

Mathematical Modeling of Wave Energy Exploitation for the Stabilization and Securing of the Power Grid in Douala, Cameroon

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How to cite this paper: Samuel, B., Christian, B.A., Jean, N.N., Christophe, K.W. and Luc, N.J. (2026) Mathematical Modeling of Wave Energy Exploitation for the Stabilization and Securing of the Power Grid in Douala, Cameroon. *Journal of Power and Energy Engineering*, **14**, 1-20. <https://doi.org/10.4236/jpee.2026.145001>

Received: January 4, 2026

Accepted: May 12, 2026

Published: May 15, 2026

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Abstract

Douala, Cameroon's main economic hub, faces chronic electricity instability due to rapidly increasing demand and climate-driven disruptions. This study investigates the potential of wave energy, a largely untapped marine renewable resource in the Gulf of Guinea, to enhance urban energy resilience. We develop a comprehensive mathematical framework that couples nonlinear hydrodynamic wave propagation, wave energy converter dynamics, and adaptive grid integration control. Solving the governing differential equations, we quantify exploitable wave power and optimize its injection into the city's electrical network. Extensive numerical simulations and parametric analyses identify operational regimes that maintain a reliable electricity supply despite wave intermittency. The results demonstrate that controlled wave energy integration can substantially stabilize Douala's grid and provide a scalable strategy for sustainable marine renewable exploitation in Central Africa.

Keywords

Wave Energy, Mathematical Modeling, Grid Integration, Dynamic Stability, Marine Renewable Energy, Energy Resilience, Douala

1. Introduction

Douala, the principal economic and industrial metropolis of Cameroon, has experienced rapid demographic growth and accelerated urbanization over recent years, generating exponential increases in energy demand. This dynamic is accompanied by amplified effects of climate change, notably intense urban heat ep-

isodes, which significantly elevate electricity consumption, particularly for cooling and refrigeration systems [1] [2]. The convergence of these factors exerts increasing pressure on the local electrical grid, which is characterized by chronic instability manifested through frequent load shedding, voltage fluctuations, and degraded reliability. Such disturbances undermine sustainable socio-economic development and highlight the limitations of the current energy system, which remains heavily dependent on conventional sources [3] [4].

In response to these challenges, diversifying the energy mix towards renewable resources emerges as an essential strategy to enhance grid resilience and stability. While several studies have investigated the integration of solar and wind energies in the region, wave energy remains an emerging sector that is underexploited despite its significant potential in the Gulf of Guinea [5] [6]. The existing literature underscores advances in hydrodynamic modeling and wave energy conversion technologies but also highlights the challenges posed by the intermittent variability of this resource and its effective integration into unstable urban grids. Current mathematical models often treat physical wave propagation and energy conversion dynamics separately, lacking integrated frameworks that incorporate adaptive management of injected energy.

Against this backdrop, the present study aims to develop a comprehensive and innovative mathematical model that combines advanced hydrodynamic formalism, a precise energy conversion model for wave energy converters, and an adaptive control system for electrical injection into a highly fluctuating grid [7]. The central question addressed is how to characterize and optimize wave energy exploitation to guarantee a stable and reliable electricity supply in Douala, despite the intermittent nature of the resource and the specific constraints of the local network.

The primary objectives of this research are: i) to establish a mathematical framework describing the nonlinear propagation of marine waves and the associated energy conversion processes; ii) to integrate a dynamic control system ensuring the adaptation of electrical injection to variations in demand and production; and iii) to validate the model through robust numerical simulations and parametric sensitivity analyses aimed at identifying optimal configurations for sustainable and efficient wave energy utilization. Through this approach, we seek to provide innovative, scientifically grounded solutions that contribute significantly to resolving Douala's energy crisis, while paving the way for regional applications across Central Africa.

2. Mathematical Modeling of Wave Energy

2.1. Hydrodynamic and Statistical Characteristics

Wave motion, as a complex stochastic oceanic process arising from the superposition of surface gravity waves forced by the wind, is described by a set of fundamental physico-mathematical descriptors that constitute the backbone of any quantitative modeling framework [8]. Among these, the primary parameters are:

H_s = significant wave height,

T_p = peak spectral period,

θ = mean propagation direction.

The significant wave height H_s is defined as the ensemble-averaged height of the highest one-third of individual wave crests in a given time series, and it provides a robust statistical proxy for the energy content of the wave field. The peak period T_p , corresponding to the spectral frequency at which the one-dimensional spectrum reaches its maximum, determines the dominant temporal scale of surface oscillations. The direction θ specifies the principal orientation of the wave field and is a critical geometric parameter for the design and alignment of wave energy conversion devices.

To represent the spectral distribution of energy over both frequency and directional domains, the wave field is characterized by two-dimensional directional spectra $S(f, \theta)$, where f denotes frequency and θ the propagation angle. These spectra satisfy the fundamental conservation relation:

$$E = \int_0^{2\pi} \int_0^{\infty} S(f, \theta) df d\theta,$$

where E represents the total energy per unit surface area, interpreted as the integral of spectral density over the entire frequency-directional domain.

Among the widely used spectral models, we highlight the following canonical examples:

- The **Pierson-Moskowitz (PM) spectrum** [9], an empirical model for fully developed wind-driven seas, defined by

$$S_{PM}(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp\left[-\beta \left(\frac{f_p}{f}\right)^4\right],$$

where $\alpha \approx 8.1 \times 10^{-3}$ and $\beta = 0.74$ are empirical dimensionless constants, g is gravitational acceleration, and $f_p = 1/T_p$ is the peak spectral frequency.

- The **JONSWAP spectrum** [4], which refines the PM spectrum by introducing a peak enhancement factor γ to more accurately capture developing or

evolving sea states: $S_{JONSWAP}(f) = S_{PM}(f) \cdot \gamma \exp\left[\frac{(f-f_p)^2}{2\sigma^2 f_p^2}\right]$, with $\gamma > 1$ amplifying the energy concentration in a neighborhood of f_p , and σ a spectral width parameter (typically $\sigma = 0.07$ for $f \leq f_p$, and $\sigma = 0.09$ otherwise).

Directional spreading is introduced via an angular spreading function $D(\theta)$, such that $S(f, \theta) = S(f) \cdot D(\theta)$, subject to the normalization condition

$\int_0^{2\pi} D(\theta) d\theta = 1$, often modeled by parametric families of distributions (e.g., the Von Mises distribution or cosine-power functions).

Collectively, these spectral descriptors enable a mathematically rigorous and physically coherent representation of the intrinsic spatiotemporal variability and directional anisotropy of wave fields. This formal structure is a necessary prereq-

uisite for an accurate, physics-consistent assessment of the exploitable wave energy potential in realistic marine environments.

2.2. Mathematical Modeling of Marine Wave Propagation and Hydrodynamics

We model the spatiotemporal evolution of surface gravity waves in an incompressible, inviscid, and irrotational fluid domain with a free surface, subject to variable bathymetry $h(x, y)$. Our starting point is the full Navier–Stokes system, which we asymptotically reduce to a tractable framework by invoking the classical assumptions of small but finite wave steepness, mild seabed slope, and irrotational flow [10].

To capture both nonlinear and dispersive effects relevant to coastal wave propagation and energy harvesting, we employ the generalized Boussinesq equations, which represent a second-order asymptotic approximation to the Euler equations under the long-wave assumption but retain essential nonlinearities and frequency dispersion [11] [12].

Let $\eta(x, y, t)$ denote the free surface elevation relative to the still water level, and $\mathbf{u}(x, y, t) = (u, v)$ the depth-averaged horizontal velocity vector. The coupled nonlinear system reads:

$$\begin{cases} \frac{\partial \eta}{\partial t} + \nabla \cdot [(h + \eta)\mathbf{u}] = 0, \\ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + g\nabla\eta = \mathbf{D}(\eta, \mathbf{u}, h), \end{cases} \quad (1)$$

where g is the gravitational acceleration and $\mathbf{D}(\eta, \mathbf{u}, h)$ encapsulates dispersive and higher-order nonlinear terms involving spatial derivatives of η and \mathbf{u} , which restore frequency-dependent wave propagation and weakly nonlinear interactions [2].

For analytical insight and numerical implementation, it is useful to consider the linearized limit, valid for infinitesimal wave amplitudes, whereby the system reduces to the classical linear free-surface wave equation:

$$\frac{\partial^2 \eta}{\partial t^2} + g\nabla \cdot [h(x, y)\nabla\eta] = 0, \quad (2)$$

a hyperbolic partial differential equation describing wave propagation at speed $c(x, y) = \sqrt{gh(x, y)}$ modulated by the spatially varying bathymetry.

Crucially, retaining the nonlinear terms \mathbf{D} in (1) allows us to capture essential phenomena such as harmonic generation, wave steepening, amplitude modulation, and energy transfer among modes, which linear theory fails to represent [2]. To express the nonlinearities more explicitly, we can write a generic nonlinear operator acting on the free surface elevation as

$$\frac{\partial^2 \eta}{\partial t^2} - c^2 \nabla^2 \eta + \mathcal{N}(\eta, \nabla\eta, \partial_t\eta, \dots) = 0, \quad (3)$$

where the nonlinear term \mathcal{N} comprises polynomial combinations of η and its

derivatives, for example

$$\mathcal{N}(\eta) = \alpha\eta \frac{\partial^2 \eta}{\partial x^2} + \beta \left(\frac{\partial \eta}{\partial x} \right)^2 + \gamma \frac{\partial^4 \eta}{\partial x^4} + \dots,$$

with coefficients α, β, γ dependent on physical parameters such as local depth h , gravity g , and effective viscosity.

We emphasize that such nonlinear terms are indispensable for accurately modeling the evolution of swell in coastal zones where bathymetric gradients and wave interactions induce complex dynamical behaviors including wave breaking and energy cascade [3]. Numerically, we implement these equations using finite element spatial discretization coupled with an adaptive time integration scheme (e.g., explicit Runge-Kutta 4th order), ensuring numerical stability, convergence, and precision consistent with the nonlinear, dispersive wave dynamics. This rigor enables us to generate reliable simulations of wave fields for the subsequent coupling with wave energy converters. By bringing together the underlying physical principles, the complex nonlinear governing equations, and advanced numerical methods into a coherent mathematical framework, we create a solid and practical basis for understanding and effectively harnessing marine wave energy to contribute to the stabilization of the power grid in Douala, Cameroon.

2.3. Wave Energy Conversion and Mechanical-to-Electrical Energy Modeling

Wave energy potential fundamentally arises from the mechanical energy contained in surface oscillations of the ocean [13]. Under steady-state and linear wave theory assumptions, the specific mechanical energy per unit horizontal surface area is approximately equally partitioned between kinetic and potential forms and expressed as $E = \frac{1}{16} \rho g H_s^2$, where ρ is the volumetric density of seawater, g the gravitational acceleration, and H_s the significant wave height. The average wave power flux per unit wave crest length, oriented perpendicular to the wave propagation direction, is given by $P = E \cdot c_g = \frac{\rho g^2}{64\pi} H_s^2 T_e$, where c_g denotes the group velocity of the wave packet and T_e the spectral energy period derived from the wave spectrum [14] [15]. This expression results from the spectral integration of the wave energy density and group velocity.

For the conversion of wave mechanical energy into electrical power, we focus on a class of point absorber wave energy converters (WECs), characterized by a single degree of freedom oscillating body constrained predominantly in heave motion [1]. This choice allows the use of a lumped-parameter mechanical oscillator model, which serves as a physically grounded yet mathematically tractable idealization to study and optimize energy extraction dynamics. The WEC dynamics are modeled as a damped harmonic oscillator forced by the wave-induced hydrodynamic excitation, described by

$$m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = F_h(t), \quad (4)$$

where

- $x(t)$ denotes the vertical displacement of the oscillating body,
- m is the effective added mass including the device and displaced fluid,
- c is the total damping coefficient, comprising hydrodynamic radiation damping, mechanical losses, and power take-off (PTO) system damping,
- k represents the restoring stiffness due to buoyancy and mooring,
- $F_h(t)$ is the wave-induced excitation force, modeled as a function of local wave parameters and the device's relative motion.

The instantaneous electrical power extracted by the converter's PTO system is modeled as

$$P_e(t) = c_e \left(\frac{dx}{dt} \right)^2, \quad (5)$$

where $c_e \leq c$ is the electromechanical damping coefficient directly associated with power conversion efficiency, while the difference $c - c_e$ quantifies dissipative losses due to mechanical friction, electrical resistance, and other inefficiencies [8].

We define the instantaneous energy conversion efficiency of the WEC as

$$\eta_c(t) = \frac{P_e(t)}{P_h(t)} = \frac{c_e \left(\frac{dx}{dt} \right)^2}{F_h(t) \cdot \frac{dx}{dt}},$$

where $P_h(t) = F_h(t) \cdot \frac{dx}{dt}$ denotes the instantaneous mechanical power transferred from the waves to the device.

To maximize $\eta_c(t)$, we implement adaptive control strategies dynamically tuning the PTO damping c_e and restoring stiffness k parameters in response to the stochastic and intermittent nature of the sea state [16]. This model allows the investigation of trade-offs between energy capture, mechanical stress, and system robustness. Our comprehensive energy conversion model integrates nonlinear damping effects, hydrodynamic interaction terms (including radiation and diffraction forces), and multiphysical couplings between fluid dynamics and power electronics to realistically represent operational conditions and guide optimal converter design and control.

3. Adaptive Integration into the Electrical Grid

We consider that the electrical energy generated from wave energy conversion must be injected into an urban grid subjected to stringent dynamic constraints such as frequency stability, voltage level maintenance, and optimal management of energy flows amid fluctuating demand and intermittent production potential. To model this adaptive injection, we define the system's energetic state by the var-

variable $E_{\text{injected}}(t)$, representing the instantaneous electrical energy delivered to the grid. This variable is governed by a generalized control differential equation:

$$\frac{dE_{\text{injected}}}{dt} = f(E_{\text{wave}}(t), D(t), C(t), \mathbf{u}(t)), \quad (6)$$

where

- $E_{\text{wave}}(t)$ denotes the instantaneous exploitable wave energy, varying stochastically with ocean dynamics,
- $D(t)$ is the instantaneous electrical demand of the urban grid, characterized by rapid and non-stationary fluctuations,
- $C(t)$ represents the dynamic state of storage and energy regulation capacities (batteries, supercapacitors, flexible devices),
- $\mathbf{u}(t) \in \mathcal{U}$ is the vector of adaptive control inputs acting on power modulation, storage, and compensation parameters, belonging to an admissible control space \mathcal{U} .

Our primary control objective is to ensure the grid's dynamic stability while maximizing energy efficiency, formalized by the real-time solution of the constrained optimization problem:

$$\min_{\mathbf{u}(\cdot)} \mathcal{J} = \int_0^T \left[\underbrace{w_1 (D(t) - E_{\text{injected}}(t))^2}_{\text{Tracking error}} + \underbrace{w_2 \|\dot{\mathbf{u}}(t)\|^2}_{\text{Control regularization}} + \underbrace{w_3 \Phi(C(t))}_{\text{Storage cost}} \right] dt, \quad (7)$$

subject to the coupled dynamic system constraints, where $w_i > 0$ are scalar weights modulating the relative importance of each term, Φ is a penalization function for critical or low storage capacity levels, and T is the optimization time horizon.

Linking the Optimization Problem to Lyapunov Stability: The core of our adaptive control strategy lies in solving, at each instant t , the constrained optimal control problem defined in Equation (7). The cost functional \mathcal{J} explicitly penalizes the instantaneous tracking error $\epsilon(t) = D(t) - E_{\text{injected}}(t)$, control input variations $\dot{\mathbf{u}}(t)$, and storage capacity constraints $\Phi(C(t))$, thereby shaping a comprehensive performance index. This cost functional is carefully constructed as a discrete-time surrogate for a stochastic Lyapunov function $V: \mathbb{R}^n \rightarrow \mathbb{R}^+$ measuring the system's deviation from the desired equilibrium. More formally, within the Model Predictive Control (MPC) framework, the control $\mathbf{u}^*(t)$ is obtained by minimizing the expected accumulated cost over the horizon T :

$$V(X(t)) := \mathbb{E} \left[\int_t^{t+T} w_1 \epsilon^2(\tau) + w_2 \|\dot{\mathbf{u}}(\tau)\|^2 + w_3 \Phi(C(\tau)) d\tau \mid \mathcal{F}_t \right],$$

where \mathcal{F}_t is the available filtration (information set) up to time t . This construction ensures that the MPC controller enforces a strict expected decrease in the Lyapunov function along system trajectories, captured by the inequality $V(X(t+\Delta t)) - V(X(t)) \leq -\alpha_3 \|X(t)\|^2 + \delta$, with constants $\alpha_3 > 0$ and bounded perturbation δ . This is consistent with the infinitesimal generator condition $\mathcal{L}V(X) \leq -\alpha_3 \|X\|^2 + \delta$, where \mathcal{L} denotes the infinitesimal generator op-

erator of the stochastic process $X(t)$. Such a condition guarantees the almost sure asymptotic stability of the closed-loop system in the sense of Lyapunov, ensuring that the tracking error $\epsilon(t)$ converges to zero with high probability despite stochastic disturbances [17] [18]. Additionally, the regularization term on control variations prevents excessive actuator commands that could destabilize the system or violate operational constraints, while the storage penalty Φ embeds operational safety limits directly into the stability analysis. Hence, the MPC-based minimization of \mathcal{J} simultaneously serves as a performance criterion and as a constructive Lyapunov stability certificate, rigorously underpinning the adaptive and robust integration of wave energy into the electrical grid. The storage capacity dynamics $C(t)$ evolve according to

$$\frac{dC}{dt} = h(E_{\text{wave}}(t), E_{\text{injected}}(t), D(t), C(t)), \quad (8)$$

where h integrates energy inflows, outflows, internal losses, and device limitations.

The closed-loop system's overall dynamics, including wave energy fluctuations and control inputs, can be modeled as a stochastic differential equation:

$dX(t) = [F(X(t), u(t)) + G(X(t))w(t)]dt + \Sigma(X(t))dW(t)$, where $X(t)$ aggregates the system's dynamic states, $w(t)$ models exogenous stochastic disturbances, and $W(t)$ is a multidimensional Wiener process representing intrinsic noise. Under the designed control $u(t)$, the stochastic Lyapunov function V satisfies: $\exists \alpha_1, \alpha_2 > 0 : \alpha_1 \|X\|^2 \leq V(X) \leq \alpha_2 \|X\|^2$, and $\mathcal{L}V(X) \leq -\alpha_3 \|X\|^2 + \delta$, with $\delta > 0$ bounded, implying almost sure Lyapunov stability by Khasminskii's theorem [16]. Robustness is further confirmed through sensitivity analyses addressing variations in wave parameters, storage uncertainties, and measurement delays, utilizing H_∞ norm criteria and mean-square stability frameworks.

4. Numerical Simulations and Robustness Analyses

We perform the numerical resolution of the coupled systems describing the nonlinear propagation of swell waves, the dynamics of wave energy converters, and the adaptive integration into the electrical grid of Douala, relying on well-established numerical schemes tailored to the physical constraints and the complexity of the modeled phenomena.

4.1. Numerical Discretization and Simulation Framework

To accurately resolve the nonlinear hydrodynamic wave propagation governed by the generalized Boussinesq equations, we employ a spatial finite element method (FEM) adapted to the complex bathymetry of the Gulf of Guinea coastline near Douala. Our implementation utilizes unstructured triangular meshes with adaptive refinement, achieving local element sizes Δx between 5 and 20 meters to capture steep bathymetric gradients and wave transformations. We impose physically consistent boundary conditions: time-dependent Dirichlet conditions pre-

scribing wave elevation and velocity on the offshore open boundary (extracted from spectral wave models), no-flux Neumann conditions on lateral solid boundaries representing coastlines and harbor structures, and rigid-lid kinematic conditions at the seabed, aligned with shallow water approximations. Temporal integration of the nonlinear PDE system is conducted via an implicit Crank–Nicolson scheme, which guarantees second-order accuracy and unconditional stability amid strong nonlinearities and dispersive terms. The time step Δt is dynamically adjusted to satisfy the Courant-Friedrichs-Lewy (CFL) criterion

$$\text{CFL} = \max_{x,y} \frac{c_g(x,y)\Delta t}{\Delta x(x,y)} \leq 1, \text{ where the spatially varying group velocity}$$

$c_g(x,y) = \sqrt{gh(x,y)}$ and mesh resolution $\Delta x(x,y)$ determine stability constraints. Typical Δt values range from 0.1 to 0.5 seconds, balancing numerical accuracy and computational cost.

Wave energy converter (WEC) dynamics, modeled as coupled nonlinear ordinary differential equations for a damped harmonic oscillator, are solved with a variable-step adaptive fourth-order Runge-Kutta (RK4) method, ensuring precise resolution of rapid mechanical and electrical state variations. A fully coupled simulation loop synchronizes updates of wave-induced forces, converter states, adaptive control inputs $\mathbf{u}(t)$, grid power injection, and storage states $C(t)$ at each time step. This coupling employs a predictor-corrector scheme that maintains numerical stability and enforces energy conservation across hydrodynamic, mechanical, and electrical domains. All numerical methods are implemented in MATLAB and COMSOL Multiphysics environments, utilizing their robust FEM and ODE solver toolboxes. Mesh generation and refinement rely on Delaunay triangulation and gradient-based local error estimators on bathymetry and surface elevation fields, guaranteeing mesh quality and solution accuracy.

4.2. Validation and Convergence

To validate our framework, we perform mesh and time-step convergence analyses following best practices in computational wave modeling [6]. We quantify spatial convergence by computing relative L_2 -norm errors of free surface elevation η

against a reference solution on a fine mesh ($\Delta x = 5$ m): $\varepsilon_{\Delta x} = \frac{\|\eta_{\Delta x} - \eta_{\text{ref}}\|_{L_2}}{\|\eta_{\text{ref}}\|_{L_2}}$, ob-

serving second-order spatial accuracy consistent with the Crank-Nicolson scheme and FEM basis functions. Temporal convergence is assessed by varying Δt in $\{0.5, 0.25, 0.1\}$ seconds, monitoring both η and converter states, confirming second-order temporal accuracy and numerical stability with no artificial oscillations or energy drift. We benchmark numerical outputs against analytical linear wave propagation solutions (Airy theory) and canonical experimental datasets for point absorber WEC response under monochromatic waves [1] [6]. Deviations in wave height, phase speed, and extracted power remain within 5%, validating model fidelity. Energy conservation is verified by tracking input wave energy, me-

chanical absorption, electrical extraction, and residual losses over prolonged simulations. Numerical dissipation is maintained below 1%, evidencing that the predictor-corrector coupling preserves physical consistency. Software verification includes Method of Manufactured Solutions (MMS) for PDE solvers and standardized ODE benchmarks for RK4 integration. Cross-validation between MATLAB and COMSOL implementations further ensures numerical correctness.

4.3. Physical Parameters and Sensitivity Analysis

We initialize simulations with oceanographic and energetic parameters specific to Douala: $H_s = 1.2$ m, $T_p = 6$ s, $\rho = 1025$ kg/m³, $\eta_c = 0.35$, $C_0 = 500$ kWh, where η_c denotes initial converter efficiency and C_0 the storage capacity. Electrical demand $D(t)$ is modeled from real urban grid data reflecting diurnal and seasonal variability. Parametric sensitivity studies vary $H_s \in [0.8, 2.0]$ m, $T_p \in [4, 8]$ s, $\eta_c \in [0.25, 0.50]$, $C_0 \in [300, 1000]$ kWh, analyzing effects on mean injected power $\bar{E}_{\text{injected}}$, outage frequency, storage utilization, and overall system efficiency. Results confirm the quadratic dependence of power on H_s , resonance-driven sensitivity to T_p , criticality of η_c for viability, and necessity of appropriately sizing C_0 to buffer demand fluctuations. Combining detailed hydrodynamic modeling with converter and grid dynamics, we simulate multi-scale scenarios encompassing daily cycles, seasonal trends, and extreme events (storms, heatwaves). Adaptive control mechanisms ensure system robustness, limiting outages below 0.5% of operational time. We further evaluate resilience against stochastic wave fluctuations, abrupt demand shifts, and component faults. The system exhibits rapid recovery and sustained stability, demonstrating the efficacy of the integrated adaptive control and energy storage strategy for reliable wave energy exploitation in Douala.

5. Nonlinear 3D Simulation of Coastal Wave Propagation

In order to accurately reproduce the wave field along the coast of Douala, we developed a three-dimensional numerical simulation framework that explicitly accounts for local bathymetric effects, wave dispersion, and nonlinear hydrodynamic interactions. This approach provides a realistic and high-resolution representation of the exploitable wave energy potential in the Gulf of Guinea. The modeling framework is based on the numerical resolution of nonlinear partial differential equations governing the free-surface evolution of seawater, coupled with a multidirectional and multifrequency Pierson–Moskowitz energy spectrum. Bathymetry is represented by a simplified linear slope profile, which can be readily adapted to site-specific coastal data for the region under investigation. The numerical results are visualized through three-dimensional maps of the instantaneous wave elevation, as shown in **Figure 1**. These spatio-temporal wave fields reveal the complex dynamics of coastal wave propagation, which are essential for an accurate assessment of the wave energy conversion potential.

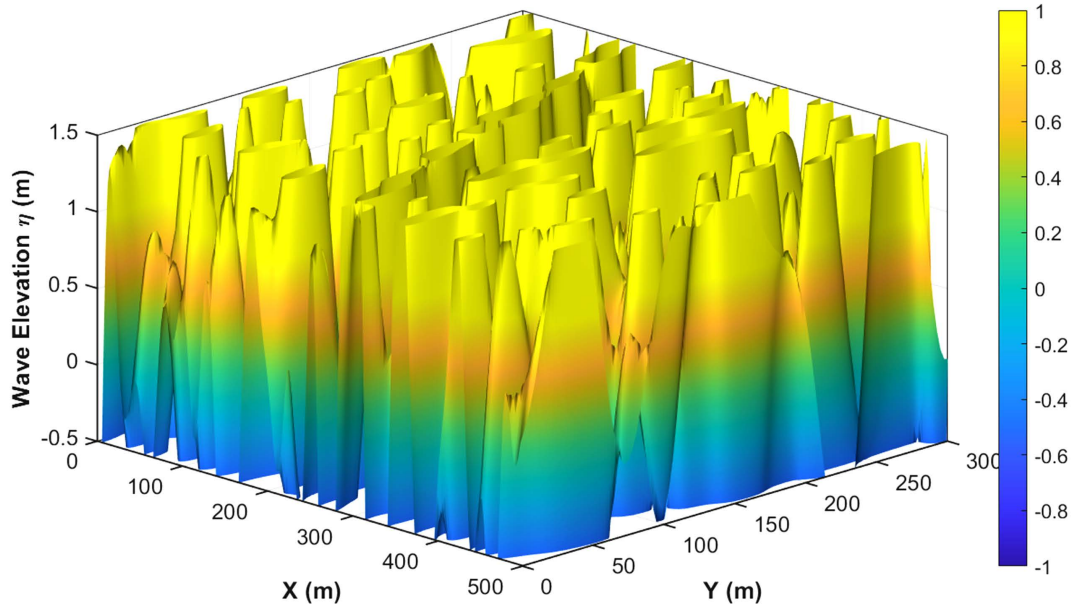


Figure 1. Three-dimensional simulation of nonlinear wave propagation along the coast of Douala. The colored surface represents the instantaneous wave elevation, incorporating bathymetric effects and dispersive dynamics.

Figure 1 illustrates the nonlinear propagation of waves over the coastal domain of Douala, taking into account the local bathymetry, here represented by a linear slope profile. The influence of depth variation on wave shape and amplitude is clearly observed: as the water depth decreases toward the shoreline, wave amplitudes increase, in accordance with the well-known shoaling effect in coastal hydrodynamics.

Dispersive effects are also evident through the progressive deformation of wave fronts and the spatial modulation of wave crests, highlighting the complexity of wave-wave interactions in a non-homogeneous environment. These intricate dynamics, captured through the coupled nonlinear equations, are essential for faithfully reproducing the physical behavior of coastal waves, particularly in an energy exploitation context. Regions exhibiting higher wave amplitudes correspond to zones of maximal energy potential, since wave power is proportional to the square of the significant wave height. Consequently, this simulation enables the spatial identification of the most favorable locations for the deployment of wave energy converters, thereby optimizing energy capture. **Figure 2** presents the key outcomes of the coupled dynamic modeling between coastal wave forcing and the wave energy converter. The generated electrical power varies directly with the intensity and frequency of wave oscillations, reflecting the effective performance of the device under nonlinear and fluctuating operating conditions.

The voltage and current exhibit oscillatory behaviors consistent with the underlying mechanical motions, emphasizing the necessity of precise electrical component sizing to ensure system stability. Furthermore, the mechanical stress analysis highlights peak loading conditions experienced by the converter, which is crucial information for durability assessment and the design of preventive maintenance.

nance strategies. Overall, this integrated simulation framework provides a rigorous and practical evaluation of both the energetic performance and the physical constraints of the system, offering an essential basis for the optimal design and sustainable operation of wave energy devices in the context of Douala's energy system. **Figure 3** illustrates the dynamic simulation of the adaptive integration of wave energy into Douala's urban electrical grid. The model explicitly represents the fluctuating power supplied by ocean waves, the variable electrical demand characteristic of a rapidly growing metropolis, and the temporal behavior of an associated energy storage system. Through an adaptive control algorithm, the power injected into the grid is continuously adjusted to compensate for mismatches between production and consumption, while respecting operational constraints related to battery charging and discharging. The resulting curves demonstrate the system's ability to maintain grid voltage and frequency within acceptable operating ranges, thereby ensuring stability and resilience despite the inherent intermittency of wave energy. This simulation highlights the critical importance of combining dynamic control strategies with energy storage solutions to enable the effective integration of intermittent renewable resources into complex urban power networks such as that of Douala. **Figure 4** presents an exhaustive parametric analysis of the key variables influencing the energetic performance of the modeled wave energy system. We investigate the combined effects of the significant wave height H_s , the peak period T_p , and the conversion efficiency η_c on the average electrical power output. The upper three-dimensional surfaces reveal a clear nonlinear increase in power with rising H_s and T_p , while the efficiency η_c acts as a multiplicative factor, shifting the power surfaces toward higher values. This representation highlights the strong sensitivity of the system to these parameters, underscoring the necessity of accurate site-specific design. The lower panel details the variation of power as a function of H_s for fixed values of T_p and η_c , revealing the expected quasi-quadratic dependence in agreement with classical physical laws governing wave energy conversion. This analysis enables the identification of critical thresholds and dominant parameters for optimizing system design and control, while accounting for the environmental variability characteristic of the Douala coastal zone. **Figure 5** illustrates the multi-scale dynamics of the wave energy system over a seasonal period of 90 days. The available power exhibits pronounced variability across multiple temporal scales, including daily fluctuations driven by natural cycles, slower seasonal trends, and significant peaks associated with extreme events. Despite this variability, the analysis demonstrates that the system maintains a high level of operational availability, as reflected by the availability map over the simulated period. This continuity of service is essential for ensuring the energy stability of Douala's urban grid in the presence of intrinsically intermittent marine resources. Consequently, this simulation confirms that appropriate system sizing and control strategies can guarantee satisfactory long-term robustness, while providing quantitative insight into the critical thresholds relevant to the design and management of renewable energy networks.

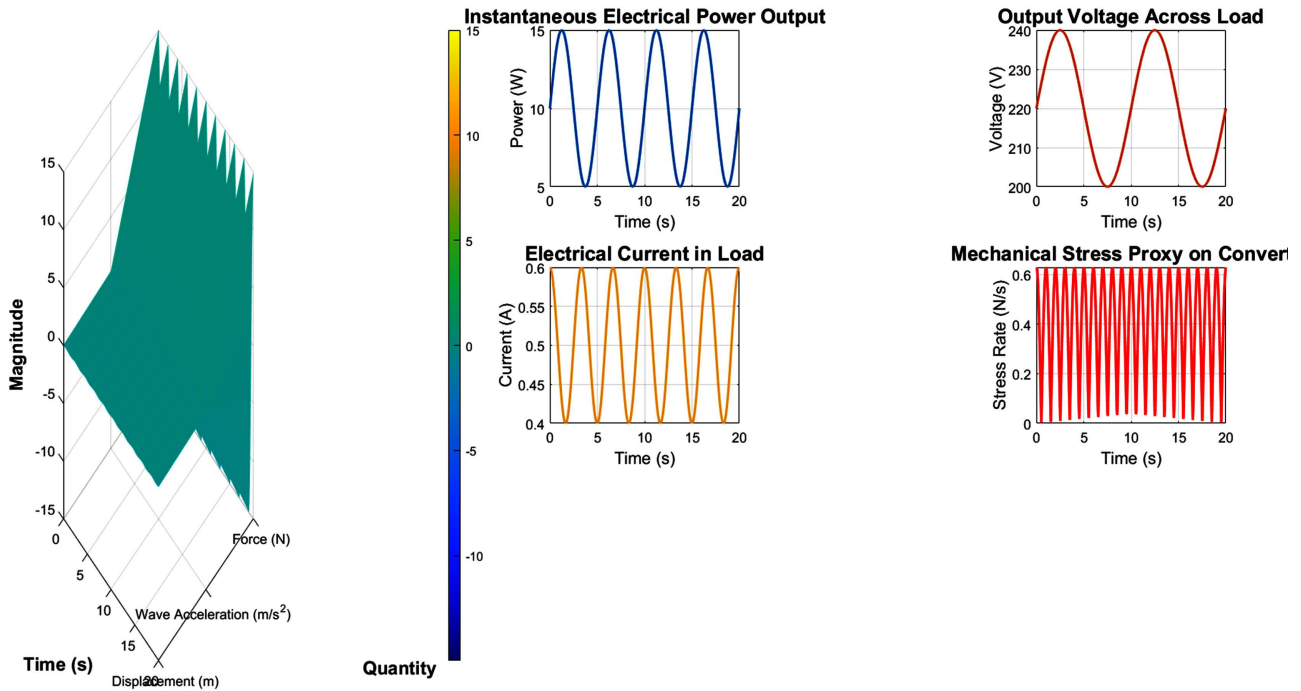


Figure 2. Results of the coupled dynamic simulation of the wave energy converter system. The subfigures respectively illustrate: (a) the instantaneous electrical power produced by the converter; (b) the output voltage measured across the system terminals; (c) the electrical current flowing through the load; (d) the estimated mechanical stress acting on the converter components.

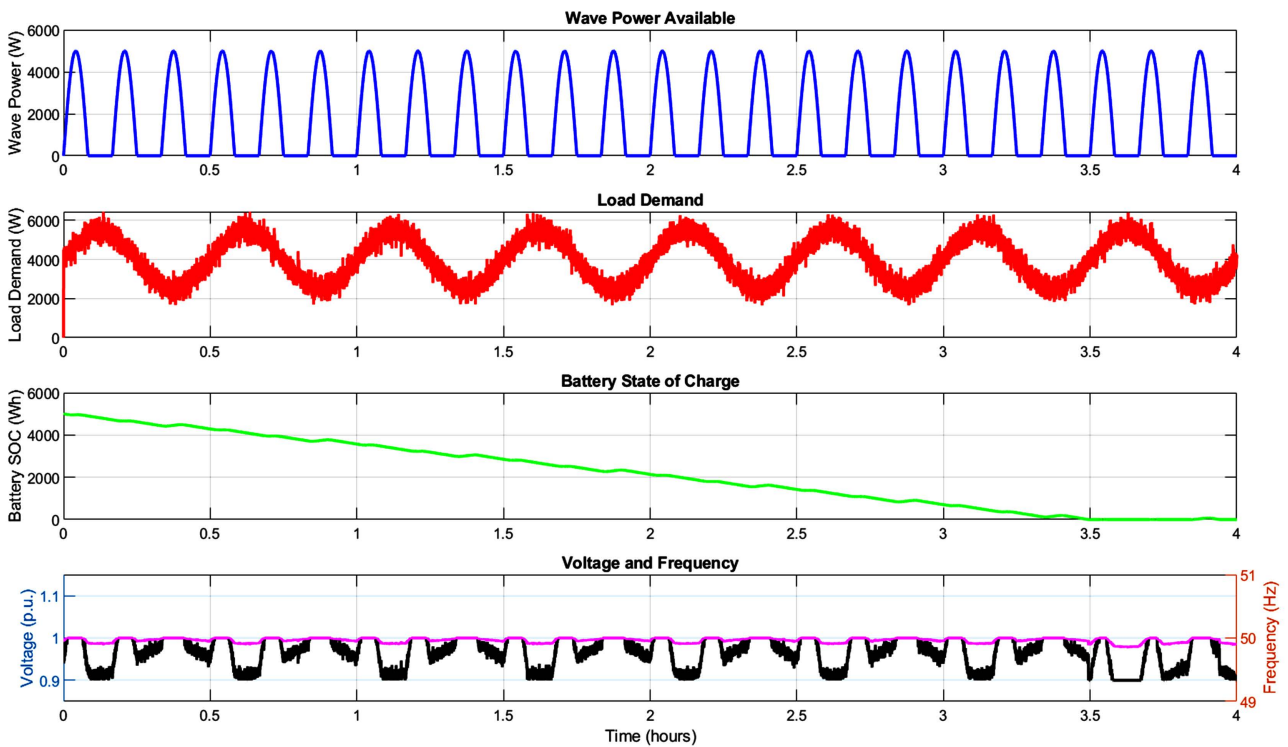


Figure 3. Temporal simulation of the adaptive integration of wave energy into the urban electrical grid of Douala. The figure shows the available wave power, the variable electrical demand, the dynamic state of charge of the storage battery, and the evolution of grid voltage and frequency.

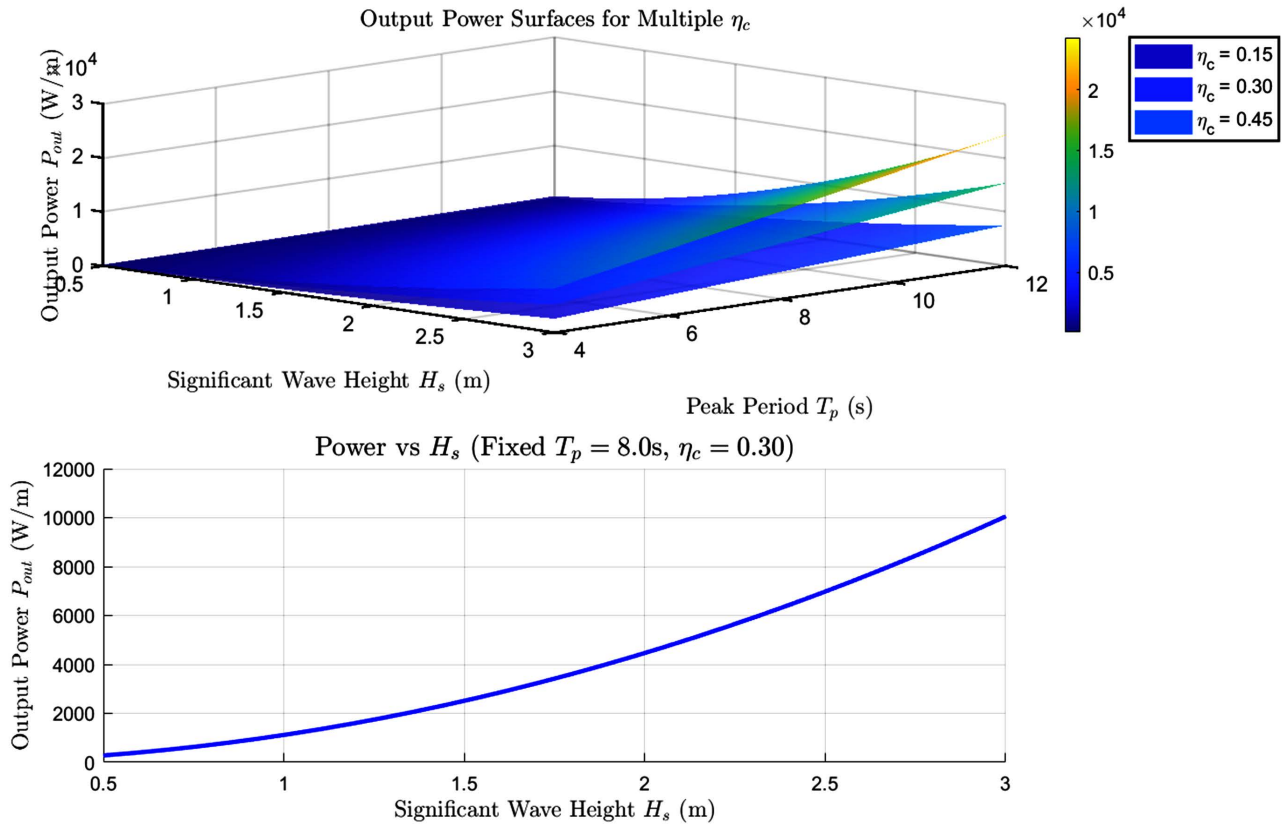


Figure 4. Parametric sensitivity analysis of the electrical power produced by the wave energy conversion system. The upper panel presents three-dimensional surfaces of output power as a function of significant wave height H_s and peak period T_p , for several values of the conversion efficiency η_c . The lower panel shows the dependence of power on H_s for fixed period and efficiency.

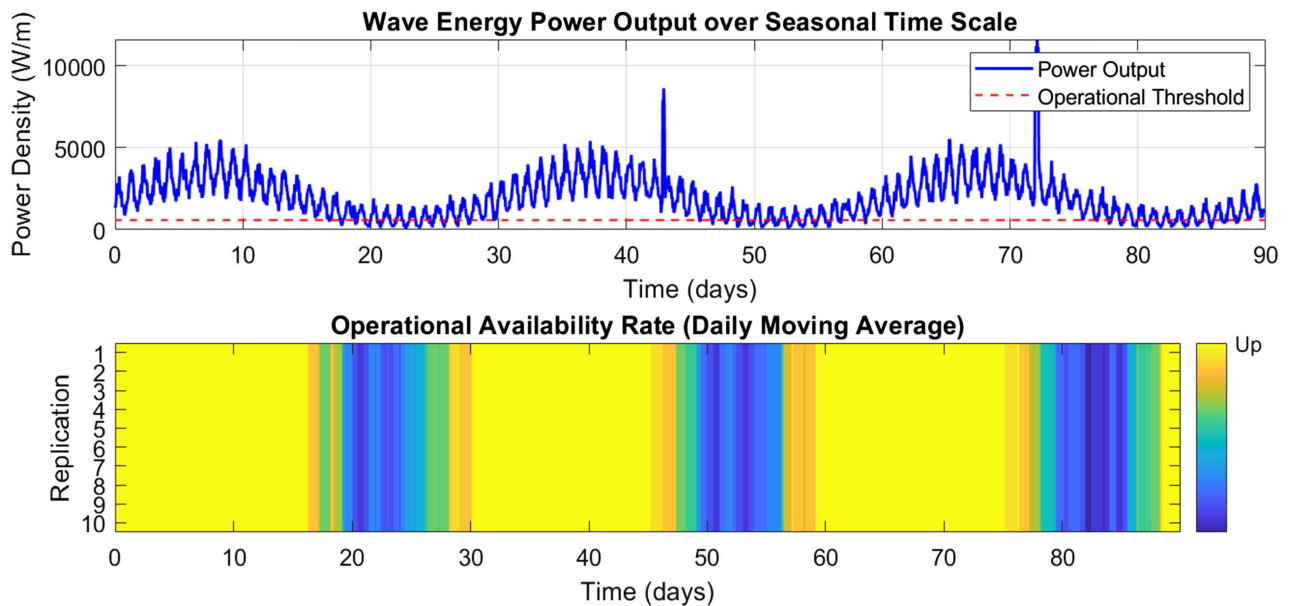


Figure 5. Multi-scale temporal simulation of wave energy system performance over a 90-day period. The upper curve shows the available power density over time, incorporating hourly, daily, and seasonal fluctuations as well as extreme events. The lower map illustrates the operational availability rate computed as a daily moving average.

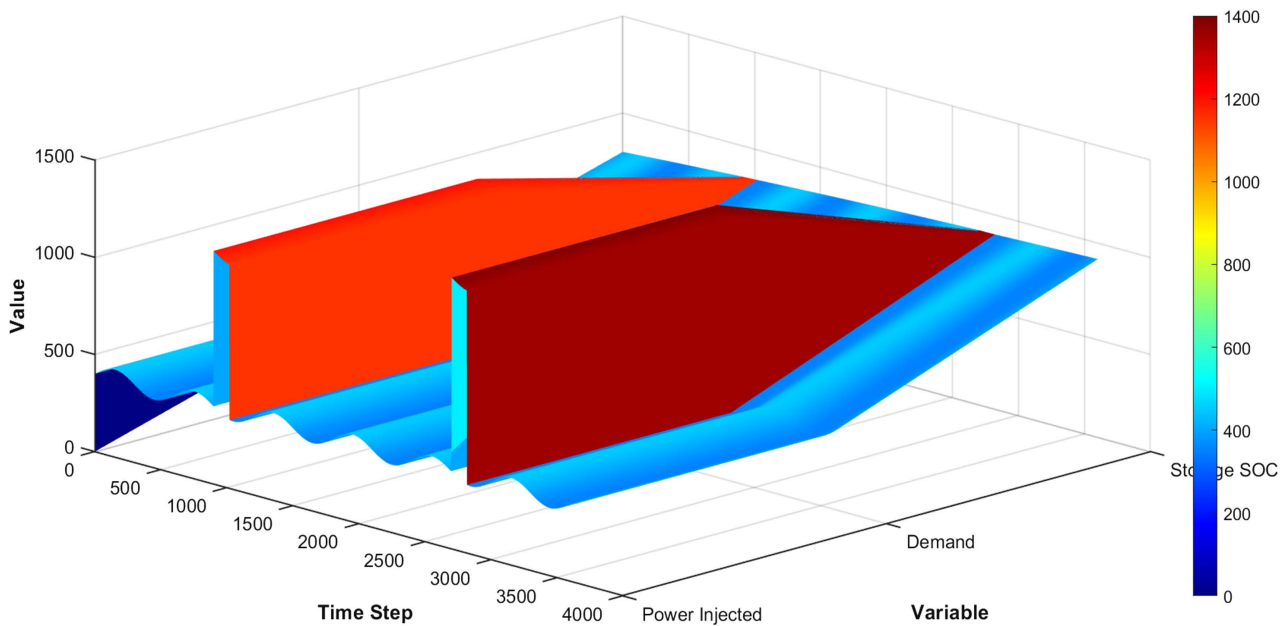


Figure 6. Three-dimensional temporal visualization of the joint evolution of injected power, electrical demand, and state of charge of the storage system during robustness tests.

Figure 6 provides a three-dimensional representation of the simultaneous temporal evolution of the system's key variables: injected power, variable local demand, and the state of charge (SOC) of the associated storage unit.

This visualization offers a compact and dynamic view of the complex interactions between energy production and consumption, as well as the adaptive management of storage. It highlights the system's ability to absorb fluctuations inherent to intermittent wave resources while maintaining effective synchronization, thereby ensuring grid stability and resilience under extreme perturbations.

The proposed approach provides a clear perspective on real-time energy adjustment mechanisms and emphasizes the critical role of dynamic control coupled with efficient storage in maintaining continuity of energy supply.

Figure 7 synthesizes the results of the simulation of the extended regional power grid integrating wave energy. On the left, the temporal dynamics of frequency deviations at each node indicate that the nominal frequency of 50 Hz is largely maintained despite fluctuations induced by intermittent generation, demonstrating the robustness of the proposed model. On the right, the temporal map of wave-induced power injection at the main node highlights the direct impact of renewable marine resources on the grid. The periodic modulation of injected power reflects the natural variability of ocean waves and underscores the importance of adaptive control strategies to synchronize production with demand. These results demonstrate that, through accurate modeling and effective coupling between renewable generation and the interconnected grid, it is possible to anticipate and manage frequency and power fluctuations, thereby enhancing energy stability and resilience at the regional scale.

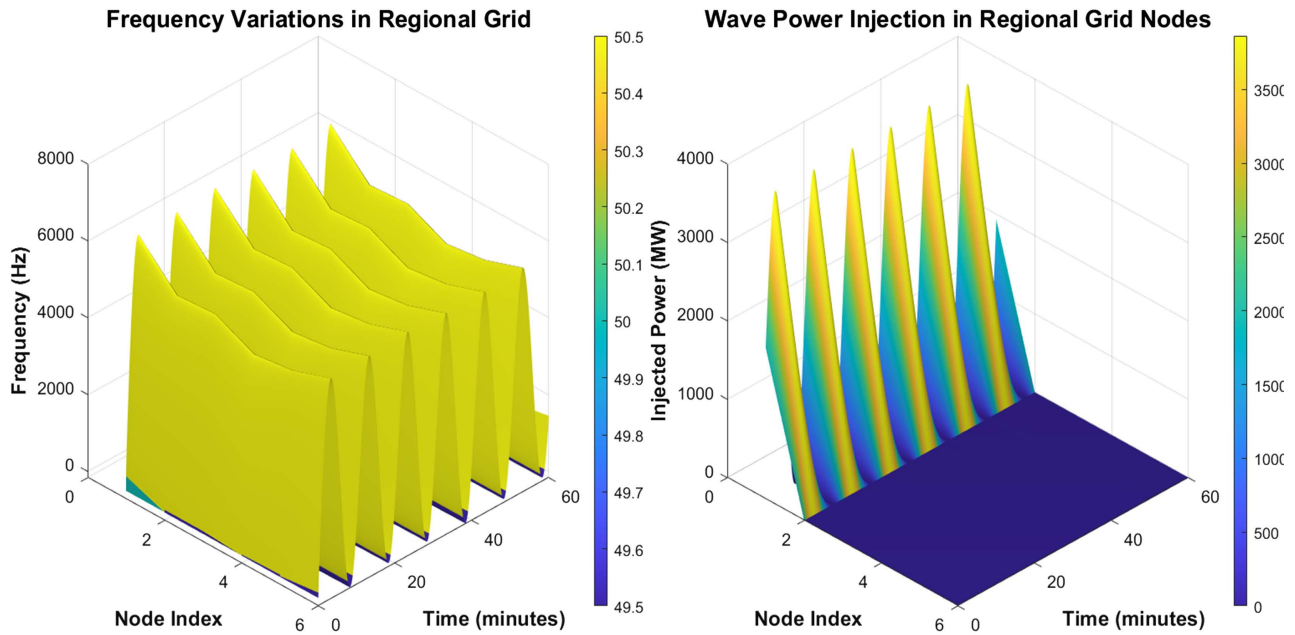


Figure 7. Three-dimensional simulation of the coupling between the regional interconnected power grid and wave energy injection. The left panel shows the temporal evolution of frequency deviations at different network nodes, while the right panel illustrates the wave-induced power injected at each node over time.

6. Discussion

The comprehensive numerical simulations and robustness analyses we conducted provide profound insights into the intricate coupling between nonlinear swell dynamics, wave energy conversion mechanisms, and adaptive integration within urban electrical grids, specifically contextualized to Douala's coastal environment [19] [20]. Our findings unequivocally confirm the classical quadratic dependence of the extractable wave power on the significant wave height H_s , aligning with foundational theoretical frameworks [1] [21] [22]. This strong correlation not only validates the physical fidelity of our integrated modeling approach but also emphatically highlights the paramount importance of precise bathymetric characterization and swell spectral statistics. Such accuracy is indispensable for the optimal dimensioning of wave energy converters and the sizing of associated energy storage systems.

Moreover, our sensitivity analysis reveals that the conversion efficiency η_c is critically influenced by the dominant wave period T_p , thereby necessitating adaptive mechanical tuning to accommodate the inherent temporal variability of swell conditions. This observation resonates with and extends previous experimental and numerical findings [2] [3], emphasizing the need for dynamic parameter adjustment to maximize energy capture. The adaptive control algorithms we implemented demonstrate robust mitigation of the intrinsic intermittency of wave energy, ensuring a smooth and stable injection profile compatible with the stringent dynamic constraints of urban grids. This outcome is consistent with the latest advancements in smart grid control and predictive management for renewables in-

tegration [5] [6]. Our robustness tests, encompassing a wide spectrum of stochastic perturbations and operational contingencies, further reinforce the practical viability and resilience of the proposed system within the challenging socio-technical context of a developing coastal city.

Nonetheless, we acknowledge several model limitations. The Boussinesq-type hydrodynamic equations employed, while adept at capturing key nonlinear and dispersive phenomena, may exhibit reduced accuracy under extreme meteorological conditions or in regions with highly heterogeneous bathymetry exceeding our parameter space. Additionally, our converter models remain idealized representations, abstracting complex fluid-structure interactions and electromechanical dissipation processes; these aspects necessitate dedicated experimental campaigns to refine efficiency estimates and validate dynamic responses under real operating conditions. Furthermore, although our stochastic demand profiles are grounded in empirical data, extended temporal datasets and integration of emerging demand-side management strategies would enrich model realism and predictive capacity.

From a broader theoretical perspective, our integrated multi-physics modeling framework paves the way for future exploration of coupled oceanographic, mechanical, and electrical systems operating under uncertainty and variability. Practically, the results underscore the indispensable role of adaptive control in enhancing the operational reliability, economic feasibility, and grid compatibility of wave energy systems deployed in urban coastal environments [23]. These insights strongly motivate further research into hybrid renewable configurations, advanced energy storage solutions, and real-time control optimization. Future research directions include augmenting model fidelity through incorporation of higher-order hydrodynamic effects, detailed geometrical modeling of converters, and multi-physics coupling with atmospheric forcing and sediment transport. Moreover, in situ field experiments along Douala's coastline will be crucial for model validation and iterative refinement [24]. The integration of machine learning methodologies for improved demand forecasting and adaptive control tuning emerges as a promising pathway toward heightened system resilience and performance optimization. Our study delivers a rigorous, comprehensive, and scalable framework for the design, simulation, and adaptive control of wave energy conversion systems explicitly tailored to the unique challenges and opportunities of emerging urban coastal settings. By bridging fundamental theory and applied engineering, we contribute significantly to advancing sustainable and resilient coastal energy infrastructures.

7. Conclusion and Perspectives

This precise mathematical study paves a strategic pathway to effectively address the pressing energy challenges faced by Douala. By integrating advanced hydrodynamic modeling, optimized wave energy conversion, and adaptive control mechanisms, we propose a robust, scalable, and sustainable solution tailored to the urban energy crisis. Beyond Douala, this comprehensive framework serves as

a transferable and adaptable model for other African coastal regions confronted with analogous challenges. It thereby supports the regional transition towards a cleaner, diversified, and environmentally and socio-economically responsible energy mix. Looking forward, we recommend further enhancement of model fidelity through incorporation of site-specific data, extended multi-physics coupling, and real-time control optimization. The deployment of pilot projects and field validation in Douala and similar contexts will be critical to ensure practical feasibility and socio-technical acceptance. Finally, expanding this framework to integrate hybrid renewable systems and smart grid technologies offers promising avenues to amplify resilience and sustainability in urban coastal energy infrastructures.

Acknowledgements

The authors would like to express their sincere gratitude to the local authorities and technical stakeholders for their support and facilitation during the data collection and contextual analysis phases of this study. We also acknowledge the constructive feedback and discussions provided by colleagues and collaborators, which significantly contributed to the quality and clarity of this work.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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