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A Preliminary Study on the Genesis of De Broglie Wave

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Abstract

As for DE Broglie's hypothesis of matter waves, many experimental physicists have experimentally verified it by using cathode ray tube and proved that matter waves do exist, so there is a mysterious hypothesis of wave-particle duality. In this paper, according to the electric field force of electrons in the cathode ray tube and the continuity equation of electrons, Under assumptions such as inelastic approximation, it is proved that the wave-particle duality is actually the macroscopic motion of real particles in the form of waves. In this paper, it is proved theoretically that the component of electron velocity in the tube satisfies the wave equation, and the wave velocity varies with the change of electric potential in the tube. From the discussion in this paper, it can be seen that there are still some problems to be studied about the motion of electrons in the cathode ray tube, so that the special motion law of electrons in the cathode ray tube can be further explored from both theoretical and experimental aspects. At the same time, it is expected that the theory in this paper can provide theoretical guidance for the manufacture and debugging of electron microscopes and the design of large-scale integrated circuits, and thus influence other related fields... This will involve a vast area of science and technology! This paper is devoted to the in-depth analysis of the objective truth of the wave-particle duality presented by microscopic particles.

Subject Areas

Theoretical Physics

Keywords

De Broglie Wave, Newtonian Dynamics, Wave Equation

1. Introduction

Since DE Broglie put forward the hypothesis of the matter wave, many physicists have carried out experiments to verify its hypothesis and obtained meaningful results, so matter wave has been generally recognized in the physics field, and thus deduces a series of theories such as the mysterious hypothesis of wave-particle duality and “probability wave”.

2. The Electric Field Force on Electron and Its Continuity Equation are Deduced Theoretically

The cathode ray tube (hereinafter referred to as the tube) is placed horizontally along x-axis, and the screen is placed at $x=0$ for convenience of discussion, the following theoretical calculation results can show the correctness of this setting, and the cathode is placed at $x=L$ position. It can be imagined that the electron beam in the tube is axially symmetric about the x-axis, and the envelope of the electron beam is a cone. Take an arbitrary plane across the x-axis as the x-z plane, and the electrons in the tube move rapidly under the action of applied voltage. Since the mass of the electrons is very small, the influence of gravity on electron motion can be considered as a secondary factor. As the most basic discussion, we ignore the gravity of the electrons here, so the electrons are not affected by external forces in any direction of the vertical x-axis, at this time, the electron velocity $\boldsymbol{v}=\{v_x,v_y,v_z\}$, because the electron mass is very small, under the application of applied voltage, the electron can be considered to move only in the x-z plane, so $v_y=0$.

In inertial coordinates, for a single electron, the equation of motion is:

$$m_e\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t}=\sum_i \boldsymbol{F}_i \tag{1}$$

where m_e is the mass of a single electron and $\boldsymbol{F}_i$ is various external forces.

Here we only consider the motion law of the electron velocity z component. At $x=0$, the potential $-u=0$, that is, the potential at the screen of the tube is zero, at $x=l$, the potential is $-u,u>0$, and at $x=L$, the potential is $-U,U>0$. At $x=l$, the z component of the electric field strength is: $E_z=-\frac{\partial(-u)}{\partial z}=\frac{\partial u}{\partial z}$, and the z component of the electric field force on the electron is:

$$F_z=eE_z=-|e|\frac{\partial u}{\partial z} \tag{2}$$

where e is the electron charge, and $e<0$ and $|e|$ are the absolute value of the electron charge. Thus, in the rectangular coordinate system, the z component of formula (1) is:

$$m_e\frac{\mathrm{d}v_z}{\mathrm{d}t}=-|e|\frac{\partial u}{\partial z} \tag{3}$$

The mass density of electrons in the tube is ρ, and the continuity equation $\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{v})$ of electrons can be written as:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t}+\rho\nabla\cdot\boldsymbol{v}=0 \tag{4}$$

In the rectangular coordinate system, formula (4) can be written as:

$$\frac{\mathrm{d}\ln\rho}{\mathrm{d}t}+\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_z}{\partial z}\right)=0,\ v_y=0 \tag{5}$$

We take the state of the stationary electron as the background state, because of some small deviation ρ', accompanied by a motion $\boldsymbol{v}'_x,\boldsymbol{v}'_z$, because there is a potential difference in the tube along the *x*-axis, so we assume

$$\begin{cases} v_x=v'_x, v_z=v'_z \\ \rho=\rho_0+\rho' \end{cases} \tag{6}$$

For this narrow and small space in the cathode ray tube, the bulk density of electrons does not change significantly, so the $\rho_0=\text{constant}$ of $\rho=\rho_0+\rho'$ in formula (6) is called formula (6) as the basic assumption. Under the basic assumption, there is

$$\ln\rho=\ln(\rho_0+\rho')=\ln\rho_0\left(1+\frac{\rho'}{\rho_0}\right)=\ln\rho_0+\ln\left(1+\frac{\rho'}{\rho_0}\right)=\ln\rho_0+\frac{\rho'}{\rho_0}$$

Thus

$$\frac{\mathrm{d}\ln\rho}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\rho'}{\rho_0}\right)+\frac{\mathrm{d}\ln\rho_0}{\mathrm{d}t}\approx\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\rho'}{\rho_0}\right) \tag{7}$$

The continuity Equation (5) then becomes:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\rho'}{\rho_0}\right)+\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_z}{\partial z}\right)=0 \tag{8}$$

At screen $x=0$ is the ideal rigid body, where the electron velocity is satisfied

$$v_x\big|_{x=0}=0 \tag{9}$$

It is conceivable that electrons emitted from the cathode at any given moment cannot simultaneously arrive in a plane perpendicular to the *x*-axis in unison, so $x=l$ should be a function of *z* and *t*, *i.e.*: $x=l(z,t)$, because $l(z,t)$ is independent of *x*, and therefore $\partial l/\partial x=0$, and because $v_y=0$, there is

$$v_{xl}=v_x\big|_{x=l}=\frac{\mathrm{d}l}{\mathrm{d}t}=\frac{\partial l}{\partial t}+v_x\frac{\partial l}{\partial x}+v_y\frac{\partial l}{\partial y}+v_z\frac{\partial l}{\partial z}=\frac{\partial l}{\partial t}+v_z\frac{\partial l}{\partial z} \tag{10}$$

The motion is also assumed to be frictionless, and the flow of electrons is assumed to be incompressible, so the first term $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\rho'}{\rho_0}\right)=0$ on the left in (8) is an inelastic approximation [1], so (8) becomes:

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_z}{\partial z}=0 \tag{11}$$

Thus, from Equations (3) and (11), the basic equations of electron motion in the tube are:

$$\begin{cases} m_e\dfrac{\mathrm{d}v_z}{\mathrm{d}t}=-|e|\dfrac{\partial u}{\partial z}, \\ \dfrac{\partial v_x}{\partial x}+\dfrac{\partial v_z}{\partial z}=0. \end{cases} \tag{12}$$

Next, Equation (12) is changed by using Equation (8) of boundary conditions. Integrate the continuity equation in Equation (12) from $x=0$ to $x=l$:

$$\int_0^l \frac{\partial v_x}{\partial x}\delta x+\int_0^l \frac{\partial v_z}{\partial z}\delta x=0$$

By using formula (8), and noting that $\partial v_z/\partial z$ is independent of *x*, we get

$$v_{xl}=-l\frac{\partial v_z}{\partial z} \tag{13}$$

Combining the expressions (10) and (13), we have:

$$\frac{\partial l}{\partial t}+v_z\frac{\partial l}{\partial z}+l\frac{\partial v_z}{\partial z}=0 \tag{14}$$

Combining Equations (3) and (14), the system of electron motion in the tube is usually written as

$$\begin{cases} m_e\dfrac{\mathrm{d}v_z}{\mathrm{d}t}=-|e|\dfrac{\partial u}{\partial z}, \\ \dfrac{\mathrm{d}l}{\mathrm{d}t}+l\dfrac{\partial v_z}{\partial z}=0. \end{cases} \tag{15}$$

In formula (15), each term of the previous formula is multiplied by l/m_e, and each term of the next formula is multiplied by $u|e|/m_e$, yielding:

$$\begin{cases} l\dfrac{\mathrm{d}v_z}{\mathrm{d}t}=-|e|\dfrac{l}{m_z}\dfrac{\partial u}{\partial z}, \\ \dfrac{|e|u}{m_e}\dfrac{\mathrm{d}l}{\mathrm{d}t}+l\dfrac{|e|u}{m_e}\dfrac{\partial v_z}{\partial z}=0. \end{cases} \tag{16}$$

Using

$$\begin{cases} l\dfrac{\mathrm{d}v_z}{\mathrm{d}t}=\dfrac{\mathrm{d}(lv_z)}{\mathrm{d}t}-v_z\dfrac{\mathrm{d}l}{\mathrm{d}t}, \\ \dfrac{l|e|}{m_e}\dfrac{\partial u}{\partial z}=\dfrac{|e|}{m_e}\dfrac{\partial(lu)}{\partial z}-\dfrac{|e|u}{m_e}\dfrac{\partial l}{\partial z}, \\ \dfrac{|e|u}{m_e}\dfrac{\mathrm{d}l}{\mathrm{d}t}=\dfrac{|e|}{m_e}\dfrac{\mathrm{d}(lu)}{\mathrm{d}t}-\dfrac{|e|l}{m_e}\dfrac{\mathrm{d}u}{\mathrm{d}t}, \\ \dfrac{|e|lu}{m_e}\dfrac{\partial v_z}{\partial z}=\dfrac{|e|u}{m_e}\dfrac{\partial(lv_z)}{\partial z}-v_z\dfrac{|e|u}{m_e}\dfrac{\partial l}{\partial z}. \end{cases}$$

formula (16) can then be written as:

$$\begin{cases} \dfrac{\mathrm{d}(lv_z)}{\mathrm{d}t}-v_z\dfrac{\mathrm{d}l}{\mathrm{d}t}=-\dfrac{|e|}{m_e}\dfrac{\partial(lu)}{\partial z}+\dfrac{|e|u}{m_e}\dfrac{\partial l}{\partial z}, \\ \dfrac{|e|}{m_e}\dfrac{\mathrm{d}(lu)}{\mathrm{d}t}-\dfrac{|e|l}{m_e}\dfrac{\mathrm{d}u}{\mathrm{d}t}+\dfrac{|e|u}{m_e}\dfrac{\partial(lv_z)}{\partial z}-v_z\dfrac{|e|u}{m_e}\dfrac{\partial l}{\partial z}=0. \end{cases} \tag{17}$$

Next, we will simplify Equation (17). In view of the overall movement of the electron beam in the tube is towards the anode, so there is no convection and other complex movements, so the following simplification is reasonable and feasible. Changing $\mathrm{d}/\mathrm{d}t=\partial/\partial t+\boldsymbol{v}\cdot\nabla$ to $\partial/\partial t$ in this formula is equivalent to ignoring the nonlinear term $\boldsymbol{v}\cdot\nabla$ in $\mathrm{d}/\mathrm{d}t$, that is, the convection and transport of energy flow are not considered, but the conversion of various forms of energy in volume V is not affected [1]. Thus, $\mathrm{d}/\mathrm{d}t$ is changed to $\partial/\partial t$, and the formula (17) becomes:

$$\begin{cases}\frac{\partial(lv_z)}{\partial t}-v_z\frac{\partial l}{\partial t}=-\frac{|e|}{m_e}\frac{\partial(lu)}{\partial z}+\frac{|e|u}{m_e}\frac{\partial l}{\partial z},\\ \frac{|e|}{m_e}\frac{\partial(lu)}{\partial t}-\frac{|e|l}{m_e}\frac{\partial u}{\partial t}+\frac{|e|u}{m_e}\frac{\partial(lv_z)}{\partial z}-v_z\frac{|e|u}{m_e}\frac{\partial l}{\partial z}=0.\end{cases} \tag{18}$$

In formula (18), each term of the previous formula takes a partial derivative with respect to time *t*, and each term of the next formula takes a partial derivative with respect to coordinate *z*, and we get:

$$\begin{cases}\frac{\partial^2(lv_z)}{\partial t^2}-\frac{\partial}{\partial t}\left(v_z\frac{\partial l}{\partial t}\right)=-\frac{|e|}{m_e}\frac{\partial^2(lu)}{\partial z\partial t}+\frac{|e|}{m_e}\frac{\partial}{\partial t}\left(u\frac{\partial l}{\partial z}\right),\\ \frac{|e|}{m_e}\frac{\partial^2(lu)}{\partial t\partial z}-\frac{|e|}{m_e}\frac{\partial}{\partial z}\left(l\frac{\partial u}{\partial t}\right)+\frac{|e|}{m_e}\frac{\partial}{\partial z}\left[u\frac{\partial(lv_z)}{\partial z}\right]-\frac{|e|}{m_e}\frac{\partial}{\partial z}\left(uv_z\frac{\partial l}{\partial z}\right)=0.\end{cases}$$

From this we have:

$$\begin{cases}\frac{\partial^2(lv_z)}{\partial t^2}-\frac{\partial}{\partial t}\left(v_z\frac{\partial l}{\partial t}\right)=-\frac{|e|}{m_e}\frac{\partial^2(lu)}{\partial z\partial t}+\frac{|e|}{m_e}\frac{\partial}{\partial t}\left(u\frac{\partial l}{\partial z}\right),\\ \frac{|e|}{m_e}\frac{\partial^2(lu)}{\partial t\partial z}-\frac{|e|}{m_e}\frac{\partial}{\partial z}\left(l\frac{\partial u}{\partial t}\right)+\frac{|e|u}{m_e}\frac{\partial^2(lv_z)}{\partial z^2}+\frac{|e|}{m_e}\frac{\partial u}{\partial z}\left[\frac{\partial(lv_z)}{\partial z}\right]-\frac{|e|}{m_e}\frac{\partial}{\partial z}\left(uv_z\frac{\partial l}{\partial z}\right)=0.\end{cases} \tag{19}$$

In formula (19) we command

$$c^2=\frac{|e|u}{m_e} \tag{20}$$

Since the electric charge of the electron is very small, the effect of this electric charge on the potential of each point in the tube is negligible, so the potential u of each point in the tube has nothing to do with time *t*, that is: $\partial u/\partial t=0$, and then the formula (20) is used, so (19) becomes:

$$\begin{cases}\frac{\partial^2(lv_z)}{\partial t^2}-\frac{\partial}{\partial t}\left(v_z\frac{\partial l}{\partial t}\right)=-\frac{|e|}{m_e}\frac{\partial^2(lu)}{\partial z\partial t}+c^2\frac{\partial^2 l}{\partial t\partial z},\\ \frac{|e|}{m_e}\frac{\partial^2(lu)}{\partial t\partial z}+c^2\frac{\partial^2(lv_z)}{\partial z^2}+\frac{|e|}{m_e}\frac{\partial u}{\partial z}\left[\frac{\partial(lv_z)}{\partial z}\right]-\frac{|e|}{m_e}\frac{\partial}{\partial z}\left(uv_z\frac{\partial l}{\partial z}\right)=0.\end{cases} \tag{21}$$

Find $\frac{|e|}{m_e}\frac{\partial^2(lu)}{\partial z\partial t}$ from the previous formula in Equation (21)

$$\frac{|e|}{m_e}\frac{\partial^2(lu)}{\partial z\partial t}=c^2\frac{\partial^2 l}{\partial t\partial z}-\frac{\partial^2(lv_z)}{\partial t^2}+\frac{\partial}{\partial t}\left(v_z\frac{\partial l}{\partial t}\right) \tag{22}$$

Then substitute the next formula in formula (21), and tidy up:

$$\frac{\partial^2(lv_z)}{\partial t^2}-c^2\frac{\partial^2(lv_z)}{\partial z^2}=\frac{|e|}{m_e}\frac{\partial u}{\partial z}\left[\frac{\partial(lv_z)}{\partial z}\right]-\frac{|e|}{m_e}\frac{\partial}{\partial z}\left(uv_z\frac{\partial l}{\partial z}\right)+\frac{\partial}{\partial t}\left(c^2\frac{\partial l}{\partial z}+v_z\frac{\partial l}{\partial t}\right) \tag{23}$$

In $c^2=|e|u/e_m$ of formula (20), $|e|u$ is the potential energy obtained by the electron moving in the tube, and $|e|u/e_m$ is the square of the electron moving speed at the potential u. Equation (23) shows that this is a nonlinear non-homogeneous wave equation, where c is the wave velocity. From Equation (20) we can see that the wave velocity is not a constant, it is related to the electric potential u at a certain point in the tube! Where u is large, the wave speed is large, where u is small, the wave speed is small, and where u is equal to zero, the wave speed is zero, so at the screen of $x = 0$'s tube, the electrons stop moving and the wave speed is zero! This is the reason for placing the screen of the tube at the origin of the coordinates. Among

$$c = \sqrt{\frac{|e|u}{m_e}}\text{m•s}^{-1} = 419378.82\sqrt{u}\text{m•s}^{-1} \tag{24}$$

Obviously, the electron fluctuation in the tube has a shear wave component.According to the expression (24) of wave velocity, when the mass of particles is large, the wave velocity is small; When the mass increases to a certain value, the wave velocity can be considered equal to zero! Therefore, particles with large mass have no wave property when moving.

3. Conclusions and Prospects

We have assumed that the movement of electrons in the tube is symmetric about the x-axis, so in any plane passing through the x-axis, the component velocity of electrons in the direction perpendicular to the x-axis moves according to the wave rule of (23), and because of the interference of the wave, the electrons are emitted to the screen to get a circular interference spot!

As mentioned above, this paper only discusses the motion law of the transverse component of electron motion in the tube, while the motion law of the longitudinal component remains to be studied, and the wave speed has unusual characteristics, which need to be verified by experiments, and this wave speed may also have unexpected physical effects! At the same time, it is expected that the theory of this paper can provide theoretical guidance for the manufacture and debugging of electron microscopes [2] and the design of large-scale integrated circuits and other applications [3], and thus influence other related fields [4], ... This will involve a vast area of science and technology!

From the above discussion, if microscopic particles in macroscopic motion are not acted on by external forces, will they still have fluctuations?

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Conflicts of Interest

The author declares no conflicts of interest.

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