}m_v + \int_{v=0}^{v}\int_{v=0}^{v} v\mathrm{d}v\mathrm{d}m_v \tag{41}$$

$$= \int_{v=0}^{v} m_v v\mathrm{d}v + \int_{v=0}^{v} v^2\mathrm{d}m_v + \int_{v=0}^{v} \frac{v^2}{2}\mathrm{d}m_v \tag{42}$$

$$= \left(-\Delta mc^2 + m_v v^2\right) + \left(2\Delta mc^2 - m_v v^2\right) + \left(\Delta mc^2 - \frac{1}{2}m_v v^2\right) \tag{43}$$

$$= 2\Delta mc^2 - \frac{1}{2}m_v v^2 \tag{44}$$

6. Interpretation of the Energy Equation of Motion Based on the Concept of Space-Time Constraint

The energy equation of motion derived from the proposed theory can be interpreted by dividing the result into three components:

1) The kinetic energy of the body, equal to Δmc^2.

2) The energy responsible for the increase of the body's mass, also equal to Δmc^2.

3) The negative energy that occurs, equal to $-\frac{1}{2}m_v v^2$.

This result demonstrates consistency with the **law of conservation of energy**. It can be analyzed in three cases:

Case 1: The active force (F_A) and the constraint force (F_C) are conservative

$$\Delta E_T = \Delta E_P + \Delta E_K + \Delta E_m \tag{45}$$

Since the work by conservative forces is:

$$W_{con} = -\Delta E_P$$

we have:

$$2\Delta mc^2 - \frac{1}{2}m_v v^2 = -\Delta E_P \tag{46}$$

By considering that the inertial force (F_i) causes a change in kinetic energy (as it changes velocity), then $\Delta E_K = \Delta mc^2$. Similarly, when the body gains its mass, the energy due to the increase of mass is $\Delta E_m = \Delta mc^2$. Thus, from (46) we get,

$$\Delta E_K + \Delta E_m + \left(\Delta E_P - \frac{1}{2} m_v v^2 \right) = 0 \tag{47}$$

The term ($\Delta E_P - \frac{1}{2} m_v v^2$) may be interpreted as a reduction of the body's potential energy below its arbitrary reference value (normally set to be zero). This indicates the occurrence of **negative energy in space-time**.

Note also that the constraint force (F_C), which resists motion, behaves similarly to friction in mechanics. Thus, it can be considered a **non-conservative force**. In such a case, the work done by the constraint force will be non-conservative work and it will be treated in Case 2 and 3. Additionally, the kinetic energy in this space-time constraint concept may be defined as the work performed by the inertial force (F_i).

Case 2: The constraint force (F_C) is non-conservative but the inertial force (F_i) is conservative

Work done by conservative force:

$$W_{con} = -\Delta E_{P1} = \Delta E_K = \Delta mc^2 \tag{48}$$

Substituting the above value of W_{con} and assigning $\Delta E_P = \Delta E_{P2}$ into Equation (16), we have:

$$W = \Delta mc^2 + W_{non-con} = \Delta E_{P2} + \Delta E_K + \Delta E_m \tag{49}$$

Thus:

$$W_{non-con} = \left(2\Delta mc^2 - \frac{1}{2} m_v v^2 \right) - \Delta mc^2 = \Delta mc^2 - \frac{1}{2} m_v v^2 \tag{50}$$

and

$$\Delta mc^2 + \Delta mc^2 - \frac{1}{2} m_v v^2 = \Delta E_{P2} + \Delta E_K + \Delta E_m \tag{51}$$

Since the body's mass increases with velocity, so ΔE_m should be equal to Δmc^2, and from Equation (48), $\Delta E_K = \Delta mc^2$, therefore:

$$\Delta E_{P2} = -\frac{1}{2} m_v v^2 \tag{52}$$

Equation (52) can be interpreted in the same way as the former case.

Case 3: The active force (F_A) and the constraint force (F_C) are non-conservative

In this case we get:

$$W = W_{non\text{-}con} = \Delta E_P + \Delta E_K + \Delta E_m \tag{53}$$

$$2\Delta mc^2 - \frac{1}{2} m_v v^2 = \Delta E_P + \Delta E_K + \Delta E_m \tag{54}$$

Since ΔE_m should be equal to Δmc^2, and if the inertial force changes kinetic energy without changing potential energy, then $\Delta E_K = \Delta mc^2$. Hence:

$$\Delta E_P = -\frac{1}{2} m_v v^2 \tag{55}$$

which can be interpreted in the same way as the two former cases.

So far it could be concluded that the work or energy supplied to a body increases its velocity, thereby increasing its mass. However, due to the space-time constraint, a resistive force arises, which in turn produces negative energy. If the mass-energy relationship also applies to negative energy, then negative mass can be expressed as:

$$m_{neg} = -\frac{m_v}{2}\left(\frac{v}{c}\right)^2$$

Furthermore, Equation (44) shows that when the velocity is small, the result remains consistent with Newtonian or classical mechanics as follows:

$$v \ll c;\ \Delta mc^2 = \frac{1}{2}m_o v^2 \tag{56}$$

$$E - E_o = 2\Delta mc^2 - \frac{1}{2}m_v v^2 = 2\left(\frac{1}{2}m_o v^2\right) - \frac{1}{2}m_o v^2 \tag{57}$$

$$E - E_o = \frac{1}{2}m_o v^2 \tag{58}$$

Thus, at low velocities relative to the speed of light, the proposed theory reduces to the classical mechanics results.

7. The Change of Mass When the Velocity of a Body Decreases

Considering a body moving with velocity $\boldsymbol{v}$ that experiences a resistive applied force $\boldsymbol{F}_R$ in the opposite direction of its motion. The inertial force ($\boldsymbol{F}_i$) is then given by:

$$\boldsymbol{F}_i = -\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}$$

Thus, the equation of motion of the body can be written as:

$$\boldsymbol{F}_R - \boldsymbol{F}_c = -\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} \tag{59}$$

$$\boldsymbol{F}_R = -\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} + \int_{v_1}^{v_2}\frac{\mathrm{d}m_v \mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \tag{60}$$

$$= -\frac{\mathrm{d}(m_v \boldsymbol{v})}{\mathrm{d}t} + \int_{v_1}^{v_2}\frac{\mathrm{d}m_v \mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \tag{61}$$

For rectilinear motion (one-dimensional or in three-dimensional space with the axis aligned along the direction of motion), when the body moves a distance $\mathrm{d}\boldsymbol{s}$ during a small time interval $\mathrm{d}t$, the energy change of the body will be:

$$\mathrm{d}E = -\boldsymbol{F}_R \cdot \mathrm{d}\boldsymbol{s} \tag{62}$$

$$\mathrm{d}E = -\left(-\frac{\mathrm{d}(m_v v)}{\mathrm{d}t} + \int_{v_1}^{v_2}\frac{\mathrm{d}m_v \mathrm{d}\boldsymbol{v}}{\mathrm{d}t}\right)\cdot \mathrm{d}\boldsymbol{s} \tag{63}$$

$$= \left(m_v\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} + \boldsymbol{v}\frac{\mathrm{d}m_v}{\mathrm{d}t} - \int_{v_1}^{v_2}\frac{\mathrm{d}m_v \mathrm{d}\boldsymbol{v}}{\mathrm{d}t}\right)\cdot \mathrm{d}\boldsymbol{s} \tag{64}$$

$$= m_v \frac{\mathrm{d}v}{\mathrm{d}t}\mathrm{d}s + v\frac{\mathrm{d}m_v}{\mathrm{d}t}\mathrm{d}s - \int_{v_1}^{v_2} \frac{\mathrm{d}m_v \mathrm{d}v}{\mathrm{d}t}\mathrm{d}s \tag{65}$$

$$= m_v v\mathrm{d}v + v^2\mathrm{d}m_v - \int_{v_1}^{v_2} v\mathrm{d}v\mathrm{d}m_v \tag{66}$$

If the resistive force causes the body to decelerate until it comes to rest, the energy of the body can be expressed as:

$$E_o - E = \int_{v=0}^{v} m_v v\mathrm{d}v + \int_{v=0}^{v} v^2\mathrm{d}m_v - \int\int_{v=0}^{v} v\mathrm{d}v\mathrm{d}m_v \tag{67}$$

$$= \int_{v=0}^{v} m_v v\mathrm{d}v + \int_{v=0}^{v} v^2\mathrm{d}m_v + \int_{v=0}^{v} \frac{v^2}{2}\mathrm{d}m_v \tag{68}$$

$$= \left(\Delta mc^2 - m_v v^2\right) + \left(-2\Delta mc^2 + m_v v^2\right) + \left(-\Delta mc^2 + \frac{1}{2}m_v v^2\right) \tag{69}$$

$$= -2\Delta mc^2 + \frac{1}{2}m_v v^2 \tag{70}$$

The result can be interpreted by dividing into three parts, analogous to the case of increasing velocity, but in the opposite sense as follows:

1) A decrease in kinetic energy of the body, equal to $-\Delta mc^2$.

2) A decrease in the energy associated with the body's mass, equal to $-\Delta mc^2$.

3) The return of energy to the negative energy field in space-time, equal to $+\frac{1}{2}m_v v^2$.

This shows that when a body slows down, its mass decreases along with the disappearance of negative energy. When the body decelerates completely to rest, its mass returns to the original rest mass.

8. Rectilinear Motion in Three Dimensions

In Cartesian coordinate system, a body moving in rectilinear with the velocity of magnitude v in the directions making the angle of θ_x with the positive direction of the $\boldsymbol{X}$ axis, θ_y with the positive direction of the $\boldsymbol{Y}$ axis, and θ_z with the positive direction of the $\boldsymbol{Z}$ axis, its velocity vector and components can be shown in Equations (71)-(74), where v_x and $\boldsymbol{i}$ are the components of velocity magnitude and unit vector in the direction of the positive $\boldsymbol{X}$ axis; v_y and $\boldsymbol{j}$ are the components of velocity magnitude and unit vector in the direction of the positive $\boldsymbol{Y}$ axis; v_z and $\boldsymbol{k}$ are the components of velocity magnitude and unit vector in the direction of the positive $\boldsymbol{Z}$ axis.

$$\boldsymbol{v} = v_x\boldsymbol{i} + v_y\boldsymbol{j} + v_z\boldsymbol{k} \tag{71}$$

$$v_x = v\cos\theta_x;\ v_y = v\cos\theta_y;\ v_z = v\cos\theta_z \tag{72}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2 \tag{73}$$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1 \tag{74}$$

When the body moves a distance $\mathrm{d}\boldsymbol{s}$ in a duration $\mathrm{d}t$, the energy change of the body can be written as in Equations (75)-(120) as follows:

$$\mathrm{d}E = \left(\boldsymbol{F}_i + \boldsymbol{F}_c\right)\cdot\mathrm{d}\boldsymbol{s} \tag{75}$$

$$\Delta E = \int \mathrm{d}E = \int \left(\boldsymbol{F}_i + \boldsymbol{F}_c \right) \cdot \mathrm{d}\boldsymbol{s} \tag{76}$$

$$\mathrm{d}\boldsymbol{s} = \mathrm{d}x\boldsymbol{i} + \mathrm{d}y\boldsymbol{j} + \mathrm{d}z\boldsymbol{k} \tag{77}$$

$$\boldsymbol{F}_i = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = F_{ix}\boldsymbol{i} + F_{iy}\boldsymbol{j} + F_{iz}\boldsymbol{k} \tag{78}$$

$$\boldsymbol{p} = m_v \boldsymbol{v} = m_o \gamma \boldsymbol{v} \tag{79}$$

$$= m_o \gamma v_x \boldsymbol{i} + m_o \gamma v_y \boldsymbol{j} + m_o \gamma v_z \boldsymbol{k} \tag{80}$$

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \frac{\mathrm{d}p_x}{\mathrm{d}t}\boldsymbol{i} + \frac{\mathrm{d}p_y}{\mathrm{d}t}\boldsymbol{j} + \frac{\mathrm{d}p_z}{\mathrm{d}t}\boldsymbol{k} \tag{81}$$

$$F_{ix} = \frac{\mathrm{d}p_x}{\mathrm{d}t} \tag{82}$$

$$= \frac{\mathrm{d}\left(m_o \gamma v_x \right)}{\mathrm{d}t} \tag{83}$$

$$= m_o \gamma \frac{\mathrm{d}v_x}{\mathrm{d}t} + m_o v_x \frac{\gamma^3}{c^2} \left(v_x \frac{\mathrm{d}v_x}{\mathrm{d}t} + v_y \frac{\mathrm{d}v_y}{\mathrm{d}t} + v_z \frac{\mathrm{d}v_z}{\mathrm{d}t} \right) \tag{84}$$

$$F_{iy} = \frac{\mathrm{d}p_y}{\mathrm{d}t} \tag{85}$$

$$= \frac{\mathrm{d}\left(m_o \gamma v_y \right)}{\mathrm{d}t} \tag{86}$$

$$= m_o \gamma \frac{\mathrm{d}v_y}{\mathrm{d}t} + m_o v_y \frac{\gamma^3}{c^2} \left(v_x \frac{\mathrm{d}v_x}{\mathrm{d}t} + v_y \frac{\mathrm{d}v_y}{\mathrm{d}t} + v_z \frac{\mathrm{d}v_z}{\mathrm{d}t} \right) \tag{87}$$

$$F_{iz} = \frac{\mathrm{d}p_z}{\mathrm{d}t} \tag{88}$$

$$= \frac{\mathrm{d}\left(m_o \gamma v_z \right)}{\mathrm{d}t} \tag{89}$$

$$= m_o \gamma \frac{\mathrm{d}v_z}{\mathrm{d}t} + m_o v_z \frac{\gamma^3}{c^2} \left(v_x \frac{\mathrm{d}v_x}{\mathrm{d}t} + v_y \frac{\mathrm{d}v_y}{\mathrm{d}t} + v_z \frac{\mathrm{d}v_z}{\mathrm{d}t} \right) \tag{90}$$

Work done by the inertial force can be expressed as:

$$\boldsymbol{F}_i \cdot \mathrm{d}\boldsymbol{s} = F_{ix}\mathrm{d}x + F_{iy}\mathrm{d}y + F_{iz}\mathrm{d}z \tag{91}$$

Substituting F_{ix} from Equation (84) we will get the work done by inertial force in x direction as:

$$F_{ix}\mathrm{d}x = m_o \gamma \frac{\mathrm{d}v_x}{\mathrm{d}t}\mathrm{d}x + m_o v_x \frac{\gamma^3}{c^2} \left(v_x \frac{\mathrm{d}v_x}{\mathrm{d}t} + v_y \frac{\mathrm{d}v_y}{\mathrm{d}t} + v_z \frac{\mathrm{d}v_z}{\mathrm{d}t} \right)\mathrm{d}x \tag{92}$$

$$= m_o \gamma v_x \mathrm{d}v_x + m_o v_x^2 \frac{\gamma^3}{c^2} \left(v_x \mathrm{d}v_x + v_y \mathrm{d}v_y + v_z \mathrm{d}v_z \right) \tag{93}$$

$$= m_o \gamma v \cos^2 \theta_x \mathrm{d}v + m_o \frac{\gamma^3}{c^2} v^2 \cos^2 \theta_x v \mathrm{d}v \tag{94}$$

Energy change of the body due to the inertial force in x direction from being at rest to any velocity v can be determined as shown in Equations (95)-(98)

$$\int F_{ix}\mathrm{d}x = \int_{v=0}^{v} m_o \gamma v \cos^2\theta_x \mathrm{d}v + \int_{v=0}^{v} m_o \frac{\gamma^3}{c^2} v^2 \cos^2\theta_x v\mathrm{d}v \tag{95}$$

$$= m_o \cos^2\theta_x \left(-c^2(\gamma-1)+\gamma v^2\right) + m_o \cos^2\theta_x \left(2c^2(\gamma-1)-\gamma v^2\right) \tag{96}$$

$$= m_o \cos^2\theta_x c^2 (\gamma-1) \tag{97}$$

$$= \Delta m c^2 \cos^2\theta_x \tag{98}$$

In the same way as described above, the energy change of the body due to the inertial force in y and z directions can be determined as shown in Equations (99) and (100) respectively.

$$\int F_{iy}\mathrm{d}y = \Delta m c^2 \cos^2\theta_y \tag{99}$$

$$\int F_{iz}\mathrm{d}z = \Delta m c^2 \cos^2\theta_z \tag{100}$$

Therefore, the energy change of the body due to the inertial force is formulated as shown in Equation (101)

$$\int F_{ix}\mathrm{d}x + \int F_{iy}\mathrm{d}y + \int F_{iz}\mathrm{d}z = \Delta m c^2 \left(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z\right) \tag{101}$$

Substituting the value of $\left(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z\right)$ from Equation (74) yields:

$$\int F_{ix}\mathrm{d}x + \int F_{iy}\mathrm{d}y + \int F_{iz}\mathrm{d}z = \Delta m c^2 \tag{102}$$

For determination of energy change due to the constraint force, it can be shown in Equations (103)-(120) as follows:

$$\boldsymbol{F}_c = F_{cx}\boldsymbol{i} + F_{cy}\boldsymbol{j} + F_{cz}\boldsymbol{k} \tag{103}$$

$$F_{cx} = \int_{v_1}^{v_2} \frac{\mathrm{d}m_v \mathrm{d}v_x}{\mathrm{d}t} \tag{104}$$

$$F_{cy} = \int_{v_1}^{v_2} \frac{\mathrm{d}m_v \mathrm{d}v_y}{\mathrm{d}t} \tag{105}$$

$$F_{cz} = \int_{v_1}^{v_2} \frac{\mathrm{d}m_v \mathrm{d}v_z}{\mathrm{d}t} \tag{106}$$

Work done by the constraint force can be expressed as:

$$\boldsymbol{F}_c \cdot \mathrm{d}\boldsymbol{s} = F_{cx}\mathrm{d}x + F_{cy}\mathrm{d}y + F_{cz}\mathrm{d}z \tag{107}$$

Substituting F_{cx} from Equation (104) we will get work done by the constraint force in x direction as follows:

$$F_{cx}\mathrm{d}x = \int_{v_1}^{v_2} \frac{\mathrm{d}m_v \mathrm{d}v_x}{\mathrm{d}t}\mathrm{d}x \tag{108}$$

$$= \int_{v_1}^{v_2} v_x \mathrm{d}v_x \mathrm{d}m_v \tag{109}$$

Energy change of the body due to the constraint force in x direction from being at rest to any velocity v can be determined as shown in Equations (110)-(115)

$$\int F_{cx}\mathrm{d}x = \int_{v=0}^{v}\int_{v=0}^{v} v_x \mathrm{d}v_x \mathrm{d}m_v \tag{110}$$

$$= \int \int_{v=0}^{v} m_o v \cos^2 \theta_x \mathrm{d}v \mathrm{d}\gamma \tag{111}$$

$$= \int \int_{v=0}^{v} \frac{m_o}{2} \cos^2 \theta_x \mathrm{d}v^2 \mathrm{d}\gamma \tag{112}$$

$$= \int_{v=0}^{v} m_o \frac{\gamma^3}{2c^2} v^2 \cos^2 \theta_x v \mathrm{d}v \tag{113}$$

$$= \frac{m_o}{2} \cos^2 \theta_x \left(2c^2 (\gamma - 1) - \gamma v^2\right) \tag{114}$$

$$= \left(\Delta m c^2 - \frac{1}{2} m_v v^2 \right) \cos^2 \theta_x \tag{115}$$

In the same way as described above, the energy change of the body due to the constraint forces in y and z directions can be determined as shown in Equations (116) and (117) respectively.

$$\int F_{cy} \mathrm{d}y = \left(\Delta m c^2 - \frac{1}{2} m_v v^2 \right) \cos^2 \theta_y \tag{116}$$

$$\int F_{cz} \mathrm{d}z = \left(\Delta m c^2 - \frac{1}{2} m_v v^2 \right) \cos^2 \theta_z \tag{117}$$

Therefore, the energy change of the body due to the constraint force is formulated as shown in Equation (118)

$$\int F_{cx} \mathrm{d}x + \int F_{cy} \mathrm{d}y + \int F_{cz} \mathrm{d}z = \left(\Delta m c^2 - \frac{1}{2} m_v v^2 \right) \left(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z \right) \tag{118}$$

Substituting the value of $\left(\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z\right)$ from Equation (74) yields:

$$\int F_{cx} \mathrm{d}x + \int F_{cy} \mathrm{d}y + \int F_{cz} \mathrm{d}z = \left(\Delta m c^2 - \frac{1}{2} m_v v^2 \right) \tag{119}$$

By Equations (102) and (119) the energy equation of the body is formulated as shown in Equation (120)

$$\Delta E = \int \boldsymbol{F}_i \cdot \mathrm{d}\boldsymbol{s} + \int \boldsymbol{F}_c \cdot \mathrm{d}\boldsymbol{s} = 2\Delta m c^2 - \frac{1}{2} m_v v^2 \tag{120}$$

Which is exactly the same as the energy equation of the motion in one dimension shown in Equation (44).

9. Conclusion

In derivation of the energy equation of motion by applying the principle of Newtonian or classical mechanics and the experimental fact that the mass of moving matter changes with its velocity, combined with the new perspective of space-time by considering that the space-time is not absolutely empty or nothing, but its existence could interact and affect a body in motion with the space-time constraint, which introduces a resistive constraint force opposing motion due to changes in mass with velocity. The result indicates that the mass of a body increases with its velocity or being relativistic mass; m_v equal to $\frac{m_o}{\sqrt{1-(v/c)^2}}$ and the mass increase will take place together with the occurrence of negative energy in spacetime equal to $-\frac{1}{2}m_v v^2$, or that is, in other words, negative mass. And it is the increase of mass that prevents matter from reaching the speed of light. The energy stored in the increased mass and the existence of negative energy resolve the contradiction posed by the traditional interpretation of relativistic mass.

Acknowledgements

With help, encouragement and valuable discussion feedback of Chana's great friend and beloved wife, Piyanart Benjakarn Pongpothakul, this article is therefore finished successfully. Special thanks to her. Our thanks also go to Wisoot Vaeteeprasit, an ex-colleague who offered valuable help in preparing the concise abstract. We also extend our appreciation to all those who contributed to this work and provided assistance that enabled this paper to be successfully published.

For the value of this achievement, we dedicate it with respect and gratitude to our parents, teachers, and the Metropolitan Waterworks Authority.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Einstein, A. (1905) On the Electrodynamics of Moving Bodies. https://users.physics.ox.ac.uk/~rtaylor/teaching/specrel.pdf

[2] Okun, L.B. (1989) The Concept of Mass. Physics Today. http://www.itep.ru/science/doctors/okun/publishing_eng/em_3.pdf

[3] Petkov, V. (2023) Relativistic Mass Is an Experimental Fact. https://philsci-archive.pitt.edu/id/eprint/21667

[4] Wikipedia (2024) Kaufmann-Bucherer-Neumann Experiments. https://en.wikipedia.org/wiki/Kaufmann-Bucherer-Neumann_experiments

[5] Rogers, M.M., McReynolds, A.W., and Rogers, F.T. (1940) A Determination of the Masses and Velocities of Three Radium B β-Particles. The Relativistic Mass of the Electron. https://repository.rice.edu/items/b59ed767-5d8b-4e38-b328-24c24f1507bc

[6] (2023) Tests of Relativistic Energy and Momentum. Wikipedia. https://en.wikipedia.org/wiki/Tests_of_relativistic_energy_and_momentum

[7] Marvel, R.E., and Vineyard, M.F. (2011) Relativistic Electron Experiment for the Undergraduate Laboratory. https://www.researchgate.net/publication/51934357