Scientific Research Publishing

Journal of Software Engineering and Applications, 2025, 18(11), 427-445
https://www.scirp.org/journal/jsea
ISSN Online: 1945-3124
ISSN Print: 1945-3116

Some Linguistic Scaling Issues of Fuzzy Sets Based Decision-Making Systems

Alex Tserkovny

Applied AI Services, Brookline, USA
Email: atserkovny@yahoo.com

How to cite this paper: Tserkovny, A. (2025) Some Linguistic Scaling Issues of Fuzzy Sets Based Decision-Making Systems. *Journal of Software Engineering and Applications*, **18**, 427-445.
https://doi.org/10.4236/jsea.2025.1811026

Received: October 4, 2025
Accepted: November 15, 2025
Published: November 18, 2025

Abstract

In this study we are examining expert assessment (***EA***) non-linear scaled based approach to map original physical scale into a linguistical one. We introduce fuzzification of linguistic clustering mechanism for both input and output of a model under investigation, followed by appropriate decision-making apparatus. We also offered defuzzification approach for presented fuzzy model output. All results are thoroughly illustrated by proper experimental results.

Keywords

Linguistic Scale, Cluster, Centroid, Fuzzy Logic, Modus Ponnens, Fuzzy Conditional Inference

1. Introduction

The original *fuzzy clustering* (also referred to as *soft clustering or soft k-means*) [1]-[6] is known as a form of clustering in which each *data point* can belong to more than one cluster. Usually, this technique involves assigning data points to clusters such that items in the same cluster are as similar as possible, while items belonging to different clusters are as dissimilar as possible. Clusters are identified via similarity measures, which include distance, connectivity, and intensity. Different similarity measures may be chosen based on the data or the application.

In this study we are investigating very different dimensions of *fuzzy clustering*, related to linguistic determination of each cluster by *EA*s, associated with corresponding physical values. Note, that in all our previous studies [7]-[13] we always suggested that there is linear (monotonic) mapping between considered physical scale and its linguistic counterpart. It always meant, that if you have a scale of certain physical component X, situated between its marginal $\left[X^{\min}, X^{\max}\right]$ values, then it is very "obvious" to consider dividing this scale by *equal intervals* with further assignment of certain linguistic label to each of them. Thus, if there is N number of linguistic labels, then the way to map them into a scale of X is to simply divide its length by ($N-1$), *i.e.* $\Delta X=\frac{X}{N-1}$ and assign each element of this set $S=\{j\cdot\Delta X\}\mid\forall j\in[0,N-1]$ to its linguistic counterpart (label) from $L=\{l_j\}\mid\forall j\in[0,N-1]$. Such linguistic approach permitted to use simple triangular shaped membership functions (***MF***).

In this investigation, in contrast, we use a different approach, based on ***EA***s, which allows to assign *linguistic labels* not to *evenly divided* physical scale, but rather to *arbitrary intervals* within it, associated with expert's knowledge and considerations about a nature of represented physical component. Thus, considered physical scale would be divided by certain number of *linguistically labeled clusts*. This ah makes *fuzzy decision-making* mechanism to be more properly applied to a nature of real-world behavior. In this study we cover issues of scaling, building fuzzy clusters, labeling, fuzzification of a model input/output, aggregation of fuzzy elements of a model and appropriate fuzzy decision-making mechanism. We also present corresponding defuzzification mechanism.

2. Scaling and Clustering

2.1. Scales

Let us consider the fact that both input x and output y values of our model situate within certain min/max boundaries (in some abstract physical units), called *scales*.

$$x\in\left[x^{\min},x^{\max}\right],\ y\in\left[y^{\min},y^{\max}\right]$$

We also suggest that both scales could consist of certain number of sub-scales $x_i\mid\forall i\in[0,N]$ and $y_m\mid\forall m\in[0,N]$, each of which represents some semantic values, proposed by certain ***EA***, made based on an expert knowledge about physical nature of a scale. (For simplification's sake we suppose the same number of sub-scales N for both input and output). Above mentioned semantic values must be presented for experts as a set of linguistic terms, associated with corresponding sub-scales, *i.e.* intervals of certain physical values. These terms usually form linguistic scales such as [***"ow"…"medium"…"high"***], [***"small"…"average"…"large"***] or [***"near"…"in between"…"far"***] etc.

$$x_i\in\left[x_i^{\min},x_i^{\max}\right]\mid\forall i\in[0,N],\quad y_m\in\left[y_m^{\min},y_m^{\max}\right]\mid\forall m\in[0,N],\tag{2.1}$$

The following feature for both input and output scales is taking place, *i.e.* the min value of each subsequent sub-scale is equal to max value of its predecessor.

$$x_i^{\max}=x_{i+1}^{\min}\mid\forall i\in[0,N-1],\quad y_m^{\max}=y_{m+1}^{\min}\mid\forall m\in[0,N-1]$$

Thus, both input and output scales would be represented in terms of their correspondent sub-scales

$$x=\left\{\left[x_0^{\min},x_0^{\max}\right],\left[x_1^{\min},x_1^{\max}\right],\cdots,\left[x_N^{\min},x_N^{\max}\right]\right\},$$

$$y = \left\{ \left[y_0^{\min}, y_0^{\max} \right], \left[y_1^{\min}, y_1^{\max} \right], \cdots, \left[y_N^{\min}, y_N^{\max} \right] \right\}.$$

We could estimate the length of each sub-scale for the case when the original scale divided *equally*.

For an input we have

$$x_{even} = \frac{x^{\max} - x^{\min}}{N} \tag{2.2}$$

For an output we have

$$y_{even} = \frac{y^{\max} - y^{\min}}{N} \tag{2.3}$$

From (2.2) and (2.3) we define the *relative weight* of each sub-scale length the following way.

$$\alpha_i = \frac{x_i^{\max} - x_i^{\min}}{x_{even}} \mid \forall i \in [0, N], \tag{2.4}$$

$$\beta_m = \frac{y_m^{\max} - y_m^{\min}}{y_{even}} \mid \forall m \in [0, N] \tag{2.5}$$

Note that both $\alpha_i \mid \forall i \in [0, N]$ and $\beta_m \mid \forall m \in [0, N]$ sets define the relative value of each sub-scale involvement in decision making process, described down below. The *larger* this relative value, the *lower* the degree of its importance and vice versa. By using (2.4) and (2.5) we introduce the *minimal* value of *relative weights* over all sub-scales as the following

$$\alpha_{\min} = \min_{i \in [0,N]} \{\alpha_i\} \tag{2.6}$$

$$\beta_{\min} = \min_{m \in [0,N]} \{\beta_m\} \tag{2.7}$$

Both $\alpha_{\min}$ and $\beta_{\min}$ will define the shape of membership functions (*MF*) for both input and output of a model, represented down below as corresponding fuzzy sets.

2.2. Clusters

We redefine each sub-scale $x_i \mid \forall i \in [0, N]$ and $y_m \mid \forall m \in [0, N]$, as a set of clusters, each of which could be characterized by its *median value* or so-called a ***centroid*** of the following type.

$$icl_i = \frac{x_i^{\max} + x_i^{\min}}{2} \mid \forall i \in [0, N], \quad ocl_m = \frac{y_m^{\max} + y_m^{\min}}{2} \mid \forall m \in [0, N] \tag{2.8}$$

2.3. Generated Experimental Data

To sustain objectivity of our experiments we use the generator of random numbers to produce experimental data. First, we created the following "physical" scales (in abstract unit values).

Input Scale: [**0, 12960**], Output Scale: [**0, 108**].

We will use the number of sub-scales/clusters $N = 8$.

Then from (2.1) and (2.2) we are getting $x_{\text{even}} = 1440.0$, $y_{\text{even}} = 12.0$

By using a random number generator, we found the following *sub-scales/clusters* for an input.

$x_0 \in [0,1440]$; $x_1 \in [1440,1800]$; $x_2 \in [1800,4320]$; $x_3 \in [4320,7200]$; $x_4 \in [7200,9000]$; $x_5 \in [9000,11160]$; $x_6 \in [11160,12240]$; $x_7 \in [12240,12960]$.

For each input *sub-scale/cluster* we define an abstract simplified linguistic term from the following set $T(x)$ = {**"in0", "in1", "in2", "in3", "in4", "in5", "in6", "in7"**}.

We also found the following sub-scales for an output.

$y_0 \in [0,21]$; $y_1 \in [21,36]$; $y_2 \in [36,60]$; $y_3 \in [60,63]$; $y_4 \in [63,69]$; $y_5 \in [69,87]$; $y_6 \in [87,99]$; $y_7 \in [99,108]$.

By analogy we define linguistic terms for each output *sub-scale*/*cluster*, *i.e.* $T(y)$ = {**"out0", " out1", "out2", "out3", "out4", "out5", "out6", "out7"**}.

Following Table 1 depicts corresponding results.

Table 1. Linguistic terms for input/output clusters.

Value of variable		$icl_i \in U, i = \overline{0,7}$
ICL	***OCL***	$ocl_j \in V, j = \overline{0,7}$
in0	**out0**	**0**
in1	**out1**	**1**
in2	**out2**	**2**
in3	**out3**	**3**
in4	**out4**	**4**
in5	**out5**	**5**
in6	**out6**	**6**
in7	**out7**	**7**

From (2.8) by using $x_i \mid \forall i \in [0,7]$ and $y_m \mid \forall m \in [0,7]$ we are getting two sets of *centroids ICL* = {**720, 1620, 3060, 5760, 8100, 10080, 11700, 12600**} – for the input and ***OCL*** = {**10, 28, 48, 61, 66, 78, 93, 103**} – for the output.

By using (2.4) and (2.5) we are getting values for $\alpha_i \mid \forall i \in [0,N]$, and $\beta_i \mid \forall m \in [0,N]$.

Following Table 2 depicts corresponding results.

Table 2. Experimental data.

Input Term	$x^{\min}$	ICL centroid	$x^{\max}$	α	Output Term	$y^{\min}$	OCL centroid	$y^{\max}$	β
"in0"	0	720	1440	1.0	"out0"	0	10	21	1.75
"in1"	1440	1620	1800	0.25	"out1"	21	28	36	1.25
"in2"	1800	3060	4320	1.75	"out2"	36	48	60	2.0

Continued

"in3"	4320	5760	7200	2.0	"out3"	60	61	63	0.25
"in4"	7200	8100	9000	1.25	"out4"	63	66	69	0.5
"in5"	9000	10080	11160	1.5	"out5"	69	78	87	1.5
"in6"	11160	11700	12240	0.75	"out6"	87	93	99	1.0
"in7"	12240	12600	12960	0.5	"out7"	99	103	108	0.75

3. Fuzzy Reasoning

3.1. Fuzzification of Input/Output Centroids

We represent each input centroid of *i*-th cluster $icl_i \mid \forall i \in [0, N]$ as a fuzzy set, forming linguistic variable, described by a triplet of the form

$$ICL_i = \left\{ \left\langle icl_i, U, \widetilde{ICL_i} \right\rangle \right\}, icl_i \in T(u), \quad \forall i \in [0, N],$$

where U is a universe of discourse of a model input, $T(u)$ is extended term set of the linguistic variable "*ICL*" from **Table 1**,

$$T(u) = \left\{ t_0^{in}, t_1^{in}, t_2^{in}, \cdots, t_N^{in} \right\}.$$

$\widetilde{ICL_i} \mid \forall i \in [0, N]$ is normal fuzzy set with correspondent *MF* $\mu_{icl_i} : U \to [0,1] \mid \forall i \in [0, N]$. To normalize values of $icl_i \mid \forall i \in [0, N]$ we use the following

$$icl_i^{norm} = \frac{icl_i - icl_i^{\min}}{icl_i^{\max} - icl_i^{\min}}, \tag{3.1}$$

We will use the following mapping $\partial : \widetilde{ICL_i} \to U \mid u = Ent\left[(CardU - 1) \cdot icl_i^{norm} \right]$, where

$$\widetilde{ICL_i} = \int_U \mu_{icl_i}(u)/u, \quad \forall i \in [0, N] \tag{3.2}$$

On the other hand, to determine the estimates of the *MF* in terms of singletons from (3.2) in the form $\mu_{icl_i}(u_j)/u_j \mid \forall j \in [0, CardU], \forall i \in [0, N]$, given (3.1) we propose the following procedure.

$$\mu_{icl_i}(u_j) = \left\{ 1 - \frac{1}{CardU - 1} \cdot \left| j - Ent\left[(CardU - 1) \cdot icl_i^{norm} \right] \right| \right\}^{\alpha_i}, \tag{3.3}$$
$$\forall j \in [0, CardU], \forall i \in [0, N]$$

where α_i is a weight coefficient of the *i*-th input cluster from (2.4). Notice that we define all values of a linguistic variable over entire physical scale of input/output parameters via normalization mechanism and therefore mathematically reject the notion of interval based *MF*s.

Each output centroid of a cluster $ocl_m \mid \forall m \in [0, N]$ we represent as a fuzzy set, forming linguistic variable, described by a triplet of the form

$$OCL_m = \left\{ \left\langle ocl_m, V, \widetilde{OCL_m} \right\rangle \right\}, ocl_m \in T(v), \forall m \in [0, N],$$

where V is a universe of discourse of a model output, $T(v)$ is extended term set of the linguistic variable "OCL" from Table 1,

$$T(v)=\left\{t_0^{out},t_1^{out},t_2^{out},\cdots,t_N^{out}\right\}.$$

$\widetilde{OCL}$ is nomal fuzzy set with correspondent MF $\mu_{ocl_m}:V\to[0,1]\,|\,\forall m\in[0,N]$. We use the same normalization procedure

$$ocl_m^{norm}=\frac{ocl_m-ocl_m^{\min}}{ocl_m^{\max}-ocl_m^{\min}},\forall m\in[0,N] \tag{3.4}$$

With the following mapping $\Omega:\widetilde{OCL_m}\to V\,|\,v=Ent\left[(CardV-1)\cdot ocl_m^{norm}\right]$, where

$$\widetilde{OCL_m}=\int_V \mu_{ocl_m}(v)/v,\forall m\in[0,N] \tag{3.5}$$

On the other hand, similarly to the previous cases, to determine the estimates of the MF in terms of singletons from (3.5) in the form $\mu_{ocl_m}(v_k)/v_k\,|\,\forall k\in[0,CardV],\forall m\in[0,N]$, given (3.4) we propose the following procedure.

$$\mu_{ocl_m}(v_k)=\left\{1-\frac{1}{CardV-1}\cdot\left|k-Ent\left[(CardV-1)\cdot ocl_m^{norm}\right]\right|\right\}^{\beta_m}$$
$$\forall k\in[0,CardV],\forall m\in[0,N] \tag{3.6}$$

where β_m is a weight coefficient of m-th output cluster from (2.5). Note that the use of both α_i in (3.3) and β_m in (3.6) changes a shape of MF from ordinary triangular form into variety of shapes, which could adequately map each cluster into aggregation form, described down below.

3.2. MFs of Input/Output Centroids

Using data from Table 1 and Table 2, given (3.3) we have the following MFs for input centroids

μ_{icl} ("**in0**") = 1.00/0 + 0.88/1 + 0.75/2 + 0.63/3 + 0.50/4 + 0.38/5 + 0.25/6 + 0.13/7

μ_{icl} ("**in1**") = 0.97/0 + 1.00/1 + 0.97/2 + 0.93/3 + 0.89/4 + 0.84/5 + 0.78/6 + 0.71/7

μ_{icl} ("**in2**") = 0.79/0 + 1.00/1 + 0.79/2 + 0.60/3 + 0.44/4 + 0.30/5 + 0.18/6 + 0.09/7

μ_{icl} ("**in3**") = 0.39/0 + 0.56/1 + 0.77/2 + 1.00/3 + 0.77/4 + 0.56/5 + 0.39/6 + 0.25/7

μ_{icl} ("**in4**") = 0.29/0 + 0.42/1 + 0.56/2 + 0.70/3 + 0.85/4 + 1.00/5 + 0.85/6 + 0.70/7

μ_{icl} ("**in5**") = 0.13/0 + 0.23/1 + 0.35/2 + 0.49/3 + 0.65/4 + 0.82/5 + 1.00/6 + 0.82/7

μ_{icl} ("**in6**") = 0.21/0 + 0.35/1 + 0.48/2 + 0.59/3 + 0.70/4 + 0.81/5 + 0.90/6 + 1.00/7

μ_{icl} ("**in7**") = 0.35/0 + 0.50/1 + 0.61/2 + 0.71/3 + 0.79/4 + 0.87/5 + 0.94/6 + 1.00/7

Here is correspondent Figure 1 depiction

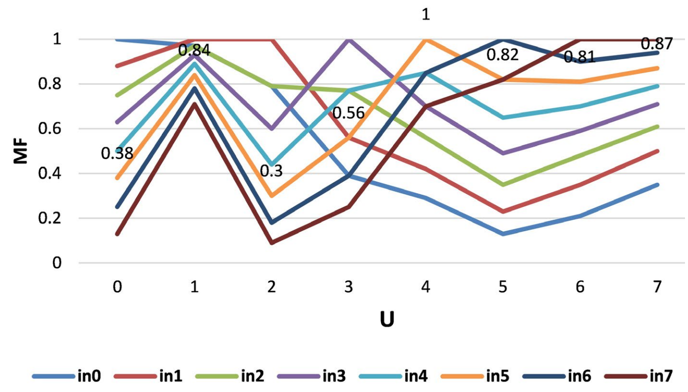

Figure 1. *MF*s of input centroids.

Using data from Table 1 and Table 2, given (3.6) we have the following *MF*s for output centroids

μ_{ocl} (“**out0**”) = 1.00/0 + 0.79/1 + 0.60/2 + 0.44/3 + 0.30/4 + 0.18/5 + 0.09/6 +0.03/7

μ_{ocl} (“**out1**”) = 0.70/0 + 0.85/1 + 1.00/2 + 0.85/3 + 0.70/4 + 0.56/5 + 0.42/6 + .29/7

μ_{ocl} (“**out2**”) = 0.39/0 + 0.56/1 + 0.77/2 + 1.00/3 + 0.77/4 + 0.56/5 + 0.39/6 + 0.25/7

μ_{ocl} (“**out3**”) = 0.84/0 + 0.89/1 + 0.93/2 + 0.97/3 + 1.00/4 + 0.97/5 + 0.93/6 + 0.89/7

μ_{ocl} (“**out4**”) = 0.71/0 + 0.79/1 + 0.87/2 + 0.94/3 + 1.00/4 + 0.94/5 + 0.87/6 + 0.79/7

μ_{ocl} (“**out5**”) = 0.23/0 + 0.35/1 + 0.49/2 + 0.65/3 + 0.82/4 + 1.00/5 + 0.82/6 + 0.65/7

μ_{ocl} (“**out6**”) = 0.25/0 + 0.38/1 + 0.50/2 + 0.63/3 + 0.75/4 + 0.88/5 + 1.00/6 + 0.88/7

μ_{ocl} (“**out7**”) = 0.21/0 + 0.35/1 + 0.48/2 + 0.59/3 + 0.70/4 + 0.81/5 + 0.90/6 + 1.00/7

Here is correspondent Figure 2 depiction

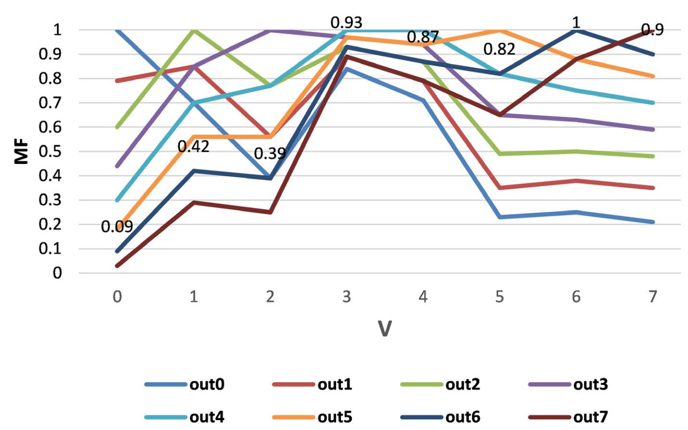

Figure 2. *MF*s of output centroids.

3.3. Fuzzification of Input/Output

We represent each input value x as a fuzzy set, forming linguistic variable, described by a triplet of the form

$$X = \left\{ \left\langle x, U, \tilde{X} \right\rangle \right\}, x \in T(u),$$

where $\tilde{X}$ is normal fuzzy set with correspondent *MF* $\mu_X : U \to [0,1]$. $T(u)$ is extended term set of the linguistic variable "*Aggregate Input*", which corresponds to fuzzy set (3.16), described down below. Each *term* of the linguistic variable "*Aggregate Input*" would be the *term* from **Table 1**. for variable "*ICL*" or *combination* of them. To normalize values of x we use the following

$$x^{norm} = \frac{x - x^{\min}}{x^{\max} - x^{\min}}, \tag{3.7}$$

We will use the following mapping $\gamma : \tilde{X} \to U \mid u = Ent\left[(CardU - 1) \cdot x^{norm} \right]$, where

$$\tilde{X} = \int_U \mu_x(u) / u, \tag{3.8}$$

On the other hand, to determine the estimates of the *MF* in terms of singletons $\mu_X(u_j)/u_j \mid \forall j \in [0, CardU]$ from (3.8), given (3.7) we propose the following procedure.

$$\mu_X(u_j) = \left\{ 1 - \frac{1}{CardU - 1} \cdot \left| j - Ent\left[(CardU - 1) \cdot x^{norm} \right] \right| \right\}^{\alpha_{\min}}, \forall j \in [0, CardU], \tag{3.9}$$

where $\alpha_{\min}$ is a smallest weight coefficient of all input clusters from (2.6). The use of $\alpha_{\min}$ in (3.9) changes a shape of original *MF* ($\alpha_{\min} = 1$) from usual *triangular* form into *parabola* one, which guarantees more uniform value distributions within set of singletons.

Similarly, we represent each output value y as a fuzzy set, forming linguistic variable, described by a triplet of the form

$$Y = \left\{ \left\langle y, V, \tilde{Y} \right\rangle \right\}, y \in T(v), \quad \mu_Y : V \to [0,1]$$

where $\tilde{Y}$ is normal fuzzy set with correspondent *MF* $\mu_Y : V \to [0,1]$. $T(v)$ is extended term set of the linguistic variable "*Aggregate Output*", which corresponds to fuzzy set (3.23), described down below. Each *term* of the linguistic variable "*Aggregate Output*" would be the *term* from **Table 1**. for variable "*OCL*" or *combination* of them. To normalize values of y we use the following

$$y^{norm} = \frac{y - y^{\min}}{y^{\max} - y^{\min}}. \tag{3.10}$$

We will use the following mapping $\omega : \tilde{Y} \to V \mid v = Ent\left[(CardV - 1) \cdot y^{norm} \right]$, where

$$\tilde{Y} = \int_V \mu_Y(v) / v, \tag{3.11}$$

And again, to determine the estimates of the *MF* in terms of singletons $\mu_Y(u_k)/u_k \mid \forall k \in [0, CardV]$ from (3.11), given (3.10) we propose the following procedure.

$$\mu_Y(u_k)=\left\{1-\frac{1}{CardV-1}\cdot\left|k-Ent\left[(CardV-1)\cdot y^{norm}\right]\right|\right\}^{\beta_{\min}},\forall k\in[0,CardV] \quad (3.12)$$

3.4. Aggregation of Fuzzy Input

The aggregation procedure consists of two steps.

1) On the first step we define the set of fuzzy sets as intersections $\tilde{X}_i^{inter}$ of a current fuzzy set of input $\tilde{X}$ with each fuzzy set of input centroid $\widetilde{ICL}_i \mid \forall i\in[0,N]$.

Thus, we define the following

$$\tilde{X}_i^{inter}=\tilde{X}\cap\widetilde{ICL}_i,\forall i\in[0,N]. \quad (3.13)$$

The set of fuzzy sets $\tilde{X}_i^{inter}$ is presented in familiar way as

$$\tilde{X}_i^{inter}=\int_U \mu_{X_i^{inter}}(u)/u \mid \forall i\in[0,N], \quad (3.14)$$

And we determine correspondent *MF*s from (3.14) in terms of singletons as $\mu_{X_i^{inter}}(u_j)/u_j \mid \forall i\in[0,CardU],\forall j\in[0,CardU]$

2) On the second step we define a fuzzy set $\tilde{X}^{aggr}$ as a union of all fuzzy sets $\tilde{X}_i^{inter}$, *i.e.*

$$\tilde{X}^{aggr}=\bigcup_{i=0}^{N}\tilde{X}_i^{inter},\quad \tilde{X}_i^{inter}\subseteq U, \quad (3.15)$$

We also present the fuzzy sets $\tilde{X}^{aggr}$ as follows

$$\tilde{X}^{aggr}=\int_U \mu_{X^{aggr}}(u)/u, \quad (3.16)$$

And we determine correspondent *MF* from (3.16) in terms of singletons as $\mu_{X^{aggr}}(u_j)/u_j \mid \forall j\in[0,CardU]$

3.5. Fuzzy Inference

To convert (3.17) - (3.18) into fuzzy logic-based statement and terms from Table 1. we use a *Fuzzy Conditional Inference Rule* (*FCIR*), formulated by means of "common sense" as a following conditional clause:

$$P=\text{“IF } (x \text{ is } X)\text{, THEN } (y \text{ is } Y)\text{”} \quad (3.17)$$

In other words, we use fuzzy conditional inference of the following type [8]:

$$\begin{array}{c}\text{Ant 1: If Input is } X\text{, then Output is } Y\\ \text{Ant 2: Input is } X'\\ \text{Cons: Output is } Y'\end{array} \quad (3.18)$$

where $X,X'\subseteq U$ *and* $Y,Y'\subseteq V$.

Note that statements (3.17) and (3.18) represent "modus-ponens" syllogism. Given that we use the following type of implication [1]

$$X\to Y=\begin{cases}(1-x)\cdot y, x>y,\\ 1, x\le y\end{cases} \quad (3.19)$$

For practical purposes, described down below, we will use *Fuzzy Conditional Rule* (*FCR*) [14] of the following type

$$\begin{aligned} R\left(A_1(x),A_2(y)\right) &= (X\times U\to V\times Y)\cap(\neg X\times U\to V\times\neg Y) \\ &= \int_{U\times V}\left(\mu_x(u)\to\mu_y(v)\right)\wedge\left(\left(1-\mu_x(u)\right)\to\left(1-\mu_y(v)\right)\right)\Big/(u,v) \end{aligned} \tag{3.20}$$

Given (3.19) from (3.20) we are getting

$$\begin{aligned} R\left(A_1(x),A_2(y)\right) &= \left(\mu_x(u)\to\mu_y(v)\right)\wedge\left(\left(1-\mu_x(u)\right)\to\left(1-\mu_y(v)\right)\right) \\ &= \begin{cases} \left(1-\mu_x(u)\right)\cdot\mu_y(v), \mu_x(u)<\mu_y(v), \\ 1, \mu_x(u)=\mu_y(v), \\ \left(1-\mu_y(v)\right)\cdot\mu_x(u), \mu_y(v)<\mu_x(u). \end{cases} \end{aligned} \tag{3.21}$$

Given a unary relationship $R\left(A_1(x')\right)=\tilde{X}^{aggr}$ one can obtain the consequence $R\left(A_2(y')\right)$ by *CRI* to $R\left(A_1(x')\right)$ and $R\left(A_1(x),A_2(y)\right)$ of type (3.21):

$$\begin{aligned} & R\left(A_2(y')\right)=\tilde{X}^{aggr}\circ R\left(A_1(x),A_2(y)\right) \\ & =\int_U \mu_{X^{aggr}}(u)/u\circ\int_{U\times V}\left(\mu_x(u)\to\mu_y(v)\right)\wedge\left(\left(1-\mu_x(u)\right)\to\left(1-\mu_y(v)\right)\right)\Big/(u,v) \\ & =\int_V \bigcup_{X^{aggr}\in U}\mu_{X^{aggr}}(u)\wedge\left(\mu_x(u)\to\mu_y(v)\right)\wedge\left(\left(1-\mu_x(u)\right)\to\left(1-\mu_z(v)\right)\right)\Big]\Big/v \end{aligned} \tag{3.22}$$

The value of an output fuzzy set $\tilde{Y}'$ could be consider as

$$\tilde{Y}'=R\left(A_2(y')\right) \tag{3.23}$$

3.6. Aggregation of Fuzzy Output

The aggregation procedure consists of two steps.

1) On the first step we define the set of fuzzy sets as intersections $\tilde{Y}_j^{inter}$ of a current fuzzy set of input $\tilde{Y}$ with each fuzzy set of input centroid $\widetilde{OCL}_j \mid \forall j\in[0,N]$.

Thus, we define the following

$$\tilde{Y}_j^{inter}=\tilde{Y}'\cap\widetilde{OCL}_j, \forall j\in[0,N] \tag{3.24}$$

The set of fuzzy sets $\tilde{Y}_j^{inter}$ is presented in familiar way as

$$\tilde{Y}_j^{inter}=\int_V \mu_{Y_j^{inter}}(v)\Big/v \mid \forall j\in[0,N], \tag{3.25}$$

And we determine correspondent *MF*s from (3.14) in terms of singletons as $\mu_{Y_j^{inter}}(v_k)\Big/v_k \mid \forall j\in[0,CardV], \forall k\in[0,CardV]$

2) On the second step we define a fuzzy set $\tilde{Y}^{aggr}$ as a union of all fuzzy sets $\tilde{Y}_j^{inter}$, *i.e.*

$$\tilde{Y}^{aggr}=\bigcup_{j=0}^{N}\tilde{Y}_j^{inter}, \quad \tilde{Y}_j^{inter}\subseteq V, \quad \forall j\in[0,N] \tag{3.26}$$

We also present the fuzzy sets $\tilde{Y}^{aggr}$ as follows

$$\tilde{Y}^{aggr}=\int_U \mu_{X^{aggr}}(u)\Big/u \tag{3.27}$$

3.7. Linguistic Classification and Defuzzification of Aggregated of Fuzzy Sets

3.7.1. Linguistic Classification

It is well known that any fuzzy set from (3.11) $\tilde{P}=\int_W \mu_P(w)\Big/w$ could be represented as the following sum of singletons

$$\begin{aligned}&\tilde{P}=\sum_{i=\overline{0,CardW}}\mu_P\left(w_i\right)/w_i\\&\Rightarrow\exists W_1\subseteq W\mid CardW_1<CardW;\forall k\in\left[0,CardW_1\right]\\&\Rightarrow\mu_P\left(w_k\right)/w_k=\underline{\max}_{i=0,CardW}\left\{\mu_P\left(w_i\right)/w_i\right\}\end{aligned}\tag{3.28}$$

There are two cases of *MF* in use.

1) Case of *unimodal MF* (semantical non ambiguity) for fuzzy set $\tilde{P}$, *i.e.* $CardW_1=1\Rightarrow\exists k^*!\in W_1\mid\mu_P\left(w_{k^*}\right)/w_{k^*}=\underline{\max}_{i=0,CardW}\left\{\mu_P\left(w_i\right)/w_i\right\}\Rightarrow p\in T\left(w\right)=t_{k^*}^{out}$. Thus, there is unique linguistic term $t_{k^*}^{out}$, associated with fuzzy set $\tilde{P}$. $CardW_1=1$.

2) Case of *polymodal MF* (semantical ambiguity) for fuzzy set $\tilde{P}$, *i.e.* $CardW_1>1$ $\Rightarrow\exists i^*,j^*,\cdots!\in W_1\mid\mu_P\left(w_{i^*}\right)/w_{i^*}=\mu_P\left(w_{j^*}\right)/w_{j^*}=\cdots=\underline{\max}_{k=0,CardW}\left\{\mu_P\left(w_k\right)/w_k\right\}$, $\forall k\in\left[0,CardW\right]\Rightarrow p\in T\left(w\right)=t_{i^*}^{out},t_{j^*}^{out},\cdots$. Thus, there are few linguistic terms $t_{i^*}^{out},t_{j^*}^{out},\cdots$, associated with fuzzy set $\tilde{Y}$.

3.7.2. Defuzzification

From (2.8) we have the following defuzzied values for an input

$$x_{Def}=\frac{1}{CardW_1}\sum_{\forall k\in W_1}icl_k,\tag{3.29}$$

And for output

$$y_{Def}=\frac{1}{CardW_1}\sum_{\forall k\in W_1}ocl_k\tag{3.30}$$

3.8. Formalization of Knowledge Base

In [7]-[13] we always consider the fact that for *FCR* (3.20) its fuzzy *binary relationship matrix* must be formulated by conditional clause, which contain terms, characterized by *margins* of physical scales in use. It means, that semantic knowledge about a system input/output relationship must be formulated as one of the following clauses (3.17):

"IF (x is "**lowest**"), THEN (y is "**highest**")"

"IF (x is "**lowest**"), THEN (y is "**lowest**")"

"IF (x is "**highest**"), THEN (y is "**lowest**")"

"IF (x is "**highest**"), THEN (y is "**highest**")"

In our case "**lowest**"/"**highest**" pair for an input is "**in0**"/"**in7**" and "**out0**"/"**out7**"—for an output similarly.

Such an approach guarantees logical *non-contradiction* of FCR (3.21), *i.e.* appropriate *binary relationship matrix* has one of major diagonals all of *singles* [15]. To achieve this goal, we use the following triangular shape *MF*

$$\mu_X\left(t_{k^*}^{in}\right)=\left\{1-\frac{1}{CardU-1}\cdot\left|j-k^*\right|\right\},\forall j\in\left[0,CardU\right],\exists k^*!\in\left[0,CardU\right]\tag{3.31}$$

$$\mu_Y\left(t_{m^*}^{out}\right)=\left\{1-\frac{1}{CardV-1}\cdot\left|j-m^*\right|\right\},\forall j\in[0,CardV],\exists m^*!\in[0,CardV] \quad (3.32)$$

In (3.31) we use term for $k^*=0\,|\,t_0^{in}$ **= “in0”** and in (3.32) we use term for $m^*=7\,|\,t_7^{out}$ **= “out7”**. Here are correspondent *MF*s

μ_X (“**in0**”) = 1.00/0 + 0.88/1 + 0.75/2 + 0.63/3 + 0.50/4 + 0.38/5 + 0.25/6 + 0.13/7

μ_Y (“**out7**”) = 0.13/0 + 0.25/1 + 0.38/2 + 0.50/3 + 0.63/4 + 0.75/5 + 0.88/6 + 1.00/7

Thus, as a result we will use the following binary relationship matrix

$$R_1\left(A_1(x),A_2(y)\right)=\left(\mu_X(\text{“in0”})\rightarrow\mu_Y(\text{“out7”})\right). \quad (3.33)$$

Applying (3.21) and (3.33) we get matrix from Table 3.

Table 3. Binary relationship matrix.

X→*Y*	0	1	2	3	4	5	6	7
0	0.00	0.02	0.03	0.05	0.06	0.08	0.09	**1.00**
1	0.00	0.03	0.06	0.09	0.13	0.16	**1.00**	0.09
2	0.00	0.05	0.09	0.14	0.19	**1.00**	0.16	0.08
3	0.00	0.06	0.13	0.19	**1.00**	0.19	0.13	0.06
4	0.00	0.08	0.16	**1.00**	0.19	0.14	0.09	0.05
5	0.00	0.09	**1.00**	0.16	0.13	0.09	0.06	0.03
6	0.00	**1.00**	0.09	0.08	0.06	0.05	0.03	0.02
7	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00	0.00

3.9. Experimental Results for Fuzzification of Input/Output

1) The case of unimodal aggregated input *MF*

We use input value X' = **1080**.

From (3.9), given (3.7) we have the following input *MF*

$\mu_{X'}$ (“**1080**”) = 1.00/0 + 0.97/1 + 0.93/2 + 0.89/3 + 0.84/4 + 0.78/5 + 0.71/6 + 0.59/7

Here is correspondent Figure 3 depiction.

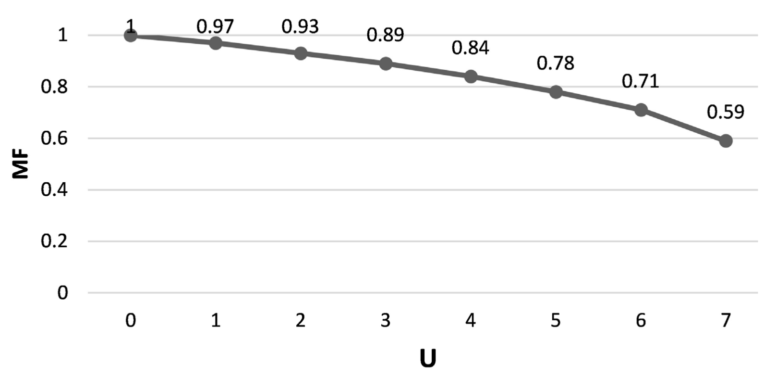

Figure 3. MF of “**1080**” input value.

From (3.13) we are getting *MF*s for intersection of input *MF* of "**1800**" with the set of *MF*s of input centroid **Table 2**.

$\mu_{X_0^{inter}}$ ("with **720**") = 1.00/0 + 0.88/1 + 0.75/2 + 0.63/3 + 0.50/4 + 0.38/5 + 0.25/6 + 0.13/7

$\mu_{X_1^{inter}}$ ("with **1620**") = 0.97/0 + 0.97/1 + 0.93/2 + 0.89/3 + 0.84/4 + 0.78/5 + 0.71/6 + 0.59/7

$\mu_{X_2^{inter}}$ ("with **3060**") = 0.79/0 + 0.97/1 + 0.79/2 + 0.60/3 + 0.44/4 + 0.30/5 + 0.18/6 + 0.09/7

$\mu_{X_3^{inter}}$ ("with **5760**") = 0.39/0 + 0.56/1 + 0.77/2 + 0.89/3 + 0.77/4 + 0.56/5 + 0.39/6 + 0.25/7

$\mu_{X_4^{inter}}$ ("with **8100**") = 0.29/0 + 0.42/1 + 0.56/2 + 0.70/3 + 0.84/4 + 0.78/5 + 0.71/6 + 0.59/7

$\mu_{X_5^{inter}}$ ("with **10080**") = 0.13/0 + 0.23/1 + 0.35/2 + 0.49/3 + 0.65/4 + 0.78/5 + 0.71/6 + 0.59/7

$\mu_{X_6^{inter}}$ ("with **11700**") = 0.21/0 + 0.35/1 + 0.48/2 + 0.59/3 + 0.70/4 + 0.78/5 + 0.71/6 + 0.59/7

$\mu_{X_7^{inter}}$ ("with **12600**") = 0.35/0 + 0.50/1 + 0.61/2 + 0.71/3 + 0.79/4 + 0.78/5 + 0.71/6 + 0.59/7

We depicted results in **Figure 4**.

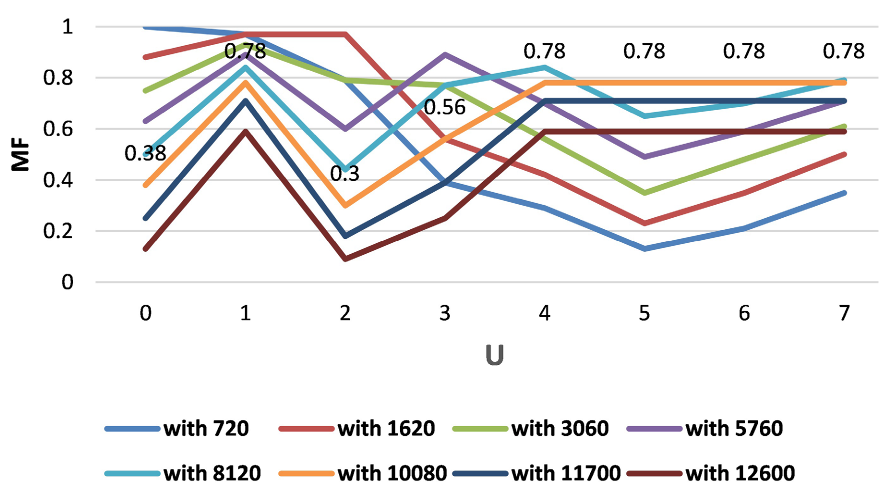

Figure 4. Intersection of "**1080**" input value *MF* with *MF*s of centroids.

From (3.16), given (3.15) we are getting the following fuzzy set $\tilde{X}^{aggr}$

$\mu_{X^{aggr}}$ ("**1080**") = 1.00/0 + 0.97/1 + 0.93/2 + 0.89/3 + 0.84/4 + 0.78/5 + 0.71/6 + 0.59/7

Here is correspondent **Figure 5** depiction.

Linguistic classification of *aggregated* fuzzy set, represented by $\mu_{X^{aggr}}$ ("**1080**").

2) The Case of Polymodal Aggregated Input *MF*

We use input value X' = **2430**.

By using (3.8), given (3.7) we have the following input *MF*

$\mu_{X'}$ (**"2430"**) = 0.93/0 + 0.97/1 + **1.00**/2 + 0.97/3 + 0.93/4 + 0.89/5 + 0.84/6 +

0.78/7

Here is correspondent **Figure 6** depiction.

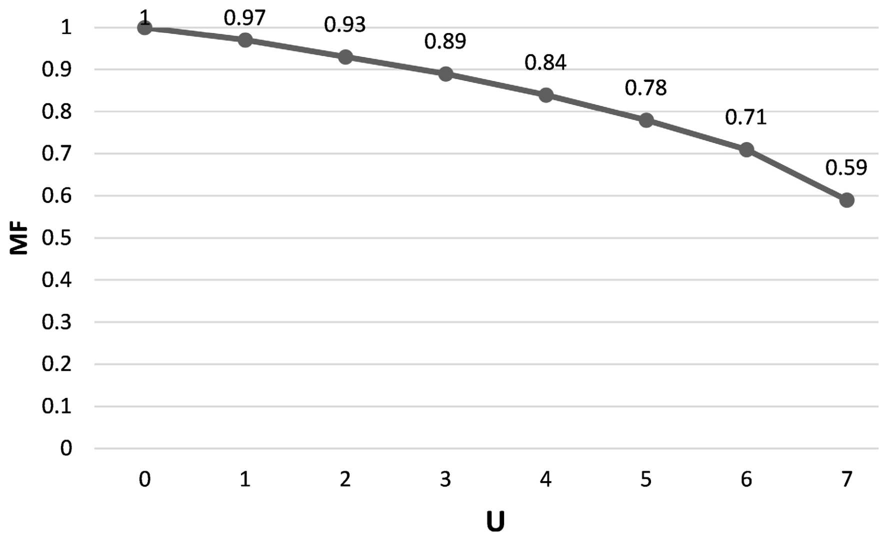

Figure 5. Aggregate input *MF* for "**1080**".

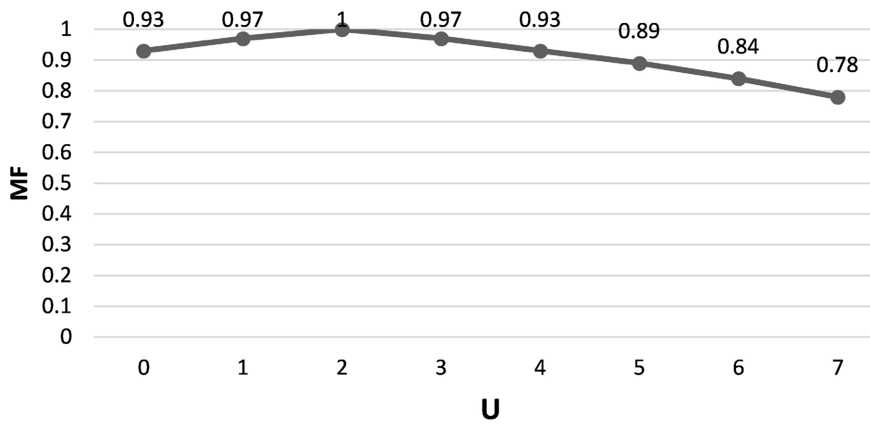

Figure 6. MF of "**2430**" input value.

Again from (3.11) we are getting *MF*s for intersection of input *MF* of "**2430**" with the set of *MF*s of input centroids from **Table 2.**

$\mu_{X_0^{inter}}$ ("with **720**") = 0.93/0 + 0.88/1 + 0.75/2 + 0.63/3 + 0.50/4 + 0.38/5 + 0.25/6 + 0.13/7

$\mu_{X_1^{inter}}$ ("with **1620**") = 0.93/0 + 0.97/1 + 0.97/2 + 0.93/3 + 0.89/4 + 0.84/5 + 0.78/6 + 0.71/7

$\mu_{X_2^{inter}}$ ("with **3060**") = 0.79/0 + 0.97/1 + 0.79/2 + 0.60/3 + 0.44/4 + 0.30/5 + 0.18/6 + 0.09/7

$\mu_{X_3^{inter}}$ ("with **5760**") = 0.39/0 + 0.56/1 + 0.77/2 + 0.97/3 + 0.77/4 + 0.56/5 + 0.39/6 + 0.25/7

$\mu_{X_4^{inter}}$ ("with **8100**") = 0.29/0 + 0.42/1 + 0.56/2 + 0.70/3 + 0.85/4 + 0.89/5 + 0.84/6 + 0.70/7

$\mu_{X_5^{inter}}$ ("with **10080**") = 0.13/0 + 0.23/1 + 0.35/2 + 0.49/3 + 0.65/4 + 0.82/5 + 0.84/6 + 0.78/7

$\mu_{X_6^{inter}}$ ("with **11700**") = 0.21/0 + 0.35/1 + 0.48/2 + 0.59/3 + 0.70/4 + 0.81/5 + 0.84/6 + 0.78/7

$\mu_{X_7^{inter}}$ ("with **12600**") = 0.35/0 + 0.50/1 + 0.61/2 + 0.71/3 + 0.79/4 + 0.87/5 + 0.84/6 + 0.78/7

Here is correspondent Figure 7 depiction.

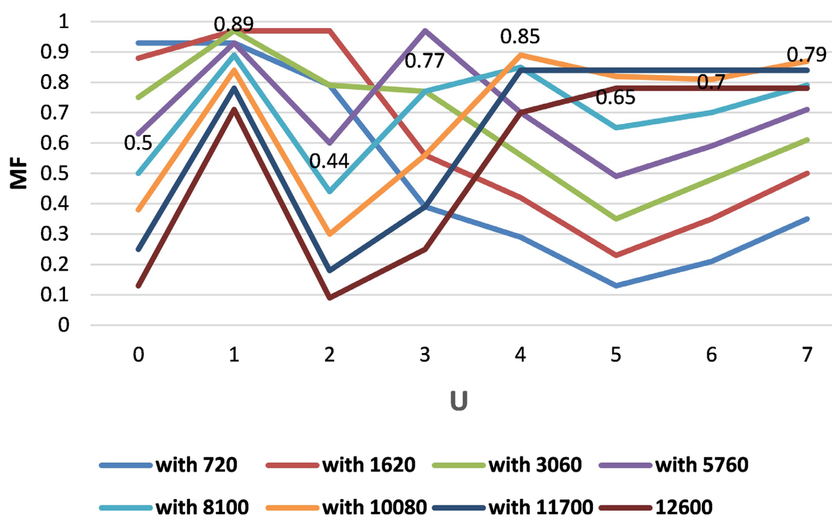

Figure 7. Intersection of "**2430**" Input Value *MF* with *MF*s of centroids.

One more time from (3.13), given (3.14) we are getting *MF* for the following fuzzy set $\tilde{X}^{aggr}$

$\mu_{X^{aggr}}$ ("**2430**") $=$ 0.93/0 + 0.97/1 + 0.97/2 + 0.97/3 + 0.89/4 + 0.89/5 + 0.84/6 + 0.78/7

Here is correspondent Figure 8 depiction.

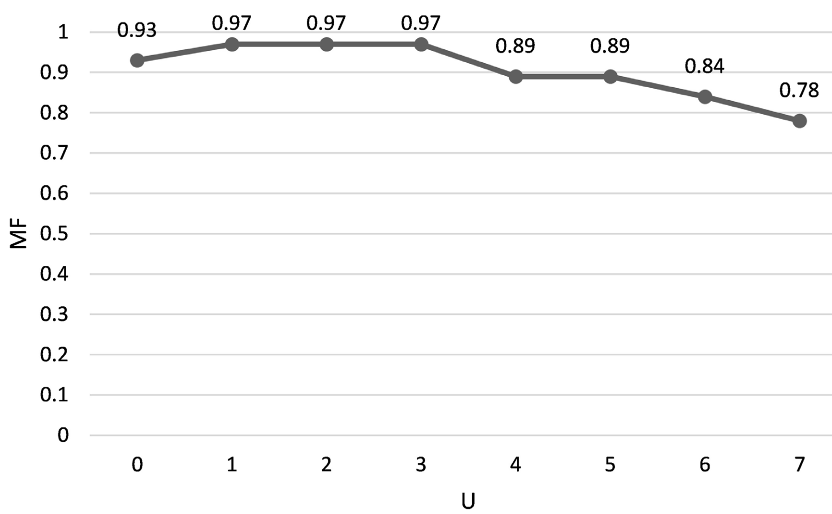

Figure 8. Aggregate input *MF* for "**2430**".

Linguistic classification of *aggregated* fuzzy set, represented by $\mu_{X^{aggr}}$ ("**2430**") from (3.28) means: X^{aggr} = "**in1**" OR "**in2**" OR "**in3**".

3.10. Use of Fuzzy Inference

1) The case of unimodal aggregated input *MF*

We use input value X' = **1080**.

From (3.9), given (3.7) we have the following input *MF*

$\mu_{X'}$ ("**1080**") = 1.00/0 + 0.97/1 + 0.93/2 + 0.89/3 + 0.84/4 + 0.78/5 + 0.71/6 + 0.59/7

We use binary relationship matrix from Table 3 in *FCR* (3.22) and getting the following fuzzy set $\tilde{Y}'$.

$\mu_{Y'}$ = 0.59/0 + 0.71/1 + 0.78/2 + 0.84/3 + 0.89/4 + 0.93/5 + 0.97/6 + 1.00/7

By use of the integration (3.24) and (3.25) we are getting

$\mu_{Y_0^{inter}}$ ("with **10**") = 0.78/0 + 0.79/1 + 0.60/2 + 0.44/3 + 0.30/4 + 0.18/5 + 0.09/6 + 0.03/7

$\mu_{Y_1^{inter}}$ ("with **28**") = 0.70/0 + 0.84/1 + 0.89/2 + 0.85/3 + 0.70/4 + 0.56/5 + 0.42/6 + 0.29/7

$\mu_{Y_2^{inter}}$ ("with **48**") = 0.39/0 + 0.56/1 + 0.77/2 + 0.89/3 + 0.77/4 + 0.56/5 + 0.39/6 + 0.25/7

$\mu_{Y_3^{inter}}$ ("with **61**") = 0.78/0 + 0.84/1 + 0.89/2 + 0.89/3 + 0.97/4 + 0.97/5 + 0.93/6 + 0.89/7

$\mu_{Y_4^{inter}}$ ("with **66**") = 0.71/0 + 0.79/1 + 0.87/2 + 0.89/3 + 0.97/4 + 0.94/5 + 0.87/6 + 0.79/7

$\mu_{Y_5^{inter}}$ ("with **78**") = 0.23/0 + 0.35/1 + 0.49/2 + 0.65/3 + 0.82/4 + 0.97/5 + 0.82/6 + 0.65/7

$\mu_{Y_6^{inter}}$ ("with **93**") = 0.25/0 + 0.38/1 + 0.50/2 + 0.63/3 + 0.75/4 + 0.88/5 + 0.97/6 + 0.88/7

$\mu_{Y_7^{inter}}$ ("with **103**") = 0.21/0 + 0.35/1 + 0.48/2 + 0.59/3 + 0.70/4 + 0.81/5 + 0.90/6 + 0.93/7

By use of the aggregation (3.26) and (3.27) we are getting in terms of singletons $\mu_{Y^{aggr}}\left(v_j\right)/v_j \mid \forall j \in \left[0, CardV\right]$. See Figure 9.

$\mu_{Y^{aggr}}$ = 0.59/0 + 0.71/1 + 0.78/2 + 0.84/3 + 0.89/4 + 0.93/5 + 0.97/6 + **1.00**/7

Thus, from (3.28) linguistical-semantically it means: Y^{aggr} = "**out7**".

Here is how fuzzy conditional inference (3.18) transformed due to use of fuzzy clustering mechanism.

Ant 1: If x is "**in0**", then y is "**out7**"

Ant2: x' is "**in0**"

Cons: y^{aggr} is "**out7**"

We use defuzzification procedure, given, that $\exists V_1 \subseteq V = \{7\}, CardV_1 = 1$ and from Table 2 $ocl_7 = 103$.

Thus, from (3.30) we are getting $y_{Def} = 103$.

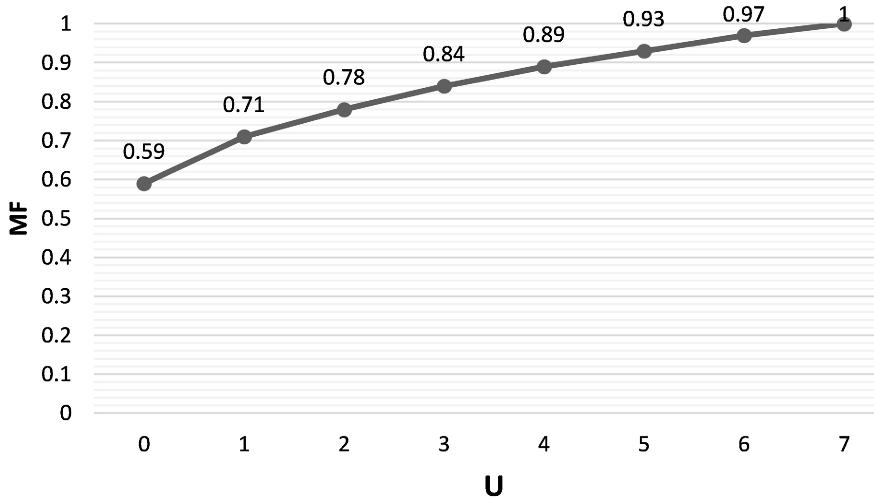

Figure 9. Aggregate output *MF* for "**103**".

2) The Case of Polymodal Aggregated Input *MF*

We use input value X' = **2430.**

By using (3.8), given (3.7) we have the following input *MF*.

One more time from (3.13), given (3.12) we are getting the following fuzzy set $\tilde{X}^{aggr}$

$\mu_{X^{aggr}}$ ("**2430**") = 0.93/0 + **0.97/1** + **0.97/2** + **0.97/3** + 0.89/4 + 0.89/5 + 0.84/6 + 0.78/7

from (3.28) linguistical-semantically it means: X^{aggr} **=** "**in1**" OR "**in2**" OR "**in3**".

Again, we use binary relationship matrix from Table 3 in *FCR* (3.22) and getting the following fuzzy set $\tilde{Y}'$

$\mu_{Y'}(v)$ = 0.78/0 + 0.84/1 + 0.89/2 + 0.89/3 + 0.97/4 + 0.97/5 + 0.97/6 + 0.93/7

One more time by use of the integration (3.24) and (3.25) we are getting

$\mu_{Y_0^{inter}}$ ("with **10**") = 0.78/0 + 0.79/1 + 0.60/2 + 0.44/3 + 0.30/4 + 0.18/5 + 0.09/6 + 0.03/7

$\mu_{Y_1^{inter}}$ ("with **28**") = 0.70/0 + 0.84/1 + 0.89/2 + 0.85/3 + 0.70/4 + 0.56/5 + 0.42/6 + 0.29/7

$\mu_{Y_2^{inter}}$ ("with **48**") = 0.39/0 + 0.56/1 + 0.77/2 + 0.89/3 + 0.77/4 + 0.56/5 + 0.39/6 + 0.25/7

$\mu_{Y_3^{inter}}$ ("with **61**") = 0.78/0 + 0.84/1 + 0.89/2 + 0.89/3 + 0.97/4 + 0.97/5 + 0.93/6 + 0.89/7

$\mu_{Y_4^{inter}}$ ("with **66**") = 0.71/0 + 0.79/1 + 0.87/2 + 0.89/3 + 0.97/4 + 0.94/5 + 0.87/6 + 0.79/7

$\mu_{Y_5^{inter}}$ ("with **78**") = 0.23/0 + 0.35/1 + 0.49/2 + 0.65/3 + 0.82/4 + 0.97/5 + 0.82/6 + 0.65/7

$\mu_{Y_6^{inter}}$ ("with **93**") = 0.25/0 + 0.38/1 + 0.50/2 + 0.63/3 + 0.75/4 + 0.88/5 + 0.97/6 + 0.88/7

$\mu_{Y_7^{inter}}$ ("with **103**") = 0.21/0 + 0.35/1 + 0.48/2 + 0.59/3 + 0.70/4 + 0.81/5 + 0.90/6 + 0.93/7

By use of the aggregation (3.26) and (3.27) we are getting in terms of singletons $\mu_{Y^{aggr}}\left(v_j\right)/v_j \mid \forall j \in \left[0, CardV\right]$. See Figure 10.

$\mu_{Y^{aggr}}$ () = 0.78/0 + 0.84/1 + 0.89/2 + 0.89/3 + **0.97**/4 + **0.97**/5 + **0.97**/6 + 0.93/7

Thus, from (3.28) linguistical-semantically it means: Y^{aggr} = "**out4**" OR "**out5**" OR "**out6**".

Here is how fuzzy conditional inference (3.18) transformed due to use of fuzzy clustering mechanism.

Ant 1: If x is "**in0**", then y is "**out7**"

Ant2: x' is "**in2**"

Cons: y^{aggr} is "**out4**" OR "**out5**" OR "**out6**"

We use defuzzification procedure, given, that $\exists V_1 \subseteq V = \{4,5,6\}, CardV_1 = 3$ and from Table 2:

$$ocl_4 = 66,\ ocl_5 = 78 \text{ and } ocl_6 = 93.$$

Thus, from (3.30) we are getting $y_{Def} = 79$.

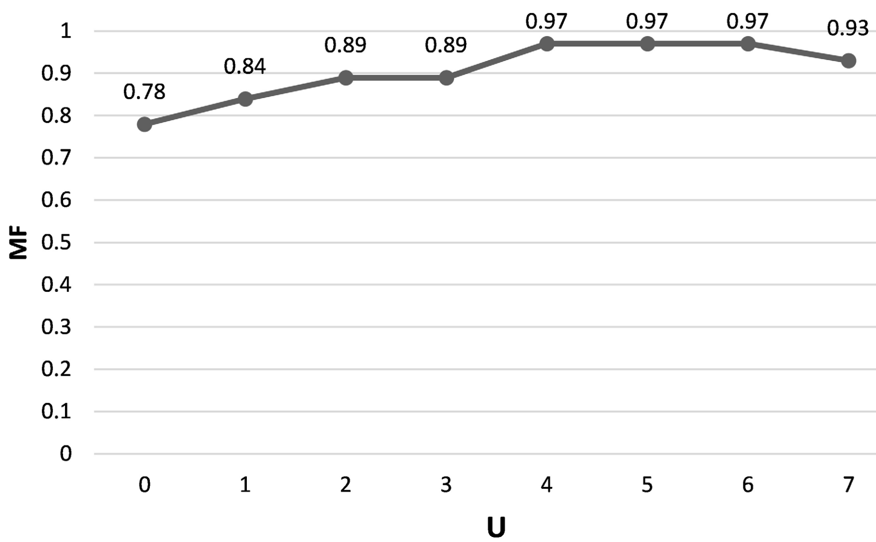

Figure 10. Aggregate output *MF* for "79".

4. Conclusion

In this study we examined ***EA*** non-linear scaled based approach to map original physical scale into a linguistical one. This approach allows to assign *linguistic labels* not to an evenly divided physical scale, but rather to *arbitrary intervals* within it, associated with expert's knowledge and considerations about the nature of represented physical component. Thus, considered physical scale would be divided by certain number of *linguistically labeled clusters*. This approach makes *fuzzy decision-making* mechanism more properly applied to a nature of real-world behavior. In this study we covered issues of scaling, building fuzzy clusters, linguistic labeling, the fuzzification of a model's input/output, the aggregation of fuzzy elements of a model and appropriate fuzzy decision-making mechanism. We also presented corresponding defuzzification mechanism.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Bezdek, J. (1981) C. Pattern Recognition with Fuzzy Objective Function Algorithms. Springer.

[2] Chiu, S.L. (1994) Fuzzy Model Identification Based on Cluster Estimation. *Journal of Intelligent & Fuzzy Systems*, **2**, 267-278. https://doi.org/10.3233/ifs-1994-2306

[3] Yager, R.R. and Filev, D.P. (1994) Generation of Fuzzy Rules by Mountain Clustering. *Journal of Intelligent & Fuzzy Systems*, **2**, 209-219. https://doi.org/10.3233/ifs-1994-2301

[4] Gustafson, D. and Kessel, W. (1978) Fuzzy Clustering with a Fuzzy Covariance Matrix. 1978 *IEEE Conference on Decision and Control Including the* 17*th Symposium on Adaptive Processes*, San Diego, 10-12 January 1979, 761-766. https://doi.org/10.1109/cdc.1978.268028

[5] Gath, I. and Geva, A.B. (1989) Unsupervised Optimal Fuzzy Clustering. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **11**, 773-780. https://doi.org/10.1109/34.192473

[6] Höppner, F. (1999) Fuzzy Cluster Analysis: Methods for Classification, Data Analysis, and Image Recognition. Wiley, 49-53.

[7] Tserkovny, A. (2017) A T-Norm Fuzzy Logic for Approximate Reasoning. *Journal of Software Engineering and Applications*, **10**, 639-662. https://doi.org/10.4236/jsea.2017.107035

[8] Aliev, R. and Tserkovny, A. (2011) Systemic Approach to Fuzzy Logic Formalization for Approximate Reasoning. *Information Sciences*, **181**, 1045-1059. https://doi.org/10.1016/j.ins.2010.11.021

[9] Tserkovny, A. (2016) Fuzzy Logic for Incidence Geometry. *The Scientific World Journal*, **2016**, Article ID: 9057263. https://doi.org/10.1155/2016/9057263

[10] Tserkovny, A. (2017) A Fuzzy Logic Based Resolution Principal for Approximate Reasoning. *Journal of Software Engineering and Applications*, **10**, 793-823. https://doi.org/10.4236/jsea.2017.1010045

[11] Tserkovny, A. (2022) Some Considerations about Fuzzy Logic Based Decision Making by Autonomous Intelligent Actor. *Journal of Software Engineering and Applications*, **15**, 19-58. https://doi.org/10.4236/jsea.2022.152002

[12] Tserkovny, A. (2023) Dynamically Scaled Fuzzy Control of Autonomous Intelligent Actor. *Journal of Software Engineering and Applications*, **16**, 301-313. https://doi.org/10.4236/jsea.2023.167016

[13] Tserkovny, A. (2024) A Neuro T-Norm Fuzzy Logic Based System. *Journal of Software Engineering and Applications*, **17**, 638-663. https://doi.org/10.4236/jsea.2024.178035

[14] Fukami, S., Mizumoto, M. and Tanaka, K. (1980) Some Considerations on Fuzzy Conditional Inference. *Fuzzy Sets and Systems*, **4**, 243-273. https://doi.org/10.1016/0165-0114(80)90014-7

[15] Tserkovny, A. (1987) Non-Contradiction in Fuzzy Production Knowledge Bases. *The Conference* "*Informatics*-87", Yerevan, October 1987, 192-194.