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Why Oracle-Based Quantum Search Cannot Use Deep Loops: Physical Limits on Sequential Operations

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Abstract

Oracle-based quantum algorithms cannot use deep loops because quantum states exist only as mathematical amplitudes in Hilbert space with no physical substrate. Critically, quantum wave functions are inherently self-destructive. Quantum information must complete all operations within decoherence time, T_2, before the state is destroyed, fundamentally limiting implementable circuit depth. This constraint is particularly severe for algorithms like Grover's search, which requires $O\left(\sqrt{N}\right)$ sequential iterations. We identify what we term the Loop Depth Barrier (or Quantum Loop Barrier): any quantum algorithm requiring $f(N)$ sequential operations faces the physical constraint $f(N) \times t_{gate} < T_2$, where t_{gate} is the gate operation time. We analyze a concrete reinforcement learning problem to quantify these limits. For this problem requiring search over 3^{20} states, we systematically compare multiple physical factors limiting quantum loop depth. We identify that the most restrictive limits arise from coherence time and gate fidelity constraints. The coherence time constraint limits implementable loops to approximately 10^4 iterations on superconducting hardware and 10^6 iterations on ion traps. The gate fidelity constraint, accounting for cumulative errors over deep sequential circuits, limits reliable loops to approximately 1000 iterations. Both constraints are severely restrictive, falling short of requirements by many applications. Critically, we identify that these limits arise from fundamental physics—quantum states have no persistent physical substrate and decohere on timescale T_2, bounded by the Margolus-Levitin limit on operation speed and the Heisenberg uncertainty principle on state fidelity. These cannot be overcome by technological advancement; they represent absolute physical constraints on sequential quantum operations. We demonstrate a fundamental asymmetry in scaling: For Grover-type algorithms with problem size $N = k^T$, qubit requirements scale linearly as $O(T)$ (manageable through engineering), while sequential operation requirements scale as $O\left(\sqrt{k^T}\right)=O\left(k^{T/2}\right)$, exponentially exceeding available coherence time T_2. Decoherence time improvements are logarithmic while algorithm requirements are exponential in problem size, which suggests the barrier is fundamental to quantum physics rather than a temporary engineering challenge.

Keywords

Quantum Algorithms, Coherence Time, Decoherence, Gate Fidelity, Loop Depth Barrier, Grover's Algorithm, Oracle, Oracle-Based Algorithm, Quantum Search, Margolus-Levitin Bound, Amplitude Amplification

1. Introduction

Quantum computing promises exponential speedups for certain computational problems through exploitation of quantum superposition and interference. Grover's algorithm [1], for instance, claims to search an unsorted database of N items in $O\left(\sqrt{N}\right)$ queries, offering a quadratic speedup over classical algorithms requiring $O(N)$ queries. Quantum machine learning algorithms similarly promise speedups for optimization and learning tasks by operating on superpositions of exponentially many states [2] [3]. While other quantum algorithms such as Shor's factoring algorithm achieve polynomial circuit depth, algorithms based on amplitude amplification and oracle-based quantum search require circuit depths that scale as $O\left(\sqrt{N}\right)$, creating particularly severe physical constraints.

Classical algorithms can use loops with arbitrary depth because classical bits persist in physical memory—the state of computation exists in transistors or magnetic storage that maintain their values indefinitely. Oracle-based quantum algorithms cannot use deep loops because quantum states exist only as mathematical amplitudes in Hilbert space with no physical substrate. Critically, quantum wave functions are inherently self-destructive: all wave functions have a limited lifetime. Quantum information must complete all operations within decoherence time T_2 before the state is destroyed, fundamentally limiting implementable circuit depth.

Consider the Grover algorithm:

Input: Search space of size N, Oracle O

Output: Target item x*

1) Initialization: Uniform superposition in Equation (1)

2) For ($i = 0$; $i <$ O(√N); i++)

- Oracle
- Amplitude Amplification

3) Measurement: Measure to obtain target x*

The total query complexity is $O\left(\sqrt{N}\right)$, providing a quadratic speedup over classical search requiring O(N) queries [1].

Oracle-based algorithms requiring deep sequential operations share a common implicit assumption: quantum computation can be performed with arbitrary circuit depth. This assumption is physically invalid because it ignores the decoherence constraint. Consider the quantum loop in Grover's algorithm:

$$\text{for } (i = 0;\ i < O(\sqrt{N});\ i{+}{+})$$

Physical reality imposes a fundamental constraint. For all quantum algorithms, this must be:

$$\text{for } (i = 0;\ i < \min\{D, O(\sqrt{N})\};\ i{+}{+})$$

where D is the maximum implementable loop depth determined by physical constraints. When $O\left(\sqrt{N}\right) > D$, the algorithm becomes physically infeasible regardless of qubit count or gate quality.

In this paper, we systematically analyze multiple factors that limit D. After examining all factors, we identify the most restrictive limits through rigorous comparison. We demonstrate that for large-scale problems where $O\left(\sqrt{N}\right)$ exceeds physically achievable D, oracle-based quantum search cannot provide practical advantage—not due to engineering limitations, but due to fundamental physical barriers.

We identify what we term the **Loop Depth Barrier** [4] [5] (or **Quantum Loop Barrier**): any quantum algorithm requiring $f(N)$ sequential operations faces the physical constraint $f(N) \times t_{\text{gate}} < T_2$, where t_{gate} is the gate operation time and T_2 is the decoherence time. To illustrate this barrier concretely, we use reinforcement learning for financial trading [6] as a motivating example throughout this paper. Our analysis considers the physical constraints of current quantum computing platforms: superconducting qubits [7], trapped ions [8], photonic systems [9], and neutral atoms [10].

This barrier manifests through five distinct physical constraints:

1) **Superposition maintenance constraint** [11]: Large superpositions cannot be maintained during sequential operations exceeding T_2.

2) **Coherence time constraint** [11]**:** Quantum coherence cannot be preserved for $O\left(\sqrt{N}\right)$ sequential operations when $\sqrt{N} \times t_{\text{gate}} > T_2$.

3) **Cumulative decoherence**: Sequential operations experience cumulative degradation unlike classical operations on persistent memory.

4) **Effective measurement**: Long sequential evolution causes decoherence that effectively measures the quantum state.

5) **Verification impossibility**: Results cannot be verified without time comparable to classical computation.

While quantum error correction (QEC) [12] [13] can theoretically extend coherence time, it introduces substantial overhead that scales with circuit depth, transforming the coherence constraint into an execution time constraint (Section 5.3).

A fundamental distinction exists between classical and quantum computing. Classical algorithm analysis focuses primarily on time complexity (number of operations) and space complexity (memory requirements). Once these resources are available, classical computation proceeds reliably—bits remain stable in physical memory, gates operate with near-perfect fidelity, and intermediate states can be observed without affecting the computation. The key enabler is persistent physical substrate: classical bits stored in transistors maintain their values throughout arbitrarily long sequential operations.

Quantum computing, in contrast, faces a vastly more complex constraint space. Quantum states are probability amplitudes in abstract Hilbert space, not physical objects that can be stored. Without persistent physical substrate, quantum information decays via decoherence on timescale T_2, typically microseconds to milliseconds. This creates fundamental constraints: superposition stability limited by T_2, quantum coherence time restricting sequential operation count, gate fidelity degradation accumulating over time, measurement-induced collapse from environmental coupling, and the impossibility of verifying intermediate states without destroying them. These are not merely engineering challenges but arise from the fundamental physics of quantum systems lacking persistent storage substrate. The Loop Depth Barrier encapsulates these constraints, and our analysis reveals that satisfying all of them simultaneously is physically impossible for algorithms requiring deep sequential operations on large-scale problems.

A central contribution of this work is distinguishing between engineering limitations (which improve with technology) and absolute physical limits (which cannot be overcome). While the Bekenstein Bound (Information Density Limit) [14] and Landauer's Limit (Thermodynamic Bound) [15] can be ignored, we identify fundamental barriers—including the Margolus-Levitin quantum speed limit, Heisenberg uncertainty bounds on gate fidelity [11] [16], and unavoidable decoherence sources [16]—that constrain sequential quantum operations regardless of technological advancement. Current quantum platforms [17] and error correction schemes [18] [19] face significant engineering challenges; however, our analysis reveals that even if all engineering challenges were solved, fundamental physics would still prevent algorithms requiring $O\left(\sqrt{N}\right)$ or deeper sequential operations from achieving practical quantum advantage for large N.

Critically, we demonstrate a fundamental asymmetry in scaling: For Grover-type algorithms with problem size $N = k^T$, qubit requirements scale linearly as $O(T)$ (manageable through engineering), while sequential operation requirements scale as $O\left(\sqrt{k^T}\right) = O\left(k^{T/2}\right)$, exponentially exceeding available coherence time T_2. Decoherence time improvements are logarithmic (10 μs → 100 μs → 1 ms) while algorithm requirements are exponential in problem size. This exponential divergence between what can be built (qubits) and what must be maintained (coherence during deep sequential operations) suggests the barrier is fundamental to quantum physics rather than a temporary engineering challenge.

We systematically analyze the following factors that limit quantum loop depth:

- Superposition size constraint (Section 3): We derive fundamental requirements on physical qubits and analyze implications for deep sequential operations using a concrete reinforcement learning example.

- Operator size constraint (Section 4): We derive fundamental requirements on physical operators.
- Temporal coherence constraint (Section 5): We establish the Loop Depth Barrier, showing precisely how T_2 limits maximum implementable sequential depth.
- Gate fidelity requirements (Section 6): We derive minimum fidelity bounds [20] [21] and show how errors accumulate during deep loops [22]-[24], imposing strict limits on sequential operations.
- Measurement boundary problem (Section 7): We analyze three independent mechanisms [25]-[28]—weak measurement accumulation, the Margolus-Levitin quantum speed limit [29], and information leakage—each imposing distinct bound on achievable sequential operations.

In the rest of the paper:

- **Synthesis (Section 8):** We unify these findings into a comprehensive framework showing how these constraints collectively prevent deep sequential operations.
- **Engineering vs. absolute limits (Section 9):** We distinguish between technological limitations and fundamental physical barriers, ranking constraints by their impact on sequential operation depth.
- **Future directions (Section 10):** We discuss implications for quantum algorithm design, comparisons with prior work on quantum computational limits, and open questions.
- **Complete barrier framework (Section 11):** We integrate the physical barriers identified in this work with the computational/logical barriers from companion work, demonstrating that oracle-based quantum search must avoid four independent barriers simultaneously to achieve practical advantage. We analyze why even the narrow theoretical exception (cryptographic primitives) fails due to physical implementation constraints.
- **Conclusion (Section 12):** We summarize our findings and chart the path forward for quantum computing paradigms that avoid the Loop Depth Barrier.

2. Motivating Example: Reinforcement Learning Trading

Consider a concrete reinforcement learning problem [6]: finding an optimal trading policy for a financial instrument. This problem serves as an excellent test case for oracle-based quantum search because quantum results can be easily verified against efficient classical solutions.

Stock Trading Problem Setup: Consider an automated trading agent that must decide actions based on recent stock price history:

- **Price history:** $D = [p_0, p_1, \cdots, p_{T+N-1}]$
- **Price windows:** $t_i = [p_i, p_{i+1}, \cdots, p_{i+N-1}]$ for $i = 0, \cdots, T-1$
- **Price Vectors:** $[t_0, t_1, \cdots, t_{T-1}]$
- **Actions:** A = {Buy, Sell, Hold}

- **State:** $h \in \{0, 1\}$ (holding 0 or 1 share)
- **Trading history:** $[a_0, a_1, \cdots, a_{T-1}]$, where $a_i \in$ {Buy, Sell, Hold}
- **Policy π:** Decision rule mapping $a_i = \pi(t_i)$ for $i = 0, \cdots, T-1$
- **Objective:** Maximize total profit over trading period

An optimal policy π^* is one that achieves the highest possible expected profit:

$$\pi^* = \arg\max_{\pi} \{E(D, \pi)\} \tag{1}$$

where $E(D, \pi)$ is total profit when following policy, π, for data, D, and argmax is the argument of the maximum.

Although this problem can be solved efficiently ($O(T \times S \times A)$) using dynamic programming with a value function and Bellman recursion from $i = T - 1$ backward to $i = 0$, we adopt an exhaustive search approach to examine quantum computational limits. The efficient classical solution serves as verification for quantum search results.

Classical Complexity: To find the optimal policy π^* through exhaustive search, we must examine all possible action sequences $\{a_0, a_1, \cdots, a_{T-1}\}$. Since each action can be one of 3 choices (Buy, Sell, Hold) and there are T time steps, the search space has size 3^T. Evaluating each action sequence requires computing profit over T steps. Therefore, the classical exhaustive search time complexity is $O(T \times 3^T)$.

The optimal policy π^* must be learned from the optimal action sequence: $\{a_0^*, a_1^*, \cdots, a_{T-1}^*\}$, *where* $a_i = \pi^*(t_i)$, for $i = 0, \cdots, T-1$.

Quantum Approach: A quantum algorithm using Grover's search would search through the space of 3^T possible action sequences, requiring $O\left(\sqrt{3^T}\right)$ sequential iterations, with each iteration evaluating the oracle on a superposition of action sequences.

This example uses exhaustive search strictly to illustrate quantum algorithm limitations, not because it represents optimal problem-solving.

Why this example matters:

1) Quantum algorithm proposals: Several papers have proposed quantum algorithms for reinforcement learning [30]-[32] and portfolio optimization [33] [34] claiming quantum advantage through Grover-style search or quantum sampling methods.

2) Illustrates general barrier: The physical depth barrier demonstrated here applies to any problem requiring deep sequential quantum operations, regardless of whether classical alternatives exist.

3) Clear cost accounting: Trading provides concrete parameters ($T = 20$, $N = 3^{20}$) enabling precise calculation of required qubit count, circuit depth, and physical requirements."

For concreteness, consider $T = 20$ (a 20-step trading horizon):

- Search space size: $N = 3^{20} \approx 3.5 \times 10^9$ action sequences
- Classical complexity: $O\left(20 \times 3^{20}\right) \approx 7 \times 10^{10}$ operations
- Quantum complexity (theoretical): $O\left(20 \times \sqrt{3^{20}}\right) \approx 1.2 \times 10^6$ operations

- Required sequential Grover iterations: $\sqrt{3^{20}} \approx 55000$ or $(\pi/2)\sqrt{N} \approx 59000$

  This example will be used throughout this paper.

3. Superposition Size Constraint

The fundamental question: Can quantum systems create and maintain superpositions large enough to enable deep sequential operations that provide quantum advantage?

Superposition size limit: There exists a physical limit, $N_{\max}$, on the dimension of maintainable quantum superpositions, such that superpositions over state spaces with $N > N_{\max}$ cannot be reliably created or maintained throughout the sequential operations required by algorithms like Grover's search.

3.1. Current Experimental Bounds

The largest quantum superpositions demonstrated experimentally involve (See Table 1):

- **Superconducting qubits** [7]: Google's Sycamore processor achieved 53-qubit random circuit sampling, corresponding to superpositions over $2^{53} \approx 9 \times 10^{15}$ states
- **Trapped ions** [8]: Up to 32 qubits in fully connected architectures, giving $2^{32} \approx 4.3 \times 10^{9}$ states
- **Photonic systems** [9]: Up to 20 photons, with states in $O(10^{14})$ dimensional Hilbert spaces
- **Neutral atoms** [10]: Arrays of up to 256 qubits reported, though with limited connectivity

Table 1. Current experimental superposition limits.

Platform	Maximum qubits	Superposition size	Notes
Superconducting	53	9×10^{15} states	Google Sycamore [7]
Trapped ions	32	4.3×10^{9} states	Fully connected [8]
Photonic systems	20 photons	$\sim 10^{14}$ states	Boson sampling [9]
Neutral atoms	256	$2^{256} \approx 10^{77}$ states	Limited connectivity [10]

Note: While neutral atom systems have demonstrated the largest qubit counts (256), the limited connectivity means not all qubits can interact, reducing the effective size of achievable superpositions for many algorithms. For our trading example requiring full connectivity among 32 logical qubits (~80,000 physical qubits with error correction), trapped ions and superconducting platforms are more relevant benchmarks.

3.2. The Scaling Challenge

Oracle-based quantum algorithms for practical problems require superpositions over much larger spaces than currently demonstrated. Critically, these large superpositions must remain coherent throughout all sequential operations—typically $O(\sqrt{N})$ iterations for Grover-type algorithms.

PROPOSITION 3.1 (Exponential Resource Growth): A quantum system representing k^T action sequences requires $n = T\log_2(k)$ qubits. For $k = 3$, this gives $n \approx 1.585T$ qubits.

EXAMPLE 3.1 (Trading Problem Requirements): For $T = 20$, $k = 3$: logical qubits needed: $n \approx 32$ qubits.

In addition to basic qubit requirements:

1) **Encoding overhead:** Representing ternary choices (three actions) in binary qubits requires additional qubits and gates for basis transformations [11].

2) **Ancilla requirements:** Oracle implementation typically requires $O(n)$ ancilla qubits [11].

3) **Error correction:** Fault-tolerant computation requires 10^3 - 10^4 physical qubits per logical qubit [12].

THEOREM 3.1 (Effective Qubit Requirement): Including encoding overhead ($\alpha \approx 1.5$), ancilla qubits ($\beta \approx 1$) [11], and error correction ($\gamma \approx 10^3$) [12], the total physical qubit requirement is:

$$n_{\text{phys}} = \gamma \cdot (\alpha + \beta) \cdot T \log^2(k) \quad (2)$$

where $\gamma = 1$ if no error correction is required.

EXAMPLE 3.2 (Without Error Correction): For $T = 20$, $k = 3$, assuming no error correction ($\gamma = 1$): $n_{\text{phys}} \approx 1 \cdot 2.5 \cdot 32 \approx 80$ physical qubits. This would be achievable with current technology **if error-free operation throughout 55,000 sequential iterations were possible**.

EXAMPLE 3.3 (With Error Correction): For $T = 20$, $k = 3$, including error correction ($\gamma \approx 10^3$): $n_{\text{phys}} \approx 80 \times 10^3 = 80{,}000$ physical qubits, see **Table 2**.

Table 2. Gap between available and required qubits (with error correction).

Platform	Maximum qubits [7]-[10]	Required	Gap
Superconducting	53	80,000	~1500×
Trapped ions	32	80,000	~2500×
Photonic systems	20 photons	80,000	~4000×
Neutral atoms	256	80,000	~310×

EXAMPLE 3.4 (Longer Trading Windows with Error Correction):

- $T = 30$ steps: ~120,000 physical qubits and $\sqrt{3^{30}}$ ≈ 8.5 million sequential iterations
- $T = 60$ steps: ~240,000 physical qubits and $\sqrt{3^{60}}$ ≈ 6 billion sequential iterations
- $T = 100$ steps: ~400,000 physical qubits and $\sqrt{3^{100}} \approx 7 \times 10^{23}$ sequential iterations

The double barrier: As problem size T increases, both the superposition size **and** the required sequential depth grow exponentially, while hardware improvements grow logarithmically.

REMARK: These calculations assume only representing the action sequence space. Adding the state representation (price windows) would require additional qubits. Current systems provide ~500 physical qubits, while practical problems typically require 10^4 - 10^6 physical qubits, creating gaps of 20× - 2000× that grow exponentially with problem size T.

Critically: Without error correction ($\gamma = 1$), qubit requirements would be modest (~80 qubits for typical problems), but the necessity of fault-tolerant operation imposes a ~1000× overhead that transforms achievable problems into infeasible ones. However, without error correction, the 55,000 sequential operations would accumulate errors making the result meaningless (see Section 5).

PROPOSITION 3.2 (Unverified Scaling Assumption): Grover's algorithms assume superposition size scales exponentially with qubit count indefinitely, and that these superpositions remain coherent throughout deep sequential operations. This has been experimentally verified only up to $n \approx 50$ - 60 qubits with shallow circuits. Extrapolation to $n = 1000+$ qubits with $O\left(\sqrt{N}\right)$ sequential depth required for practical algorithms is unverified by experiment.

3.3. Summary: The Superposition-Depth Coupling

Oracle-based quantum algorithms face a fundamental superposition size constraint that is intrinsically coupled to the Loop Depth Barrier. For algorithms searching over k^T possible configurations (such as action sequences), physical qubit requirement is given in Equation (2), where:

- $\gamma \approx 10^3$ (error correction overhead)
- $\alpha \approx 1.5$ (encoding overhead)
- $\beta \approx 1$ (ancilla qubits)

Sequential operation requirement is $O\left(\sqrt{k^T}\right) = O\left(k^{T/2}\right)$ iterations. For quantum advantage, both constraints must be satisfied simultaneously:

1) **Superposition constraint:** $n_{\text{phys}} < n_{\text{available}}$

2) **Loop depth constraint:** $O(k^{T/2}) \times t_{\text{gate}} < T_2$ (next two sections)

The fundamental asymmetry: Qubit requirements scale as $O(T)$ (linear), which is potentially achievable through engineering. Sequential operation requirements scale as $O(k^{T/2})$ (exponential Loop Depth), which faces the absolute physical limit T_2. As T increases, the gap between what can be built and what must be maintained during deep sequential operations grows exponentially.

Connection to Loop Depth Barrier: Even if sufficient qubits are available to create the initial superposition, maintaining coherence throughout the exponentially many sequential operations required by Grover-type algorithms remains physically impossible for large problem sizes. The superposition must not only be created, but must survive intact through thousands to millions of sequential gate operations—this is where the Loop Depth Barrier becomes insurmountable.

4. Operator Depth Constraint

This section makes an estimate: how deep are the quantum circuits required for each iteration of Grover's algorithm?

Grover's algorithm requires $O\left(\sqrt{N}\right)$ iterations, but each iteration involves applying two operators: the oracle, O_f, and the diffusion operator, $O_{\text{diffusion}}$, for amplitude amplification, with depth, $d_{\text{diffusion}}$. Understanding operator depth is critical because total circuit depth is $\sqrt{N} \times d_{\text{iter}}$:

$$d_{\text{iter}} = d_{\text{oracle}} + d_{\text{diffusion}} \tag{3}$$

where d_{iter}, d_{oracle}, and $d_{\text{diffusion}}$ are iteration, oracle, and diffusion depth, respectively.

4.1. Oracle Circuit Depth Analysis

DEFINITION 4.1 (Oracle Depth): The oracle depth, d_{oracle}, is the minimum number of sequential gate layers required to implement the oracle operation O_f that marks target states.

Any unitary operator U on an n-qubit system is a $2^n \times 2^n$ complex matrix. The oracle operator must be implemented as a quantum circuit consisting of elementary gates (single-qubit rotations and two-qubit controlled gates).

Oracle Parameter Complexity: Any n-qubit unitary operator can be decomposed into $O(4^n)$ elementary gates [11]. A general n-qubit unitary has 4^n independent real parameters. Each elementary gate provides $O(1)$ degrees of freedom. Therefore, $O(4^n)$ gates are required in the worst case. However, practical oracles exploit problem structure and require far fewer gates.

THEOREM 4.1 (Oracle Depth Bounds): For a quantum oracle marking solutions to a problem on n qubits, the circuit depth depends on problem structure:

1) **Simple marking oracle** (marks one specific state): $d_{\text{oracle}} = O(n)$

2) **Function evaluation oracle** (evaluates $f: \{0,1\}^n \rightarrow \{0,1\}$): $d_{\text{oracle}} = O(n)$ to $O(n^2)$

3) **Complex predicate oracle** (evaluates compound Boolean formula): $d_{\text{oracle}} = O(n^2 \log n)$

Proof: We establish these bounds through concrete examples representing each oracle class:

- Simple marking: Original Grover's algorithm oracle (marks a single target state from the search space)
- Function evaluation: Subset sum optimization oracle (evaluates arithmetic conditions on bit strings)
- Complex predicate: SAT problem oracle (evaluates Boolean formulas with multiple clauses)

These examples illustrate the general scaling behavior for their respective oracle classes.

1) Simple marking: An oracle marking state $|x\rangle$ requires checking if all n qubits match pattern x. This is implemented as an n-controlled NOT gate, which decomposes into $O(n)$ two-qubit gates using ancilla qubits [11] (Nielsen & Chuang, Ch. 4). Since these gates must be applied sequentially, depth is $O(n)$.

Example: For $n = 3$, marking target $|101\rangle$: the oracle O marks target states by phase inversion:

$$O|x\rangle = (-1)^{f(x)}|x\rangle \tag{4}$$

where $f(x) = 1$ if $x = 101$, else 0.

a) apply X (NOT Gate) to middle qubit, transforming $|101\rangle \rightarrow |111\rangle$, b) apply multi-controlled-Z gate: this gate checks if all qubits equal $|1\rangle$ and flips the phase only for that state ($|111\rangle \rightarrow -|111\rangle$), leaving all other states unchanged—since only our target $|101\rangle$ became $|111\rangle$ in step (a), only it gets marked, (c) undo the X gate. Total depth: O(n).

2) Function evaluation: Computing $f(x)$ requires:

- Arithmetic operations: $O(n)$ to $O(n^2)$ depth
- Comparison operations: $O(\log n)$ depth
- Total: $O(n^2)$ in typical cases

Example: For $n = 3$, consider an oracle that marks states where the sum of bits exceeds 1: Oracle definition is Equation (4) where $f(x_0x_1x_2) = 1$, if $(x_0 + x_1 + x_2 > 1)$, else 0. Oracle circuit implementation:

(a) Compute sum (depth $O(n)$):

- Apply quantum adder: $|x_0\rangle|x_1\rangle|x_2\rangle|0\rangle|0\rangle \rightarrow |x_0\rangle|x_1\rangle|x_2\rangle|\text{sum}\rangle|\text{carry}\rangle$
- For 3 bits: sum = $x_0 + x_1 + x_2$ ($O(n)$)

(b) Compare sum > 1 (depth $O(\log n)$):

- Apply comparator circuit checking if sum > 1
- Stores result in ancilla: $|\text{result}\rangle = |1\rangle$ if sum > 1, else $|0\rangle$

(c) Apply phase flip (depth $O(1)$):

- Apply controlled-Z on $|\text{result}\rangle$ ancilla
- This implements $(-1)^{f(x)}$ phase flip

(d) Uncompute (depth $O(n)$):

- Reverse steps (a) and (b) to clean up ancillas

Total oracle depth: $O(n) + O(\log n) + O(1) + O(n) = O(n)$ for this function. For more complex functions like $f(x)$ = "product of first two bits plus third bit exceeds threshold" (requiring multiplication), depth becomes $O(n^2)$ due to cascaded arithmetic operations.

3) Complex predicates: Evaluating Boolean formula with m clauses over n variables requires $O(n\log m)$, typically $O(n^2\log n)$ when $m = O(n)$.

Example: For $n = 3$, consider an oracle marking satisfying assignments of:

$$\varphi = \left(x^0 \vee \neg x^1 \vee x^2\right) \wedge \left(\neg x^0 \vee x^1 \vee x^2\right) \wedge \left(x^0 \vee x^1 \vee \neg x^2\right) \tag{5}$$

Oracle definition is Equation (4), where $f(x) = 1$ if $\varphi(x)$ = TRUE, else 0.

Test case $|x\rangle = |110\rangle$:

- Clause 1: $(1 \vee \neg 1 \vee 0) = (1 \vee 0 \vee 0) = 1$ ✓
- Clause 2: $(\neg 1 \vee 1 \vee 0) = (0 \vee 1 \vee 0) = 1$ ✓
- Clause 3: $(1 \vee 1 \vee \neg 0) = (1 \vee 1 \vee 1) = 1$ ✓
- Result: $f(110) = 1$, so $O|110\rangle = -|110\rangle$

Circuit depth (sequential architecture):

- Each clause with n literals evaluated sequentially: $O(n)$ depth per clause
- m clauses evaluated sequentially: $m \times O(n) = O(n) \times O(n) = O(n^2)$ depth
- Combining results and uncomputation adds $O(\log n)$ overhead

Circuit depth:

- Parallel evaluation (tree architecture): Each n-literal clause requires $O(\log n)$ depth, combining m clauses requires $O(\log m)$ depth. Total: $O(n\log n)$ when $m = O(n)$.
- Sequential evaluation (linear architecture): Each n-literal clause requires $O(n)$ depth (sequential OR gates), evaluating $m = O(n)$ clauses sequentially requires $m \times O(n) = O(n^2)$ depth, plus $O(\log n)$ for combining. Total: $O(n^2\log n)$.

EXAMPLE 4.1 (Trading Oracle Depth): For our trading problem with $T = 20$ steps, $n \approx 32$ qubits, the oracle must evaluate: "Does action sequence $\{a_0, \ldots, a_{19}\}$ yield profit above threshold?"

Oracle computation:

1) Decode binary to action sequence: $O(n)$ depth
2) Simulate T trading steps: $O(T \times n) = O(640)$ depth
3) Compare profit to threshold: $O(\log T)$ depth

Total oracle depth estimate: $d_{\text{oracle}} = O(T \times n) \approx 640$ gates.

4.2. Diffusion Operator Depth

The diffusion operator D performs inversion about the mean: $D = 2|\psi\rangle\langle\psi| - I$.

PROPOSITION (Diffusion Depth): The diffusion operator can be implemented with circuit depth $d_{\text{diffusion}} = O(n)$.

Proof: D decomposes as:

- Computing the mean, and shifting the amplitude coordinate system, $O(n)$;
- Reflecting all amplitudes, $O(n)$;
- Shifting the amplitude coordinate system back, $O(n)$.

Total depth: $O(n)$.

For $n = 32$ qubits: $d_{\text{diffusion}} \approx 96$ gates.

4.3. Estimating the Conservative Bound

Total Iteration Depth: Each Grover iteration requires, d_{iter}, in Equation (3). For our problems:

- Best case (simple marking): $d_{\text{iter}} = O(n) \approx 100$ gates
- Typical case (function evaluation): $d_{\text{iter}} = O(n^2) \approx 500$ - 2000 gates
- Our trading example: $d_{\text{iter}} \approx 1000$ gates

Conservative Bound for Analysis: For subsequent quantitative analysis, we adopt:

$$d_{\text{iter}} \approx d_{\text{oracle}} \approx 1000 \text{ gates per Grover iteration.}$$

The Oracle-based circuit depth is $\sqrt{N} \times d_{\text{iter}}$; where $\sqrt{N}$ can only use sequential evaluation and d_{iter} can use parallel evaluation, which can have:

$$d_{\text{iter}} \approx d_{\text{oracle}} \approx 100 \text{ gates per Grover iteration.}$$

This is the key parameter for coherence analysis in Section 5.

4.4. Summary

Oracle depth is not negligible. Key findings:

1) **Oracle depth varies:** From $O(n)$ for simple marking to $O(n^2)$ for function evaluation.

2) **Trading example iteration depth:** $d_{\text{iter}} = d_{\text{oracle}} + d_{\text{diffusion}} \approx 1000$ gates total.

For Grover's algorithm searching N items, the circuit depth is given in Equation (6):

$$d_{\text{Grover}} = O\left(\sqrt{N}\right) \times d_{\text{iter}} \sim \sqrt{N} \times 1000 \text{ gates} \tag{6}$$

This amplifies the Loop Depth Barrier by ~1000× compared to naive analysis assuming unit-depth operations.

Section 5 analyzes how coherence time T_2 limits this total circuit depth, establishing quantitative bounds on maximum searchable N.

5. Temporal Constrains on Coherence

Quantum coherence is the ability of a quantum system to maintain well-defined phase relationships between its quantum states—the essence of being "quantum" [16] [26]. Decoherence occurs when a system interacts with its environment, which effectively "measures" or randomizes the phase information through entanglement with environmental degrees of freedom. Quantum coherence (the off-diagonal terms of the density matrix) typically decays exponentially with time ($\rho(t) = \rho(0)\mathrm{e}^{-t/T_2}$), characterized by the coherence time T_2. This exponential decay sets fundamental practical limits on:

- **Circuit depth**: How many sequential gate operations can be performed before coherence is lost.
- **Algorithm execution time**: How long quantum information can be stored and manipulated coherently.
- **Computational capacity**: The total number of operations achievable within the coherence window.

For Oracle-based quantum algorithms requiring many sequential operations—such as Grover's algorithm with $O\left(\sqrt{N}\right)$ iterations—coherence time becomes a critical bottleneck that may be insurmountable even with technological improvements.

Decoherence Bound: Quantum coherence time T_2 imposes a hard limit on achievable circuit depth $d_{\max} \propto T_2/t_{\text{gate}}$ [17], where t_{gate} is single-gate execution time. If $d_{\max} < d_{\text{alg}}$ (required algorithm depth), the algorithm cannot complete within coherence time T_2, causing the quantum state to decohere and eliminating quantum advantage.

5.1. Current Coherence Times

State-of-the-art coherence times and single-gate execution times vary by platform [17], as shown in Table 3.

THEOREM 5.1 (Coherence-Depth Constraint) [17]: The maximum achievable circuit depth on a quantum computing platform is limited by:

Table 3. State-of-the-art coherence times, single-gate execution times by platform [17], and maximum achievable circuit depths ($d_{max} = T_2/t_{gate}$).

Platform	T_2 (coherence)	t_{gate}	d_{max}
Superconducting	100 - 200 μs	20 - 50 ns	2,000 - 10,000
Trapped ions	1 - 10 s	10 - 100 μs	10,000 - 1,000,000
Neutral atoms	1 - 10 s	1 - 10 μs	100,000 - 10,000,000
Silicon spin	0.1 - 1 s	1 - 10 μs	10,000 - 1,000,000

$$d_{max} = \frac{T_2}{t_{gate}} \tag{7}$$

where T_2 is the coherence time and t_{gate} is the single-gate execution time (given in Table 3).

Note: Grover's algorithm for searching N items requires the circuit depth, $\boldsymbol{d}_{\textbf{Grover}}$, in Equation (7). The maximum achievable circuit depths ($d_{max} = T_2/t_{gate}$) are computed in the last column of Table 3.

COROLLARY 5.1 (Coherence Requirement for Quantum Advantage): For the coherence bound not to eliminate quantum advantage in Grover's algorithm, we require:

$$T_2 > T_{Grover} = t_{gate} \cdot \sqrt{N} \cdot d_{iter} \tag{8}$$

where N is the search space size, t_{gate} is the gate time, and d_{iter} is the circuit depth for each iteration.

5.2. The Coherence Gap and Time Gap Estimates

EXAMPLE: For $N = 3^{20} \approx 3.5 \times 10^9$ action sequences:

$$\text{Grover iterations} = (\pi/4)\sqrt{N} \approx 59000$$

Assuming iteration depth, $d_{iter} \approx 1000$ gates (see Section 4), the total Grover's algorithm depth is:

$$d_{total} \approx 59000 \times 1000 = 59000000 \approx 6 \times 10^7 \text{ gates}$$

Assuming some parallelization exist and iteration depth, $d_{iter} \approx 100$ gates (see Section 4), the total Grover's algorithm depth is:

$$d_{total} \approx 59000 \times 100 = 5900000 \approx 6 \times 10^6 \text{ gates}$$

Note on circuit depth: Circuit depth measures the longest sequential path through the circuit, not the total gate count. While many gates can operate in parallel on different qubits, the oracle depth, d_{oracle}, represents the minimum number of sequential layers required to evaluate the trading policy. The 59,000 Grover iterations are inherently sequential (one iteration must be completed before the next begins). Thus, $d_{total} = 59{,}000 \times d_{iter} \approx 59{,}000 \times d_{oracle}$ represents the total sequential depth of the algorithm.

EXAMPLE (Coherence Gap): For the depth, $d_{total} \approx 6 \times 10^7$ gates, the coherence gap is given in Table 4.

Table 4. Coherence gap between achievable and required circuit depths for Grover search example, $N = 3^{20}$.

Platform	d_{max} (gates)	Required	Gap
Superconducting	~10^4	6×10^7	6000×
Trapped ions	~10^6	6×10^7	60×
Neutral atoms	~10^7	6×10^7	6×
Silicon spin	~10^6	6×10^7	60×

Even the best current platforms (neutral atoms) fall short by a factor of 6, while most platforms fall short by 2 - 3 orders of magnitude.

EXAMPLE (Time Gap): For $N = 3^{20}$, $d_{oracle} = 1000$, the required coherence time varies by platform gate speed:

$$T_2 > T_{\text{Grover}} = t_{\text{gate}} \cdot \sqrt{N} \cdot d_{\text{iter}} = t_{\text{gate}} \times 59000 \times 1000 \quad (9)$$

The time gap is given in Table 5.

Table 5. Time gap between achievable and required circuit depths for Grover search example, $N = 3^{20}$.

Platform	t_{gate}	Required T_2	Current T_2	Gap
Superconducting	50 ns	2950 s (49 min)	200 μs	~15 million×
Trapped ions	100 μs	5.9×10^6 s (68 days)	10 s	~590,000×
Neutral atoms	10 μs	5.9×10^5 s (6.8 days)	10 s	~59,000×
Silicon spin	10 μs	5.9×10^5 s (6.8 days)	1 s	~590,000×

Key insight: Faster gates (superconducting) reduce the required coherence time but still exceed current capabilities by orders of magnitude. Slower gates (trapped ions) require impossibly long coherence times—68 days for trapped ions—making the problem even more severe.

5.3. Quantum Error Correction

Quantum error correction (QEC) can extend effective coherence time, but at significant cost [18] [19]:

- **Qubit overhead**: Surface codes require ~10^3 physical qubits per logical qubit
- **Time overhead**: Error correction cycles add latency, increasing effective t_{gate}
- **Depth overhead**: Additional gates for syndrome measurement and correction

PROPOSITION (QEC Depth Penalty) [12] [18] [19]: With surface code error correction achieving logical error rate $p_L = p_{\text{phys}}^{d/2}$ (where d is code distance), the effective gate time becomes:

$$t_{\text{gate,logical}} = d \cdot t_{\text{gate,phys}} + t_{\text{syndrome}}$$

EXAMPLE (QEC Time Overhead): For $t_{gate} = 50$ ns (superconducting), code distance $d = 31$, and $t_{syndrome} = 1$ μs [12] [18] [19]:

$$t_{\text{gate,logical}} \approx 31 \times 50 \text{ ns} + 1 \text{ μs} \approx 2.5 \text{ μs}$$

This QEC increases required coherence time by 50×, from 50 ns to 2.5 μs. Thus, QEC transforms the requirement to:

$$T_2 > 2.5 \text{ μs} \times 59000 \times 1000 \approx 147500 \text{ seconds} \approx 41 \text{ hours}$$

This exceeds *any* demonstrated coherence time by orders of magnitude.

Important distinction: Quantum Error Correction (QEC) is theoretically capable of extending logical coherence time indefinitely through repeated error correction cycles. However, QEC introduces substantial overhead: each logical gate requires multiple physical gates, and error correction cycles must occur faster than error accumulation. For the trading example requiring 10^9 sequential logical operations, even with QEC:

1) **Practical barrier:** If each logical operation requires 10^3 physical operations (conservative estimate), total physical depth becomes $10^3 \times 10^9$ operations. At 1 μs per operation, execution time is many days—during which environmental drift, calibration errors, and other non-correctable noise accumulate.

2) **Overhead scaling:** QEC overhead grows with circuit depth, potentially requiring $O(d^2)$ physical operations for d logical operations in worst case.

3) **Threshold requirements:** QEC only works if physical error rates fall below fault-tolerance threshold (~10^{-3} to 10^{-4}). Current systems approach but don't reliably maintain this threshold.

The barrier is not physical impossibility of maintaining coherence (QEC addresses this theoretically), but the practical impossibility of executing exponentially many operations within reasonable timeframes while maintaining error rates below fault-tolerance thresholds. This distinction is important: our argument is about practical feasibility given physical overhead, not theoretical impossibility.

Summary: Quantum coherence time fundamentally limits both achievable circuit depth and total algorithm execution time. The exponential decay of coherence creates a race between computation speed and decoherence: algorithms must complete before the quantum advantage dissipates. Current platforms support circuit depths of ~10^4 gates (superconducting) to ~10^7 gates (neutral atoms). Two fundamental coherence constraints must be satisfied:

1) **Circuit depth constraint**: The maximum achievable circuit depth is limited by: $d_{max} = T_2 / t_{gate}$.

2) **Algorithm time constraint**: For Grover-type algorithms with $O\left(\sqrt{N}\right)$ iterations, coherence must persist through: $T_2 > t_{\text{gate}} \cdot \sqrt{N} \cdot d_{\text{oracle}}$.

These constraints are particularly severe for algorithms requiring deep circuits (large d) or many iterations (large $\sqrt{N}$). Quantum error correction can extend effective coherence time but introduces overhead that increases the required T_2 by additional orders of magnitude, suggesting coherence may represent a fundamental rather than merely engineering barrier for large-scale quantum algorithms.

6. Gate Fidelity Requirements

Quantum gate fidelity measures how closely an implemented quantum gate matches the ideal unitary transformation—it is the fundamental accuracy metric for quantum operations [20] [21].

Cumulative Degradation: Sequential quantum operations cause cumulative fidelity loss [11]. After d operations with per-gate fidelity, F_{gate}, system fidelity, $F(d) = F_{\text{gate}}^{d}$, may fall below the threshold for quantum advantage [22] [23].

6.1. Gate Fidelity Accumulation

Current gate fidelities are given in **Table 6** [20] [21]:

Table 6. Gate fidelities.

Platform	Single-qubit	Two-qubit
Superconducting	99.9% - 99.99%	99% - 99.5%
Trapped ions	100.00%	99.90%
Silicon spin	99.90%	99%

THEOREM 6.1 (Fidelity Decay): For a circuit with circuit depth, d, and two-qubit gate fidelity, $F_{\text{gate}} = 1 - \varepsilon$ [11], the system fidelity is:

$$F(d) = (1-\varepsilon)^{d} \approx e^{-\varepsilon d} \tag{10}$$

Here, $F(d) = (1 - \varepsilon)^d$ follows from the definition and $(1 - \varepsilon)^d \approx e^{-\varepsilon d}$ is an identity.

PROPOSITION (Minimum Required Fidelity): For quantum advantage with circuit depth, d, we require:

$$F_{\text{gate}} > 1 - \frac{\ln(2)}{d} \approx 1 - \frac{0.693}{d} \tag{11}$$

Equivalently, for a given gate fidelity F_{gate}, the maximum achievable circuit depth is:

$$d_{\max} = \frac{\ln(2)}{1-F_{\text{gate}}} \approx \frac{0.693}{1-F_{\text{gate}}} \tag{12}$$

For quantum advantage, we typically require $F(d) > 0.5$ (better than random guessing) [22]:

$$F(d) = e^{-\varepsilon d} > \frac{1}{2}, \quad -\varepsilon d = -\ln(2), \quad \varepsilon = \frac{\ln 2}{d}$$

therefore,

$$F_{\text{gate}} = 1 - \varepsilon = 1 - \ln(2)/d.$$

Solving this equation,

$$d_{\max} = \frac{\ln(2)}{1-F_{gate}}$$

Maximum achievable operations for current platforms: Using two-qubit gate fidelities from Table 7 and $d_{max} = 0.693/(1 - F_{gate})$, Table 7 gives the maximum depth.

Table 7. Maximum depth, $d_{max} = 0.693/(1 - F_{gate})$.

Platform	Two-qubit F_{gate}	$1 - F_{gate}$	d_{max} (operations)
Superconducting (best)	99.50%	0.005	139
Superconducting (typical)	99%	0.01	69
Trapped ions	99.90%	0.001	693
Silicon spin	99%	0.01	69

Note: These d_{max} values represent the maximum number of sequential operations before system fidelity drops below 50% ($F(d) < 0.5$). For the trading example, all platforms violate fidelity requirements. Even the best platform (trapped ions at 99.9% fidelity) can only support ~700 operations while maintaining quantum advantage. This is the most restrictive condition in this paper.

6.2. Depth-Limited Qubit Constraints

Both Section 5.1 and Section 6.1 introduce the maximum number of sequential operations, d_{max}, which in turn will introduce restrictions on qubits.

THEOREM 6.2 (Depth-Limited Qubit Constraint): The maximum achievable circuit depth, d_{max}, imposes a fundamental limit on usable qubits for Grover search:

$$n_{usable} \leq 2\log_2\left(\frac{d_{max}}{g}\right) \tag{13}$$

where g is the gate count per Grover iteration. Equivalently, the maximum searchable space is:

$$N_{max} \leq \left(\frac{d_{max}}{g}\right)^2 \tag{14}$$

Proof: Grover's algorithm requires $\sqrt{N}$ iterations, each with g gates, giving total depth

$$d_{alg} = g\sqrt{N}$$

For executability,

$$d_{alg} \leq d_{max}$$

$$d_{alg} = g\sqrt{N} \leq d_{max}, \quad \sqrt{N} \leq \frac{d_{max}}{g} \; N_{max} \leq \left(\frac{d_{max}}{g}\right)^2$$

Since N states are represented by n qubits, $N = 2^n$, we have:

$$n_{usable} \leq 2\log_2\left(\frac{d_{max}}{g}\right)$$

Implications:

1) **Quadratic depth-to-space relationship:** Doubling d_{max} only quadruples searchable space N:

$$N_{max} \le \left(\frac{2d_{max}}{g}\right)^2 = 4\left(\frac{d_{max}}{g}\right)^2$$

2) **Logarithmic qubit scaling:** Doubling d_{max} adds only two usable qubits:

$$n_{usable} \le 2\log_2\left(\frac{2d_{max}}{g}\right) = 2\log_2\left(\frac{d_{max}}{g}\right) + 2$$

3) **Oracle overhead:** For complex oracles where $g = O(n^2)$, usable qubits become:

$$n_{usable} \le 2\log_2\left(\frac{d_{max}}{n^2}\right)$$

This creates negative feedback: more qubits → larger g → more restrictions on usable qubits. Building systems with more qubits provides no advantage for Grover search if circuit depth remains bounded by decoherence and fidelity constraints.

6.3. Gate Fidelity Gap

EXAMPLE (Fidelity Loss): With $d = 6 \times 10^7$ gates and $F_{gate} = 0.995$ (optimistic two-qubit fidelity):

$$F(d) = 0.995^{6\times10^7} \approx e^{-0.005\times6\times10^7} = e^{-300000} \approx 0$$

The system fidelity becomes negligibly small, completely eliminating quantum advantage.

EXAMPLE (Required Fidelity): For depth, $d = 6 \times 10^7$:

$$F_{gate} > 1 - \ln(2)/d \approx 1 - 0.693/d = 0.999999988$$

The current two-qubit gate fidelities are ~1000× below this requirement.

EXAMPLE (d_{max} Gap): The gate fidelity gap is given in Table 8.

Table 8. Gate fidelity gap.

Platform	d_{max} (operations)	Required	Gap
Superconducting (best)	139	6×10^7	430,000×
Superconducting (typical)	69	6×10^7	870,000×
Trapped ions	693	6×10^7	87,000×
Silicon spin	69	6×10^7	870,000×

REMARK: For algorithms with $d = 10^7$ - 10^8 operations, this requires $F_{gate} > 1 - 10^{-8}$ (gate fidelities better than 99.999999%), which is approximately 1000× better than current two-qubit gate fidelities of 99% - 99.5%. Current gates, at 99.5% fidelity, would yield system fidelity $F(d) \approx 0$ after 6×10^7 operations, completely eliminating quantum advantage.

6.4. Error Correction Cannot Fully Solve This

While QEC addresses random errors, it faces limitations:

1) **Threshold requirement**: QEC works only if $\varepsilon < \varepsilon_{\text{threshold}} \approx 10^{-2}$ to 10^{-3} [22] [23].

2) **Overhead multiplication**: Each logical gate requires $O(d^2)$ physical gates, where d is code distance [22] [23].

3) **Coherent errors**: Systematic errors may not be correctable [24].

THEOREM (QEC Overhead) [12] [18] [22]: With code distance d and syndrome measurement cycles, logical gate count becomes:

$$d_{\text{logical}} \times \left(d^2 \text{ to } d^3\right) = d_{\text{physical}} \tag{15}$$

EXAMPLE (QEC): For $d_{\text{logical}} = 6 \times 10^7$ with $d = 31$:

$$d_{\text{physical}} \approx 6 \times 10^7 \times 31^2 \approx 5.8 \times 10^{10} \text{ physical gates}$$

This depth further exacerbates the coherence time requirement.

REMARK: The longest demonstrated quantum circuits have depth $d \approx 100$ - 1000 [7] [25]. The assumption that fidelity can be maintained for $d \sim 10^7$ - 10^8 operations is extrapolated by 4 - 6 orders of magnitude beyond experimental verification. Even with quantum error correction, the overhead scales as $O(d^2)$ to $O(d^3)$, further exacerbating depth and coherence requirements.

Summary: Cumulative gate errors impose exponential fidelity decay—after d sequential operations with per-gate fidelity, F_{gate}, system fidelity decays as $F(d) = F_{\text{gate}}^d \approx \mathrm{e}^{-\varepsilon d}$, where $\varepsilon = 1 - F_{\text{gate}}$. For quantum advantage, we require $F(d) > 0.5$, which imposes a fundamental constraint:

$$F_{\text{gate}} > 1 - \ln(2)/d \approx 1 - 0.693/d$$

Equivalently,

$$d_{\max} = \frac{\ln(2)}{1 - F_{\text{gate}}} \approx \frac{0.693}{1 - F_{\text{gate}}}$$

7. Measurement Boundary Problem

After Section 5 and 6, this section continues to analyze various factors that limit circuit depth. Kraus operators mathematically describe how a quantum system evolves when it interacts with an environment—they encode all possible environmental effects (noise, decoherence, dissipation) in a compact operator-sum representation [11] [16]. These environmental interactions manifest through multiple physical mechanisms that blur the boundary between unitary quantum evolution and measurement:

1) weak measurement accumulation (where each gate operation slightly “observes” the system) [27] [28],

2) fundamental time-energy constraints from the Margolus-Levitin quantum speed limit [29],

3) information leakage to the environment [11] [16], and

4) quantum Zeno effect [35] [36].

Each mechanism independently imposes an upper bound on the number of sequential operations a quantum algorithm can perform before accumulated decoherence destroys the superposition. This section examines these three fundamental constraints and demonstrates that accumulated quantum operations—even in the absence of intentional measurement—constitute unavoidable decoherence that destroys quantum advantage.

Measurement Boundary: There exists a threshold on the number of sequential operations, $d_{\max}$, beyond which accumulated quantum operations constitute **effective decoherence**, **degrading** the superposition and destroying quantum advantage. Section 5, 6, and 7 all point to quantum computers with limited $d_{\max}$ [37]-[41]. In Section 9, more fundamental physical limits on $d_{\max}$ will be introduced, such as Heisenberg Uncertainty Principle [42]-[44].

7.1. What Constitutes Measurement?

In standard quantum mechanics, measurement is defined operationally: an interaction that:

1) Collapses the wave function to an eigenstate;

2) Is irreversible (increases entropy);

3) Yields classical information.

However, the boundary between unitary evolution and measurement is not sharp [16] [26].

DEFINITION (Decoherence): Decoherence is the process by which a quantum system loses coherence through interaction with its environment:

$$\rho(t) = \sum_k E_k \rho(0) E_k^+ \quad (16)$$

where ρ is the density matrix (describing the quantum state of the system), E_k are Kraus operators representing environmental interactions [11], and $\rho(t)$ is the density matrix at time t after environmental interaction. Decoherence causes the off-diagonal elements of ρ (which encode quantum superposition) to decay exponentially, transforming pure quantum states into classical mixtures without requiring measurement.

7.2. Decoherence from Sequential Operations (Weak Measurements)

Each quantum-gate operation couples qubits to their environment through unavoidable interactions. While gates are designed as unitary evolutions, physical implementation requires control fields (electromagnetic or optical pulses) that interact with qubits and inevitably couple them to external degrees of freedom, causing information leakage to the environment.

Decoherence mechanism: The accumulated decoherence from d sequential operations follows [27] [28]:

$$\rho_{\text{final}} = \varepsilon^d \rho_{\text{initial}} + \left(1 - \varepsilon^d\right) \rho_{\text{mixed}} \quad (17)$$

where ε is the fidelity per operation, ρ_{initial} is the initial density matrix representing the quantum state, $\rho_{\text{mixed}} = I/2^n$ is the maximally mixed state (where I is the identity matrix and n is the number of qubits), and ε^d represents the exponential decay of quantum coherence after d sequential operations. This equation is the discrete-time approximation of the Lindblad master equation [27] [28] [45] [46], which governs the continuous evolution of open quantum systems interacting with their environment. Decoherence manifests as exponential decay of the off-diagonal elements of ρ, which encode quantum coherence and superposition: the off-diagonal elements decay as

$$\rho_{ij}(d) \approx \varepsilon^d \rho_{ij}(0), \; i \neq j \tag{18}$$

This is not measurement-induced collapse, but rather continuous information leakage to the environment.

Quantifying decoherence accumulation: For a sequence of d operations, each with gate fidelity $\varepsilon = 1 - \gamma$ (where γ represents the error rate per operation), the probability that the system remains in a coherent quantum state is:

$$P_{\text{coherent}}(d) \approx \varepsilon^d \approx \mathrm{e}^{-d\gamma} \tag{19}$$

Here, $P_{\text{coherent}}(d) \approx \varepsilon^d$ follows from the definition and $\varepsilon^d \approx \mathrm{e}^{-d\gamma}$ ($\varepsilon = 1 - \gamma$) is an identity. Equivalently, the probability of losing quantum coherence is:

$$P_{\text{decohere}}(d) \approx 1 - \mathrm{e}^{-d\gamma} \tag{20}$$

Practical limit on operations: For quantum advantage, we require $P_{\text{decohere}} < 1/2$ (superposition maintained with >50% probability). This gives:

$$d < 0.693/\gamma$$

Derivation: $1 - \mathrm{e}^{-d\gamma} < 1/2$, implies, $\mathrm{e}^{-d\gamma} > 1/2$, thus, $-d\gamma > \ln(1/2) \approx -0.693$, and $d < 0.693/\gamma$.

EXAMPLE 7.2.1 (Maximum operation count): If each gate operation has error rate $\gamma = 10^{-6}$ (representing minimal environmental coupling achievable with current technology), then:

$$d < 0.693/10^{-6} \approx 693000 \approx 7 \times 10^5 \text{ operations}$$

This sets an upper bound of approximately 7×10^5 sequential operations before the superposition is more likely to decohere than remain coherent.

EXAMPLE 7.2.2 (Trading problem): For the reinforcement learning trading problem requiring $d = 2.4 \times 10^9$ operations with gate fidelity $\varepsilon = 0.999$ (error rate $\gamma = 0.001$):

$$P_{\text{decohere}}(d) \approx 1 - \mathrm{e}^{-0.001 \times 2.4 \times 10^9} \approx 1 - \mathrm{e}^{-2.4 \times 10^6} \approx 1$$

The system is essentially guaranteed to decohere completely before computation completes. Even with optimistic gate fidelity $\varepsilon = 0.9999$ ($\gamma = 10^{-4}$):

$$P_{\text{decohere}}(d) \approx 1 - \mathrm{e}^{-10^{-4} \times 2.4 \times 10^9} = 1 - \mathrm{e}^{-2.4 \times 10^5} \approx 1$$

Complete decoherence remains inevitable.

REMARK (Additive effect of fidelity and environmental decoherence): Both gate fidelity errors and environmental decoherence contribute to system degradation. The combined effect is multiplicative:

- System fidelity from gate errors: $F(d) \approx e^{-\varepsilon d}$ where $\varepsilon = 1 - F_{gate}$
- Environmental decoherence: $1 - P_{collapse}(d) \approx e^{-\gamma d}$
- Combined system quality: $e^{-\varepsilon d} \times e^{-\gamma d} = e^{-(\varepsilon+\gamma)d}$

The effective error rate is the sum: $\varepsilon + \gamma$.

Important distinction: This analysis describes decoherence (gradual information loss to environment) rather than measurement (instantaneous wavefunction collapse). While some early literature framed environmental interactions as "weak measurements" [27] [28] [47] [48], the modern understanding treats this as continuous decoherence governed by the Lindblad formalism [27] [28] [45] [46] introduced above. The mathematical form (exponential decay) is the same, but the physical mechanism differs: decoherence is gradual entanglement with environmental degrees of freedom, not repeated projection onto measurement eigenstates. Our argument relies on decoherence theory, which is well-established and does not require invoking measurement formalism.

7.3. The Energy-Time Uncertainty Relation

The energy-time uncertainty relation provides another bound:

THEOREM (Margolus-Levitin Quantum Speed Limit) [29]: The minimum time for a quantum state to evolve orthogonally is:

$$\tau \geq \frac{\pi\hbar}{2\Delta E} \tag{21}$$

where ΔE is the energy uncertainty of the state.

Fundamental speed limits by platform: Typical energy scales by platform [29]:

Superconducting qubits:

- $\Delta E \approx h \times 5\ \text{GHz} \approx 6.6\times10^{-34} \times 5\times10^{9} \approx 3.3\times10^{-24}\ \text{J}$
- $\tau_{min} \geq \pi\hbar/(2\Delta E) \approx \pi\times1.05\times10^{-34}/(2\times3.3\times10^{-24}) \approx 50\ \text{ps}$ **(picoseconds)**

Trapped ions:

- $\Delta E \approx h \times 1\ \text{MHz} \approx 6.6\times10^{-34} \times 10^{6} \approx 6.6\times10^{-28}\ \text{J}$
- $\tau_{min} \geq \pi\hbar/(2\Delta E) \approx \pi\times1.05\times10^{-34}/(2\times6.6\times10^{-28}) \approx 250\ \text{ns}$

Neutral atoms:

- $\Delta E \approx h \times 10\ \text{MHz} \approx 6.6\times10^{-34} \times 10^{7} \approx 6.6\times10^{-27}\ \text{J}$
- $\tau_{min} \geq \pi\hbar/(2\Delta E) \approx \pi\times1.05\times10^{-34}/(2\times6.6\times10^{-27}) \approx 25\ \text{ns}$

Silicon spin:

- $\Delta E \approx h \times 10\ \text{MHz} \approx 6.6\times10^{-34} \times 10^{7} \approx 6.6\times10^{-27}\ \text{J}$
- $\tau_{min} \geq \pi\hbar/(2\Delta E) \approx 25\ \text{ns}$

Minimum gate times from Margolus-Levitin Bounds are given in **Table 9**.

Table 9. Minimum gate time from Margolus-Levitin Bounds.

Platform	Typical ΔE	τ_{min} (M-L bound)	Current t_{gate}	Ratio (t_{gate}/τ_{min})
Superconducting	~h × 5 GHz	~50 ps	20 - 50 ns	~400 - 1000×
Trapped ions	~h × 1 MHz	~250 ns	10 - 100 μs	~40 - 400×
Neutral atoms	~h × 10 MHz	~25 ns	1 - 10 μs	~40 - 400×
Silicon spin	~h × 10 MHz	~25 ns	1 - 10 μs	~40 - 400×

Key insight: Current gate times are already within 40 - 1000× of the fundamental Margolus-Levitin bound. This means gate speeds cannot be arbitrarily improved—we are already approaching fundamental physical limits. Even if engineering advances improve gate times by another order of magnitude, the required coherence times for our trading example (49 minutes to 68 days) would still be orders of magnitude beyond current capabilities.

Maximum achievable operations before decoherence (at M-L bound, based on current coherence times): Using

$$d_{max} = \frac{T_2}{\tau_{min}} \tag{22}$$

under the current **decoherence** time, T_2, the absolute maximum depths (if gates operated at fundamental speed) are given in Table 10.

Table 10. The absolute maximum depths if gates operated at fundamental speed.

Platform	T_2	τ_{min}	d_{max}
Superconducting	200 μs	50 ps	4×10^6
Trapped ions	10 s	250 ns	4×10^7
Neutral atoms	10 s	25 ns	4×10^8
Silicon spin	1 s	25 ns	4×10^7

EXAMPLE (Time Gap) Even at the fundamental speed limit, the problem remains insurmountable. For d sequential operations at the Margolus-Levitin bound, the minimum total time is:

$$t_{total,min} \geq d \times \tau_{min} \tag{23}$$

For our trading example with $d = 6 \times 10^7$ operations, the coherence time gaps are given in Table 11.

Table 11. The coherence time gaps for $d = 6 \times 10^7$ operations.

Platform	τ_{min} (M-L bound)	Minimum time (6×10^7 ops)	Current T_2	Gap
Superconducting	50 ps	3 milliseconds	200 μs	15×
Trapped ions	250 ns	15 seconds	10 s	
Neutral atoms	25 ns	1.5 seconds	10 s	
Silicon spin	25 ns	1.5 seconds	1 s	

Even if gates could operate at the fundamental Margolus-Levitin speed limit (impossible in practice), the computation time would still conflict with current coherence limits for superconducting under the current **decoherence** time, T_2.

EXAMPLE (Depth Gap): For our trading example with $d = 6 \times 10^7$ operations, the depth gaps are given in Table 12.

Table 12. The depth gaps for $d = 6 \times 10^7$ operations.

Platform	d_{max} (at M-L limit)	Required	Gap
Superconducting	4×10^6	6×10^7	15×
Trapped ions	4×10^7	6×10^7	
Neutral atoms	4×10^8	6×10^7	
Silicon spin	4×10^7	6×10^7	

Even at the theoretical speed limit, the computation depth would still conflict with the current coherence limits for superconducting.

7.4. Interaction with Environment

Each gate operation involves [16] [17] [26]:

- **Control fields**: External electromagnetic fields that could leak information.
- **Thermal bath**: Coupling to phonon modes in the substrate.
- **Control electronics**: Classical systems that must remain correlated with quantum state.

PROPOSITION (Information Leakage Bound) [11] [16]: If each operation leaks ΔI bits of information to the environment, then after d operations:

$$I_{\text{leaked}} = d \cdot \Delta I \tag{24}$$

Current experimental techniques can characterize information leakage at best to ~10^{-3} to 10^{-4} bits per operation through process tomography and randomized benchmarking [24].

EXAMPLE For quantum advantage, we require $I_{\text{leaked}} < \log_2(N)$ (less information leaked than gained from quantum speedup). For $N = 3^{20}$ ($\log_2(N) \approx 32$ bits) and $d = 6 \times 10^7$:

$$\Delta I = I_{\text{leaked}} / d < 32 / \left(6 \times 10^7\right) \approx 5 \times 10^{-7} \quad \text{bits per operation}$$

REMARK: This requires each gate operation to leak less than 1 part in 10^7 of a bit of information, leaving a gap of 3 - 4 orders of magnitude in measurement precision. Whether gates actually achieve $\Delta I < 5 \times 10^{-7}$ bits per operation remains experimentally unverified.

Maximum allowable operations before information leakage destroys quantum advantage: Using the above example for an order of magnitude estimate, we require $I_{\text{leaked}} < 32$ bits. Using the inequality, $I_{\text{leaked}} = d \cdot \Delta I < 32$, we can compute the maximum number of operations for different platforms:

$$d_{\max} = \frac{32}{\Delta I} \tag{25}$$

Assuming current best-case information leakage rates $\Delta I \approx 10^{-4}$ bits per operation (optimistic), the restrictions on depth are given in Table 13.

Table 13. maximum achievable circuit depths from informaation leak.

Platform	ΔI (bits/op)	$d_{\max}$ (operations)
Superconducting	$\sim 10^{-4}$	320,000
Trapped ions	$\sim 10^{-4}$	320,000
Neutral atoms	$\sim 10^{-4}$	320,000
Silicon spin	$\sim 10^{-4}$	320,000

Note: These estimates assume the best-case measured information leakage rate of $\Delta I \approx 10^{-4}$ bits per operation. If actual leakage rates are higher ($\Delta I \approx 10^{-3}$), the maximum allowable operations drops to $d_{\max} \approx 32,000$.

7.5. The Zeno Effect

Frequent interactions can lead to the quantum Zeno effect [35] [36], where continuous measurement prevents evolution:

THEOREM (Quantum Zeno Regime): If the rate of environmental interactions exceeds the characteristic evolution rate ω of the system, the system remains in its initial state:

$$\Gamma_{\text{env}} > \omega \Longrightarrow \text{Evolution suppressed}$$

EXAMPLE For $d = 6 \times 10^7$ operations, superconducting qubits, 50 ps for the Margolus-Levitin bound, the minimum time is: $t = d \times \tau_{\min} = 6 \times 10^7 \times 50$ ps $= 3 \times 10^{-3}$, the operation rate is:

$$\Gamma_{\text{op}} = d/t \approx \left(6\times10^{7}\right)/\left(3\times10^{-3}\ \text{s}\right) = 2\times10^{10}\ \text{Hz}$$

If environmental interactions occur at comparable rates, Zeno freezing may prevent the quantum algorithm from executing.

Summary: Each quantum gate operation constitutes a physical interaction that may act as weak measurement—accumulated over many operations, these interactions risk collapsing the superposition before computation completes. Similar limits on maximum achievable operations arise from multiple independent physical constraints examined in this section, based on current coherence.

Comparison of fundamental operation limits are given in Table 14.

Table 14. Limits from Section 5, 6, and 7.

Platform	Weak measurement (7.2)	M-L + coherence (7.3)	Info leakage (7.4)	Fidelity (6.2)	Coherence (5.1)
Superconducting	7×10^5	4×10^6	3.2×10^5	139	2,000 - 10,000
Trapped ions	7×10^5	4×10^7	3.2×10^5	693	10,000 - 1,000,000
Neutral atoms	7×10^5	4×10^8	3.2×10^5	~1000*	100,000 - 10,000,000
Silicon spin	7×10^5	4×10^7	3.2×10^5	69	10,000 - 1,000,000

*Estimated for neutral atoms assuming similar two-qubit fidelity to trapped ions.

Key insight: Multiple independent physical mechanisms are:

- Coherence
- Weak measurement accumulation
- The quantum speed limit combined with decoherence
- Information leakage, and
- Gate fidelity decay

All impose upper bounds on the number of sequential operations that can be performed while maintaining quantum advantage (**Table 14**). Remarkably, gate fidelity (Section 6.2) is the most restrictive constraint for all platforms**,** limiting achievable operations to 100 - 1000 even under ideal conditions. While weak measurement (7×10^5), Margolus-Levitin bounds (4×10^6 to 4×10^8), and information leakage (3.2×10^5) also create significant barriers, the exponential decay of fidelity with circuit depth ($F(d) = F_{\text{gate}}^{d}$) represents the most immediate and severe limitation. These are no alternatives; the cumulative constraints must all be satisfied simultaneously.

8. The Unified Constraint Framework

8.1. The Combined Constraint

The five variables in the last section impose joint constraints that may be impossible to satisfy simultaneously for Grover-Type Algorithms:

THEOREM (Quantum Loop Satisfaction Conditions): For a Grover-Type quantum algorithm with problem size $N = k^T$, to achieve practical quantum advantage, ALL of the following must hold:

1) **Superposition Size**:

- Physical qubits available: $n_{\text{phys}} = \gamma \cdot (\alpha + \beta) \cdot T \log_2(k)$
- where $\gamma \approx 10^3$ (error correction), $\alpha \approx 1.5$ (encoding), $\beta \approx 1$ (ancilla)

2) **Coherence Time**:

- Algorithm completion: $T_2 > t_{\text{gate}} \cdot \sqrt{N} \cdot d_{\text{oracle}}$
- Circuit depth limit: $d_{\max} = T_2 / t_{\text{gate}}$

3) **Gate Fidelity**:

- For algorithms with depth d: $F_{\text{gate}} > 1 - \ln(2)/d$, where $d = \sqrt{N} \cdot d_{\text{oracle}}$ for Grover-type algorithms
- Equivalently, maximum achievable depth: $d_{\max} = \ln(2)/(1 - F_{\text{gate}})$

4) **Measurement Boundary**:

- Weak measurement: $P_{\text{decohere}}(d) \approx 1 - e^{-d\gamma}$
- Quantum speed limit: $d_{\max} = T_2 / \tau_{\text{gate}}$ (Margolus-Levitin bound)
- Information leakage: $I_{\text{leaked}} = d \cdot \Delta I < \log_2(N)$ (where ΔI is bits leaked per operation)

All categories of constraints must be satisfied simultaneously for quantum advantage to be achievable.

8.2. Current Status

We analyze a concrete reinforcement learning problem for financial trading to quantify these limits. For this problem requiring search over 3^{20} states, we systematically compare multiple physical factors limiting quantum loop depth. We identify that the most restrictive limits arise from coherence time and gate fidelity constraints. The coherence time constraint limits implementable loops to approximately 10^4 iterations on superconducting hardware and 10^6 iterations on ion traps—while our problem requires far more iterations. The gate fidelity constraint, accounting for cumulative errors over deep sequential circuits, limits reliable loops to approximately 1000 iterations. Both constraints are severely restrictive, falling short of requirements by many applications.

8.3. Scaling Behavior

As problem size grows, requirements become more stringent:

PROPOSITION (Scaling Exacerbation for Grover-Type Algorithms): For Grover-type algorithms with search space size $N = k^T$:

1) **Qubit requirement**:

- $n_{\text{phys}} = \gamma \cdot (\alpha + \beta) \cdot T \log_2(k)$
- Scales as $\boldsymbol{O(T)}$—linear in problem size T

2) **Coherence requirement**:

- $T_2 > t_{\text{gate}} \cdot \sqrt{N} \cdot d_{\text{oracle}} = t_{\text{gate}} \cdot \sqrt{k^T} \cdot d_{\text{oracle}}$
- Scales as $O\left(\sqrt{N}\right) = O\left(k^{T/2}\right)$—exponential in problem size T

3) **Fidelity requirement**:

- $F_{\text{gate}} > 1 - \ln(2)/d$ where $d = \sqrt{N} \cdot d_{\text{oracle}}$
- Equivalently: $d_{\max} = \ln(2)/(1 - F_{\text{gate}})$
- Scales as $F(d) = (1-\varepsilon)^d \approx (1-\varepsilon)^{k^{T/2}}$ —exponentially stringent in T

COROLLARY: The Quantum Loop Barrier becomes *exponentially harder to satisfy* as problem size T increases for Grover-type algorithms, suggesting a fundamental scaling barrier rather than mere engineering challenges. While qubit requirements grow linearly (manageable), coherence time requirements and fidelity requirements become exponentially more demanding, creating an insurmountable barrier for large T.

REMARK: Algorithms with polynomial circuit depth in problem size n (such as Shor's algorithm with $d = O(n^3)$) face less severe constraints. For example, factoring a 2048-bit number requires $d \approx 2048^3 \approx 8.6 \times 10^9$ operations, which still violates condition (2), Coherence Time, but by less than our trading example. The key distinction is whether circuit depth scales polynomially (in n) or exponentially (in $\sqrt{k^n}$).

9. Engineering vs. Absolute Physical Limits

An important question arises: as technology advances, which Quantum Loop Barrier requirements could theoretically be satisfied, and which face fundamental physical limits that no engineering improvement can overcome?

9.1. Technological Progress Projections

Technology projections are given in Table 15.

Table 15. Technology projections.

Constraint	Current (2024)	Near-term (5 - 10 years)	Engineering limit (20 - 50 years)	Absolute physical limit
Qubits	~500	1000 - 10,000	100,000 - 1,000,000	None known (Bekenstein bound [14] at ~10^{69} for room-sized system)
T_2	10 s (ions), 200 μs (SC)	Minutes (ions), 1 ms (SC)	Hours-Days	Yes: CMB radiation, gravitational waves, vacuum fluctuations (~years max)
F_{gate}	0.995 - 0.999	0.9999	0.999999	Yes: Heisenberg uncertainty [11] (~1 – 10^{-10} per operation)

Here:

- SC = Superconducting qubits
- CMB = Cosmic Microwave Background

9.2. Fundamental Physical Barriers

Several absolute limits exist that cannot be overcome by any technology:

1) The Margolus-Levitin Bound (Quantum Speed Limit) [29]

The minimum time for a quantum state to evolve orthogonally is:

$$\tau \geq \frac{\pi\hbar}{2E} \tag{26}$$

This is an **absolute physical limit** derived from energy-time uncertainty. For d sequential operations (Section 5.3):

$$t_{\min} = d \times \frac{\pi\hbar}{2E} < T_2 \tag{27}$$

2) The Bekenstein Bound (Information Density Limit) [14]

Maximum information content in a finite volume is [14]:

$$I_{\max} \leq \frac{2\pi RE}{\hbar c \ln 2} \tag{28}$$

where R is the radius and E is the total energy.

Example calculation: For a room-sized quantum computer (R = 5 m) with energy $E = 10^6$ J:

$$I_{\max} \approx 2\pi \times 5 \times 10^6 / \left(1.054 \times 10^{-34} \times 3 \times 10^8 \times \ln 2\right) \approx 10^{51} \text{ bits}$$

This corresponds to ~10^{51} qubits - far beyond any practical need. This limit can be ignored. While not limiting on quantum computers, this sets an ultimate bound on information processing in finite space.

3) Landauer's Limit (Thermodynamic Bound) [15]

Minimum energy per irreversible bit operation:

$$E_{\min} = k_B T \ln 2 \approx 3 \times 10^{-21} \text{ J}, \text{ at } T = 300 \text{ K} \tag{29}$$

While quantum gates are theoretically reversible, practical implementation involves irreversible operations (measurement, error correction, classical control). If a fraction, ε, of the d operations are irreversible, the minimum energy dissipation is:

$$E_{\text{total}} \geq \varepsilon \times d \times E_{\min} = \varepsilon \times d \times k_B T \ln 2$$

EXAMPLE: For $d = 6 \times 10^7$ operations at $T = 300$ K, if $\varepsilon = 1\%$ (1% of operations irreversible): $E_{\text{total}} \geq 0.01 \times 6 \times 10^7 \times 3 \times 10^{-21}$ J $\approx 1.8 \times 10^{-15}$ J ≈ 2 femtojoules; At $d = 6 \times 10^7$ operations completing in seconds, power dissipation is: $P \approx E_{\text{total}}/t \approx 10^{-15}$ to 10^{-14} watts (femtowatts to tens of femtowatts). This limit can be ignored.

Implication: While Landauer's limit is not currently restrictive (femtowatts are manageable), energy dissipation scales linearly with operation count. For algorithms requiring $d \sim 10^9$ - 10^{12} operations or higher repetition rates, heat dissipation could become a fundamental constraint, particularly when combined with cooling requirements for superconducting qubits.

4) Heisenberg Uncertainty Principle (Measurement Precision Limit)

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2} \tag{30}$$

This fundamentally limits the precision with which we can control quantum operations.

Gate fidelity limit derivation: Consider a quantum gate operation with:

- **Energy scale E:** The characteristic energy of the quantum operation (e.g., qubit transition energy)
- **Gate time t_{gate}:** Duration of the gate operation
- **Energy uncertainty:** From Heisenberg uncertainty, $\delta E \geq \hbar/(2t_{\text{gate}})$

Phase error mechanism: In quantum mechanics, a quantum state with energy E evolves in time according to [11] [42]-[44]:

$$\left|\psi(t)\right\rangle = \mathrm{e}^{-\frac{i(Et)}{\hbar}} \left|\psi(0)\right\rangle \tag{31}$$

The phase accumulated over gate time, t_{gate}, is:

$$\varphi = \frac{E \cdot t_{\text{gate}}}{\hbar} \tag{32}$$

When there is energy uncertainty δE (from the Heisenberg principle), the actual energy is $E \pm \delta E$, leading to phase uncertainty:

$$\delta\varphi = (\delta E) \cdot \frac{t_{gate}}{\hbar} \tag{33}$$

The **relative phase error** is:

$$\frac{\delta\varphi}{\varphi}=\frac{\left[(\delta E)\cdot\frac{t_{gate}}{\hbar}\right]}{\left[E\cdot\frac{t_{gate}}{\hbar}\right]}=\frac{\delta E}{E} \tag{34}$$

Connection to gate fidelity: Phase errors translate to gate infidelity. A state with phase error $\delta\varphi$ evolves as:

$$\left|\psi'(t)\right\rangle=\mathrm{e}^{-i(\varphi+\delta\varphi)}\left|\psi(0)\right\rangle=\mathrm{e}^{-i\varphi}\cdot\mathrm{e}^{-i\delta\varphi}\left|\psi(0)\right\rangle$$

For small $\delta\varphi$, we can expand: $\mathrm{e}^{-i\delta\varphi}\approx 1-i\delta\varphi$, the **fidelity** (overlap with the ideal state) is:

$$F=\left|\left\langle\psi'(t)\middle|\psi'(t)\right\rangle\right|^2\approx\left|\left\langle\psi(t)\middle|\psi(t)\right\rangle\right|^2\left(1-(\delta\varphi)^2\right) \tag{35}$$

Therefore, the **gate error** is:

$$\varepsilon_{\text{fundamental}}\approx(\delta\varphi)^2=\left(\frac{\delta\varphi}{\varphi}\right)^2\cdot\varphi^2$$

Now:

$$\varepsilon_{\text{fundamental}}\propto\left(\frac{\delta\varphi}{\varphi}\right)^2\cdot\varphi^2\propto\left(\frac{\delta E}{E}\right)^2\cdot\varphi^2 \tag{36}$$

Fundamental limit calculation: For any quantum state with energy uncertainty ΔE evolving over time Δt, Equation (29) must hold. During a quantum gate operation:

- $\boldsymbol{\Delta E}$ = energy uncertainty in the control Hamiltonian
- $\boldsymbol{\Delta t}$ = t_{gate} = duration of the gate operation

A quantum gate is implemented by a time-dependent Hamiltonian $H(t)$ that acts for duration t_{gate}. This Hamiltonian has characteristic energy scale E and inherent uncertainty δE in how precisely this energy is defined/controlled. Therefore:

1) Energy control precision: $\Delta E=\delta E$

2) Gate operation time: $\Delta t=t_{\text{gate}}$

$$\delta E\cdot t_{\text{gate}}\geq\frac{\hbar}{2} \tag{37}$$

For a gate operating at the quantum speed limit (Margolus-Levitin bound), the minimum gate time is:

$$t_{\min}=\frac{\pi\hbar}{2E} \tag{38}$$

Substituting into the Heisenberg bound:

$$\delta E\geq\frac{\hbar}{2t_{\min}}=\frac{\hbar}{2\cdot\frac{\pi\hbar}{2E}}=\frac{E}{\pi}$$

Thus, the **minimum relative energy uncertainty** is:

$$\frac{\delta E}{E} \geq \frac{1}{\pi} \tag{39}$$

This yields a **fundamental error floor**:

$$\varepsilon_{\text{fundamental}} \approx \left(\frac{1}{\pi}\right)^2 \cdot \varphi^2 \approx \frac{\varphi^2}{10} \text{ per operation} \tag{40}$$

Interpretation: This naive estimate assumes:

1) Operations at the quantum speed limit (shortest possible gate time)

2) Maximum Heisenberg uncertainty (no error suppression)

3) No control engineering or optimization

Therefore, 10% per gate represents an absolute worst case, not a realistic limit.

More realistic estimate: In practice, quantum control systems operate with multiple error suppression mechanisms (dynamical decoupling, optimal control pulses, composite pulse sequences, etc.) that can suppress this fundamental uncertainty by several orders of magnitude. The achievable fidelity is limited not by the crude Heisenberg bound above, but by:

1) **Control Hamiltonian precision:** How accurately can we engineer the desired quantum operation?

2) **Systematic error cancellation:** Pulse shaping and concatenation techniques.

3) **Environmental isolation:** Shielding from external noise sources.

Current theoretical analyses [11] [17] [20] and experimental extrapolations suggest that with optimal control engineering, fundamental fidelity limits are in the range:

$$F_{\text{gate}} \approx 1 - 10^{-10} \text{ to } 1 - 10^{-12} \text{ per operation}$$

Equivalently, the fundamental error rate is:

$$\varepsilon_{\text{fundamental}} \approx 10^{-10} \text{ to } 10^{-12} \text{ per operation}$$

This represents the ultimate physical limit achievable with perfect control engineering, beyond which quantum uncertainty prevents further improvement. This limit constrains the maximum achievable sequential depth $d_{\max}$.

Relationship to control theory:

Advanced quantum control techniques (optimal control, dynamical decoupling, composite pulses) can suppress errors below the naive Heisenberg limit, $\Delta E \cdot \Delta t \geq \hbar/2$, for individual gates by [11] [30] [49]:

1) Averaging over multiple pulses

2) Engineering pulse shapes to cancel systematic errors

3) Using concatenated gate sequences

However, these techniques face fundamental limits for deep sequential circuits:

1) No free lunch: Error suppression for individual gates requires additional operations (multiple pulses, decoupling sequences), increasing total depth d.

2) Overhead scaling: Achieving n times better fidelity per gate requires $O(n)$ additional operations, increasing total depth by factor of n.

3) Net effect: For deep circuits ($d = 10^9$), improving individual gate fidelity through control techniques increases total operation count proportionally, maintaining or worsening the overall $(1 - \varepsilon)^d$ fidelity decay.

4) Heisenberg bound applies to aggregate: While individual gates can surpass naive limits through control engineering, the total energy-time budget for the entire circuit remains Heisenberg-bounded: $\Delta t_{\text{total}} \geq \hbar/(2\Delta E_{\text{available}})$.

Thus, the Heisenberg bound is not absolute for individual gates but becomes absolute in aggregate for exponentially deep circuits [29]. Control theory provides constant-factor improvements but cannot overcome exponential scaling.

EXAMPLE: Assumes:

1) **Perfect achievement of fundamental limits** (no engineering imperfections beyond Heisenberg uncertainty)

2) **No other error sources** (environmental decoherence, control errors, cross-talk)

3) **Ideal error correction** (if QEC overhead is included, requirements become more stringent)

For $d = 6 \times 10^7$ operations, even at the fundamental limit of $F_{\text{gate}} = 1 - 10^{-10}$:

$$F(d) = \left(1 - 10^{-10}\right)^{6\times10^7} \approx e^{-6\times10^{-3}} \approx 0.994$$

While this example exceeds the absolute minimum threshold ($F > 0.5$), $F(d)$ grows with d.

REMARK: Even with perfect future technology achieving absolute physical limits, large-scale Oracle-based quantum algorithms operating near $d \sim 10^7$ - 10^8 operations are at the edge of physical feasibility. Any additional error sources beyond the Heisenberg limit would push the system below the quantum advantage threshold.

5) Fundamental Decoherence Sources

Even with perfect engineering isolation, unavoidable interactions exist:

- **Cosmic microwave background (CMB)**: $T \approx 2.7$ K, photon density ~400/cm^3
- **Gravitational wave fluctuations**: Strain ~10^{-21} at frequencies relevant to quantum gates
- **Vacuum fluctuations**: Zero-point energy of quantum fields
- **Neutrino background**: Cosmic neutrino flux ~10^6/(cm^2·s)

Estimated fundamental coherence time: While these effects are extraordinarily weak, they set an absolute upper bound on coherence times, estimated to be on the order of years to centuries for perfectly isolated systems [16]. This limit can be ignored.

Implication: For our example requiring $T_2 > 49$ minutes (without QEC) or 41 hours (with QEC), fundamental decoherence is not the limiting factor. However, it represents an ultimate ceiling.

Summary: Among the absolute physical limits identified in this section, the Margolus-Levitin bound (quantum speed limit) and Heisenberg uncertainty principle (gate fidelity limit) impose the most restrictive constraints on Oracle-based quantum algorithms requiring deep circuits. The Heisenberg limit on gate fidelity ($\varepsilon_{\text{fundamental}}$ ~10^{-10} to 10^{-12}) allows only $F(d) \approx 0.994$ for $d = 6 \times 10^7$ operations even under ideal conditions, creating a narrow margin above the quantum advantage threshold. The Margolus-Levitin bound constrains the minimum time per operation, and even at this fundamental limit, current coherence times remain insufficient for algorithms requiring ~10^7 - 10^8 sequential operations.

In contrast, the Bekenstein bound (information density), Landauer's limit (thermodynamic energy dissipation), and fundamental decoherence sources (cosmic background, vacuum fluctuations) do not currently constrain practical quantum algorithms. The Bekenstein bound allows ~10^{51} qubits in room-sized systems (far exceeding any practical need), Landauer's limit permits femtowatt-scale operation (manageable), and fundamental decoherence sets coherence time limits of years to centuries (far beyond current technological coherence of seconds).

9.3. The Fundamental Tradeoff

The critical insight is that multiple fundamental limits interact:

1) Need more operations → Requires minimum time (Margolus-Levitin).

2) Each operation has maximum fidelity (Heisenberg) → Cumulative errors.

3) Long computation time → Decoherence accumulates (even fundamental sources).

Even with perfect future technology**,** these fundamental limits create a constraint space that may not contain a solution for large-scale Oracle-based quantum algorithms requiring deep circuits.

THEOREM (Fundamental Feasibility Bound): For a quantum algorithm with d operations to be fundamentally feasible, it must satisfy ALL of:

1) $t_{\min} = d \times \pi\hbar/(2E) < T_2$ (Margolus-Levitin vs decoherence)

2) $F(d) = (1 - \varepsilon_{\min})^d > 0.5$ where $\varepsilon_{\min} \approx 10^{-10}$ (Heisenberg limit)

3) Total energy < Bekenstein bound for system volume

4) Heat dissipation < Landauer limit × operation rate

For $d = 6 \times 10^7$, the Heisenberg limit alone gives: $F(d) \approx e^{-6\times10^{-3}} \approx 0.994$. This suggests that even with unlimited technological advancement, the trading RL example (and similar large-scale Oracle-based quantum algorithms requiring $O(\sqrt{N})$ or deeper circuits) may be fundamentally infeasible due to the interaction of absolute physical limits.

10. Future Directions

Implications for Quantum Algorithm Design:

1) **Depth awareness**: Algorithms should minimize circuit depth, not just gate count.

2) **Verification strategy**: Include built-in verification that doesn't negate quantum advantage.

3) **Realistic problem sizing**: Target problems where $\sqrt{N}$ is within demonstrable coherence bounds.

4) **Hybrid approaches**: Use quantum subroutines only where Quantum Loop Barrier can be satisfied.

A Path Forward: Modular Quantum Computation

The constraints identified in this work—particularly the exponential growth in coherence and fidelity requirements with circuit depth—suggest that viable Oracle-based quantum algorithms must adopt a fundamentally different structure. Instead of fighting the exponential barrier with deeper circuits, we must embrace the constraints by decomposing problems into shallow phases separated by measurements. This transforms an impossible problem (exponential depth) into a potentially tractable one (many polynomial phases). Rather than attempting deep circuits with $O\left(\sqrt{N}\right)$ sequential operations, practical Oracle-based quantum algorithms may need to follow a modular measurement-based architecture**:**

Repeat as necessary:

Phase i: Polynomial-depth quantum computation (within physical bounds)

Measurement: Extract result m_i

End repeat

Classical post-processing: Combine measurements $\{m_1, m_2, \ldots, m_k\}$

Key principles of this approach:

1) **Bounded depth per phase**: Each quantum phase operates within demonstrated coherence times ($d_{\max} \approx 10^4$ - 10^7 operations depending on platform).

2) **Intermediate measurement**: Each phase terminates with measurement, collapsing the superposition and extracting classical information before decoherence destroys quantum advantage.

3) **Progressive computation**: Each phase's measurement outcome informs subsequent phases, allowing complex problems to be decomposed into manageable quantum subroutines.

4) **Classical coordination**: Post-processing combines measurement results to solve the overall problem, similar to MapReduce parallelization in classical computing.

Advantages of this architecture:

- **Respects physical bounds**: Each phase stays within coherence time, fidelity, and measurement boundary constraints
- **Fault tolerance**: Errors are contained within individual phases rather than accumulating exponentially
- **Verifiable**: Intermediate measurements provide checkpoints for verification
- **Scalable**: Problem size can grow by adding more phases (linear overhead) rather than deeper circuits (exponential overhead)

Challenges:

- Not all problems decompose naturally into this structure
- May lose quantum advantage if too many measurements are required
- Requires new algorithm design paradigms

We will explore this modular quantum computation framework in future work, investigating which problem classes can benefit from this architecture while maintaining quantum advantage within physical constraints.

Open Questions: Several fundamental questions remain:

1) **Experimental limits**: What is the maximum demonstrated circuit depth with maintained fidelity?

2) **Measurement boundary**: Can we quantify the weak measurement strength of gate operations?

3) **Fundamental bounds**: Are there other information-theoretic limits on superposition size or operation count?

4) **Operator depth limits**: Are there theoretical estimates on the maximum implementable operator complexity (circuit depth d_{oracle}) for specific problem classes? While we assumed $d_{\text{oracle}} \approx 100$ - 1000 for our trading example, a rigorous lower bound on oracle complexity for learning problems would strengthen our analysis.

5) **Alternative architectures**: Do topological quantum computers [39] or measurement-based quantum computation [40] avoid these limitations?

11. The Complete Barrier Framework: Computational and Physical Constraints

Our companion work [4] established three independent computational/logical barriers affecting oracle-based quantum search algorithms: the Grover Dilemma, Setup Cost Dilemma, and Oracle Circularity. That analysis identified a narrow exception—cryptographic primitives with polynomial-size specifications and no structural weaknesses—that theoretically avoids all three computational barriers and achieves asymptotic quantum advantage $O(2^{k/2})$ versus classical $O(2^k)$.

This work demonstrates that even this narrow exception faces insurmountable physical barriers. The combination reveals that oracle-based quantum search faces fundamental limitations from two independent directions: computational/logical barriers [4] and physical implementation barriers (this work).

11.1. The Four Independent Barriers

A complete evaluation of oracle-based quantum search algorithms must consider all four barriers:

Computational/Logical Barriers [4]:

Barrier 1 - Grover Dilemma: For Type A problems where computational space exceeds valid data space, constructing superposition over only valid states requires $O(|D|)$ or $O(|S|)$ classical preprocessing, eliminating quantum advantage.

Barrier 2 - Setup Cost Dilemma: Oracle construction or data loading may require classical computation with cost $C_{\text{setup}} \geq C_{\text{classical}}$, eliminating quantum advantage for single-query scenarios.

Barrier 3 - Oracle Circularity: Oracle construction or specification may require solving the target problem, creating circular dependency where $C(\text{oracle}) \geq C(\text{problem})$.

Physical Implementation Barrier (This Work):

Barrier 4 - Loop Depth Barrier: Quantum coherence time T_2 and gate fidelity constraints impose fundamental limits on achievable sequential circuit depth. For algorithms requiring d_{alg} sequential operations, quantum advantage is eliminated when:

- **Coherence constraint:** $d_{\text{alg}} \times t_{\text{gate}} > T_2$
- **Fidelity constraint:** $(1-\varepsilon)^{d_{\text{alg}}}$ < threshold for reliable computation
- **Fundamental limits:** Margolus-Levitin bound, Heisenberg uncertainty, unavoidable decoherence

Key Property: These four barriers are logically and physically independent. An algorithm must avoid ALL FOUR simultaneously to achieve practical quantum advantage.

11.2. Why the Barriers are Independent

The independence of these barriers is crucial for understanding why quantum advantage is so rare:

Computational barriers are problem-dependent [4]:

- Grover Dilemma affects Type A problems (structured data) but not Type B (natural encoding).
- Setup Cost affects problems requiring expensive oracle construction or data loading.
- Oracle Circularity affects optimization/learning problems where oracles must identify solutions.

Physical barriers are implementation-dependent:

- Loop Depth Barrier affects ALL algorithms requiring deep sequential operations.
- Applies regardless of problem structure or oracle specification.
- Determined by fundamental physics, not problem formulation.

Critical insight: An algorithm can avoid Barriers 1-3 (computational) but still fail due to Barrier 4 (physical), or vice versa. All four must be avoided simultaneously.

11.3. Cryptographic Key Search: The Narrow Exception of Barrier 1, 2, and 3

Reference [4] identified cryptographic primitives (e.g., AES key search) as avoiding Barriers 1-3:

Barrier 1 (Grover Dilemma) - AVOIDED:

- Type B encoding: All 2^k keys are valid computational states
- No invalid states requiring filtering
- Superposition construction: $O(k)$ Hadamard gates

Barrier 2 (Setup Cost) - AVOIDED:

- AES specification is public and compact: $O(\text{poly}(k))$ circuit complexity
- Both classical and quantum implement same algorithm with polynomial setup
- Symmetric setup cost for both approaches

Barrier 3 (Oracle Circularity) - AVOIDED:

- Oracle evaluates "Does this key produce correct ciphertext?"
- This evaluation does NOT require knowing which key is correct
- The oracle checks a condition without encoding the solution

Asymptotic Advantage:
- Classical: $O(\text{poly}(k)) + O(2^k) = O(2^k)$
- Quantum: $O(\text{poly}(k)) + O(2^{k/2}) = O(2^{k/2})$
- **Theoretical advantage exists**

11.4. Barrier 4 Eliminates the Exception

However, this narrow exception faces the Loop Depth Barrier (Barrier 4) analyzed in this work:

AES-128 ($k = 128$ bits): Required sequential operations:
- Grover iterations: $O\left(\sqrt{2^{128}}\right) = O\left(2^{64}\right) \approx 1.8 \times 10^{19}$ iterations
- Each iteration: Oracle evaluation + Diffusion operator ≈ 40,000 gates (AES circuit)
- Total sequential depth: $d_{\text{alg}} \approx 1.8 \times 10^{19} \times 4 \times 10^4 \approx 7.2 \times 10^{23}$ gates

Physical constraints (best case - trapped ions):
- Coherence time: $T_2 \approx 1000$ seconds (optimistic future projection)
- Gate time: $t_{\text{gate}} \approx 10^{-6}$ seconds
- Maximum achievable depth: $d_{\max} = T_2 / t_{\text{gate}} \approx 10^9$ gates

The gap:
- Required: 7.2×10^{23} gates
- Achievable: 10^9 gates
- **Shortfall: 7.2×10^{14} × (14 orders of magnitude)**

Fundamental limits prevent closing this gap:

1) **Margolus-Levitin bound:** Quantum operations require minimum time $\Delta t \geq \hbar/(2\Delta E)$. For available energy budgets, cannot achieve required operation speed.

2) **Heisenberg uncertainty:** Gate fidelity limited by $\Delta E \Delta t \geq \hbar/2$. Required fidelity $(1-\varepsilon)^{7.2\times10^{23}} \approx 1-10^{-23}$ per gate is physically impossible.

3) **Unavoidable decoherence:** Cosmic microwave background radiation, gravitational wave fluctuations, and vacuum fluctuations impose ultimate T_2 ceiling regardless of shielding.

4) **Exponential scaling:** For AES-256 ($k = 256$), required depth becomes $2^{128} \approx 10^{38}$ gates—astronomically beyond any conceivable technology.

Conclusion for AES: While asymptotic complexity shows $O(2^{k/2})$ advantage, physical implementation barriers make this advantage unachievable for any practical key length.

11.5. The Complete Picture

For oracle-based quantum search algorithms to achieve practical advantage, they must simultaneously:

1) Avoid Grover Dilemma (Type B encoding)

2) Avoid Setup Cost (polynomial oracle construction for both quantum and classical)

3) Avoid Oracle Circularity (oracle specifiable without solving problem)

4) Avoid Loop Depth Barrier ($d_{alg} < d_{max}$ with reliable fidelity)

Classification of all analyzed problems are given in **Table 16** [4].

Table 16. All four barrier: no oracle-based quantum search algorithm provides genuine computational advantage [4].

Algorithm	Barrier 1 (Grover Dilemma)	Barrier 2 (Setup Cost)	Barrier 3 (Oracle Circularity)	Barrier 4 (Loop Depth)	**Advantage?**
Deutsch (general case)	✓ Avoids	**✗ Fails**	✓ Avoids	✓ Avoids	NO
Deutsch-Jozsa (black-box)	✓ Avoids	**✗ Fails**	✓ Avoids	✓ Avoids	NO
Deutsch-Jozsa (explicit formula)	✓ Avoids	**~ Neutral**	✓ Avoids	✓ Avoids	NO*
Simon (black-box)	✓ Avoids	**✗ Fails**	✓ Avoids	✓ Avoids	NO
Simon (structured/linear)	✓ Avoids	**~ Neutral**	✓ Avoids	✓ Avoids	NO*
Database search (Type A)	**✗ Fails**	✗ Fails	✓ Avoids	✗ Fails	NO
Subset Sum	✓ Avoids	✓ Avoids	**✗ Fails**	✗ Fails	NO
SAT, TSP, NP-complete	✓ Avoids	✓ Avoids	**✗ Fails**	✗ Fails	NO
Reinforcement Learning	✓ Avoids	✓ Avoids	**✗ Fails**	✗ Fails	NO
AES-128 key search	✓ Avoids	✓ Avoids	✓ Avoids	**✗ FAILS**	NO
AES-256 key search	✓ Avoids	✓ Avoids	✓ Avoids	**✗ FAILS**	NO

Universal finding: Through systematic analysis of major problem classes and specific quantum algorithms, we found **no practical cases** where oracle-based quantum search provides genuine computational advantage over optimal classical approaches when all four barriers are properly accounted for.

Even the narrow exception of cryptographic primitives—which avoids the first three computational/logical barriers—faces the fourth barrier of physical implementation constraints, making practical advantage impossible despite theoretical asymptotic improvements.

11.6. The Fundamental Asymmetry

The combination of computational and physical barriers reveals a fundamental asymmetry in quantum vs. classical computing:

Classical computing:

- Bits persist in physical memory indefinitely
- Can perform arbitrarily deep sequential operations
- Fidelity constraints are not engineering challenges (99.9999...% achievable)
- Intermediate states can be verified without destruction

Quantum computing:

- Qubits exist as probability amplitudes without persistent substrate
- Sequential operations limited by T_2 (microseconds to seconds)

- Fidelity constraints are fundamental physics (Heisenberg uncertainty)
- Measurement destroys superposition

For shallow circuits (polynomial depth): Quantum can provide exponential advantages (Shor's, simulation)

For deep sequential operations ($O(\sqrt{N})$ or deeper): Physical barriers become insurmountable as N grows, while classical faces no such fundamental depth limit

11.7. Summary

The complete four-barrier framework establishes that oracle-based quantum search faces fundamental limitations from two independent directions:

Computational direction [4]: Three barriers eliminate advantage for most problem classes, with only cryptographic primitives as narrow theoretical exception.

Physical direction (This work): Loop Depth Barrier eliminates advantage even for the narrow theoretical exception, due to exponential scaling of sequential operation requirements versus logarithmic improvements in coherence time.

Combined result: No practical quantum advantage exists for oracle-based quantum search when all four barriers are properly accounted for. This is not a temporary limitation of current technology but a fundamental constraint arising from the combination of:

Computational and Logical requirements of oracle specification (Barriers 1-3).

Physical limits of quantum state coherence (Barrier 4).

Exponential divergence between algorithm requirements and physical capabilities.

The path forward for quantum computing lies not in overcoming these barriers (which is physically impossible for deep sequential operations) but in exploiting paradigms that naturally avoid them.

12. Conclusions

We have identified and analyzed the Quantum Loop Barrier affecting oracle-based quantum algorithms requiring deep sequential operations. These algorithms share a foundational assumption: that computation can be performed in superposition over exponentially large state spaces with arbitrary circuit depth. This assumption decomposes into five distinct physical claims:

1) Arbitrarily large superpositions can be created

2) Arbitrarily large operators can be created

3) Coherence persists through $O(\sqrt{N})$ sequential operations (Grover-type algorithms)

4) Sequential operations don't cause cumulative degradation

5) Sequential operations don't constitute measurement

Using reinforcement learning for financial trading as a concrete example, we demonstrated that:

- Current systems fall short of Loop Barrier requirements by 160× (qubits), 300× (coherence for best platforms), 1000× (fidelity)
- For superconducting qubits, coherence shortfall is ~15 million times
- The search space (3^{20} action sequences for $T = 20$) requires superpositions far beyond current capabilities
- Requirements scale exponentially with problem size (trading horizon T)
- Verification faces fundamental epistemological barriers
- These physical constraints are independent of the computational/logical barriers (Grover Dilemma, Setup Cost, Oracle Circularity) identified in companion work [4]

Most significantly, we have identified absolute physical limits that cannot be overcome by **any** technological advancement:

- **Margolus-Levitin bound [29]:** Imposes fundamental time-energy tradeoff for quantum operations
- **Heisenberg uncertainty [11]:** Limits achievable gate fidelity
- **Fundamental decoherence [16]:** Sets ultimate ceiling on coherence times from unavoidable cosmic interactions
- **Bekenstein bound:** Limits information density in finite volume
- **Landauer limit:** Constrains minimum energy per operation

These fundamental limits interact to create a constraint space that provably does not permit solutions for large-scale oracle-based quantum search algorithms. It is not merely that current technology is insufficient—fundamental physics **prevents** these algorithms from ever working at scale for problems requiring $O\left(\sqrt{N}\right)$ or deeper sequential operations when $\sqrt{N}$ exceeds physically achievable limits.

Critical Finding - Fundamental Asymmetry in Scaling:

For Grover-type algorithms with problem size $N = k^T$:

- **Qubit requirements:** Grow linearly as $O(T)$—a manageable engineering challenge
- **Coherence time requirements:** Grow exponentially as $O(k^{T/2})$—fundamental barrier
- **Gate fidelity requirements:** Become exponentially stringent as $(1-\varepsilon)^{k^{T/2}}$ must remain above threshold—fundamental barrier

This exponential divergence means that while we can build systems with more qubits through engineering advances, the exponentially growing demands on coherence and fidelity create an insurmountable barrier. The Loop Depth Barrier becomes exponentially harder to satisfy as problem size increases, revealing a fundamental scaling limit on Grover's algorithm rooted in quantum physics rather than engineering limitations.

Implications:

The combination of computational barriers [4] and physical barriers (this work) suggests that oracle-based quantum search faces fundamental obstacles from multiple independent directions:

1) **Computational barriers:** Grover Dilemma, Setup Cost, Oracle Circularity

2) **Physical barriers:** Loop Depth Barrier, coherence constraints, fidelity requirements

An algorithm must avoid ALL barriers simultaneously to achieve practical quantum advantage. For the current oracle-based quantum search on large-scale problems, this appears physically impossible.

The Path Forward: Quantum computing's future lies in paradigms that avoid deep sequential operations:

- **Shor's algorithm:** Exploits mathematical structure with polynomial circuit depth
- **Quantum simulation:** Direct physical implementation without deep loops
- **Variational algorithms:** Shallow circuits with classical optimization loops [37] [38]
- **Future paradigms:** Yet to be discovered approaches avoiding the Loop Depth Barrier

Our analysis clarifies that the limitation is not quantum computing itself, but specifically the paradigm of oracle-based amplitude amplification requiring $O(\sqrt{N})$ or deeper sequential operations on large N.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Grover, L.K. (1996) A Fast Quantum Mechanical Algorithm for Database Search. *Proceedings of the* 28*th Annual ACM Symposium on Theory of Computing*, Philadelphia, 22-24 May 1996, 212-219. https://doi.org/10.1145/237814.237866

[2] Biamonte, J., Wittek, P., Pancotti, N., Rebentrost, P., Wiebe, N. and Lloyd, S. (2017) Quantum Machine Learning. *Nature*, **549**, 195-202. https://doi.org/10.1038/nature23474

[3] Preskill, J. (2018) Quantum Computing in the NISQ Era and Beyond. *Quantum*, **2**, Article No. 79. https://doi.org/10.22331/q-2018-08-06-79

[4] Liu, Y. (2026) The Grover Dilemma and Three Fundamental Barriers to Oracle-Based Quantum Search Algorithms. *Journal of Quantum Information Science*, **16**, 1300507.

[5] Liu, Y. (2026) The Oracle Impossibility Problem: Why Oracle-Based Quantum Algorithms Cannot Solve RL Learning Problems.

[6] Sutton, R.S. and Barto, A.G. (2018) Reinforcement Learning: An Introduction. 2nd Edition, MIT Press.

[7] Arute, F., *et al.* (2019) Quantum Supremacy Using a Programmable Superconducting Processor. *Nature*, **574**, 505-510.

[8] Monroe, C., Campbell, W.C., Duan, L., Gong, Z., Gorshkov, A.V., Hess, P.W., *et al.*

(2021) Programmable Quantum Simulations of Spin Systems with Trapped Ions. *Reviews of Modern Physics*, **93**, Article ID: 025001. https://doi.org/10.1103/revmodphys.93.025001

[9] Zhong, H.-S., Wang, H., Deng, Y., Chen, M., Peng, L., Luo, Y., *et al.* (2020) Quantum Computational Advantage Using Photons. *Science*, **370**, 1460-1463. https://doi.org/10.1126/science.abe8770

[10] Ebadi, S., Wang, T.T., Levine, H., Keesling, A., Semeghini, G., Omran, A., *et al.* (2021) Quantum Phases of Matter on a 256-Atom Programmable Quantum Simulator. *Nature*, **595**, 227-232. https://doi.org/10.1038/s41586-021-03582-4

[11] Nielsen, M.A. and Chuang, I.L. (2010) Quantum Computation and Quantum Information. Cambridge University Press.

[12] Fowler, A.G., Mariantoni, M., Martinis, J.M. and Cleland, A.N. (2012) Surface Codes: Towards Practical Large-Scale Quantum Computation. *Physical Review A*, **86**, Article ID: 032324. https://doi.org/10.1103/physreva.86.032324

[13] Kim, Y., Eddins, A., Anand, S., Wei, K.X., van den Berg, E., Rosenblatt, S., *et al.* (2023) Evidence for the Utility of Quantum Computing before Fault Tolerance. *Nature*, **618**, 500-505. https://doi.org/10.1038/s41586-023-06096-3

[14] Bekenstein, J.D. (1981) Universal Upper Bound on the Entropy-to-Energy Ratio for Bounded Systems. *Physical Review D*, **23**, 287-298. https://doi.org/10.1103/physrevd.23.287

[15] Landauer, R. (1961) Irreversibility and Heat Generation in the Computing Process. *IBM Journal of Research and Development*, **5**, 183-191. https://doi.org/10.1147/rd.53.0183

[16] Zurek, W.H. (2003) Decoherence, Einselection, and the Quantum Origins of the Classical. *Reviews of Modern Physics*, **75**, 715-775. https://doi.org/10.1103/revmodphys.75.715

[17] Kjaergaard, M., Schwartz, M.E., Braumüller, J., Krantz, P., Wang, J.I., Gustavsson, S., *et al.* (2020) Superconducting Qubits: Current State of Play. *Annual Review of Condensed Matter Physics*, **11**, 369-395. https://doi.org/10.1146/annurev-conmatphys-031119-050605

[18] Gottesman, D. (1997) Stabilizer Codes and Quantum Error Correction. PhD Thesis, California Institute of Technology.

[19] Calderbank, A.R. and Shor, P.W. (1996) Good Quantum Error-Correcting Codes Exist. *Physical Review A*, **54**, 1098-1105. https://doi.org/10.1103/physreva.54.1098

[20] Barends, R., *et al.* (2014) Logic Gates at the Surface Code Threshold: Superconducting Qubits Poised for Fault-Tolerant Quantum Computing. *Nature*, **508**, 500-503.

[21] Ballance, C.J., Harty, T.P., Linke, N.M., Sepiol, M.A. and Lucas, D.M. (2016) High-Fidelity Quantum Logic Gates Using Trapped-Ion Hyperfine Qubits. *Physical Review Letters*, **117**, Article ID: 060504. https://doi.org/10.1103/physrevlett.117.060504

[22] Aharonov, D. and Ben-Or, M. (2008) Fault-Tolerant Quantum Computation with Constant Error Rate. *SIAM Journal on Computing*, **38**, 1207-1282. https://doi.org/10.1137/s0097539799359385

[23] Knill, E., Laflamme, R. and Zurek, W.H. (1998) Resilient Quantum Computation. *Science*, **279**, 342-345. https://doi.org/10.1126/science.279.5349.342

[24] Wallman, J.J. and Emerson, J. (2016) Noise Tailoring for Scalable Quantum Computation via Randomized Compiling. *Physical Review A*, **94**, Article ID: 052325. https://doi.org/10.1103/physreva.94.052325

[25] Wu, Y., *et al.* (2021) Strong Quantum Computational Advantage Using a Superconducting Quantum Processor. *Physical Review Letters*, **127**, Article ID: 180501.

[26] Schlosshauer, M. (2007) Decoherence and the Quantum-to-Classical Transition. Springer.

[27] Aharonov, Y., Albert, D.Z. and Vaidman, L. (1988) How the Result of a Measurement of a Component of the Spin of a Spin-1/2 Particle Can Turn out to Be 100. *Physical Review Letters*, **60**, 1351-1354. https://doi.org/10.1103/physrevlett.60.1351

[28] Korotkov, A.N. (2016) Quantum Bayesian Approach to Circuit QED Measurement with Moderate Bandwidth. *Physical Review A*, **94**, Article ID: 042326. https://doi.org/10.1103/physreva.94.042326

[29] Margolus, N. and Levitin, L.B. (1998) The Maximum Speed of Dynamical Evolution. *Physica D: Nonlinear Phenomena*, **120**, 188-195. https://doi.org/10.1016/s0167-2789(98)00054-2

[30] Dong, D., Chen, C., Li, H. and Tarn, T.J. (2008) Quantum Reinforcement Learning. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, **38**, 1207-1220. https://doi.org/10.1109/tsmcb.2008.925743

[31] Dunjko, V., Taylor, J.M. and Briegel, H.J. (2016) Quantum-Enhanced Machine Learning. *Physical Review Letters*, **117**, Article ID: 130501. https://doi.org/10.1103/physrevlett.117.130501

[32] Crawford, D., Levit, A., Ghadermarzy, N., Oberoi, J.S. and Ronagh, P. (2018) Reinforcement Learning Using Quantum Boltzmann Machines. *Quantum Information and Computation*, **18**, 51-74. https://doi.org/10.26421/qic18.1-2-3

[33] Rebentrost, P., Gupt, B. and Bromley, T.R. (2018) Quantum Computational Finance: Monte Carlo Pricing of Financial Derivatives. *Physical Review A*, **98**, Article ID: 022321. https://doi.org/10.1103/physreva.98.022321

[34] Orús, R., Mugel, S. and Lizaso, E. (2019) Quantum Computing for Finance: Overview and Prospects. *Reviews in Physics*, **4**, Article ID: 100028. https://doi.org/10.1016/j.revip.2019.100028

[35] Misra, B. and Sudarshan, E.C.G. (1977) The Zeno's Paradox in Quantum Theory. *Journal of Mathematical Physics*, **18**, 756-763. https://doi.org/10.1063/1.523304

[36] Facchi, P. and Pascazio, S. (2008) Quantum Zeno Dynamics: Mathematical and Physical Aspects. *Journal of Physics A: Mathematical and Theoretical*, **41**, Article ID: 493001. https://doi.org/10.1088/1751-8113/41/49/493001

[37] Farhi, E., Goldstone, J. and Gutmann, S. (2014) A Quantum Approximate Optimization Algorithm.

[38] Peruzzo, A., McClean, J., Shadbolt, P., Yung, M., Zhou, X., Love, P.J., *et al.* (2014) A Variational Eigenvalue Solver on a Photonic Quantum Processor. *Nature Communications*, **5**, Article No. 4213. https://doi.org/10.1038/ncomms5213

[39] Nayak, C., Simon, S.H., Stern, A., Freedman, M. and Das Sarma, S. (2008) Non-Abelian Anyons and Topological Quantum Computation. *Reviews of Modern Physics*, **80**, 1083-1159. https://doi.org/10.1103/revmodphys.80.1083

[40] Raussendorf, R. and Briegel, H.J. (2001) A One-Way Quantum Computer. *Physical Review Letters*, **86**, 5188-5191. https://doi.org/10.1103/physrevlett.86.5188

[41] Hoeffding, W. (1963) Probability Inequalities for Sums of Bounded Random Variables. *Journal of the American Statistical Association*, **58**, 13-30. https://doi.org/10.1080/01621459.1963.10500830

[42] Knill, E., Leibfried, D., Reichle, R., Britton, J., Blakestad, R.B., Jost, J.D., *et al.* (2008)

Randomized Benchmarking of Quantum Gates. *Physical Review A*, **77**, Article ID: 012307. https://doi.org/10.1103/physreva.77.012307

[43] Devitt, S.J., Munro, W.J. and Nemoto, K. (2013) Quantum Error Correction for Beginners. *Reports on Progress in Physics*, **76**, Article ID: 076001. https://doi.org/10.1088/0034-4885/76/7/076001

[44] Emerson, J., Alicki, R. and Życzkowski, K. (2005) Scalable Noise Estimation with Random Unitary Operators. *Journal of Optics B*: *Quantum and Semiclassical Optics*, **7**, S347-S352. https://doi.org/10.1088/1464-4266/7/10/021

[45] Breuer, H.P. and Petruccione, F. (2002) The Theory of Open Quantum Systems. Oxford University Press.

[46] Preskill, J. (1998) Lecture Notes on Quantum Computation. California Institute of Technology, Chapter 3. https://www.preskill.caltech.edu/ph229/

[47] Wiseman, H.M. and Milburn, G.J. (2009). Quantum Measurement and Control. Cambridge University Press. https://doi.org/10.1017/cbo9780511813948

[48] Viola, L., Knill, E. and Lloyd, S. (1999) Dynamical Decoupling of Open Quantum Systems. *Physical Review Letters*, **82**, 2417-2421. https://doi.org/10.1103/physrevlett.82.2417

[49] Khaneja, N., Reiss, T., Kehlet, C., Schulte-Herbrüggen, T. and Glaser, S.J. (2005) Optimal Control of Coupled Spin Dynamics: Design of NMR Pulse Sequences by Gradient Ascent Algorithms. *Journal of Magnetic Resonance*, **172**, 296-305. https://doi.org/10.1016/j.jmr.2004.11.004