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Risk Averse Members Coordination with Extended Buy-Back Contract

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ABSTRACT

This paper considers how to coordinate a supply chain (SC) consisted of one supplier and one retailer who possess different risk aversion preference with a contract. Based on the classical buy-back contract, this paper presents an extended buy-back contract. In addition to the member's objective of maximizing his expected profit, downside risk constraint is used to represent the SC member's risk aversion preference. Under different risk aversion preference combination, the SC perfect solution existence conditions are identified and the specific contract is provided accordingly. This research finds out that, with a low risk aversion supplier and a high risk aversion retailer, the supplier as the SC coordinator can give the retailer incentive to increase the order quantity so as to reach SC perfect coordination. Finally a numerical analysis verifies the effectiveness of the extended buy-back contract.

1. Introduction

Supply chain (SC) performance is affected by many uncertainty factors. In the supply chain contract (SCC) literatures two major uncertainty factors receives most focus. These factors include market demand uncertainty and manufacture's production capacity uncertainty. These uncertainty factors make direct impacts on the SC performance and incur SC members' profit risks. The SC members' profit risks are different from the SC members' risk preferences. The former are objective while the latter are subjective. Many SCC literatures consider the objective profit risks. In these literatures SCCs are designed to share profit risks through transfer payment between SC members. The SC members are assumed to be risk neutral, though they may have different risk preferences in reality. The risk preferences have great impacts on the members' decisions. Therefore some papers that consider the members' risk preferences are presented in recent years.

Eeckhoudt *et al*. research the effect of risk aversion in the single period newsboy problem. They examine what changes in price and cost parameters relate to the member's risk aversion [1]. Agrawal and Seshadri study the impacts of demand uncertainty and retailer's risk aversion on the retailer's pricing and ordering decision. It is proved that with different models that depict the relationship between price and demand distribution, the risk

aversion retailer will price higher or price lower than the risk neutral retailer [2]. Lau and Lau model a two echelon SC consisted of different risk attitude members in a single period. They use mean–variance utility function as the member's objective. It is found that the optimal return policy may change from "no return allowed" to "unlimited return with full credit" based on the member's risk attitude [3]. Gan *et al*. consider the channel coordination issue that the SC is consisted of a risk–neutral supplier and a downside-risk retailer. They put forward a downside risk formulation and design a risk-sharing contract that can achieve channel coordination [4]. Choi *et al*. study the channel coordination and risk control issues of a SC. They make comparison of the SC expected performance between the model with risk control and one without risk control. A mean-variance framework is used in [5]. Gan *et al*. consider a two echelon SC consisted of risk aversion members. They use mean-variance trade-off and expected utility as the member's objective. The set of Pareto optimal solutions are identified [6]. Wei and Choi use the mean-variance framework to analyze the supply chain coordination issue with wholesale pricing and profit sharing scheme (WPPS). In the decentralized case, they find that there exists a unique equilibrium of the Stackelberg game with WPPS [7]. Mastui research a SC consisted of one manufacture and one retailer. The two players are risk averse. He presents a new contract that is a trade-off between outright sales contract and

full-credit return policy and investigates the economic outcome differences [8]. Xiao and Choi use dynamic game theory to analyze a more complicated structure SC consisted of two manufactures and two retailers who are all risk averse. Their research focuses on the impacts of the retailer's risk sensitivity and other properties on the channel structure strategies and wholesale prices of the manufactures [9].

The above mentioned papers all involve risk aversion SC members. Only [4–7] consider the issue of SC coordination. The impacts of the risk aversion of the retailer on the SC coordination contract are considered in [4]. The impacts of the risk aversion of both the retailer and the supplier are researched and the existence conditions of Pareto optimal solution are identified in [6]. The numerical analysis of the channel coordination contract properties is presented for a SC consisted of different risk aversion members in [5]. SC coordination contract WPPS in [7] is different from the contract researched in this paper.

Unlike the previous literatures, this paper is devoted to coordination of a supply chain consisting of a supplier and a retailer with different risk aversion preference. The SC member wishes to maximize his expected profit and has to satisfy the risk aversion constraint. The major purposes of this paper include: a) Identify the conditions that may achieve SC perfect coordination; b) Design SCC for the decentralized SC to achieve perfect coordination; c) Analysis the impacts of member's risk aversion on the SC efficiency.

The remainder of this paper is organized as follows. In Section 2, we describe the basic models of the problems facing with the supplier, the retailer and the supply chain as a whole. In Section 3, we give the detail of supply chain contract that can achieve perfect coordination under SC members' different risk aversion preference combination. In Section 4, we provide a numerical analysis to verify the contract. Finally in Section 5, we summarize the main findings of this paper and possible future research of this study.

2. Assumptions, Notations and Models

2.1 Assumptions, Notations

As stated earlier, we consider a SC consisted of one supplier (she) and one retailer (he). The SC members have different risk aversion preferences. The supplier provides the retailer with a kind of short life cycle products. There is only one selling season. Prior to the selling season, the retailer should decide the order quantity and submit it to the supplier. The supply capacity of the supplier is unlimited. The supplier will ship the ordered products before the selling season. Market demand is stochastic and with a specific known distribution. Both the supplier and the retailer have the complete knowledge about the demand, profits, costs, risk-aversion preferences and prices.

We analyze this problem as a Stackelberg game in which the supplier acts as the leader while the retailer acts as the follower. The supplier provides the contract. Then the retailer decides order quantity that maximizes his expected profit as well as satisfies his risk aversion constraint according to the contract. The supplier as the leader may give the retailer incentive to increase his order quantity.

Notations used in this paper are defined as follows. Let q denote the retailer's order quantity and q_i^* denote the optimal order/supply quantity. The subscript i might be r(retailer), s(supplier), sc(supply chain). The following notations' subscripts are defined in the same way. Let X denote the stochastic market demand with distribution $F(\cdot)$ and density $f(\cdot)$. Let c denote the supplier's unit supplying cost; ω denote the unit wholesale price charging the retailer; b denote the unit buy back price paying the retailer for all returned products at the end of the selling season; p denote the unit retail price selling to the end customers. Let $\Pi_i(\cdot)$ be the members' profit. Let $\pi_i(\cdot)$ denote the members' expected profit, therefore $\pi_i(\cdot) = E[\Pi_i(\cdot)]$. Let α_i be the target profit; β_i ($0 \le \beta_i \le 1$) be the maximum risk level that the member will accept. We define γ_i ($0 \le \gamma_i \le 1$) as the member's downside risk. We assume there is a relationship among the parameters of c, b, ω, p that is $0 < c \prec b \prec \omega \prec p$.

2.2 Definition of the Member Risk Aversion Preferences

We use downside risk to measure SC member's risk level. The downside risk is used in [4,10]. Downside risk of the member is the probability that his realized profit is less than or equal to his specified target profit. The member's risk aversion preference is defined as the downside risk must be less than or equal to his or her maximum risk level.

The member's downside risk definition formula is shown as below:

$$\gamma_i \equiv P\{\Pi_i \le \alpha_i\} \tag{1}$$

The member's risk aversion preference definition formula is shown as below:

$$\gamma_i \le \beta_i \tag{2}$$

2.3 Modeling on Supply Chain Consisted of Members with Risk Aversion Preferences

Gan *et al*. provide the definition of the supply chain coordination of one risk neutral supplier and one downside risk aversion retailer [4]. We reference their definition and make adjustment in line with our conditions.

Definition 1. The supply chain is perfectly coordinated if the following conditions are satisfied:

1) the retailer and the supplier get payoffs not less than their respective reservation payoffs,

2) both the retailer's and the supplier's downside risk constraint are satisfied,

3) the supply chain's expected profit is maximized.

The model of the supply chain consisted of risk aversion members is shown as below.

For the supplier:

$$\max_{q \geq 0} \pi_s(q)$$
$$s.t. \ \gamma_s \leq \beta_s \tag{3}$$

For the retailer:

$$\max_{q \geq 0} \pi_r(q)$$
$$s.t. \ \gamma_r \leq \beta_r \tag{4}$$

For the supply chain:

$$\max_{q \geq 0} \pi_{sc}(q) \tag{5}$$

2.4 The Benchmark Model

We select the model of the buy-back contract (also called return policy) with risk neutral members as the benchmark. To differentiate the benchmark contract from the contract presented in this paper, we call the former initial buy-back contract while the latter extended buy-back contract. Pasternack has proved that in the risk neutral case, buy-back contract will coordinate the supply chain if the wholesale price ω and buy-back price b satisfy the Formula (6). The retailer's optimal order quantity to maximize the supply chain profit is shown as the Formula (7) [11].

$$\frac{p-b}{p} = \frac{w-b}{c} \tag{6}$$

$$\hat{q} = F^{-1}\left(\frac{p-c}{p}\right) \tag{7}$$

2.5 The relationship between order/supply quantity and member's downside risk under initial buy-back contract.

SC member's minimum supply or order quantity equals to the least amount of products to be sold so as to reach her or his target profit α_i.

Lemma 1. Assumed the member's target profit is $\alpha_i \ (i:s,r)$, the supplier' minimum supply quantity q_s^0 and the retailer's minimum order quantity q_r^0 are shown as the formulas below.

$$q_s^0 = \frac{\alpha_s}{\omega - c} \tag{8}$$

$$q_r^0 = \frac{\alpha_r}{p - \omega} \tag{9}$$

Proof: see the appendix A.1.

Theorem 1. If the supply quantity q_s is less than or equal to the minimum supply quantity q_s^0 then the supplier's downside risk γ_s equals to 1; if q_s is larger than q_s^0 then γ_s equals to $F\left(\dfrac{\alpha_s - (\omega - c - b)q_s}{b}\right)$, where $\alpha_s \geq (\omega - c - b)q_s$.

Further description about Theorem 1: Let $\Delta \omega = \omega - c$, $\Delta \omega$ approximately equals to the net profit of the supplier. b is unit buy-back price. $\Delta \omega$ and b represent the relative bargaining power between the supplier and the retailer. We have assumed that the supplier is the SC leader and the retailer is the follower. Therefore it is reasonable to assume the supplier is dominant and thus $\Delta \omega \succ b$. $\Delta \omega \leq b$ contradicts our assumptions. We only consider the case of $\Delta \omega \succ b$ in the rest of this paper.
Proof: see the appendix A.2.

Theorem 2. If the order quantity q_r is less than or equal to the minimum order quantity q_r^0 then the retailer's downside risk γ_r equals to 1; if q_r is larger than q_r^0 then γ_r equals to $F\left(\dfrac{\alpha_r + (\omega - b)q_r}{p - b}\right)$.

Proof: see the appendix A.3.

2.6 Threshold of the Member's Downside Risk

Definition 2. The threshold of the SC member's downside risk is defined as the maximum value of the member's downside risk when $q_i = \hat{q}$ and

$$\alpha_i = \pi_i(\hat{q}) \quad i:s,r.$$

Theorem 3. The downside risk threshold of the supplier equals to that of the retailer and the value equals to $F(\hat{q} - \int_0^{\hat{q}} F(x)dx)$.

For notation convenient we represent this value as BN.

That is $BN = F(\hat{q} - \int_0^{\hat{q}} F(x)dx)$.

Proof: see the appendix A.4.

2.7 The Relationship between the Member Risk Aversion Preference and the Order/Supply Quantity

Corollary 1 describes the relationship between the supplier's risk aversion preference and her optimal supply quantity. It can be inferred from Theorem 1 and Theorem 3.

Corollary 1. If $\beta_s \prec F\left[\dfrac{\pi_s(\hat{q}) - (\omega - c - b)q_s^0}{b}\right]$ then the

supplier's optimal supply quantity that maximize her expected profit while satisfy her risk aversion constraint is defined by Formula (10).

$$q_s^* = \begin{cases} \hat{q} & BN \leq \beta_s \prec F\left[\dfrac{\pi_s(\hat{q}) - (\omega - c - b)q_s^0}{b}\right] \\[2ex] \dfrac{\pi_s(\hat{q}) - bF^{-1}(\beta_s)}{\omega - c - b} & \beta_s \prec BN \end{cases} \quad (10)$$

Similar to Corollary 1, Corollary 2 describes the relationship between the retailer's risk aversion preference and his optimal order quantity.

Corollary 2. If $\beta_r \succ F\left[\dfrac{\pi_r(\hat{q}) - (\omega - b)q_r^0}{p - b}\right]$ then the retailer's optimal order quantity that maximize his expected profit while satisfy his risk aversion constraint is defined by Formula (11).

$$q_r^* = \begin{cases} \hat{q} & \beta_r \geq BN \\[2ex] \dfrac{(p - b)F^{-1}(\beta_r) - \pi_r(\hat{q})}{\omega - b} & F\left[\dfrac{\pi_r(\hat{q}) - (\omega - b)q_r^0}{p - b}\right] \prec \beta_r \prec BN \end{cases} \quad (11)$$

3. Supply Chain Contracts with Different Risk Aversion Preference Members

In this section we will give specific supply chain contracts for three scenarios that have different risk aversion preferences combination. The first scenario depicts that both the supplier and retailer show low risk aversion preferences. The second scenario has a low risk aversion supplier and a high risk aversion retailer. In the third scenario the supplier is high risk aversion while the retailer's risk aversion might be high or low.

3.1 Both the Supplier and the Retailer are Low Risk Aversion ($\beta_s \geq BN$ and $\beta_r \geq BN$)

Step 1. Compute the optimal order/supply quantity of the member with risk aversion preference.

It can be inferred from Corollary 1 and Corollary 2 that both the retailer's optimal order quantity q_r^* and the supplier's optimal supply quantity q_s^* equal to the supply chain optimal order quantity $\hat{q}$ that can achieve the maximum expected profit of the supply chain.

Step 2. Design supply chain coordination contract.

It is obviously that the SC member's risk aversion preference constraint doesn't impact on the optimal solution when her (his) risk aversion preference is low. The supply chain contract in this scenario is the same as that with risk neutral SC members. The supply chain contract can be buy-back contract, revenue sharing contract or other contracts. For the purpose of comparison we select buy-back contract here.

The initial buy-back contract that the wholesale price

ω and buy-back price b satisfy the Formula (6) will coordinate the supply chain with low risk aversion preference SC members. In that the supply chain is perfectly coordinated.

3.2 The Supplier is Low Risk Aversion while the Retailer is High Risk Aversion ($\beta_s \geq BN$ and $\beta_r \prec BN$)

Step 1. Compute the optimal order/supply quantity of the SC member with risk aversion preferences.

It can be inferred from Corollary 1 and Corollary 2 that the retailer's optimal order quantity q_r^* is less than the supply chain optimal order quantity $\hat{q}$ while the supplier's optimal supply quantity q_s^* equal to $\hat{q}$.

Step 2. Design supply chain coordination contract.

For the sake of high risk aversion preference, the retailer's optimal order quantity q_r^* is less than the supply chain optimal order quantity $\hat{q}$. In order to improve the SC efficiency the supplier as the SC coordinator has to give incentive to the retailer to increase the order quantity.

We present an extended buy-back contract to coordinate the SC in this scenario. The main contents of the contract are shown as follows.

1) If the retailer's order quantity q is less than or equal to q_r^* then the supplier provides the retailer with the initial buy-back contract;

2) If the retailer's order quantity q is larger than q_r^* but no larger than $\hat{q}$ then the supplier charges the retailer with the wholesale price ω' and gives full refund of ω' for all unsold for the part of order quantity that is equal to $q - q_r^*$ and the supplier provides the retailer with initial buy-back contract for the part of order quantity that is equal to q_r^*;

3) If the retailer's order quantity q is larger than $\hat{q}$ then the supplier treat the retailer the same as that in (ii) but with a maximum full refund return quantity being equal to $\hat{q} - q_r^*$.

Proof: the extended buy-back contract will satisfy all requirements of Definition 1.

Let $\Pi_r^*(q, X)$, $\Pi_s^*(q, X)$ and $\pi_r^*(q)$, $\pi_s^*(q)$ denote the profit and expected profit of the retailer and the supplier respectively with extended buy-back contract.

1) Firstly, we'll prove that the SC member's expected profit is no less than her (his) original expected profit with the extended buy-back contract.

a) If $q = \hat{q}$ then the retailer's profit is

$$\Pi_r^*(\hat{q}, X) = \begin{cases} \Pi_r\left(q_r^*, X\right) & X \leq q_r^* \\ \Pi_r\left(q_r^*, X\right) + (p - \omega')(X - q_r^*) & q_r^* \prec X \leq \hat{q} \\ \Pi_r\left(q_r^*, X\right) + (p - \omega')(\hat{q} - q_r^*) & X \succ \hat{q} \end{cases}$$

(12)

If $q = \hat{q}$ then the retailer's expected profit is:

$$\pi_r^*(\hat{q}) = \int_0^{q_r^*} \Pi_r\left(q_r^*, x\right) f(x) dx$$

$$+ \int_{q_r^*}^{\hat{q}} \left[\Pi_r\left(q_r^*, x\right) + (p - \omega')(X - q_r^*)\right] f(x) dx$$

$$+ \int_{\hat{q}}^{\infty} \left[\Pi_r\left(q_r^*, x\right) + (p - \omega')(\hat{q} - q_r^*)\right] f(x) dx$$

$$\pi_r^*(\hat{q}) - \pi_r\left(q_r^*\right) = (p - \omega')\left[\int_{q_r^*}^{\hat{q}}(x - q_r^*) f(x) dx \right.$$

$$\left. + (1 - F(\hat{q}))(\hat{q} - q_r^*)\right]$$

If $\omega' \leq p$ then $\pi_r^*(\hat{q}) - \pi_r\left(q_r^*\right) \geq 0$.

A sample comparison between the retailer's profit with initial buy-back contract Π_r and the profit with extended buy-back contract Π_r^* is made with the parameters that are declared in Section 4. The comparison is illustrated in Figure 1.

If $q = \hat{q}$ then the supplier's profit is:

$$\Pi_s^*(\hat{q}, X) = \begin{cases} \Pi_s\left(q_r^*, X\right) - c(\hat{q} - q_r^*) & X \leq q_r^* \\ \Pi_s\left(q_r^*, X\right) + \omega'(X - q_r^*) - c(\hat{q} - q_r^*) & q_r^* \prec X \leq \hat{q} \\ \Pi_s\left(q_r^*, X\right) + (\omega' - c)(\hat{q} - q_r^*) & X \succ \hat{q} \end{cases}$$

(13)

If $q = \hat{q}$ then the supplier's expected profit is:

$$\pi_s^*(\hat{q}) = \int_0^{q_r^*} [\Pi_s\left(q_r^*, x\right) - c(\hat{q} - q_r^*)] f(x) dx$$

$$+ \int_{q_r^*}^{\hat{q}}\left[\Pi_s\left(q_r^*, x\right) + \omega'(x - q_r^*) - c(\hat{q} - q_r^*)\right] f(x) dx$$

$$+ \int_{\hat{q}}^{\infty}\left[\Pi_s\left(q_r^*, x\right) + (\omega' - c)(\hat{q} - q_r^*)\right] f(x) dx$$

$$\pi_s^*(\hat{q}) - \pi_s\left(q_r^*\right) = \omega'\left[\int_{q_r^*}^{\hat{q}}(x - q_r^*) f(x) dx\right.$$

$$\left. + (1 - F(\hat{q}))(\hat{q} - q_r^*)\right] - c(\hat{q} - q_r^*)$$

If $\omega' \geq \dfrac{c(\hat{q} - q_r^*)}{\int_{q_r^*}^{\hat{q}}(x - q_r^*) f(x) dx + (1 - F(q))(\hat{q} - q_r^*)}$ then

$$\pi_s^*(\hat{q}) - \pi_s\left(q_r^*\right) \geq 0.$$

If $\dfrac{c(\hat{q} - q_r^*)}{\int_{q_r^*}^{\hat{q}}(x - q_r^*) f(x) dx + (1 - F(q))(\hat{q} - q_r^*)} \leq \omega' \leq p$

then the SC member's payoff will be no less than his or her reservation payoff.

A sample comparison between the supplier's profit with initial buy-back contract Π_s and the profit with extended buy-back contract Π_s^* is illustrated in Figure 2.

2) Secondly it is proved that with the extended buy-back contract, the SC member's risk aversion constraint is satisfied.

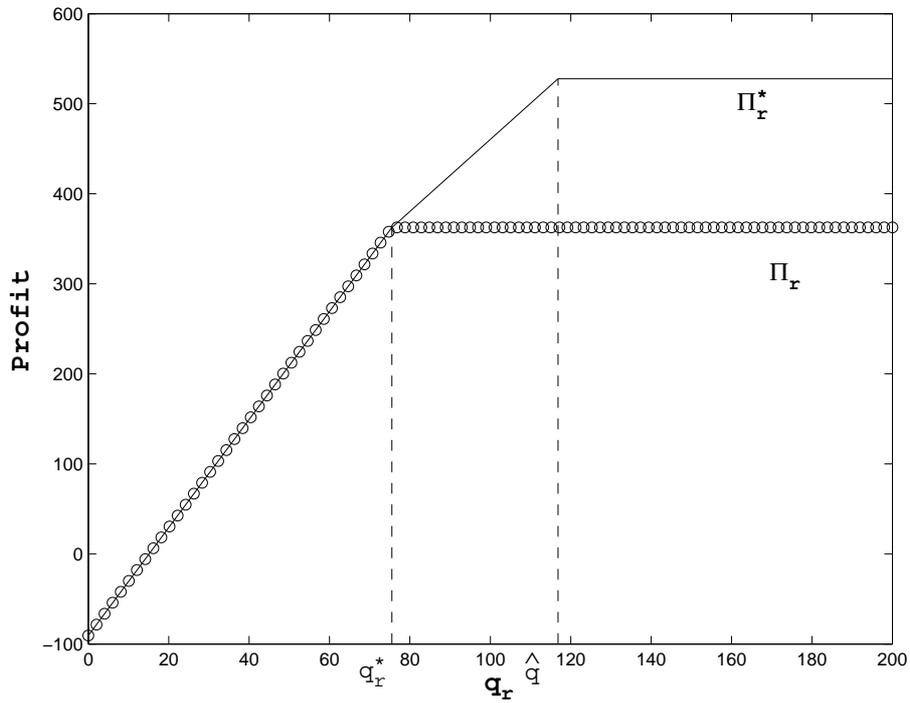a) The retailer's risk aversion constraint is satisfied.

According to Formula (12):

If $X \leq q_r^*$, $\Pi_r^*(\hat{q}, X) = \Pi_r\left(q_r^*, X\right)$

then $P\{\Pi_r(q_r^*, x) \leq \pi_r(\hat{q})\} = \beta_r$;

If $X \succ q_r^*$, $\Pi_r^*(\hat{q}, X) \succ \Pi_r\left(q_r^*, X\right)$

then $P\{\Pi_r^*(\hat{q}, x) \leq \pi_r(\hat{q})\} \prec \beta_r$.

Therefore the retailer's risk aversion constraint is satisfied under the extended buy-back contract.

b) The supplier's risk aversion constraint is satisfied.

It can be proved that $\Pi_s^*(\hat{q}, \hat{q}) \succ \pi_s(\hat{q})$. The proving is simple and is omitted here. Then $\ni q_B$, $q_B \in (0, \hat{q})$, q_B meets the requirement that $\Pi_s^*(\hat{q}, q_B) = \pi_s(\hat{q})$. If $\beta_s \geq F(q_B)$ then the supplier's risk aversion constraint is satisfied.

3) The supply chain order quantity is consistent with the system perfect solution i.e. $q = \hat{q}$ with the extended buy-back contract.

Summarize 1 to 3, we have the result that the supplier can give the retailer incentive to increase the order quantity and achieve the system perfect solution by providing the extended buy-back contract when the supplier is low risk aversion e and the retailer is high risk aversion.

3.3 The Supplier is High Risk Aversion ($\beta_s \prec BN$)

According to Corollary 1, we have $q_s^* < \hat{q}$. In this case no matter what the retailer's risk aversion is the supply chain can't achieve system perfect solution.

Figure 1. Retailer's profits comparison between Initial and extended buy-back contract
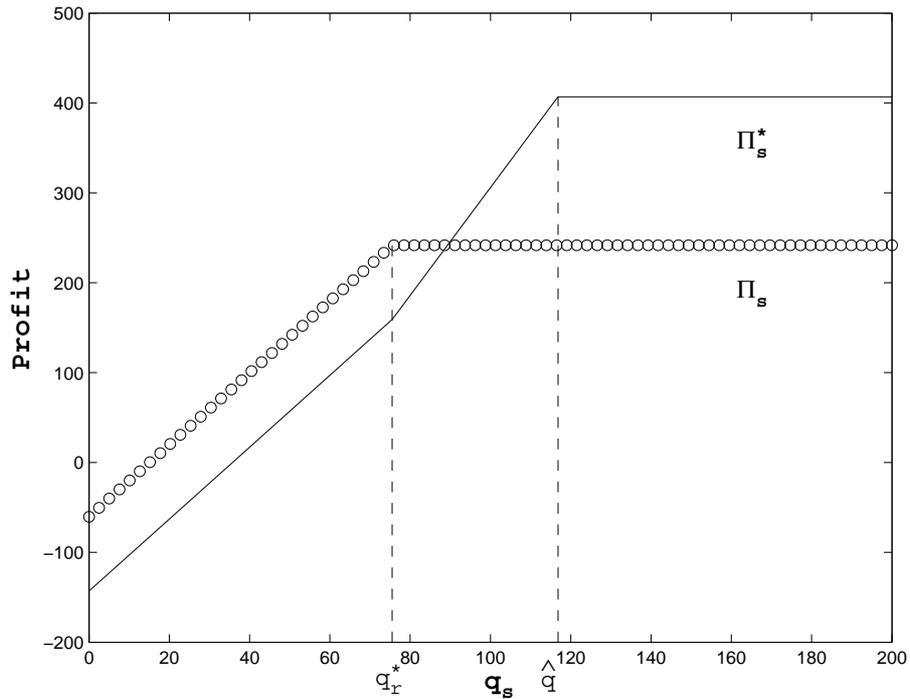

Figure 2. Supplier's profits comparison between Initial and extended buy-back contract

4. Numerical Analysis

In this section we'll give a numerical analysis for the case of low risk aversion supplier and high risk aversion retailer. It is shown that with the extended buy-back con-tract, with the retailer's risk aversion increasing, the op-timal order quantity decreases and the two SC member's expected profits get better off comparing with that of the initial buy-back contract.

Table 1. Impact of the retailer's risk aversion on the members' expected profit with extended buy-back contract

β_r	q_r^*	$\triangle\pi_r$	$\triangle\pi_s$
0.35	90.74	7.19	0.59
0.3	81.89	13.05	25.82
0.25	72.34	20.74	63.15
0.2	61.71	30.3	111.93
0.15	49.31	41.98	172.85
0.1	33.71	56.92	251.16

The parameters used in this section are setup as follows. Let the stochastic market demand $X \sim N(100, 20^2)$. Set the supplier's unit supplying cost c = 2, the retailer's unit retail price $p = 10$. With the initial buy-back contract assume the unit wholesale price $\omega = 5.2$, the unit buy back price b=4. With the extended buy-back contract the supplier provides the retailer with the unit wholesale price (buy-back price) $\omega' = 8.8$ for the amount of order quantity that is greater than his optimal quantity. Assume the retailer's risk aversion threshold $\beta_r = 0.15$, the supplier's risk aversion threshold $\beta_s = 0.7$ and BN = 4.6. Based on these parameters it can be obtained that $q_r^* = 49.31$, $\pi_r(q_r^*) = 236.46$, $\pi_s(q_r^*) = 157.65$ with the initial buy-back contract; $q_r^* = \hat{q} = 49.31$, $\pi_r(\hat{q}) = 278.44$, $\pi_s(\hat{q}) = 330.5$ with the extended buy-back contract.

The supplier gives the retailer incentive to increase the order quantity and finally increase the whole channel efficiency via the extended buy-back contract. How the role of the extended buy-back contract changes with different retailer's risk aversion levels needs to be analyzed in detail. With the retailer's risk aversion threshold changing from 0.35 to 0.1, the corresponding values of q_r^*, $\triangle\pi_r$, $\triangle\pi_s$ are presented in Table 1. q_r^* is the retailer's optimal order quantity with the initial buy-back contract. $\triangle\pi_r = \pi_r^*(\hat{q}) - \pi_r(q_r^*)$ is the retailer's expected profit increment with the extended buy-back contract. $\triangle\pi_s = \pi_s^*(\hat{q}) - \pi_s(q_r^*)$ is the supplier's expected profit increment with the extended buy-back contract. By analyzing the data in Table 1 we will find that the member's expected profit increment is increasing in the retailer's risk aversion threshold. This confirms that the extended buy-back contract definitely increase the whole channel's efficiency. Moreover it is shown that the extended buy-back contract is more attractive for the supplier than the retailer.

5. Conclusions

In this study, we consider a supply chain that is consisted of a supplier and a retailer who posses different risk aversion preferences in a single period. Based on the initial buy-back contract we provide the extended buy-back contract model. We analyze the condition under which the supply chain can achieve system perfect solution and provide the corresponding contract terms. We report in this paper that: (1) if both the supplier and the retailer are low risk aversion, then the supply chain can achieve system perfect solution with the initial buy-back contract. (2) if the supplier is low risk aversion while the retailer is high risk aversion, then the supplier can give the retailer incentive to achieve system perfect solution with the extended buy-back contract. (3) if the supplier is high risk aversion (no matter what risk aversion the retailer is then the supply chain can not achieve system perfect solution. At the end of this paper we present a numerical analysis to verify the extended buy-back contract.

There are many related problems that need to be further explored. First, the model we considered is in single period. A good extension to this research may be to consider the model in multiple periods. Secondly, the structure of the SC in this paper is simple. The structure of the supply chain is complicated in reality. It is more appreciate to investigate in a more complicated structure.

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Appendix

A.1 Proof of Lemma 1

(1) Proof of Formula 8
The supplier's profit function is

$$\prod_s(q,X) = (\omega - c)q - b(q - X)^+$$

The least amount products means all products sold in the retail market i.e. $X \geq q$.

The above formula transforms to

$$\prod_s(q,X) = (\omega - c)q$$

Substitute $\prod_s$, q with α_s, q_s^0 into above formula we get

$$\alpha_s = (\omega - c)q_s^0$$

$$q_s^0 = \frac{\alpha_s}{\omega - c}.$$

(2) Proof of Formula 9
The retailer's profit function is

$$\prod_r(q,X) = p\min\{q,X\} - \omega q + b(q - X)^+$$

All products are sold out i.e. $X \geq q$.

The above formula transforms to

$$\prod_r(q,X) = (p - \omega)q$$

Substitute $\prod_r$, q with α_r, q_r^0 into above formula we get

$$\alpha_r = (p - \omega)q_r^0$$

$$q_r^0 = \frac{\alpha_r}{p - \omega}$$

A.2 Proof of Theorem 1

1) In the case of $q_s \leq q_s^0$

$$\begin{aligned}
&\prod_s(q_s, X)\\
&= (\omega - c)q_s - b(q_s - X)^+\\
&\leq (\omega - c)q_s\\
&\prec (\omega - c)q_s^0 = \alpha_s
\end{aligned}$$

Therefore $P\{\prod_s(q_s, X) \leq \alpha_s\} = 1$

ii) In the case of $q_s \succ q_s^0$

If $X \succ q_s$

$$\begin{aligned}
&P[\{\prod_s(q_s, X) \leq \alpha_s\} \bigcap \{X \succ q_s\}]\\
&= P\{(\omega - c)q_s \leq \alpha_s\}
\end{aligned}$$

$$\because (\omega - c)q_s \succ (\omega - c)q_s^0 = \alpha_s$$

$$\therefore P\{(\omega - c)q_s \leq \alpha_s\} = 0$$

If $X \leq q_s$

$$\begin{aligned}
&P[\{\prod_s(q_s, X) \leq \alpha_s\} \bigcap \{X \leq q_s\}]\\
&= P\{(\omega - c)q_s - b(q_s - X) \leq \alpha_s\}\\
&= P\{(\omega - c - b)q_s + bX \leq \alpha_s\}\\
&= P\left\{X \leq \frac{\alpha_s - (\omega - c - b)q_s}{b}\right\}\\
&= F(\frac{\alpha_s - (\omega - c - b)q_s}{b})
\end{aligned}$$

Therefore if $q_s \succ q_s^0$ then the supplier's downside risk is $F\left(\dfrac{\alpha_s - (\omega - c - b)q_s}{b}\right)$.

A.3 Proof of Theorem 2

1) In the case of $q_r \leq q_r^0$

$$\begin{aligned}
&\prod_r(q_r, X)\\
&= p\min\{q_r, X\} - \omega q_r + b(q_r - X)^+\\
&\leq (p - \omega)q_r\\
&\prec (p - \omega)q_r^0 = \alpha_r
\end{aligned}$$

Thus $P\{\prod_r(q_r, X) \leq \alpha_r\} = 1$

Therefore if $q_r \leq q_r^0$ then γ_r equals to 1.

2) In the case of $q_r \succ q_r^0$

If $X \succ q_r$

$$\begin{aligned}
&P[\{\prod_r(q_r, X) \leq \alpha_r\} \bigcap \{X \succ q_r\}]\\
&= P\{(p - \omega)q_r \leq \alpha_r\}\\
&\because (p - \omega)q_r \succ (p - \omega)q_r^0 = \alpha_r\\
&\therefore P\{(p - \omega)q_r \leq \alpha_r\} = 0
\end{aligned}$$

If $X \leq q_r$

$$\begin{aligned}
&P[\{\prod_r(q_r, X) \leq \alpha_r\} \bigcap \{X \leq q_r\}]\\
&= P\{(p - b)X - (\omega - b)q_r \leq \alpha_r\}\\
&= P\left\{X \leq \frac{\alpha_r + (\omega - b)q_r}{p - b}\right\}\\
&= F(\frac{\alpha_r + (\omega - b)q_r}{p - b})
\end{aligned}$$

Therefore if $q_r \succ q_r^0$ then γ_r equals to

$$F(\frac{\alpha_r + (\omega - b)q_r}{p - b}).$$

A.4 Proof of Theorem 3

Firstly we'll show how to get the downside risk threshold of the supplier.

$$\pi_s(\hat{q})$$
$$= (\omega - b - c)\hat{q} + bE[\min\{\hat{q}, X\}]$$
$$= (\omega - b - c)\hat{q} + b(\hat{q} - \int_0^{\hat{q}} F(x)dx)$$

$$P\{\prod_s(\hat{q}, X) \le \pi_s(\hat{q})\}$$
$$= P\left\{\min\{\hat{q}, X\} \le \hat{q} - \int_0^{\hat{q}} F(x)dx\right\}$$

If $\hat{q} \prec X$

$$P\{\prod_s(\hat{q}, X) \le \pi_s(\hat{q})\}$$
$$= P\left\{\hat{q} \le \hat{q} - \int_0^{\hat{q}} F(x)dx\right\}$$
$$= 0$$

If $\hat{q} \ge X$

$$P\{\prod_s(\hat{q}, X) \le \pi_s(\hat{q})\}$$
$$= P\left\{X \le \hat{q} - \int_0^{\hat{q}} F(x)dx\right\}$$
$$= F(\hat{q} - \int_0^{\hat{q}} F(x)dx)$$

Therefore the downside risk threshold of the supplier equals to $F(\hat{q} - \int_0^{\hat{q}} F(x)dx)$.

In the similar way we can prove that the downside risk threshold of the retailer also equals to $F(\hat{q} - \int_0^{\hat{q}} F(x)dx)$.