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Location Choice and Entry Timing under Uncertainty: A Dynamic Hotelling Model with Location-Dependent Premium Rates

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Abstract

This paper develops a dynamic extension of the Hotelling model in which firms choose locations prior to entry and optimally determine entry timing under uncertainty. By embedding spatial competition into a real-options framework, the model allows firms to delay irreversible investment until market conditions become sufficiently favorable. A key innovation is the introduction of a location-dependent discount premium, which endogenously links firms' spatial configuration to the intertemporal valuation of future profit streams. Unlike a standard exit hazard, this premium captures gradual erosion of project value arising from benefits of agglomeration, knowledge spillovers and so on. We show that location choices affect not only post-entry price competition but also the effective discount rate governing the value of waiting and the optimal entry threshold. As a result, location choice and entry timing are jointly determined. Under reasonable conditions, this dynamic mechanism overturns the classical prediction of maximal differentiation in the static Hotelling model, leading firms to choose interior locations rather than the market endpoints. The analysis highlights a novel channel through which spatial configuration shapes dynamic investment incentives and provides a unified framework integrating spatial competition and real-options theory.

Keywords

Spatial Competition, Entry Timing, Real Options, Location-Dependent Discounting, Hotelling Model

1. Introduction

Spatial competition model introduced by Hotelling (1929) has played a central role in industrial organization and regional economics by formalizing the trade-off between location choice and price competition. In the canonical static framework, firms choose locations prior to engaging in price competition, which often yields the principle of maximal differentiation: firms locate at the endpoints of the market interval in order to soften price competition (d'Aspremont, Gabszewicz, & Thisse, 1979). This insight has been widely extended and refined in the literature on spatial competition (see, e.g., Anderson, de Palma, & Thisse, 1992; Gabszewicz & Thisse, 1996), and remains a benchmark result for analyzing location incentives.

At the same time, a separate literature has emphasized that investment and entry decisions are inherently dynamic and subject to uncertainty. The real options approach to investment under uncertainty, pioneered by McDonald and Siegel (1986) and Dixit and Pindyck (1994), shows that firms optimally delay irreversible investment until economic conditions reach a critical threshold. This framework has been applied to a wide range of economic contexts, including entry decisions (Dixit, 1989), industry dynamics (Hopenhayn, 1992) and trade liberalization (e.g., Brainard, 1997; Handley, 2014; Handley & Limão, 2015, 2017).

Despite these advances, interaction between spatial competition and optimal entry timing has remained relatively unexplored, particularly in settings where location choices themselves affect the risk profile faced by firms. This study brings together two bodies of literature, by developing a dynamic Hotelling model under uncertainty in which firms choose locations prior to entry and optimally determine entry timing as the exercise of a real option.

A key innovation of our model is the introduction of a location-dependent discount premium, which captures how spatial configuration affects the intertemporal valuation of future profit streams. Rather than representing the arrival of a terminal adverse event, this premium reflects a gradual erosion of project value driven by benefits of agglomeration, knowledge spillovers and so on. By endogenizing this discount component through firms' location choices, our model establishes a direct link between where firms locate and how they value the option to delay entry. We show that, under reasonable conditions, this dynamic mechanism overturns the classical result of maximal differentiation, leading firms to choose interior locations rather than the endpoints of the market interval.

By endogenizing the effective discount rate through location choices, this paper contributes to the literature on spatial competition and dynamic investment in three ways. First, it provides a new channel through which location decisions affect firms' dynamic incentives under uncertainty. Second, it establishes a direct theoretical link between location choice and entry timing, highlighting their strategic interdependence. Third, it offers a unified framework that integrates Hotelling-type spatial competition with the real-options theory of entry, thereby enriching both literatures.

The remainder of the paper is organized as follows. Section 2 presents basic model. Section 3 analyzes optimal pricing, entry timing, and location choices. Section 4 contrasts the dynamic model with the static Hotelling benchmark. Section

5 concludes the paper and summarizes the main findings.

2. Basic Model

Consider a linear market of unit length in which consumers are uniformly distributed. There are two firms, firm 1 and firm 2, both supplying a homogeneous good with constant marginal cost c. Each firm chooses its location in the first stage, its location timing in the second stage, and its price in the third stage. Let a and b denote the locations of firm 1 and firm 2, respectively (with $a<b$) and define d as $d \equiv |a-b|$, distance between the two firms. Let t_1 and t_2 denote their entry times, and let p_1 and p_2 denote their prices. Fixed investment cost for location is denoted by I.

Each consumer purchases one unit of the good and chooses from which firm to buy. When a consumer located at x purchases the good from firm i located at z_i, her utility is given by

$$U(x) = \overline{v} - p_i - s_t (x - z_i)^2, \tag{1}$$

where $\overline{v}$ is the reservation utility, which is assumed to be sufficiently large so that every consumer always purchases one unit from either firm. $s_t > 0$ represents the transportation cost parameter, which follows a geometric Brownian motion

$$\mathrm{d}s_t = \mu s_t \mathrm{d}t + \sigma s_t \mathrm{d}W_t, \tag{2}$$

where μ is the drift, σ is the volatility and W_t is a standard Brownian motion.

After entry, the two firms play the Hotelling price game at each moment given the current state s_t. Thus, equilibrium prices are state-contingent policies $p_i(s_t)$ ($i \in \{1,2\}$) and are continuously re-optimized rather than fixed at the entry-time level.

In addition to the standard discount rate ρ, we introduce location-dependent discount factor—by modeling it as a Poisson process with arrival rate $\lambda(d)$. Thus, the probability that the firm survives until time t is given by $\exp(-\lambda(d)t)$, and the expected present value of future profits is effectively discounted at rate $\rho+\lambda$. Thus, expected present value of post-entry profits of firm i is given by

$$V_i(s) = \mathbb{E}\left[\int_0^\infty \mathrm{e}^{-(\rho+\lambda(d))t} \pi_i(s_t)\,\mathrm{d}t\right], \tag{3}$$

where

$$\rho + \lambda(d) > \mu, \tag{4}$$

is assumed to ensure that $V_i(s)$ is finite.

We call $\lambda(d)$ a premium factor and assume that $\lambda(d)$ is strictly increasing in distance,

$$\lambda'(d) > 0. \tag{5}$$

and specify $\lambda(d)$ as

$$\lambda(d) = \lambda_0 + \lambda_1 d^2, \tag{6}$$

where $\lambda_0 > 0$ and $\lambda_1 \ge 0$. This specification captures the idea that firms located closer to each other—i.e., when d is smaller—are less vulnerable to common shocks due to benefits of agglomeration, knowledge spillovers and so on, and face a lower effective discount rate. In this sense, the premium rate reflects a gradual erosion of project value over time that is endogenously shaped by location choices.

Following the standard procedure for solving a multi-stage game, we first solve the third-stage problem, where optimal prices are determined simultaneously.

Suppose firm 1 is located at a and sets price p_1, while firm 2 is located at b and sets price p_2. The consumer who is indifferent between purchasing from firm 1 and firm 2 satisfies

$$\bar{v} - p_1 - s(x-a)^2 = \bar{v} - p_2 - s(x-b)^2. \tag{7}$$

Solving for the indifferent consumer's location yields

$$x^* = \frac{p_2 - p_1}{2s(b-a)} + \frac{a+b}{2}. \tag{8}$$

Since consumers are uniformly distributed, the outputs of firm 1 and firm 2 are given by

$$x_1 = x^*; \tag{9}$$

$$x_2 = 1 - x^*. \tag{10}$$

Hence, the profits of the two firms are

$$\pi_1 = (p_1 - c)x^*; \tag{11}$$

$$\pi_2 = (p_2 - c)(1 - x^*). \tag{12}$$

The profit-maximization condition for firm 1 with respect to price is therefore $\frac{\partial \pi_1}{\partial p_1} = 0$, which yields

$$p_1 = \frac{p_2 + c}{2} + s(b-a)(a+b). \tag{13}$$

Similarly, the profit-maximization condition for firm 2, $\frac{\partial \pi_2}{\partial p_2} = 0$, implies

$$p_2 = \frac{p_1 + c}{2} + s(b-a)(2-a-b). \tag{14}$$

Solving these two equations, the equilibrium prices are obtained as

$$p_1^* = c + s(b-a)(2+a+b); \tag{15}$$

$$p_2^* = c + s(b-a)(4-a-b). \tag{16}$$

Substituting these equilibrium prices, (15) and (16), into the expression for the indifferent consumer, (8), yields firm 1's demand

$$x^* = \frac{2+a+b}{4}. \tag{17}$$

Substituting this equation, (17), and the equilibrium prices, (15) and (16), into the profit functions, (11) and (12) gives

$$\pi_1 = \frac{s(b-a)(2+a+b)^2}{4}; \tag{18}$$

$$\pi_2 = \frac{s(b-a)(4-a-b)^2}{4}. \tag{19}$$

3. Optimal Location Timing and Location Choice

We next solve the second-stage problem, which is modeled as a two-player simultaneous real-options timing game in threshold strategies.

Each firm chooses an entry threshold s_i^*, and entry occurs when the stochastic state first reaches that threshold. We focus on a symmetric equilibrium in threshold strategies, where both firms enter simultaneously when $s_t = s^*$.

From (18) and (19), we have firm i's profit which is proportional to s.

$$\pi_i(s) = K_i s, \tag{20}$$

where

$$K_1(a,b) = \frac{1}{4}(b-a)(a+b+2)^2; \tag{21}$$

$$K_2(a,b) = \frac{1}{4}(b-a)(4-a-b)^2, \tag{22}$$

By substituting these profits, (21) and (22), into the post-investment value function of firm i

$$V_i(s) = \mathbb{E}\left[\int_0^\infty e^{-(\rho+\lambda(d))t}\pi_i(s_t)\,dt\right], \tag{23}$$

we have the post-investment value of firm i as

$$V_i(s) = \frac{K_i s}{\rho+\lambda(d)-\mu}. \tag{24}$$

Let $V_i(s)$ net of the investment cost I be

$$\Pi_i(s) = \frac{K_i s}{\rho+\lambda(d)-\mu} - I, \tag{25}$$

maximization of which yields the optimal entry threshold s^*. In the following, we will use this $\Pi_i(s)$ in the option-value and threshold calculations.

From the standard results on perpetual investment options, the value of waiting is given by

$$F_i(s,d) = A_i(d)s^{\beta(d)}, \tag{26}$$

where $\beta(d)$ is the positive root of

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - (\rho+\lambda(d)) = 0, \tag{27}$$

that is,

$$\beta(d)=\frac{-\left(\mu-\frac{1}{2}\sigma^2\right)+\sqrt{\left(\mu-\frac{1}{2}\sigma^2\right)^2+2\sigma^2\left(\rho+\lambda(d)\right)}}{\sigma^2}, \tag{28}$$

which is greater than 1.

The value-matching condition,

$$F_i\left(s^*\right)=\Pi_i\left(s^*\right), \tag{29}$$

together with the smooth-pasting condition,

$$F_i'\left(s^*\right)=\Pi_i'\left(s^*\right), \tag{30}$$

yields the optimal entry threshold of s as

$$s^*=\frac{\beta(d)}{\beta(d)-1}\frac{\left(\rho+\lambda(d)-\mu\right)I}{K_i}, \tag{31}$$

and the optimal location timing is given by the first hitting time of s_t to s^*.

Finally, we solve the first-stage problem, where both firms choose their locations simultaneously to maximize the ex ante option values prior to entry.

Since the two firms are symmetric and the consumers are uniformly distributed on the unit interval, we have $a=1-b$, which together with $d=b-a$ yields

$$a=\frac{1-d}{2}; \tag{32}$$

$$b=\frac{1+d}{2}. \tag{33}$$

Thus, under symmetry, the first-stage outcomes simplify substantially. By substituting these a and b, (32) and (33), into the equilibrium profits in the third stage, (18) and (19), we have the two firms' profits in the second stage as

$$\pi_1=\pi_2=ds^*. \tag{34}$$

4. Difference from the Static Hotelling Model

Now we are ready to determine each firm's optimal location in the first stage.

Firm i's post-entry value under symmetry is obtained by substituting (31), (32) and (33) into (25) as,

$$\Pi_i(d)=\frac{ds^*}{\rho+\lambda(d)-\mu}-I. \tag{35}$$

Maximizing (35) with respect to d, which is equivalent to maximizing (35) with respect to a for firm 1 and with respect to b for firm 2, determines the optimal location for each firm.

From the condition $\Pi_i'(d)=0$, we obtain

$$\frac{\beta(d)}{\beta'(d)}=\frac{d}{\beta(d)-1}. \tag{36}$$

Hence, the value of d that maximizes $\Pi_i(d)$ is determined as the d-coordinate of the intersection between the solid blue curve representing $\beta(d)/\beta'(d)$

and the dashed blue curve representing $d/(\beta(d)-1)$ in **Figure 1**.

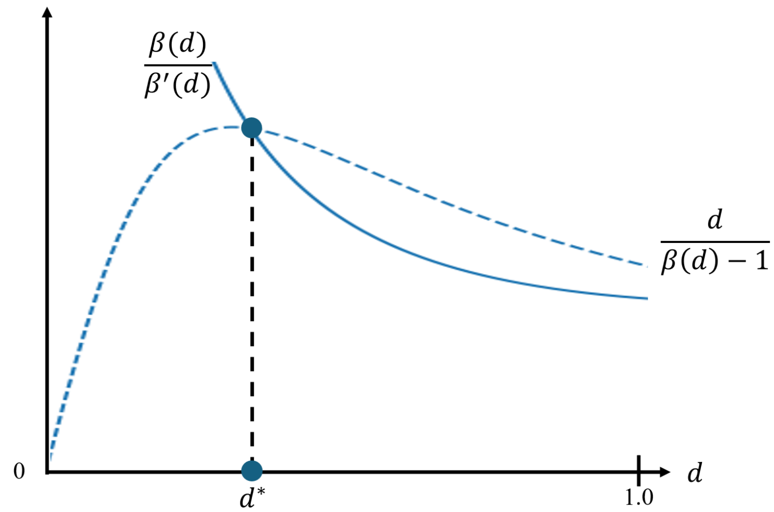

Figure 1. Determination of d^*.

The graph of $\Pi_i(d)$ is typically illustrated and is maximized at d^* as in **Figure 2**.

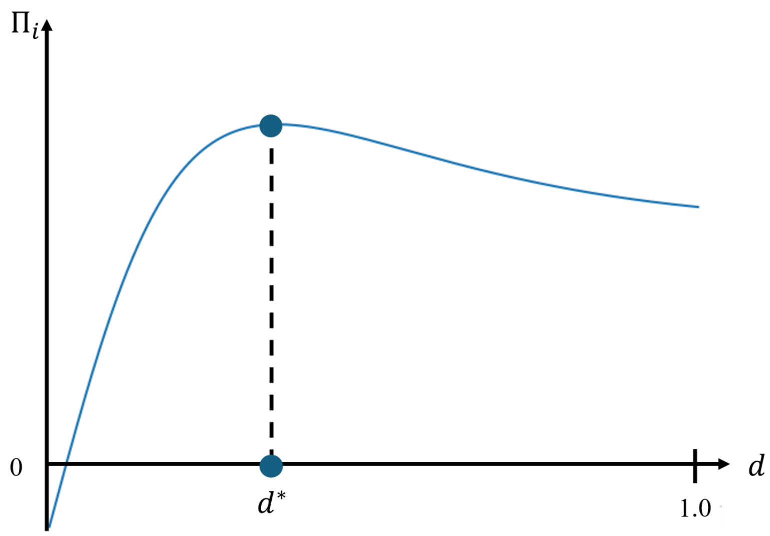

Figure 2. Shape of $\Pi_i(d)$.

From the symmetry conditions, the equilibrium locations are therefore

$$a^* = \frac{1-d^*}{2}; \quad (37)$$

$$b^* = \frac{1+d^*}{2}. \quad (38)$$

This leads to the following proposition.

Proposition

Firms do not locate at the endpoints of the interval in a dynamic extension of the static Hotelling model in which the effective discount rate is determined by a distance-dependent premium rate and firms optimally choose their entry timing.

In contrast, in the standard static Hotelling model—where firms choose locations in the first stage and prices in the second stage without entry timing decisions—each firm locates at an endpoint of the interval, as shown in the **Appendix**.

This proposition highlights that firms' location choices affect their entry incentives not only through contemporaneous price competition, as in the static Hotelling model, but also through an endogenous location-dependent discount premium that governs the intertemporal valuation of future profits. In particular, closer location mitigates competitive pressure by reducing the effective discount rate applied to post-entry profit flows, thereby shaping the value of waiting and the optimal timing of entry. As a result, location choice and entry timing are jointly determined, and the classical prediction of maximal differentiation no longer holds once firms optimally account for the dynamic consequences of spatial configuration.

5. Conclusion

This paper has developed a dynamic extension of the Hotelling model in which firms choose their locations prior to entry and optimally determine the timing of entry under uncertainty. By embedding spatial competition in a real-options framework, our analysis allows firms to delay irreversible investment until market conditions become sufficiently favorable, thereby introducing an explicitly intertemporal dimension into location choice.

A central contribution of the paper is the introduction of a location-dependent discount premium, which endogenously links firms' spatial configuration to the intertemporal valuation of future profit streams. Unlike a standard exit hazard, this premium captures how location choices affect the rate at which project value depreciates over time, reflecting mechanisms such as benefits of agglomeration, knowledge spillovers and so on. As a result, location decisions influence not only price competitions after entry but also the effective discount rate that governs the value of waiting and the optimal entry timing.

The main result shows that once firms optimally account for this dynamic effect, the classical prediction of maximal differentiation in the static Hotelling model no longer holds. Instead, firms may optimally choose interior locations, as spatial separation involves a trade-off between relaxing price competition and increasing the discount premium applied to future profits.

This finding highlights the strategic interdependence between location choice and entry timing, and demonstrates that conclusions drawn from static spatial competition models can be overturned when uncertainty and irreversible investment are explicitly incorporated. More broadly, the framework developed in this paper provides a unified approach to studying the interaction between spatial competition, uncertainty and dynamic investment decisions. It suggests that policies or institutional features that affect firms' exposure to location-specific benefits may influence not only pricing but also the timing of entry and spatial configuration of firms.

Several extensions remain for future research. Allowing for asymmetric discount premia across firms or locations would enable the analysis of heterogeneous risk environments. Endogenizing exit decisions or incorporating learning about demand or costs after entry would further enrich the dynamic structure. Finally, a welfare analysis comparing privately optimal and socially optimal location-timing outcomes would help clarify the policy implications of location-dependent discounting in spatial markets.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix: The Static Hotelling Model

In this Appendix, we briefly analyze the standard static Hotelling model in order to contrast it with the dynamic model studied in the main text.

Suppose that firms choose their locations in the first stage and prices in the second stage, and that there is no choice of entry timing. Consumers are uniformly distributed on the unit interval.

Price Competition

Given locations a and b with $a < b$, firm 1 chooses price p_1 and firm 2 chooses price p_2.

The profit functions of the two firms are given by

$$\pi_1 = (p_1 - c)x^*;$$

$$\pi_2 = (p_2 - c)(1 - x^*),$$

where the indifferent consumer x^* satisfies

$$\bar{v} - p_1 - s(x^* - a)^2 = \bar{v} - p_2 - s(x^* - b)^2.$$

Solving this condition yields

$$x^* = \frac{p_2 - p_1}{2s(b-a)} + \frac{a+b}{2}.$$

The first-order condition for firm 1's price choice is

$$\frac{\partial \pi_1}{\partial p_1} = 0,$$

which implies

$$p_1 = \frac{p_2 + c}{2} + s(b-a)(a+b).$$

Similarly, the first-order condition for firm 2 is

$$\frac{\partial \pi_2}{\partial p_2} = 0,$$

which implies

$$p_2 = \frac{p_1 + c}{2} + s(b-a)(2-a-b).$$

Solving these two equations simultaneously yields the Nash equilibrium prices

$$p_1^* = c + s(b-a)(2+a+b);$$

$$p_2^* = c + s(b-a)(4-a-b).$$

Substituting the equilibrium prices into the expression for the indifferent consumer yields firm 1's demand

$$x^* = \frac{2+a+b}{4}.$$

Substituting the equilibrium prices into the profit functions gives

$$\pi_1 = \frac{s(b-a)(2+a+b)^2}{4};$$

$$\pi_2 = \frac{s(b-a)(4-a-b)^2}{4},$$

where we assume that $b-a>0$.

Location Choice

We now consider the firms' location choices in the first stage.

Firm 1 chooses a taking b as given. Differentiating firm 1's profit with respect to a yields

$$\frac{\partial \pi_1}{\partial a} = \frac{s}{2}(b-a)(2+a+b) - \frac{s}{4}(2+a+b)^2.$$

For any interior location $a \in (0,1)$, this derivative is negative, implying that firm 1's profit decreases as it moves to the right. Therefore, firm 1's optimal location is at the left endpoint of the interval:

$$a^* = 0.$$

Similarly, firm 2 chooses b taking a as given. Differentiating firm 2's profit with respect to b yields

$$\frac{\partial \pi_2}{\partial b} = \frac{s}{2}(b-a)(4-a-b) - \frac{s}{4}(4-a-b)^2.$$

This derivative is positive for any interior $b \in (0,1)$, implying that firm 2's profit increases as it moves to the right. Therefore, firm 2's optimal location is at the right endpoint of the interval:

$$b^* = 1.$$

Equilibrium Outcome

Thus, in the standard static Hotelling model without entry timing, the unique Nash equilibrium in locations is characterized by maximal differentiation: firm 1 locates at the left endpoint $a=0$, and firm 2 locates at the right endpoint $b=1$. This result stands in sharp contrast to the outcome obtained in the dynamic model analyzed in the main text, where optimal entry timing and location-dependent premium rates lead firms to choose interior locations.