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Atomic Quantum Field Theory and AString Quantum Gravity Based on Atomic String Functions

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Abstract

Finite Atomic String Functions exactly composing the polynomials of any order can be the foundation of new AString Quantum Gravity and AString theories. They offer the common mathematical blocks to reconcile General Relativity with Quantum Field theories to contribute to the resolution of the quantum gravity problem. "Atomic theory" was envisaged by A. Einstein in 1933. The Atomic Series decomposes quantum fields into Atomic Quanta of energies for Standard Model particles, which can be treated as the distortions of spacetime. The Atomic Spacetime theory leads to gravitons and atomized quantization of gravity, upholding the quantization rules of Quantum Mechanics. A unified description of fields offers the prospects of a new AString model of string theory. AStrings can either extend into spacetime as open strings or compose "solitonic atoms" of matter as closed strings/loops. The theory offers a simple local atomic network model of nature to compose fields; for example, 9 interacting atoms produce parabolic fields.

Keywords

Atomic Function, Quantum Field Theory, Quantum Gravity, Graviton, String Theory, Unified Theory

1. Introduction and Main Ideas

1.1. Main Ideas

This overview paper continues original works [1]-[9] started in 2017 on expansion of the theory of Atomic String Functions (ASF) [1]-[19] evolving since the 1970s to the fundamental theories [20]-[43] including General Relativity (GR) [1] [8] [33], Quantum Mechanics (QM) [9], and now Quantum Field Theories (QFT), Quantum Gravity (QG) and String theories [21]-[31]. ASF are the finite functions possessing the unique feature to *exactly* compose the polynomials of *any* order from finite "solitonic atoms" resembling quanta/excitations/strings/loops. Finite ASF functions become the base of the Atomic Series [1]-[5], generalizing widely-used Taylor series, to compose a wide range of analytic functions and solutions of differential equations of mathematical physics. Like in string [28]-[31] and Loop Quantum Gravity (LQG) [26] [27], the theory offers a new theorem-based approach to introduce desired finiteness into GR, QFT, and QM theories traditionally operating with infinitely-spread functions based on space points. It offers a new Atomic Quanta model for the fundamental particles from the Standard Model (§6), with inclusion of quantum sizes into the definitions of energy while upholding the QM quantization rules. In application to GR, it leads to the Atomic Spacetime theory [1]-[8] and the new model of a graviton. Using the Atomic Series for both quantum and spacetime fields allows deriving "common blocks" to link GR and Quantum Theories within a framework of AString theory and AString Quantum Gravity, which can help to resolve the quantum gravity problem [26] [27] included in the list [41] of unresolved problems in physics. Hope this theory, coming from a mathematical school, would attract the attention of physicists and string theorists to the unique apparatus of atomic functions naturally describing the finite building blocks of nature and fields as shown in this paper.

1.2. "Atomic Theory" of A. Einstein (1933)

The long-standing problem of reconciling GR with Quantum Theories [41] is related to the mathematical incompatibility of nonlinear GR with linear QM, or simply the absence of "common mathematical blocks" between the theories [8]. The root cause of it is the use of continuous mathematical analysis based on "point in space" dots and infinitely spread functions [8], sometimes leading to unphysical infinities like GR singularities, QM wavefunction collapse into "a point", QFT infinities renormalization. As string theorist M. Kaku mentioned [31] [43], it is hard to find the commonality between "dots". String and LQG theories [26]-[31] appeared to replace them with finite strings and loops.

This problem was well understood by A. Einstein, who in his 1933 lecture [25] classified it as "*stumbling blocks*" of theories operating "…*exclusively with continuous functions of space*". Interestingly, in the same lecture he envisaged the solution—"*perfectly thinkable*" "*atomic theory*" with "*finite regions of space*" discussed in [1]-[3] and especially [8]. Development of this "atomic theory" based on atomic functions started in 2018 [1]-[9] is the focus of this paper.

2. Brief History of Atomic String Functions

2.1. Discovery of the Finite Function $up(x)$ (1967-1971)

The theory of Atomic Functions (AF) [1]-[19] has been evolving since 1967-1971 when a distinguished mathematician, the author teacher, Academician of NAS of Ukraine V.L. Rvachev[1] (Ukrainian V.L. Rvachov) had proposed a finite pulse function $up(x)$ for which derivatives (also pulses) would conveniently be similar to the original pulse shifted and stretched by the factor of 2:

$$up'(x) = 2up(2x+1) - 2up(2x-1), |x| \le 1 : up(x) = 0, |x| > 1. \quad (2.1)$$

As noted in the historical survey [14], some elements or similar functions were probably known earlier in 1935, Fabius [44] in 1966, and have been getting reinvented by different scientists even in the XXI century; the similarity of pulses seems quite an obvious idea. After describing $up(x)$ in 1971 as a "one finite function" [10], the full theory of such a new class of functions was mainly developed by V. A. Rvachev [10]-[13] and other collaborators [10]-[19] who extended the theory toward many functions called atomic functions in 1975 [10]-[19] (§4).

2.2. Discovery of Exact Composition of Polynomials and Analytic Functions from Finite Atomic Functions (1971-1979)

The main feature of AF $up(x)$ is the unique ability of *exact* composition of sections of polynomials of *any* order (§4) from superposition of a *limited* number N of neighboring "atoms" on a lattice of size a

$$P_n(x) \equiv \sum_{k=-N}^{k=N} C_k up\left(\frac{x - ab_k}{a}\right); \quad x^n \equiv \sum_{k=-N}^{k=+N} C_k up\left(x - k2^{-n}\right). \quad (2.2)$$

For example, due to finiteness, only 9 neighboring atoms are required to calculate the value of a parabola x^2 at a given point x

$$x^2 \equiv \sum_{k=-4}^{k=4} \left(\frac{k^2}{64} - \frac{1}{36}\right) up\left(x - \frac{k}{4}\right). \quad (2.3)$$

The most significant is that other functions like trigonometric, exponential, and other analytic functions, which by definition [32] are the converging Taylor series of polynomials, can be represented either exactly (like polynomials) or via a converging series of shifts and stretches of AFs. Like from "mathematical atoms", smooth functions can be composed of the AF superpositions, and because of that, those "atoms" have been called Atomic Functions (AF) in 1975 [10]-[19].

It means that what we perceive as continuous polynomials, lines, shapes, orbits, and trajectories are actually the stretches/interactions of a few finite neighboring atoms, with the hint that nature, which does not know what the functions are, operates in the same simple way—it just "distribute" or "shares" the energy/information in a local network of neighbors (§12).

The unique features of atomic functions attracted attention of many followers from different countries, notably by schools of V.F. Kravchenko [13]-[15], B. Gotovac, H. Gotovac [16] [45], and the author [1]-[9], with the number of papers and books observed in [14] has grown to a few hundred.

[1]Volodymyr Lohvynovych Rvachov (1926-2005), https://en.wikipedia.org/wiki/Volodymyr_Rvachov, Academician of National Academy of Sciences of Ukraine, author of 600 papers, 18 books, mentor of 80 PhDs, 20 Doctors and Professors including the author.

3. History of Atomic String Functions in Spacetime, Quantum Mechanics, and Fields Research

3.1. AString Functions and Atomic Solitons (2018)

In 2017, the author noted that AF $up(x)$ (2.1), known since the 1970s, is a composite object/solitonic atom consisting of two kink functions called AStrings [1]-[9], making them more generic and interchangeable (§4.2):

$$up(x) = AString(2x+1) - AString(2x-1) = AString'(x). \tag{3.1}$$

This expanded the 50-year history of atomic functions (AF) into Atomic String Functions (ASF) [1]-[9]. Composing AF pulse via kink-antikink pair (3.1) of nonlinear AStrings resembles topological "solitonic atoms" which led to the theory of Atomic Solitons [4] [6] (Eremenko, 2018) where AString (3.1) becomes a "solitonic kink" while $up(x)$ is a "solitonic atom" made of two AStrings. It turns out that translations of AStrings allow composing flat and curved spacetime from finite blocks resonating with quantization ideas (§6-10) and open and closed strings from string theory [28]-[31] (§11)

$$x \equiv \cdots + AString(x-1) + AString(x) + AString(x+1) + \cdots$$
$$\tilde{x}(x) = \sum_k c_k Astring\left((x-b_k)/a_k\right). \tag{3.2}$$

The ability of finite ASF functions to compose smooth polynomials, analytic functions, and solutions of differential equations, including GR, QM and QFT, has led to the novel interpretations of atomic spacetime [1]-[3] [5] and uncovering the "common blocks" between General Relativity and Quantum Field Theories within AString Quantum Gravity theory described in §9.

3.2. Atomic Series and Atomization Quantization Theorems in Spacetime and Field Theories (2022-2025)

The mathematical foundation of decomposition ("atomization") of physical fields is based on the sequence of Atomic Quantization (Atomization) Theorems summarized in [1]-[3]. It led to the Atomic Series [1]-[3] as universal as Taylor and Fourier series but based on finite functions which can be applied to many physical theories, including GR, QM, QFT (§8-10), Quantum Gravity and String (§11) theories, with a detailed overview in [8].

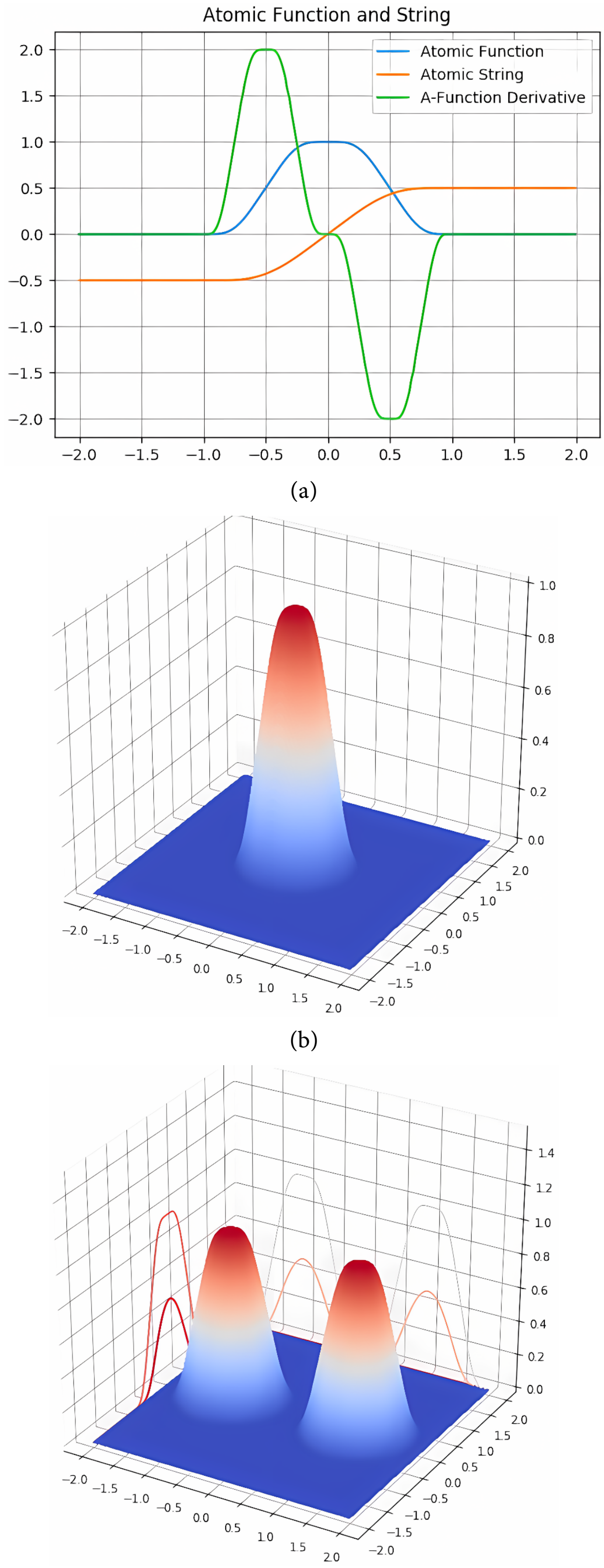
4. Atomic and AString Functions

Let's describe Atomic [10]-[19] and AString [1]-[9] Functions in more detail.

4.1. Atomic Function $up(x)$ (1967-1971)

Atomic Function (AF) (V.L. Rvachev, V.A. Rvachev, [10]) $up(x)$ was introduced in 1967-1971 as a finite compactly supported non-analytic infinitely differentiable function (**Figure 1**) with the first derivative conveniently expressed via the function itself shifted and stretched by the factor of 2:

$$up'(x) = 2up(2x+1) - 2up(2x-1), |x| \le 1; up(x) = 0, |x| > 1. \tag{4.1}$$



(a)

(b)

(c)

Figure 1. (a) Atomic Function pulse with its derivative and integral (AString); (b) Atomic Function pulse ("solitonic atom") in 3D; (c) Two Atomic Function pulses ("solitonic atoms" or "atomic solitons").

With exact Fourier series representation [1]-[13],

$$up(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{e}^{itx}\prod_{k=1}^{\infty}\frac{\sin\left(t2^{-k}\right)}{t2^{-k}}\mathrm{d}t,$$
$$\int_{-1}^{1}up(x)\mathrm{d}x=1, \tag{4.2}$$

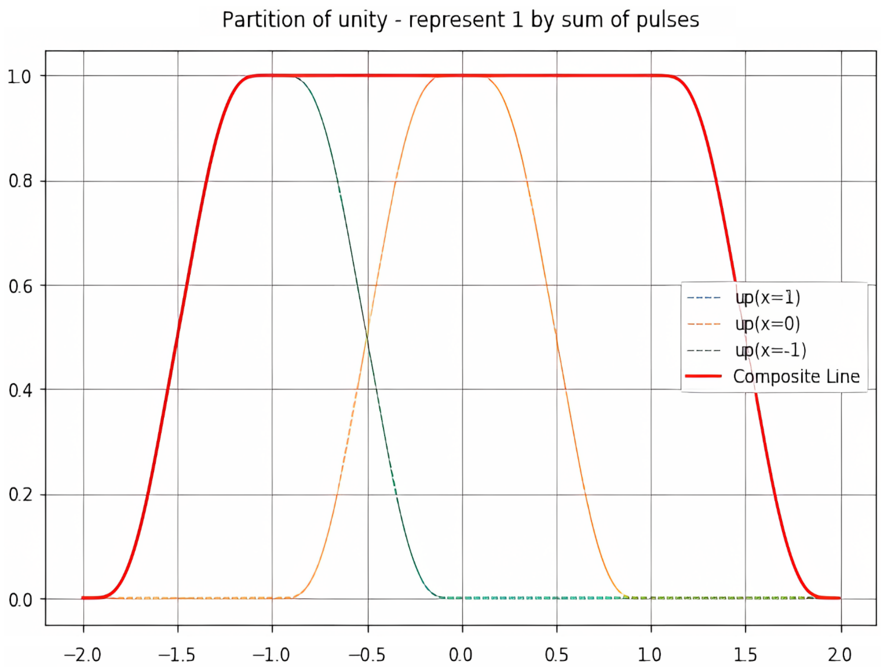
the values of $up(x)$ can be tabulated in Appendix 2 with scripts [2] [4] [19].

Higher derivatives $up^{(n)}$ and integrals I_m can also be expressed via $up(x)$ [1]-[13] [15]

$$up^{(n)}(x)=2^{\frac{n(n+1)}{2}}\sum_{k=1}^{2^n}\delta_k up\left(2^n x+2^n+1-2k\right),$$
$$\delta_{2k}=-\delta_k,\delta_{2k-1}=\delta_k,\delta_1=1;$$
$$I_m(x)=2^{C_m^2}up\left(2^{-m}x-1+2^{-m}\right),x\le 1;$$
$$I_m(x)=2^{C_m^2}up\left(2^{-m+1}-1\right)+\frac{(x-1)^{m-1}}{(m-1)!},x>1;$$
$$I_1(x)=up\left(2^{-1}x-2^{-1}\right);I_1'(x)=up(x). \tag{4.3}$$

Due to special double symmetry, AF provides a partition of unity [10]-[19] to exactly represent the number 1 by summing up individual overlapping pulses set at regular points ... -2, -1, 0, 1, 2… (**Figure 2(a)**):

$$up(x)=up(-x),x\in[-1,1];up(x)+up(1-x)=1,x\in[0,1].$$
$$\cdots+up(x-2)+up(x-1)+up(x)+up(x+1)+up(x+2)+\cdots\equiv 1. \tag{4.4}$$



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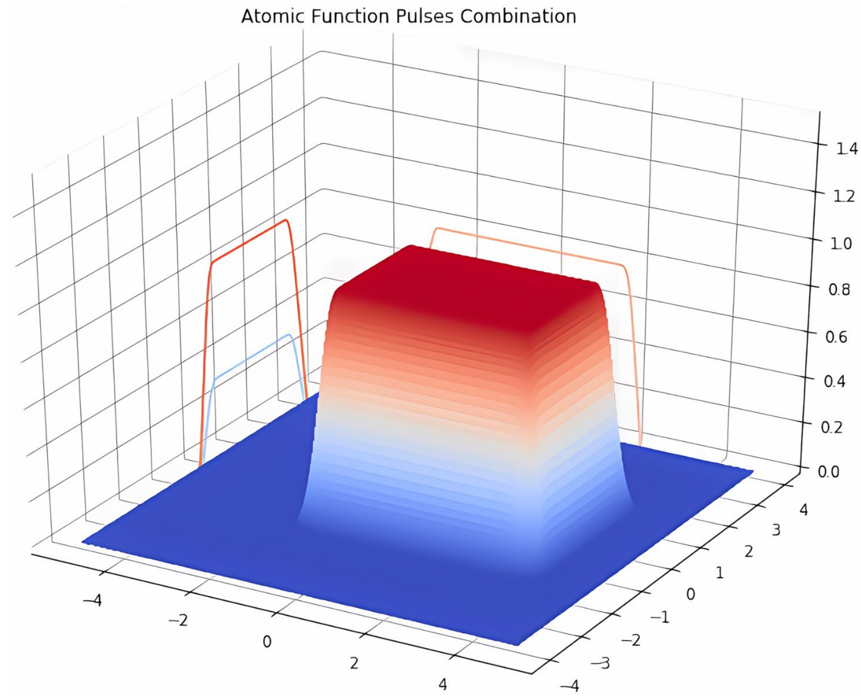

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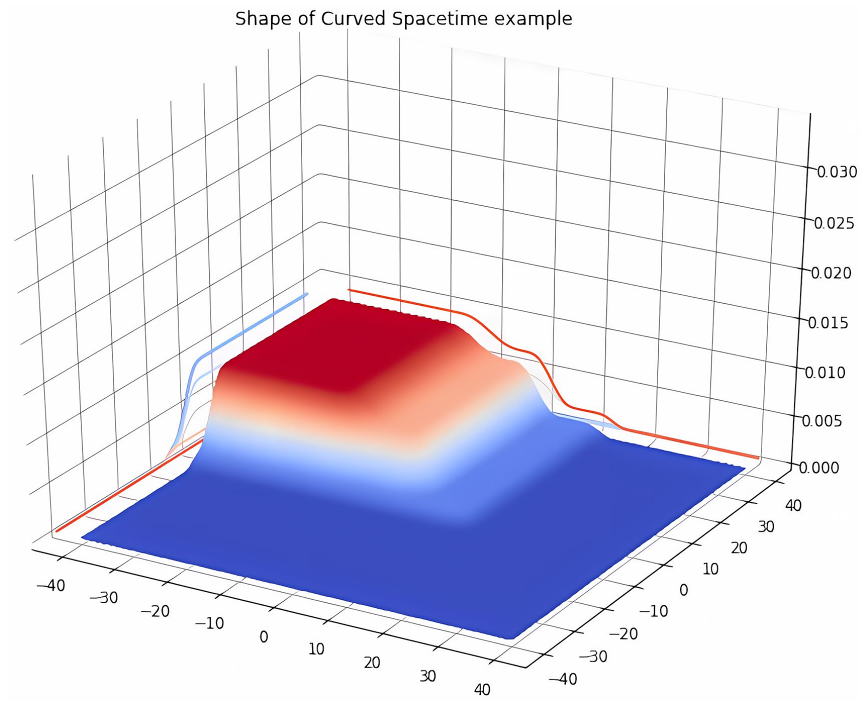


(c)

Figure 2. (a) Partition of unity with Atomic Functions; (b) Representation of flat surface via summation of Afs; (c) Curved surface as a superposition of "solitonic atoms".

Generic AF pulse of width $2a$, height c, and center positions b,d is

$$up(x,a,b,c,d=0)=d+c*up((x-b)/a).\ \int_{-a}^{a}cup(x/a)\mathrm{d}x=ca. \quad (4.5)$$

Multi-dimensional atomic functions [1]-[14] (**Figure 1**, **Figure 2**) can be constructed as either multiplications or radial atomic functions:

$$up_n(x_n) = up(x_1,\cdots,x_n) = up(x_1)\cdots up(x_n),$$
$$up(r) = up\left(\sqrt{x^2+y^2+z^2}\right),\quad \int cup_n(x_n/a)\,\mathrm{d}^n x = ca^n. \tag{4.6}$$

As a normalized function, AF possesses statistical properties [10]-[14] of being a weighted density distribution of the random variable ξ composed from independent, equally distributed variables ξ_k:

$$\xi = \sum_{k=1}^{n}\xi_k 2^{-k}, \xi_k \in [-1,1];$$
$$up(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{itx}\prod_{k=1}^{\infty}\frac{\sin(t2^{-k})}{t2^{-k}}\mathrm{d}t. \tag{4.7}$$

4.2. AString Function (2018)

$AString(x)$ function (**Figure 3**) was introduced in 2018 [1]-[9] as both the integral (4.3) and "composing branch" of $up(x)$, noting that due to double-symmetry (4.4) AF $up(x)$ can be composed from two simpler kink functions:

$$AString'(x) = AString(2x+1) - AString(2x-1) = up(x).$$
$$AString(x) = up(x/2-1/2)-1/2. \tag{4.8}$$

AString is related to the shifted Fabius function [44] but was obtained based on AFs. AString is a solitary kink (**Figure 3(a)**) which can simply compose a straight line $y = x$ as a translation of the same AString kinks (**Figure 3(c)**):

$$x \equiv AString\left(x-\frac{1}{2}\right) + AString\left(x+\frac{1}{2}\right), x\in\left[-\frac{1}{2},\frac{1}{2}\right];$$
$$x \equiv \cdots + AString(x-2) + AString(x-1) + AString(x)$$
$$+ AString(x+1) + AString(x+2) + \cdots \tag{4.9}$$

The shifted and stretched AString kink function can be generalized as

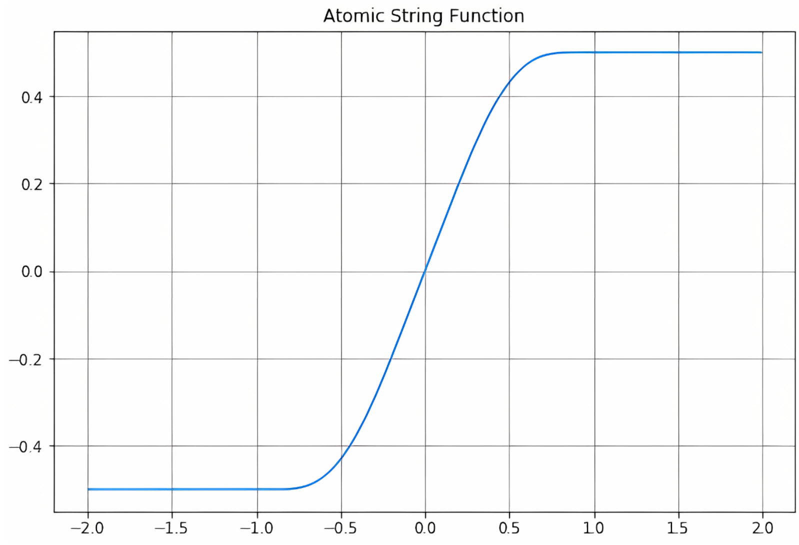
$$AString(x,a,b,c,d=0) = d + c * AString((x-b)/a) \tag{4.10}$$

and their combination can reproduce curved functions and surfaces.

Importantly, the Atomic Function pulse (4.5) can be presented as a sum of two opposite AString kinks (**Figure 3(b)**), making AStrings and AFs deeply related

$$up(x,a,b,c) = AString\left(x,\frac{a}{2},b-\frac{a}{2},c\right) + AString\left(x,\frac{a}{2},b+\frac{a}{2},-c\right). \tag{4.11}$$

AFs and AStrings, or simply *Atomics* or ASF functions [1]-[3] can compose straight and curved geodesics and fields via superposition of solitonic kinks which inspired the spacetime "atomization"/quantization research [1]-[9] leading to Atomic Spacetime theory (§8) [1]-[3] and extended here to Quantum Fields (§5-7), Quantum Gravity (§10), and string theories where AString becomes the "open string" candidate (§11).



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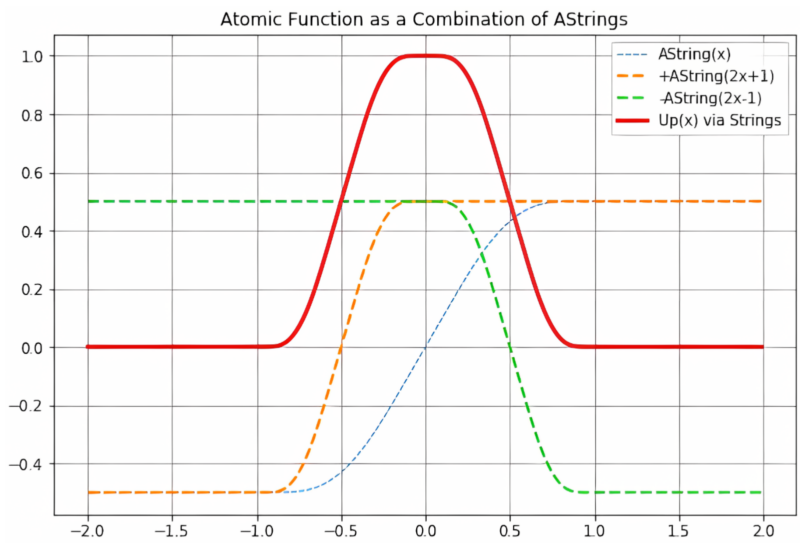

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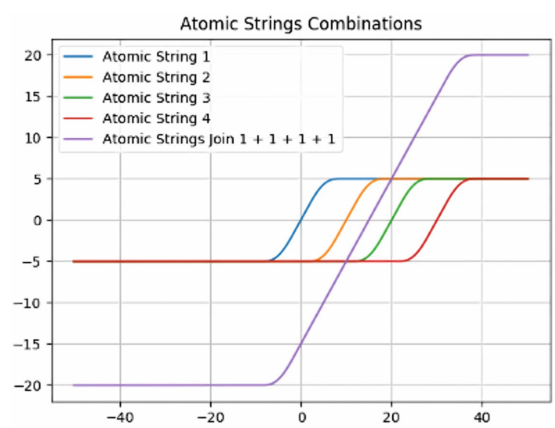


(c)

Figure 3. (a) Atomic String Function (AString); (b) Atomic function as a combination of two AStrings; (c) Representation of a straight-line segment by summing of AStrings.

4.3. Atomic Series and Atomization Theorems

The functions for which derivatives are expressed via the function itself (like exponents $\exp'(x)=\exp(x)$ or trigonometric functions $\sin'(x)=\sin(\pi/2-x)$ become especially useful in mathematics, and ASF functions are not the exception. It turns out that polynomials of *any* order can be *exactly* composed from the shifts and stretches of ASF pulses [10]-[18]

$$\frac{1}{4}\sum_{k=-N}^{k=+N} kup\left(x-\frac{k}{2}\right) \equiv \sum_{k=-N}^{k=+N} AString(x-k) \equiv x;$$

$$\sum_{k=-N}^{k=+N}\left(\frac{k^2}{64}-\frac{1}{36}\right)up\left(x-\frac{k}{4}\right) \equiv x^2;$$

$$\begin{aligned} x^n &\equiv \sum_{k=-N}^{k=+N} C_k up\left(x-k2^{-n}\right) \\ &= \sum_{k=-N}^{k=+N} C_k\left(AString\left(2\left(x-k2^{-n}\right)+1\right)-AString\left(2\left(x-k2^{-n}\right)-1\right)\right). \end{aligned} \tag{4.12}$$

Due to finiteness, only a limited number $N=2^{n+1}+1$ of neighboring "atoms" are required to calculate a n-order polynomial value at a given point x (for example, 9 for a parabola). Setting $N=\infty$ expands the series into x direction for exact representation of continuous polynomials. It means Atomics can also compose—"atomize"—any analytic function (a function representable by converging Taylor power series [32]) with calculable coefficients:

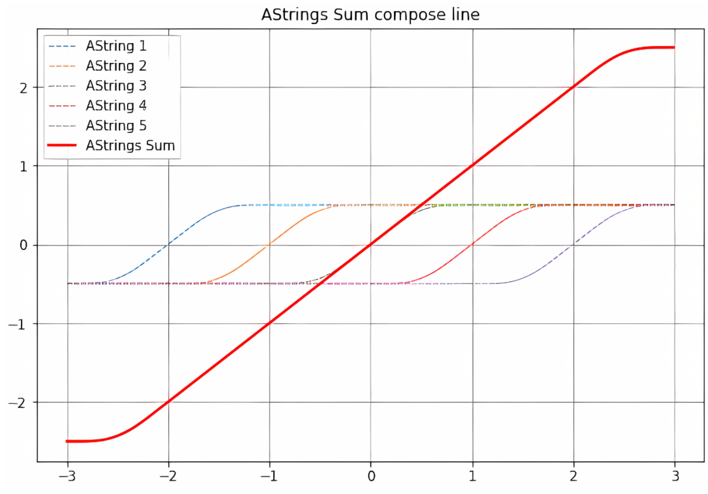
$$\begin{aligned} y(x) &= \sum_{m=0}^{\infty}\frac{y^{(m)}(0)}{m!}x^m = \sum_{m=0}^{\infty} B_m x^m = \sum_{m=0}^{\infty}\sum_{k=-\infty}^{k=+\infty} B_m C_k up\left(x-k2^{-m}\right) \\ &= \sum_{mk=-\infty}^{\infty} c_{mk} up\left(\frac{x-b_{mk}}{a_{mk}}\right) = \sum_{l=-\infty}^{l=+\infty} AString\left(x,a_l,b_l,c_l\right). \end{aligned} \tag{4.13}$$

These series called *Atomic Series* [1] [2] [7]-[9] become the foundation of Atomic Quantization (Atomization) Theorems described in [2]. They state how Analytic functions, including a wide range of polynomial, trigonometric, exponential, hyperbolic, and other functions, and their sums, derivatives, integrals, reciprocals, multiplications, and superpositions [32] can be "atomized" either exactly (like polynomials) or with predefined precision by subdividing the lattice. Examples are shown in **Figure 4** and throughout the paper.

Let's note that being non-analytic (not representable via converging Taylor series), ASF functions can also compose non-analytic functions; for example, multiplication of $y(x)$ (4.13) to the AF $up(x)$ $z(x)=up(x)y(x)$ would be non-analytic, but representable via AF combinations.

Instead of sums (4.12) and (4.13), we will be using short notation with localized atomic ASF functions $ASF_k(x)$ and function values $y^{(k)}$ at space lattice node k, assuming summation over repeated indices k:

$$y(x)=ASF_k(x)y^{(k)};\quad f(x,y,z)=ASF_k(x,y,z)f^{(k)}. \tag{4.14}$$



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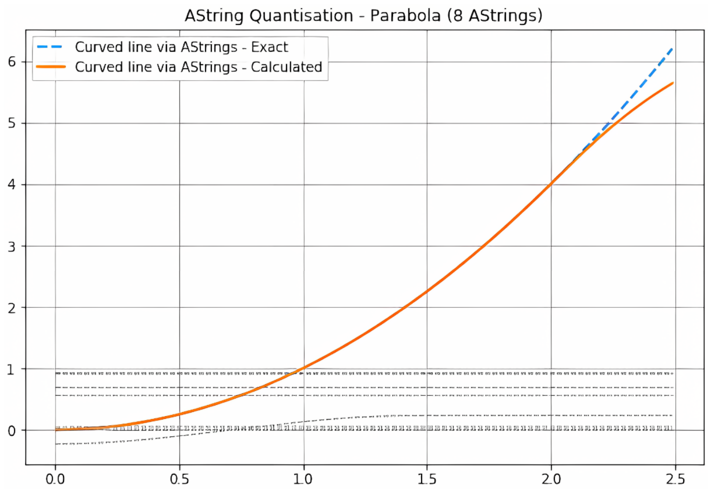

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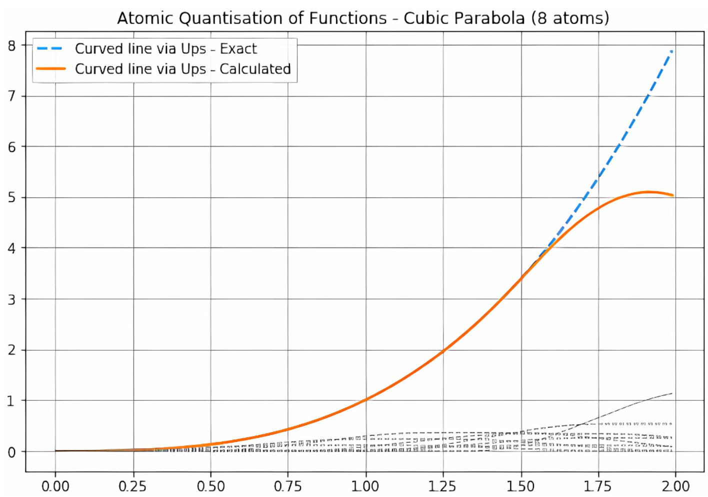


(c)

Figure 4. Representing sections of polynomials and analytic functions with AStrings and Atomic Functions: (a) Linear function via 5 AStrings; (b) Parabola via 8 AStrings; (c) Cubic parabola via 8 Atomic Functions.

4.4. Atomics as Generic Smooth Splines

Composing continuous functions from finite pieces can also be achieved with widely-used polynomial splines, but with the crucial limitations (Streng-Fix theorem [2] [3]) that n-order splines can exactly reproduce only n-order polynomials (eq cubic parabola cannot be *exactly* represented by quadratic splines [7] [17]). Infinitely smooth Atomic Splines are more generic and provide the smooth connections between overlapping splines, which is important for precise rather than approximated field theories.

4.5. Atomic Solitons, or Solitonic Atoms

Being solutions of special kinds of nonlinear differential equations with shifted arguments (4.1), (4.8), ASF Functions possess some mathematical properties of topological lattice solitons [4] [6] [46] called Atomic Solitons, described in detail in [4] [6]. AString is a solitonic kink whose particle-like properties exhibit themselves in the composition of lines (4.9), curves (4.13), and kink-antikink "atoms" (4.8) (**Figure 3**). Being a composite object (4.8) made of two AStrings, AF $up(x)$ is not a true soliton but rather a "solitonic atom", or "topological dislocation atoms" [1]-[6].

4.6. Fractal Properties and More Complex Atomic Functions

For 50 years, many other Atomic Functions more complex than $up(x)$ [10]-[18] have been discovered. In fact, for any analytic function $y(x)$ which is a solution of a linear differential equation $L(y)=0$ it is possible to construct its own Atomic Function $\varphi(x)$ from the equations with shifted arguments like

$$L(\varphi)=c_1\varphi(ax+b)+c_2\varphi(ax-b). \tag{4.15}$$

Then, the function $y(x)$ can be composed from stretches of AF $\varphi(x)$

$$y(x)=\sum_k c_k\varphi(ax-b_k). \tag{4.16}$$

So, it is possible to build AFs *exactly* composing sinusoids, exponents, gaussians, and others [10]-[19]. However, the multitude of AFs hides the importance of $up(x)$ as the simplest "mother function" [13]-[15] which, in combination, can exactly compose polynomials, hence the analytic functions which are the converging sums of polynomials [32]. For example, a function expanded via Taylor series as $\mathrm{e}^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$ can be represented by its "own" atomic function $eup(x)$ [13]-[18] but also by a series of AFs

$$x\equiv\frac{1}{4}\sum_{k=-N}^{k=+N}kup\left(x-\frac{k}{2}\right),x^2\equiv\sum_{k=-N}^{k=+N}\left(\frac{k^2}{64}-\frac{1}{36}\right)up\left(x-\frac{k}{4}\right),$$

$$x^3\equiv\sum_{k=-N}^{k=+N}C_kup\left(x-\frac{k}{8}\right);\mathrm{e}^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots \tag{4.17}$$

By simply subdividing a lattice of nodes $k/2$, we can define a lattice $k/4$ for parabolas x^2, and then $k/8$ for cubic parabolas x^3, $k/2^{-n}$ for x^n. It resonates with fractality (divisibility) of nature sizes/lengths [1] [19] [34], hinting that nature can use the same fractal mechanism to accommodate high variability fields (§12). Choosing a more granular lattice (up to Planck length) allows composing practically any fields from the simplest AF $up(x)$ without the need to invent complex atomic functions. In the future, in Atomic Series (4.14), we would assume that $ASF_k(x)$ can represent the multitude of Atomic Functions, but they ultimately can be expressed via the simplest and universal AFs $up(x)$ and $AStiring(x)$.

4.7. $up(x)$ as the Simplest Finite Atomic Function

AF theory [10]-[13] proves that with a norm

$$\left[\!\left[\varphi^{(n)}\right]\!\right] > a2^{C_{n+2}^{2}}, \varphi(x) \in [-1,1], a > 0, \tag{4.18}$$

$up(x)$, with minimal wiggles, is "the best" and "simplest" ($a = 1$) out of finite functions $\varphi(x)$ capable of composing polynomials x^n. This property allows deriving $up(x)$ from some important minimization (like least actions or shortest path) physical principles for quantization of fields (§5,8).

5. Atomic String Functions in Quantum Field Theories

This chapter shows how ASF functions can be introduced and deduced from QFT theories and describe the atomic quanta of fundamental particles from the Standard Model [20]-[24].

5.1. Atomic Fields Quantization Based on the Atomic Series and Preservation of Polynomiality

Atomic Series and Atomization Theorems (§4.3), described in detail in [2] [8], are the theoretical foundation of applying ASF functions in QFT theories. The key idea is simple—because ASF finite functions can *exactly* compose polynomials of *any* order, we need to check whether QFT differential equations or Lagrangians $L(A^\mu)$ can be resolved, at least in principle, in multi-dimensional polynomials, or generally, analytic functions which are the sums of polynomials [32]. This can be achieved in a typical power series way by analyzing whether QFT differential operators $L(A^\mu)$ "preserve polynomiality" [1]-[3] and being applied to presumably polynomial operands A^μ produce other polynomials matchable to the polynomials for F^μ on the right side of a field equation, or schematically

$$Field \to L(A^\mu) = F^\mu; A^\mu = \sum A^\mu \ Polynomials,$$

$$F^\mu = \sum F^\mu \ Polynomials; L(A^\mu) = \sum L^\mu \ Polynomials;$$

$$Match\ L^\mu \ and\ F^\mu \ polynomials\ to\ find\ A^\mu \ Polynomials;$$

$$A^\mu \ polynomials = \sum ASF\ Functions; Field = \sum ASF\ Functions. \tag{5.1}$$

As will be shown below, the majority, if not all, QFT and GR operators and

Lagrangians historically made of derivatives and multiplications [2] possess this crucial property of "preservation of polynomiality"—because derivatives, integrals, multiplications, powers, sums, inversions, and superpositions of polynomials would be the polynomials ("polynomials are hard to destroy" [2]). Finally, the resolution, at least in principle, of QFT equations in polynomials would guarantee that the solutions can be atomizable by the Atomic Series via finite ASF functions. This atomization idea can be generalised to analytic functions, which by definition [32] are the converging sum of polynomials. So, the representability of QFT solutions via finite ASF functions is based on the two key features—*preservation of polynomiality* (and generally analyticity by QFT operators) and exact *representation of polynomials by ASF functions*. The universality of the Atomic Series is related to the fact that derivatives and integrals of ASF functions are expressed via ASF themselves (4.3), so they can be matched on both sides of the field equations in finite regions of space. Let's demonstrate the fields' decomposition/atomisation idea for some theories.

5.2. Atomic Series for Main QED Equations

Quantum Electrodynamics (QED) [21], as the most successful QFT theory matching experiments with astonishing precision, deals with interactions of electron, photon, muon, and other electromagnetic (EM) fields. Apart from the non-relativistic Schrödinger equation

$$L\left(\psi\left(x_i,t\right)\right)=i\hbar\frac{\partial\psi\left(x_i,t\right)}{\partial t}+\frac{\hbar^2}{2m}\frac{\partial^2\psi\left(x_i,t\right)}{\partial x_i^{\,2}}=V\left(x_i,t\right)\psi\left(x_i,t\right),\tag{5.2}$$

QED deals with the Klein-Gordon equations for spin-0 particles for the wavefunction

$$L\left(\psi\left(x_i\right)\right)=-\hbar^2\partial^\mu\partial_\mu\psi-m^2c^2\psi=0,\tag{5.3}$$

more advanced Dirac Equation for electron/positron spinor ψ fields [21] [22]

$$L\left(\psi\left(x_i\right)\right)=i\hbar\gamma^\mu\partial_\mu\psi-mc\psi=0,\tag{5.4}$$

and relativistic Maxwell Equations for EM field components A^μ [21]

$$L\left(A^\mu\right)=\partial^\mu\partial_\mu A^\mu-\frac{4\pi}{c}J^\mu=0.\tag{5.5}$$

It is easy to observe that all these L operators preserve polynomiality and analyticity, and being applied to presumably polynomic 4D field functions would yield other polynomials (because derivatives, integrals, sums, and multiplications of polynomials are the polynomials) which can be matched on both sides to satisfy the field equations according to scheme (5.1). So, the field functions ψ, A^μ can be at least in principle resolved in polynomials, which in turn can be exactly represented by Atomic Series via overlapping ASF functions (4.14):

$$\psi\left(x_i\right)=ASF_{ik}\left(x_i\right)\psi^{(k)};A^\mu\left(x_i\right)=ASF_{\mu ik}\left(x_i\right)A^{\mu(k)}.\tag{5.6}$$

Field analytic functions, which by definition [32] are the converging sum of polynomials, follow the same rules. Physically, it means the QED fields can be decomposed into overlapping interactions of finite quantum-like objects (§6,7), like in string and LQG theories [26]-[31], leading to the AString theory (§11).

5.3. Atomic Quantization of the Known QED Solutions

The fields' atomization with the Atomic Series can be validated and demonstrated with some known QED solutions. The core QED Dirac equation (5.2) solution for a position-independent electron (and positron) [21]

$$\psi(t) = \mathrm{e}^{\pm i\left(\frac{mc^2}{\hbar}\right)t}\psi(0) = \sum\nolimits_k \psi_k up\left(\frac{t-b_k}{s}\right). \tag{5.7}$$

includes analytic exponents and trigonometric functions expandable by Taylor series over polynomials [32], exactly representable via ASF series (4.13)

$$\mathrm{e}^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \quad \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots. \tag{5.8}$$

A similar validation can be applied for electron plain-wave solution [21] for spinor ψ, including an analytic exponential function representable by sums of polynomials, hence ASF finite functions on electron field lattice of size s (§6):

$$\psi(x) = \mathrm{e}^{\pm px/\hbar}u = \sum\nolimits_k \psi_k up\left(\frac{x-b_k}{s}\right). \tag{5.9}$$

5.4. Atomic Function Envelope Photon Model

Let's demonstrate how ASF functions can be used in QFT with the known photon QED model for wavefunction components A^μ which reduces to a plain-wave solution [21], typically resolvable in analytic functions

$$A^\mu(x) = a\mathrm{e}^{-\frac{ipx}{\hbar}}\epsilon^\mu(p). \tag{5.10}$$

One photon excitation wave packet travelling in x direction can be written as a carrier wave frequently oscillating inside an envelope [21] (**Figure 5**):

$$\psi(x,t) = \mathrm{e}^{i(kx-\omega t)}Env(x,t) \tag{5.11}$$

with a traditionally used Gaussian envelope

$$Env(x,t=0) = a\mathrm{e}^{-b(x-c)^2}. \tag{5.12}$$

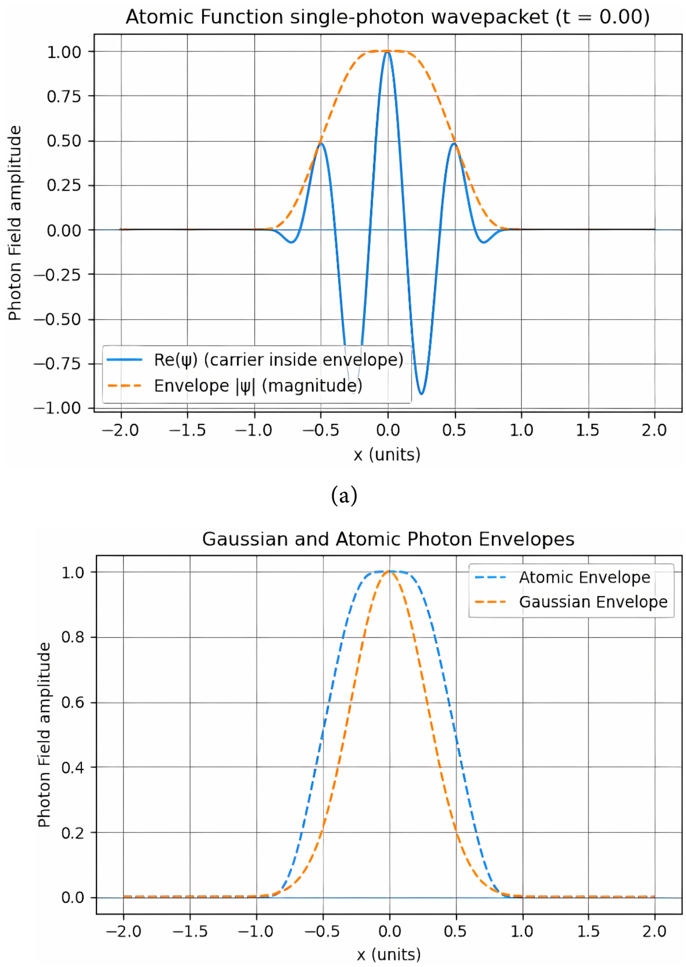
Interestingly, in QED, the envelope function is not enforced but dictated by the initial and decay conditions [21] [22], so it is not a QED violation if we choose another envelope that provides additional benefits. Assuming that a generic envelope function is analytical allows composing an envelope from Atomics:

$$Env(x,0) = \sum\nolimits_k \varepsilon_k up\left(\frac{x-b_k}{s}\right). \tag{5.13}$$

Extracting one atomic pulse from this series would yield the Atomic Photon model based on AF pulse with amplitude ε, width s, and energy/integral E

$$APhoton = e^{i(kx-\omega t)} \varepsilon up\left(\frac{x-ct}{s}\right), \; E = \int \varepsilon up\left(\frac{x}{s}\right) dx = \varepsilon s. \tag{5.14}$$

This *APhoton* pulse can serve as the simplest *elementary atomic excitation*/*quantum* of the QED photon field. More complex envelopes can be constructed by summing Atomic Envelopes (4.13) and (5.13) on some lattice. This *APhoton* model is better than the traditional Gaussian model (5.12), because from a few Gaussians it is not possible to compose even the simplest fields, like a constant or a line.



(a)

(b)

Figure 5. (a) Atomic Photon envelope wavepacket; (b) Atomic and Gaussian Envelopes.

This Atomic Photon model correlates well with the extensive faint starlight laboratory experiments [21], confirming photon as the simplest (§4.6, 4.7) extremely stable soliton-like (§4.5) electro-magnetic quantum energy excitation/pattern (5.14) moving billions of years over vast distances without disturbances and registered as a single quantum lump of energy (5.14). It can be associated with the moving rearrangement of a few spacetime atoms (§9, 12), or "intricate distortions of spacetime", as S. Hawking [37] mentioned.

Let's note that ASF functions can be applied not only to the envelope, but to the whole presumably analytic photon field function (5.11), with a schematic spatial extension sum representation

$$PhotonField = \sum_k APhotons_k . \tag{5.15}$$

This APhoton model can be the basis for the experiments to check whether a few photon envelopes are shaped (**Figure 5**) as $up(x/s)$ envelopes (5.14).

5.5. Atomizing QCD Equations and Gauge Theories

Another core QFT theory of Quantum Chromodynamics (QCD), describing strong nuclear forces and interactions of quark and gluon fields, is based on the gauge formulation with Lagrangian density [21] [22]:

$$\mathcal{L}_{QCD}\left(\psi\left(x_i\right)\right) = \left[i\hbar c\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\bar{\psi}\psi\right] - \frac{1}{16\pi}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\mu\nu} - \left(qc\bar{\psi}\gamma^{\mu}\lambda\psi\right)\boldsymbol{A}_{\mu} . \tag{5.16}$$

The variation of this Lagrangian produces the equations for three Dirac fields interacting with eight gluon fields [21] [22]. Despite the complex compactification of differential operators and spinor vectors ψ, one can see that those operators containing derivatives and multiplications preserve the polynomiality and analyticity, and, being applied to polynomial operands, would produce other polynomials for the Lagrangian density [2]. The minimization of it should ultimately produce a polynomial representation for the field ψ, which in turn can be exactly representable by Atomic Series via the finite ASF functions:

$$\psi\left(x_i\right) = ASF_{ik}\left(x_i\right)\psi^{(k)}. \tag{5.17}$$

So, the Atomic Series is applicable not only for differential equations, but also for the minimization of Lagrangian functionals typically used in QFT.

5.6. Atomic Quantization of the Higgs Field

Scalar Higgs field ϕ, which gives particles a property of mass, is described by gauge QFT theory with Lagrangian density [21]

$$\mathcal{L}_{Higgs}\left(\phi\left(x_i\right)\right) = \frac{1}{2}\left(\partial_{\mu}\phi\right)^{*}\left(\partial_{\mu}\phi\right) + \frac{1}{2}\mu^2\left(\phi^{*}\phi\right) - \frac{1}{4}\lambda^2\left(\phi^{*}\phi\right). \tag{5.18}$$

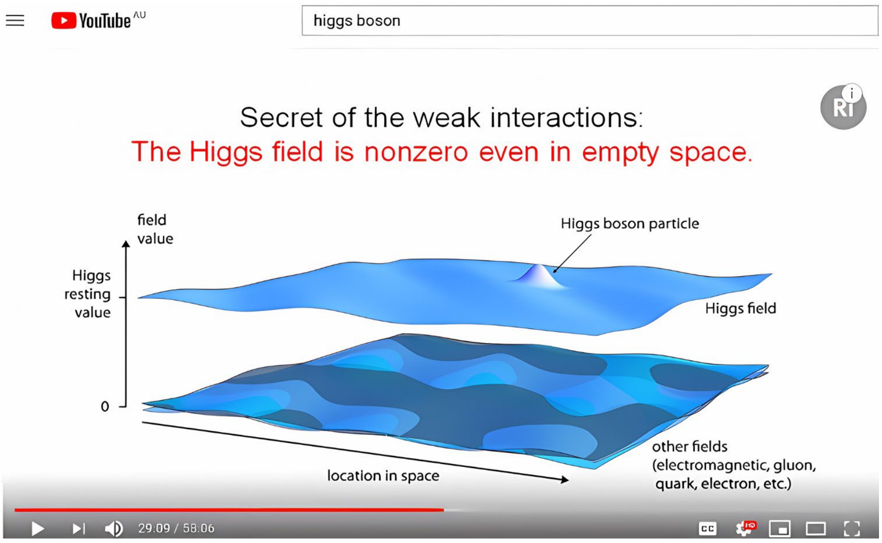
The operators contain derivatives and multiplications which preserve polynomiality and analyticity, and, being applied to polynomial operands, would produce other polynomials for the Lagrangian density and the solution ϕ, which on a lattice can be exactly atomizable with ASF functions:

$$\phi\left(x_i\right) = ASF_{ik}\left(x_i\right)\phi^{(k)} . \tag{5.19}$$

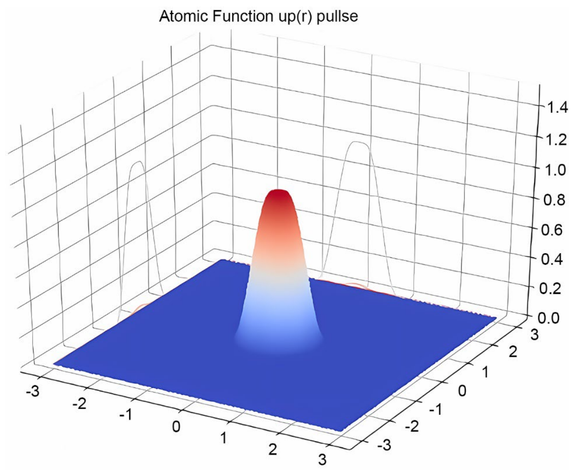
For further validation, let's note that the Higgs theory yields the well-known "Mexican hat" function [21]

$$\mathcal{U}\left(\phi\right) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4 \tag{5.20}$$

which, as a simple polynomial, is exactly representable by Atomic Functions (4.17). As described in §6, the elementary atomic pulse in series (5.19) would represent an atomic Higgs boson (**Figure 6**), often coined as "the God particle", popularised in the book [5] (Eremenko, 2020).



(a)



(b)

Figure 6. (a) Sean Carroll's [33] lecture features the Higgs boson particle in space; (b) Higgs boson representation via atomic soliton pulse $up(r)$.

5.7. Atomization of the Standard Model of Particle Physics

The triumph of QFT as the main physical theory was the formulation of the Standard Model (SM) of Particle Physics with the Lagrangian including the parts from all QFT theories describing up to 60 fundamental fields [21]-[24]

$$
\begin{aligned}
&Z = \int D\Phi \mathrm{e}^{iS_{SM}}; S_{SM} = \int \mathrm{d}^4 x \sqrt{-g} \mathcal{L}_{SM}; \\
&\mathcal{L}_{SM} = R - F_{\mu\nu}F^{\mu\nu} - G_{\mu\nu}G^{\mu\nu} - W_{\mu\nu}W^{\mu\nu} - \sum_i \bar{\psi}_i D\psi_i \\
&\quad - D_\mu H^\dagger D^\mu H - V(H) - \lambda_{ij}\bar{\psi}_i H\psi_i.
\end{aligned}
\tag{5.21}
$$

This compactified formula incorporates the minimizable action Z, the GR Ricci scalar R, electromagnetism $F^{\mu\nu}$, strong field $G^{\mu\nu}$, weak field $G^{\mu\nu}$, Higgs field H, matter fields $\bar{\psi}_i$, with an overarching Feynman quantization integral $\int D\Phi e^{iS_{SM}}$. Let's analyze what kind of additional commonality can be extracted. First, it describes the distribution of tensor/spinor/scalar field functions $f(x,y,z,t)$ coexisting at point x,y,z,t of spacetime. Second, those solutions are based on classical infinite spread functions and operations, which assume the existence of an infinitesimal point in space $dx \to 0$. Third, the functional-differential operators $F_{\mu\nu}, G_{\mu\nu}, W_{\mu\nu}$ are historically built upon the classical derivatives and multiplication operations (§5.2), for example, $i\hbar c\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\bar{\psi}\psi$ for QCD fields (5.16). Those operators preserve polynomiality of arguments due to the powerful feature of polynomials—their multiplications, derivatives, integrals, and superpositions are also polynomials. So, despite their complexity, the QFT gauge field equations (5.21) are, in principle, resolvable in multi-dimensional polynomials or their sums (analytic functions [32]). Composing the infinitely-spread polynomials with ASF functions naturally introduces the quantized smooth finiteness and replaces "a point in space" with "finite region of space"—in line with A. Einstein's 1933 perspective "atomic theory" [1] [2] [8] [25], (§1.2). In this case, up to 60 quantum fields coexisting in every "point" of space would be replaced by interacting finite pulses of energy, where a field configuration at every "point" would be defined by the interaction pattern of neighboring quanta (§12). It opens the possibility of fields unification (§9-11), including Quantum Gravity (§9), based on a common representation of different fields via the core finite function $up(x,a,b,c)$ (4.5). String theory [26]-[31] also suggests a similar common derivation of all fields from "vibrating strings" (§6.9) based on the universality of Fourier series, but here the core functions are atomic, which leads to the new variant of AString theory (§11).

Formally, introducing finite ASF functions and Atomic Series into QFT Lagrangians follows the standard procedure (§5.1)—first, we evaluate whether the operators preserve polynomiality and are potentially resolvable via polynomials or their sums, and then we engage the Atomic Series, which exactly atomize polynomials of any order. Instead of following this standard way, let's formulate a more ambitious task—to derive (rather than introduce) ASF functions from the Standard Model equations, so theoretically it could have been done on the onset of QFT and Atomic Functions in the 1980s. A similar approach of deriving Atomics from Einsteinian GR is described in [1]-[3]. This can be achieved with the following theorem.

Theorem (*Standard Model Finite Atomic Function Theorem*). Assuming the preservation of the polynomiality for SM Lagrangians (5.21), it is possible to find a finite function $up(x) \in [-1,1]$ which would represent smooth SM field solutions via sums of shifts and stretches $d + cup((x-b)/a)$ in multiple dimensions. This function should be non-analytic and infinitely differentiable, with the derivatives and integrals expressed via the function itself, with the simplest function $up'(x) = 2up(2x+1) - 2up(2x-1)$.

Proof. Preservation of polynomiality by SM Lagrangian operators (5.21) implies that multi-dimensional field functions are representable by polynomials $P_m(x_n)$ being injected into these operators would yield the other polynomials, because multiplication of derivatives of polynomials by other polynomials would also be polynomials. So, in some regions of space, the Lagrangian (5.21) can be represented as the complex superposition of many polynomials of any order. Minimization (derivation) of Lagrangians would yield the field equations and their solutions as a series of polynomials. This theorem can be proved if we find some basis spline pulse-like finite function $p(x)\in[-1,1]$ which, in translation, would exactly compose a polynomial of *any* order $x^n=\sum_{k=-N}^{k=N}c_k p\left(\frac{x-b_k}{a_k}\right)$. Evaluating the candidates for $p(x)$, we first have to eliminate the polynomial splines of some order, because they are unable to exactly compose a polynomial of *any* order ([2], §4.4). The spline function should be a polynomial of "infinite" order, so belonging to class $C(\infty)$ of absolutely smooth functions. Secondly, we have to eliminate smooth trigonometric and other exponent-based analytic functions like Gaussians or sigmoid, because by summing a few pulses, they are unable to exactly reproduce even the simplest polynomials (a line, or a constant). The choice has narrowed to infinitely differentiable finite functions which at least satisfy "partition of unity" $c=\sum_k cp(x-k)$ ith derivatives expressed via the function itself $p'(x)=F(p(x))$, or in the simplest (§4.7) linear form $p'(x)=F(p(x))=kp(ax+b)-kp(ax-b)$ which, with the symmetry condition $p(x)=p(-x)$, normalization $p(0)=1$ and finiteness $p(x)=0$, $|x|>1$ leads to the Atomic Function $p(x)=up(x)$, $up'(x)=2up(2x+1)-2up(2x-1)$ discovered in the 1970s (§4). They can compose a polynomial of any order x^n (Theorem §4.3). Due to double-symmetry (4.2), the AF $up(x)$ itself can be represented via the sum of two $AString$ kink functions (4.8) $up(x)=AString(2x+1)-AString(2x-1)=AString'(x)$ which can compose the polynomials of any order too. In summary, due to preservation of polynomiality by all Lagrangian operators, the QFT field solutions can be, at least in principle, resolved in multi-dimensional polynomials, or their sums, which in turn can be exactly representable by a series of shifts and stretches of multiplications of Atomic Functions in many dimensions (4.6), (4.13).

Proof obtained. The importance of this theorem is that it allows not only introducing but deducing finite ASF functions from QFT, noticing the crucial property of QFT operators to preserve polynomials and the ability of ASF Functions to exactly compose them. Basically, it tells that presumably smooth analytic QFT fields can be represented/atomized via sums of ASF pulses in many dimensions (§6-9). Different fields governed by their own equations would have different distributions (§6), but, like in a string theory [29]-[31], the underlying common mathematical entity would be the Atomic Functions leading to AString theory (§11). As we see, the Standard Model (5.21) can incorporate another "layer of commonality" based on universal finitization/atomization of point-like QFT field functions with finite ASF functions resembling strings/loops/excitations described hereafter.

6. Atomic or AString Quantum Field Theory

Integrating Atomic String Functions into QFT theories (§5) leads to the Atomic or AString Quantum Field Theory (AQFT) [8], described hereafter.

6.1. Introducing Finiteness into Continuous Theories

While representing the fields via some universal series like Fourier or Atomic Series (4.14) has some theoretical significance, the other opportunity is important as well—to examine the fundamental elementary processes/laws/principles based on analysis of the individual series components. For example, the universal trigonometric Fourier series gave rise to string theories where elementary "building blocks" of fields are some vibrating strings [28]-[31]. Atomic Series offers its own unique features—smooth quantized finiteness and the fields' composition from interacting elementary "building blocks". It is quite natural to assume that *finite* ASF functions would better describe *finite* strings, loops, excitations, and quanta. QFT [21]-[24] conveniently postulates that fields are made of ripples, and fundamental "particles"/quanta are the elementary excitations of quantum fields. Because AF $up(x)$ is a finite pulse (**Figures 1-4**), it is tempting to use it to describe the "elementary ripple", or an Atomic Quantum of a field, as described hereafter.

6.2. What Atomic Pulse Mean in QFT

The previous chapters provide the theoretical foundation on how the Atomic Series can represent/atomize any QFT field, with the Atomic Photon (§5.3) and Higgs (§5.6) models being the examples. The most important question is what an atomic pulse actually describes in the QFT context.

Generic 1D AF pulse (4.5) (**Figure 1**, **Figure 2**) has 4 parameters—pulse centre location $x=b$, $y=d=0$, amplitude c, and width $2a$:

$$up(x,a,b,c,d)=d+cup\left(\frac{x-b}{a}\right). \tag{6.1}$$

A few neighbouring atoms overlap and produce a continuous field

$$field(x)=c_1up\left(\frac{x-ab_1}{a}\right)+c_2up\left(\frac{x-ab_2}{a}\right)+c_3up\left(\frac{x-ab_3}{a}\right)+\cdots \tag{6.2}$$

for example, a parabolic or any order polynomial (4.12) field

$$x^2\equiv\sum\nolimits_k\left(\frac{k^2}{64}-\frac{1}{36}\right)up\left(x-\frac{k}{4}\right);\ x^n\equiv\sum\nolimits_l C_l up\left(x-l2^{-n}\right). \tag{6.3}$$

Individual pulses in series (6.2), (6.3) have different shifts and amplitudes, but the same lattice width a. It allows associating the pulse width a with some fundamental field "size" parameter s, $a=s$, upholding the main QFT [21]-[24] and string theories assumptions [29]-[31] where fields differ by different scales and frequencies of vibrating strings. For example, the size s of a spacetime field can be associated with the Planck length [21] $s=l_p=1.61\times10^{-35}\ \text{m}$ while the QFT electron field (§5.3) can be associated with either radius of an electron or better with Compton quantum wavelength [21] $s = \lambda_c = \frac{\hbar}{mc} \approx 2.426 \times 10^{-12}\ \text{m}$. In summary, in the AQFT model, the difference between fields is encapsulated in the lattice width/size $a = s$ of the elementary atomic pulse $cup\left(\frac{x-b}{a}\right)$.

6.3. Quantum Wavefunction Interpretations of Atomic Pulses

Next, let's elaborate on the atomic pulse $cup\left(\frac{x-b}{a}\right)$ amplitude c. It has units of a field function (the function $up\left(\frac{x-b}{a}\right)$ is unitless) the theory is applied for, with a few useful options.

In application to QM, the meaning and units of a pulse amplitude c is the probabilistic wavefunction ψ amplitude, so the atomic pulse and QM wavefunction field on some lattice of nodes can be conceptualized as

$$QMPulse = \psi up\left(\frac{x-b}{s}\right),$$
$$QMfield = \sum_k \psi_k up\left(\frac{x-b_k}{s}\right) = Sum\ of\ QMPulses. \tag{6.4}$$

In this way, the ASF functions were introduced into Atomic Quantum Mechanics [9].

In more advanced QFT, the QM wavefunction is replaced by a complex field operator ϕ encapsulating spinor fields for fermions, vector fields for bosons, or scalar fields for Higgs [21]. In a nutshell, ϕ is just a space-defined complex operator function which can be decomposed into finite atomic functions, so the quantum fields on some lattice of nodes can be conceptualized as

$$QFTPulse = \phi up\left(\frac{x-b}{s}\right),$$
$$Field = \sum_k \phi_k up\left(\frac{x-b_k}{s}\right) = Sum\ of\ QFTPulses. \tag{6.5}$$

For example, in application to Dirac QED electron fields, the field operator ϕ would have the meaning of a Dirac spinor ψ (5.4) decomposable into finite atomic pulses.

6.4. Atomic Quanta as the Pulses of Energy

Physically, all fundamental fields are the carriers of energy. In classical physics, energy is stored in the field configuration, like electromagnetic energy density. In QFT, the fields become quantized, meaning a field can only have discrete excitations of energy associated with "particles" [21]. To apply the Atomic Functions theory to QFT fields, it is useful to associate the atomic pulses with the pulses of energy, so the amplitude c of atomic pulses $cup\left(\frac{x-b}{s}\right)$ can be related to some energy parameter ε, $c=\varepsilon$. It can be done in the following way [8]. First, we need to understand how AFs would appear in many dimensions. Let's assume that an n-dimensional field is described by an analytic function $field(x_i)$, $i=1,\cdots,n$. By definition, it is representable by n-dimensional polynomials P_{mn} which are the multiplication of some 1D polynomials representable by 1D AFs. For example, the polynomial component x^2y^5z can be exactly atomized by the sums of multiplication of AFs in the relevant dimension:

$$x^2=\sum_k\left(\frac{k^2}{64}-\frac{1}{36}\right)up\left(x-\frac{k}{4}\right),\ y^5\equiv\sum_l C_l up\left(y-l2^{-5}\right),\ z=\sum_m\frac{1}{4}up\left(z-\frac{m}{2}\right).$$

Due to finiteness, those multiplications would be non-zero only inside the finite regions of space

$$field(x_1,\cdots,x_n)=\sum c_1 up\left(\frac{x_1-b_1}{s}\right)*\cdots*c_n up\left(\frac{x_n-b_n}{s}\right).$$

Selecting identical pulses in every dimension allows introducing n-dimensional AF (4.6) and its integral

$$up_n(x_n,s,\varepsilon)=up(x_1,\cdots,x_n)=\varepsilon up(x_1/s)\cdots up(x_n/s),$$
$$\int\varepsilon up_n(x_n/s)\mathrm{d}^n x=\varepsilon s^n,\quad s=c_1*\cdots*c_n. \tag{6.6}$$

Finally, let's make a reasonable QFT assumption that the energy E of a pulse is related to the integral of an excitation pulse, which, due to (4.6) in the 1D case, would be $E=\varepsilon s$ or $E=\varepsilon$ in n-dimensional theory:

$$E=\int_{-s}^{s}\varepsilon up(x/s)\mathrm{d}x=\varepsilon s;\quad\int\varepsilon up_n(x_n/s)\mathrm{d}^n x=\varepsilon s^n. \tag{6.7}$$

Interestingly, the atomic pulse $cup\left(\frac{x-b}{s}\right)$ amplitude c has a convenient meaning and units of energy density ε (Joules per unit of volume) frequently used in many classical and QFT theories, and, importantly, in Lagrangians. It means the atomic quantum fields representations (6.2) would have a meaning of energy density distributions $\varepsilon(x_1,\cdots,x_n)$ on some lattice, or schematically

$$\varepsilon(x_1,\cdots,x_n)=\sum_k\varepsilon_k up_{kn}(x_n/s);\ field=Sum\ of\ APulses. \tag{6.8}$$

6.5. Extracting an Elementary Atomic Quantum Pulse

From the Atomic Series sum (6.8), it is possible to extract an *elementary* pulse identical in all directions, which would describe an elementary atomic excitation/ripple of a field, which in QFT theory is typically called quantum [21] [22]. Because it is described by an Atomic Function, it is reasonable to name it as Atomic Quantum or simply $AQuant$ which in n and 1 dimensions can be written as

$$AQuant(x_1,\cdots,x_n)=\varepsilon(x_1,\cdots,x_n)=\varepsilon up_n\left(\frac{x_n}{s}\right);$$
$$AQuant(x)=\varepsilon(x)=\varepsilon up\left(\frac{x}{s}\right). \tag{6.9}$$

This pulse has the shape of a multi-dimensional Atomic Function (**Figure 1**, **Figure 3**, **Figure 6**), units of energy density, and two key parameters—constant field-dependent size/width s and energy density ε as a pulse amplitude. This $AQuant$ pulse can be defined in n dimensions (simple 1D, QFT 4D spacetime, or up to 11 in some string theories). The spatial integral (6.7) of this finite pulse has the units and meaning of energy. In summary, the description of an Atomic Quantum as an elementary excitation of a quantum field can be encapsulated in the following formula

$$\boxed{E = \int AQuant\left(x_1,\cdots,x_n\right)\mathrm{d}^n x = \int \varepsilon up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x = \varepsilon s^n.} \tag{6.10}$$

This key formula unites the energy E, size/width s, and energy density ε of an Atomic Function up_n pulse which describes an elementary field excitation/quantum with n dimensions. Fields differ from each other by the presumably constant size s of quanta (similar to different string frequencies in string theory [29]-[31]). Variable energy density ε, as the amplitude of a quantum pulse, is proportional to the energy level of a field in a given pulse region.

Atomic Quanta (6.10) are not just the mathematical abstractions but can be conveniently matched to the experimental values based on known sizes (like Compton quantum lengths [21]) and energy levels, for example, for a few Atomic Quanta particles in **Table 1**.

Table 1. Energy densities and sizes of atomic quanta for some fields.

Atomic Quantum	Energy E (GeV)	Size s (m)	ε (J/m³)
AElectron	0.000511	2.43×10^{-12}	5.7×10^{34}
AUpQuark	0.0023	5.42×10^{-13}	3.7×10^{37}
ADownQuark	0.0048	2.60×10^{-13}	8.0×10^{38}
AHiggs	125.10	1.58×10^{-18}	5.1×10^{45}

In the historical context, the formula (6.10) unites the 55-year-old theory of Atomic Functions with the 80-year history of Quantum Field Theory as the major theory of physics, enriching both theories. It not only introduces the new Atomic Quantum and fields atomization concepts into QFT theory, but also leads to AString Quantum Gravity and string theories described in §9-11.

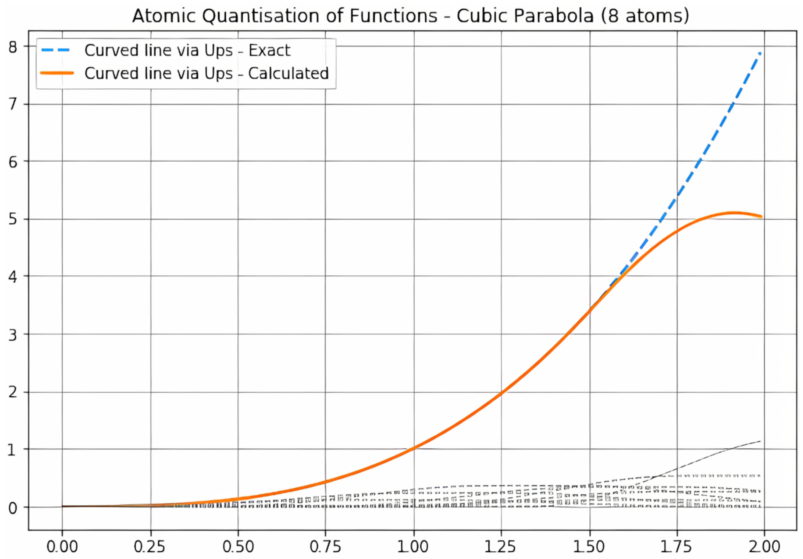
Let's highlight the novelty of Atomic Function representations (6.1)-(6.10) in comparison with traditional QFT theories based on point-in-space and infinitely spread functions, which, according to A. Einstein [25] may create the "stumbling blocks" towards unification (§1.2), notably renormalization of infinities [21]. Atomic QFT theory not only offers the easy and universal way of quantization of fields based on Atomic Series over finite ASF functions envisaged by A. Einstein in his "atomic theory" [25] but also introduces a useful field fundamental size s

parameter absent in QFT to distinguish different fields and match it to quantum, string or loop sizes from string and LQG theories [26]-[31] (§9-11).

6.6. Variable and Constant Fields Composed of Atomic Quanta

Atomic Quantum (6.10) is the elementary quantum of a field based on one AF pulse (Figure 1, Figure 3, Figure 6). In reality, fields are variable (Figure 7) and can be composed from the superposition/sum of finite quanta of different intensities located at some lattice k nodes, or schematically in the 1D case

$$\begin{gathered} AQuant = \varepsilon up\left(\frac{x - sb}{s}\right), \\ Fields = \sum_k \varepsilon_k up\left(\frac{x - sb_k}{s}\right) = Sum\ of\ AQuanta. \end{gathered} \tag{6.11}$$



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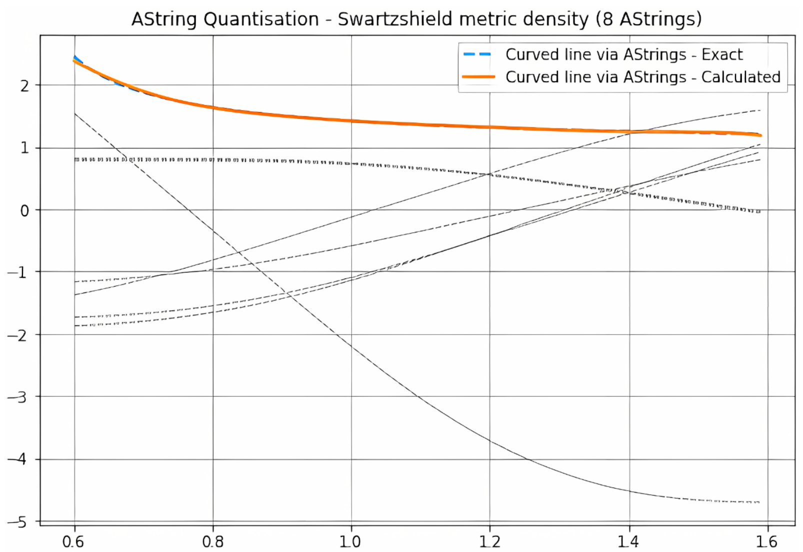

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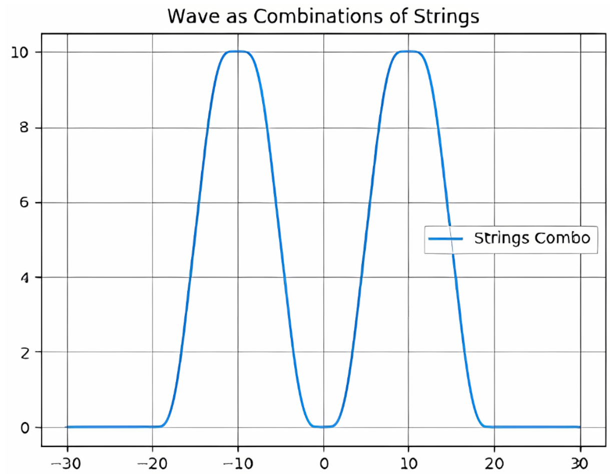

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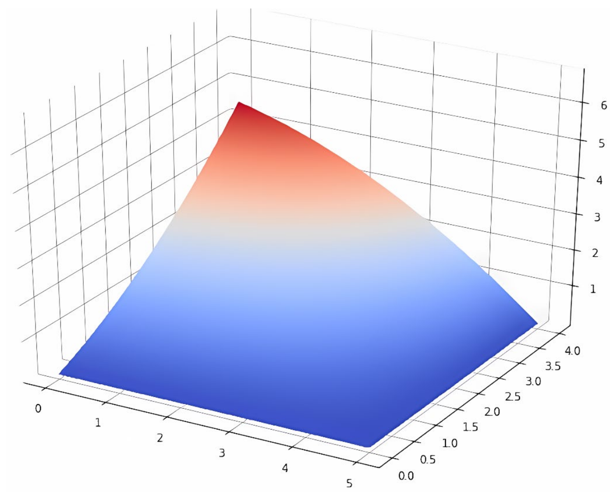


(d)

Figure 7. Different field configurations composed of atomic functions.

Due to AF partition of unity (4.4), equally spaced quanta of identical intensity $\varepsilon_k = \varepsilon$ produce a constant field (**Figure 2**) with energy (6.10):

$$ConstantField = \varepsilon \sum_k up\left(\frac{x - ks}{s}\right) = \varepsilon\ ;\quad Energy\ E = \varepsilon s\ . \tag{6.12}$$

This simulates the constant (eq., Higgs, gluon, or electron) energy field measurable in experiments, for example, on the Large Hadron Collider (see **Table 1**). But generally, the fields are varied and composed of atomic quanta of different intensities (6.11). Examples of different variable fields are shown in **Figure 7**.

6.7. Different Fields Have Different Atomic Quanta

Each field has its own size parameter s, like a Compton or other length [21], typically measured in experiments, for example, $s = 2.43 \times 10^{-12}$ m for an electron field (see **Table 1**), or Planck length $s = 1.6 \times 10^{-35}$ m for the spacetime field first described with ASF Functions [1]-[8] in 2018. So, generic $AQuant$ definition (6.10), (6.11) can be introduced for every field type, for example

$$AElectron = \varepsilon up\left(\frac{x-b}{s}\right),$$

$$ElectronField = \sum_k \varepsilon_k up\left(\frac{x-sb_k}{s}\right) = Sum\ of\ AElectrons,$$

$$s = 2.43\times 10^{-12}\ \text{m};$$

$$AHiggs = \varepsilon up\left(\frac{x-b}{s}\right),$$

$$HiggsField = \sum_k \varepsilon_k up\left(\frac{x-sb_k}{s}\right) = Sum\ of\ AHiggs,$$

$$s = 1.58\times 10^{-18}\ \text{m};$$

$$AUpQuark = \varepsilon up\left(\frac{x-b}{s}\right),$$

$$UpQuarkField = \sum_k \varepsilon_k up\left(\frac{x-sb_k}{s}\right) = Sum\ of\ AUpQuarks,$$

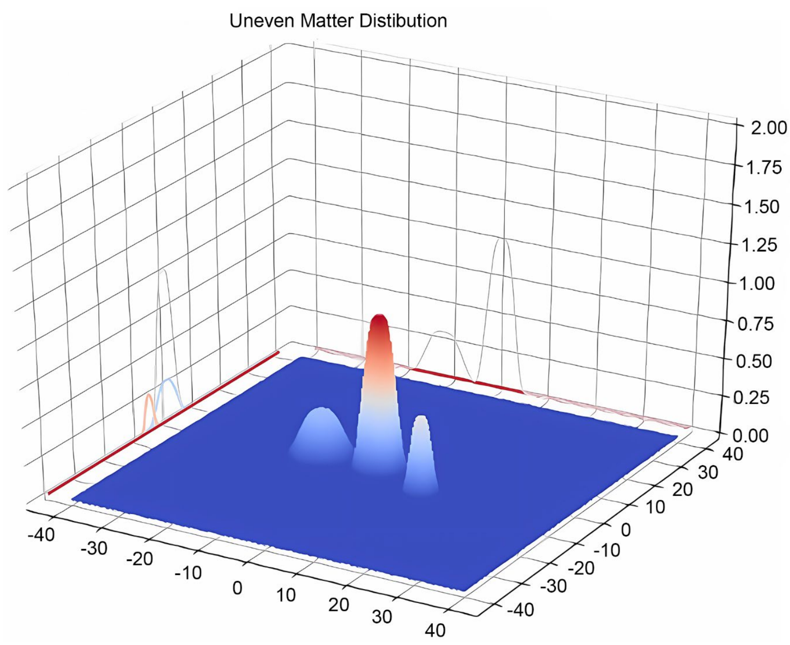
$$s = 5.42\times 10^{-13}\ \text{m};$$

$$ASpacetime = \varepsilon up\left(\frac{x-b}{s}\right),$$

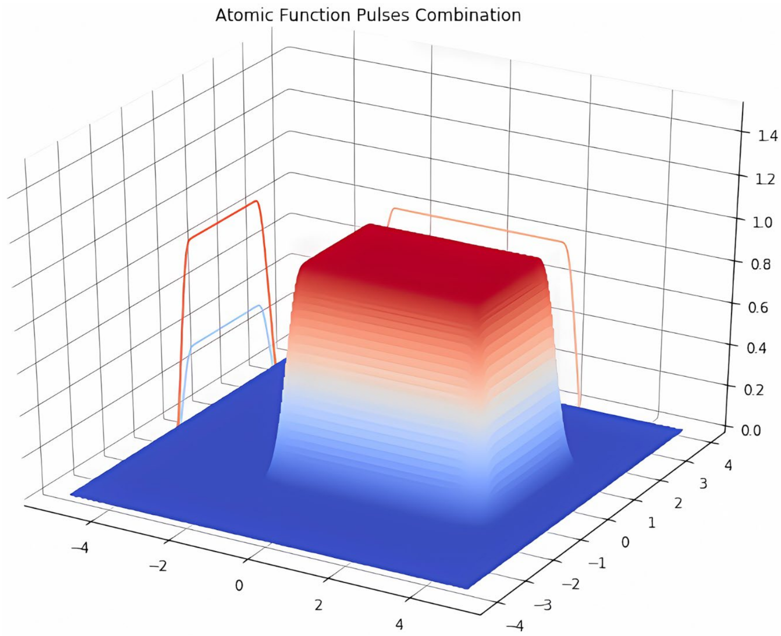
$$SpacetimeField = \sum_k \varepsilon_k up\left(\frac{x-sb_k}{s}\right) = Sum\ of\ ASpacetime, \quad (6.13)$$

$$s = 1.6\times 10^{-35}\ \text{m}.$$

Figure 8 schematically shows different energy levels for different quanta, and different distributions of fields described by Atomic Functions [1]-[9]. The benefits of introducing separate Atomic Quanta are that the engineers in each field can work with their own quanta instead of generic representations for all fields.



(a)



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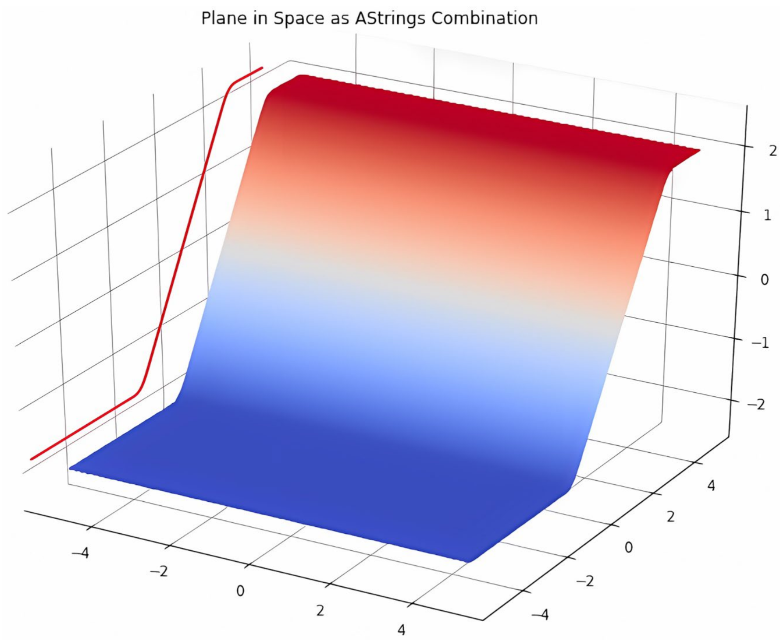


(c)

Figure 8. (a) Different fields have quanta of different energy levels; (b) Constant field as superpositions of a few atomic pulses; (c) Composing a growing field/brane from AString functions.

6.8. Energy-Mass Relationship for Atomic Quanta

The energetic definition of Atomic Quanta (§6.4) is especially useful for establishing relationships between energies and other matter characteristics. Following Einstein's energy-mass relationship $E = mc^2$, it is possible to express a field atomic quantum (6.10) in terms of mass:

$$m(x_1,\cdots,x_n)=\frac{\varepsilon up_n\left(\frac{x_n}{s}\right)}{c^2}; E=\varepsilon s^n=mc^2.$$

$$\boxed{m=\int m(x_1,\cdots,x_n)\,\mathrm{d}^n x=\int\frac{\varepsilon}{c^2}up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x=\varepsilon s^n/c^2.} \tag{6.14}$$

This formula not only unites Einsteinian Special Relativity and Atomic Functions theories but also correlates with Einstein's 1933 perspective "atomic theory" [1]-[3] [25]. It also emphasizes mass as an interaction rather than a matter collection phenomenon.

6.9. Associating Atomic Quanta with Vibrating Strings

Instead of associating Atomic Quantum energy (6.10) with mass (6.14), it is possible to choose another universal Planck–Einstein energy-frequency relation [21] $E=\hbar\nu$. In this case, Atomic energy distributions (6.8) would have the meaning of the fields of vibrating strings

$$\textit{field}=\textit{Sum of Strings}; \textit{or } \nu(x_1,\cdots,x_n)=\sum\nolimits_k \nu_k up_{kn}(x_n/s) \tag{6.15}$$

with elementary atomic pulse/quantum

$$\boxed{E=\int \textit{String}(x_1,\cdots,x_n)\,\mathrm{d}^n x=\int\varepsilon up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x=\varepsilon s^n=\hbar\nu,} \tag{6.16}$$

or, in terms of frequencies,

$$\nu=\int \nu(x_1,\cdots,x_n)\,\mathrm{d}^n x=\int\frac{\varepsilon}{\hbar}up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x=\varepsilon s^n/\hbar. \tag{6.17}$$

Let's note that we are not postulating that the coefficient $\hbar$ is a constant for every field. It just relates energy to a frequency, and we know that for a photon field, it is the Planck constant [21].

These formulae link the Atomic Functions theory with string theory [28]-[31], assuming that atomic field distributions can be expressed in terms of frequencies of vibrating strings. For example, the parabolic field around point x (4.12) can be expressed as the superposition of 9 ($N=4$) AF pulses/notes/keys

$$x^2\equiv\sum\nolimits_{k=-N}^{k=+N}\left(\frac{k^2}{64}-\frac{1}{36}\right)up\left(x-\frac{k}{4}\right). \tag{6.18}$$

with amplitudes $\left(\frac{k^2}{64}-\frac{1}{36}\right)$ which can be related to some string frequencies. The prospects of AString theory are described in §11.

6.10. Associating Atomic Quanta with Chronal Fields

Chronal fields were hypothesized in the 1980s by A. Veinik [39], who suggested that every fundamental field, including time and space, should have its own quanta—chronants and mertriants—the presence of which gives matter the properties of the "time run rate", "size", and "order of location". Later development of string theories [28]-[31] correlates with these hypotheses because vibrating strings define the frequencies of timing processes and locations of strings with size. Chronal intensity τ is also measured in frequencies [39], and replacing ν with τ in atomic quanta (6.16) and string fields (6.17) definitions allows deriving the atomic chronal fields with a quantum of energy

$$\boxed{E = \int Chronal\left(x_1,\cdots,x_n\right)\mathrm{d}^n x = \int \varepsilon up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x = \varepsilon s^n = \hbar\tau,} \tag{6.19}$$

or, in terms of chronal intensity,

$$\tau = \int \tau\left(x_1,\cdots,x_n\right)\mathrm{d}^n x = \int \frac{\varepsilon}{\hbar} up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x = \varepsilon s^n/\hbar. \tag{6.20}$$

Here, $\hbar$ is not strictly speaking a Planck constant but a proportionality coefficient to match the chronal intensity (of strings) to energy E. Interestingly, this line of research may lead to the theories where energies $E = \varepsilon s^n = \hbar\tau$ are not fundamental but emergent from chronal and metric fields, in correlation with some popularised theories where energy may be related to "time".

7. The Properties of Atomic Quanta of Fundamental Fields

Atomic Quantum (6.10) provides the novel description of elementary field excitation/ripple with Atomic Functions, with the properties described next.

7.1. The Shape of the Atomic Quantum

Atomic Quantum (6.10) has a smooth pulse/excitation/ripple shape described by the Atomic Function (**Figure 1**, **Figure 6**, **Figure 8**), generally in n dimensions. The key question is why can't we use the other pulse looking functions, for example, Gaussian $G(x) = a\mathrm{e}^{-b(x-c)^2}$, or some known spline. The answer is that from translations of a few Gaussians, it is not possible to compose even the simplest polynomial fields (like a line or parabola), while shifts and stretches of a few AF pulses compose a polynomial of *any* order, hence analytic field functions.

Another important question is the atomic function shape invariance in 3D space—the 3D pulses $\varepsilon up(x/s)up(y/s)up(z/s)$ may look different if we rotate the coordinate system x, y, z, or see the pulses from a different angle (**Figure 6**, **Figure 8**). Here, it is important to note that atomic pulses are actually the "atomized" versions of analytic functions appearing in solutions of differential QFT field equations (§5,6), typically invariant against coordinate rotations and Lorentzian transformations [21]. Final analysis of the solutions requires the selection of a coordinate system and writing the functions, "atomization" of which would lead to an elementary $\varepsilon up(x/s)up(y/s)up(z/s)$ ripple/atom/quantum. It does not mean that there is a "rigid" object like a 3D Lego block that physically exists somewhere in space. Nature does not know what is x, y, z, and the AF Quantum pulse (6.10) expresses some relationship/distribution/feature of a mathematical solution, in our case, the composition of any polynomial in any coordinate system from finite pieces.

7.2. The Energy and Mass of an Atomic Quantum

Energy of a quantum excitation is an integral (6.10) of an elementary AF excitation/quantum, matching the QFT convention. This exactly calculable integral (4.5), (4.6) is a product of two parameters—amplitude ε and size/width/volume s^n factor; in 3D, it is $V = s^3$. The bigger and more intense the excitation, the more energy/mass it has. Association of energy with mass (§6.8), based on Einsteinian $E = mc^2$, means that Atomic Quantum carries some mass $m = \varepsilon s^n / c^2$. Noting that $V = s^n$ is a volume and is a "matter" density, we can write

$$\varepsilon = \varrho c^2 ; \quad E = mc^2 = \varepsilon s^n ; \quad m/V = \varrho ; \quad V = s^n \tag{7.1}$$

It reminds the Einsteinian $E = mc^2$, but expressed in terms of an energy density ε of an Atomic Quantum pulse.

7.3. The Size of Quanta Is Defined by the Field Type

Each fundamental field (photon, gluon, electron, Higgs) has its own (minimal) atomic quantum size s. Despite the s constancy, we need to be careful about "fixing" it. According to Einstein's Special Relativity and Lorentzian transformations, the sizes of a moving object can shrink. It means the quantum pulse $\varepsilon up\left(\frac{x}{s}\right)$ width s is a "dynamic" uniformly shrinking constant, the same for all quanta. In QFT, the particles are not the "rigid" matter objects with fixed size, but the "excitations" of fields, and the Atomic Quantum model (6.10) describes exactly that.

7.4. The Energy Density of Atomic Quantum

The key intensity parameter ε of an Atomic Quantum (6.10) $\varepsilon up\left(\frac{x}{s}\right)$ have a meaning of energy density ε, which should be treated as the amplitude of a pulse that can fall into positive or negative territory [8].

Let's elaborate on the important relationship $E = \varepsilon s^n$ between Atomic Quantum energy E and quantum intensity ε, remembering that size s^n depends on the field type. For example, the Higgs field has energy levels of 125GeV, while electromagnetic fields are much weaker. It does not mean that quanta intensities ε obey the same proportions, because Atomic Quanta of different fields have different s^n sizes which are the true field-dependent constants. Using the string theory [29]-[31] analogy, the energy of strings should depend not only on frequencies but also on the sizes s^n of strings. So, energy, as a universal concept for many theories, becomes a product of two parameters, creating multiple opportunities discussed in §9-11.

7.5. The Dimension of Atomic Quantum

Another parameter of the Atomic Quantum model (6.10) is the number of dimensions n appearing in the definition of a size/volume s^n and n-dimensional

Atomic Function $up_n\left(\frac{x_n}{s}\right)$. The ASF theory does not impose the number of dimensions, and is applicable for the familiar 4D spacetime or for 11 dimensions in some string theories.

7.6. Radial Atomic Functions Can Describe Loops

Atomic Functions can also describe loops presumed to be the building blocks of spacetime in Loop Quantum Gravity (LQG) theory [26]-[28]. Radial AF (4.6) (**Figure 1**, **Figure 6**)

$$up(r,c,s) = cup\left(\frac{r}{s}\right); r = \sqrt{x^2 + y^2 + z^2} \tag{7.2}$$

is defined on the area resembling a spherical quantized loop. Atomic distributions (6.11) can be interpreted as overlapping interconnections of loops, with opportunities for research towards a common atomic LQG theory (§13).

8. Atomic Quantization of Gravity and Atomic Spacetime

General Relativity (GR) was the first theory where Atomic String Functions were introduced during 2018-2022 [1]-[8], which led to the Atomic Spacetime theory [1]-[5] briefly overviewed hereafter.

8.1. Atomic Spacetime Quantization Theorem

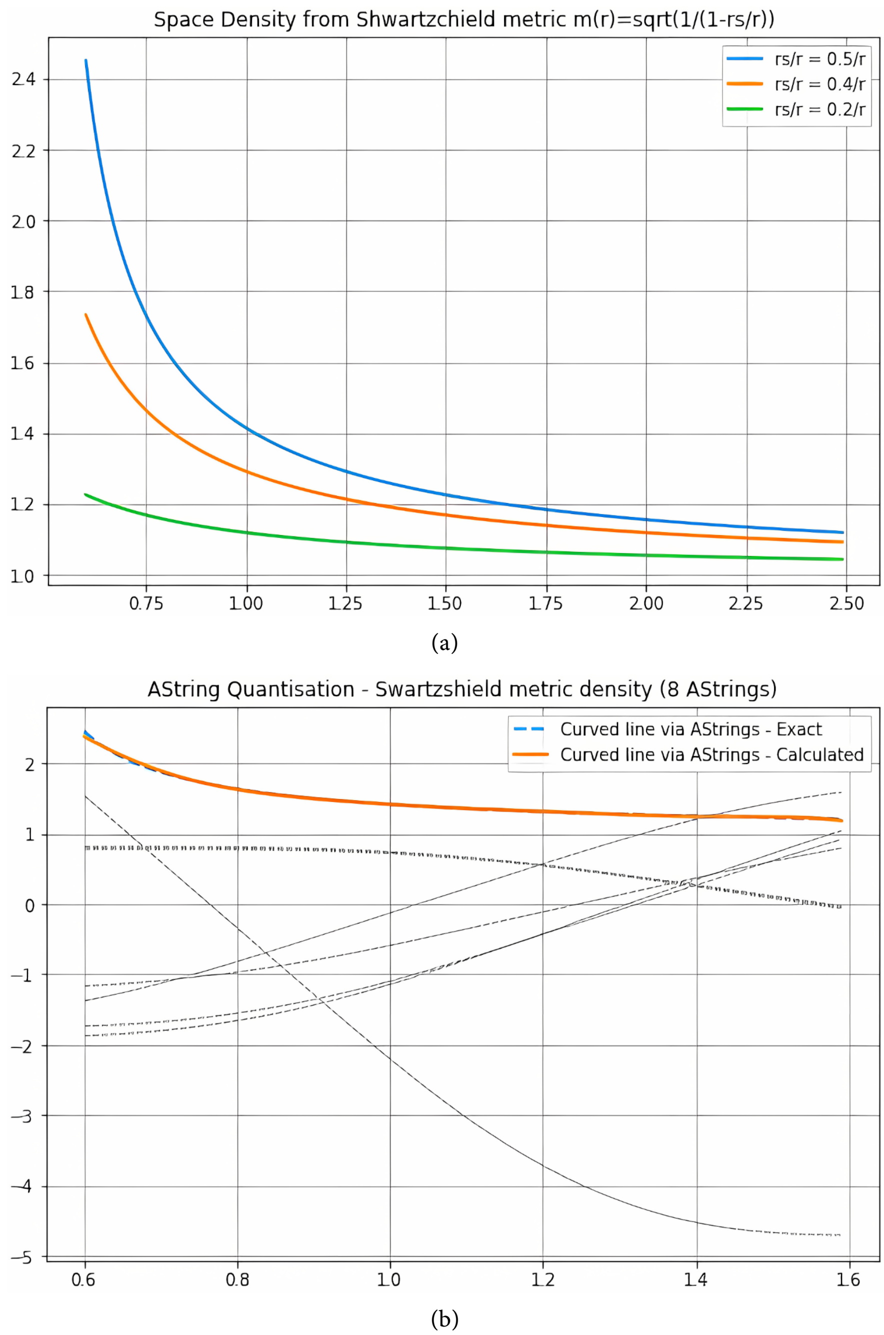
Atomic Spacetime formalism is encapsulated in the following theorem [1]-[8].

Theorem (*Atomic Spacetime Theorem*). For analytic manifolds, Einstein's curvature tensor $G_{\mu\nu}$ preserves analyticity and polynomiality and yields GR solutions for spacetime shapes, deformations, curvatures, and matter/energy tensors representable by Atomic Series via multi-dimensional Atomic AString Functions superpositions:

$$\begin{aligned} G_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}; \\ g_{\mu\nu} &= \sum\nolimits_{\mu\nu i} \boldsymbol{UP}\left(x_i, \boldsymbol{a}_{\mu\nu i}, \boldsymbol{b}_{\mu\nu i}, \boldsymbol{c}_{\mu\nu i}\right) \\ &= \sum\nolimits_{\mu\nu i} \boldsymbol{AString}\left(x_i, \boldsymbol{a}_{\mu\nu i}, \boldsymbol{b}_{\mu\nu i}, \boldsymbol{c}_{\mu\nu i}\right). \end{aligned} \tag{8.1}$$

The proof described in detail in [1] [2] is based on the following sequence. For analytic manifolds-spacetime geometries described by analytic functions $\tilde{x}_i = \tilde{x}_i\left(x_j\right)$—the metric tensors $g_{\mu\nu}$ composed of derivatives and multiplications [33] would also be some analytic functions. Injected into Christoffel operators, Ricci tensors $R_{\mu\nu}$, and scalar $R = g^{\mu\nu} R_{\mu\nu}$ [33], they would yield another set of analytic functions [2] representable by Taylor power series because the cross-multiplication of derivatives and superposition of analytic and polynomial functions would also be analytic [32]. Injected into (8.1), they produce Einstein's curvature $G_{\mu\nu}$ and energy-momentum tensors $T_{\mu\nu}$ supposedly representable by polynomials via multi-dimensional Taylor series. Because a polynomial of any order is exactly representable via Atomics (Theorem §4.3), the spacetime curvature, metric, and energy/momentum tensors can be represented as the superpositions (8.1) of multi-dimensional Atomic ***UP*** and ***AString*** functions (4.6) with derivatives expressed via themselves.

In a nutshell, this theorem tells that the smooth spacetime field, as any analytic field (4.13), can be represented as overlapping interaction of finite Atomic String Functions which can serve as "building blocks"/"mathematical atoms"/gravitons/quanta [1]-[5] of spacetime (**Figure 9**) offering Quantum Gravity, AString theory, and unification opportunities (§9-11).



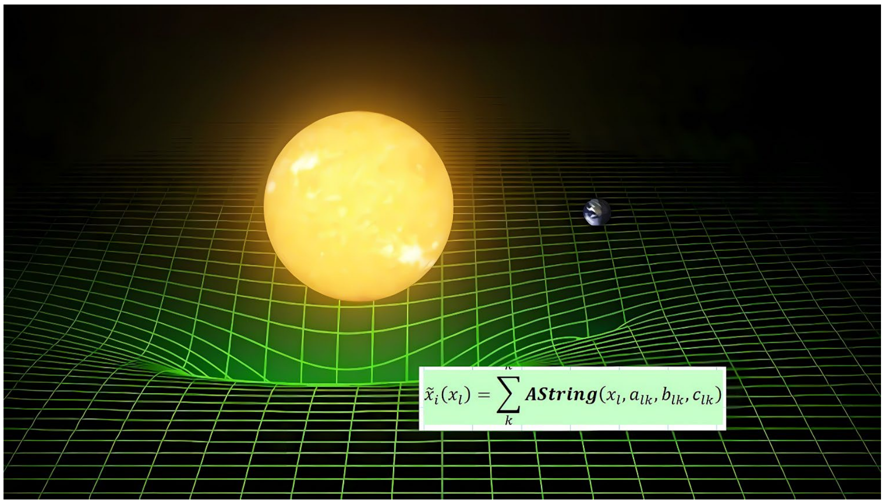


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Figure 9. (a) Space density function from Schwarzschild GR solution; (b) Representing Schwarzschild metric via AStrings; (c) Spacetime made of AStrings.

8.2. Spacetime Solutions Examples

Examples [1]-[5] are the analytic Friedmann solution [33] expandable via power series, hence Atomics (4.13):

$$d\overline{s}^2 = dr^2 + S_k(r)^2 d\Omega^2;$$

$$S_k(r) = r\,sinc\left(r\sqrt{k}\right) = r - \frac{kr^3}{6} + \frac{kr^5}{120} - \cdots = \sum_k c_k up\left(\frac{r-b_k}{a}\right). \quad (8.2)$$

Schwarzschild solution (**Figure 9**) for radial bodies and black holes also leads to analytic functions representable by Atomic Functions (4.13):

$$ds^2 = -A(r)c^2dt^2 + B(r)dr^2 + r^2d\Omega; A(r) = \left(1-\frac{r_s}{r}\right); B(r) = \left(1-\frac{r_s}{r}\right)^{-1}.$$

$$A(r) = \sum_k c_k up\left(\frac{r-b_k}{a}\right) = \sum_k AString(r, a_k, b_k, c_k), r \neq 0. \quad (8.3)$$

8.3. Spacetime as Open AStrings Interactions

Atomic Spacetime theory [1]-[5] leads to the conclusion that curved spacetime *expansion* is better described by AString kink functions (**Figure 9**), which remind "open strings" from string theory [28]-[31] as described in §11:

$$\tilde{x}(x) = \sum_k c_k AString\left((x-b_k)/a\right). \quad (8.4)$$

Spacetime "density" (metric) are some $up(x)$ pulses—"closed strings" (§11)—combinations:

$$\rho(x) = \tilde{x}'(x) = \sum_k up(x, a, b_k, c_k). \quad (8.5)$$

They are related by the fundamental AF theory Equation (4.8) (§9,11):

$$up(x) = AString'(x) = AString(2x+1) - AString(2x-1). \quad (8.6)$$

8.4. Quantization of Length and Planck Scales

Finite and fractal ASF functions are naturally good for the quantization of spacetime. AString function appears in "quantization of length" conceptualizing a distance as the overlapping interactions of "elementary pieces"/mertriants [1] [3]-[8] similar to strings [28]-[31]:

$$L(x)=\int_0^x \mathrm{d}\tilde{x}(x)=\int_0^x \rho(x)\mathrm{d}x=\sum_k AString(x,a,b_k,c_k). \tag{8.7}$$

Minimal pulse half-width parameter a in (8.4)-(8.7) can be associated with the Planck length $a=l_p=1.616\times10^{-35}$ m, graviton size [1] [3] [21] used in AString Quantum Gravity (§9), string or loop size from the string and LQG theories [26]-[31], or, due to fractality of AF and spacetime (§4.6), with some field's variability parameter.

8.5. Atomic Graviton Model

Similar to Atomic QFT fields (§6, 7), the atomic quantization of spacetime via Atomic Series (8.1) allows extracting *one* elementary pulse (6.10) typically associated with a field quantum – spacetime graviton first discussed in [1] [3]:

$$\boxed{E=\int AGraviton(x_1,\cdots,x_n)\mathrm{d}^n x=\int \varepsilon up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x=\varepsilon s^n.} \tag{8.8}$$

Here, $s=l_p=1.616\times10^{-35}$ m is a quantum size Planck length, ε is a variable energy E density of a graviton field in $n=4$ dimensions. So, in addition to (8.4)-(8.5), spacetime can be conceptualized as a lattice field made of interactions of overlapping gravitons:

$$\begin{aligned} &AGraviton=\varepsilon up\left(\frac{x-sb}{s}\right),\\ &Spacetime=\sum_k \varepsilon_k up\left(\frac{x-sb_k}{s}\right)=Sum\ of\ AGravitons. \end{aligned} \tag{8.9}$$

It also correlates well with "atomic theory" envisaged by A. Einstein in 1933 [25], with more details in [1] [3]-[8].

9. Prospects of Reconciling General Relativity and Quantum Theories Based on AString Quantum Gravity

The unified description of fields and spacetime via finite Atomic String Functions naturally leads to the new AString Quantum Gravity described hereafter. The "simplify and unite" research method is based on decomposing the fields into finite "mathematical atoms" and examining how those atoms can be united.

9.1. Quantum Gravity as an Unresolved Problem in Physics

For decades, reconciling General Relativity and Quantum theories was one of the most difficult problems in physics included in the list of unresolved problems with the following formulation [41]. "*Can quantum mechanics and general relativity be realized as a fully consistent theory* (*perhaps as a quantum field theory*)? *Is* *spacetime fundamentally continuous or discrete*? *Would a consistent theory involve a force mediated by a hypothetical graviton, or be a product of a discrete structure of spacetime itself* (*as in loop quantum gravity*)? *Are there deviations from the predictions of general relativity at very small or very large scales or in other extreme circumstances that flow from a quantum gravity mechanism*?". This chapter attempts to address some of these problems.

As discussed in §1 and [8], one of the QG problems is related to the mathematical incompatibility between complex nonlinear GR and QFT theories, or simply the absence of "common mathematical blocks". Conceptually, it was clear that the energy of QFT fields should curve spacetime as per the Einsteinian GR equation [21] [33] [35]:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}\,;\quad T_{\mu\nu} = T_{\mu\nu}^{EM} + T_{\mu\nu}^{Weak} + T_{\mu\nu}^{Strong} + T_{\mu\nu}^{Higgs} + \cdots. \tag{9.1}$$

GR Ricci scalar R is included in the definition of the QFT Lagrangian (5.21) on equal footing with other fields [21] [35]

$$\mathcal{L}_{SM} = R - F_{\mu\nu}F^{\mu\nu} - G_{\mu\nu}G^{\mu\nu} - W_{\mu\nu}W^{\mu\nu}\cdots \tag{9.2}$$

But the question remains, where are those "common mathematical blocks" on the level of quanta, and how are the quanta of fields related to the quanta of spacetime?

9.2. Prospective "Atomic Theory" of A. Einstein (1933)

Interestingly, the hint to resolve the *"quantum-riddle"* problems was expressed by A. Einstein in a 1933 [1] [2] [8] [25], where he envisaged *"atomic theory"* with "*finite regions of space*" with discrete energy levels, which are strikingly similar to Atomic Quanta (6.10) (§6, 7). The "atomic theory" described in this work addresses some QG problems by finding the finite "common blocks" which link GR and quantum theories, conceptualizing *atomized* quantum fields as *atomized* distortions of spacetime, as described next. String and LQG theories [26]-[31] also offer the unification framework, but this one is based on the ASF functions.

9.3. Reconciling General Relativity with Quantum Theories Based on Atomic String Functions

Atomic String Functions (ASF) theory does not operate with infinite-spread functions and "point-in-space" theories, which A. Einstein [25] identified as "stumbling blocks" (§1.2). Really, as M. Kaku mentioned [31] [44], it is not possible to find commonality if particles are "dots". This gave rise to the strings and LQG theories [26]-[31], where the building blocks of all fields are *finite*. But the new mathematical apparatus of *finite* ASF functions applicable for both QFT (§5-7) and GR (§8) allows discovering these "linking blocks" quite naturally with the following steps.

9.3.1. Step 1—Common Representation of Quantum and Spacetime Fields with Atomic Series

Let's start by summarizing the findings from previous chapters. ASF Functions, or Atomics (§4), can exactly compose the polynomials of any order and analytic functions from a limited number of atoms near a point x:

$$x^n \equiv \sum_{k=-N}^{k=+N} C_k up\left(x - k2^{-n}\right). \tag{9.3}$$

This allows building the universal Atomic Series (4.14), which can be applied to linear and nonlinear theories traditionally built with differential operators that "preserve polynomiality" (§5, §8). For QFT (§5-7), it decomposes the continuous energy density fields into finite AFs in n dimensions on some lattice:

$$\varepsilon\left(x_1,\cdots,x_n\right) = \sum_k \varepsilon_k up_{kn}\left(x_n/s\right), \tag{9.4}$$

or, in the 1D case (6.11),

$$\varepsilon(x) = \sum_k \varepsilon_k up\left(\frac{x - b_k s}{s}\right). \tag{9.5}$$

Taking one pulse at some point $x = 0$, $b_k = 0$ allows extracting the Atomic Quantum with energy (6.10)

$$E = \int \varepsilon up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x = \varepsilon s^n;\ E = \int \varepsilon up\left(\frac{x}{s}\right)\mathrm{d}x = \varepsilon s \ \text{ for } n = 1. \tag{9.6}$$

Knowing the quantum sizes s (like Compton lengths [21]) for different fields allows matching energies to the experiments (§6, **Table 1**), for example, for Higgs bosons ($s = 1.58\times10^{-18}\ \mathrm{m}$, $E = 125\ \mathrm{GeV}$, $\varepsilon = 5.1\times10^{45}\ \mathrm{J/m^3}$).

Next, let's describe the Einsteinian spacetime (§8) with metric $g_{\mu\nu}$ and interval $d\tilde{x}^2 = g_{\mu\nu}dx_\mu dx_\nu$ [33]. For simplification, let's consider the curved 1D spatial dilaton-like manifold [42], where it reduces to

$$d\tilde{x}^2 = g_{11}dx^2;\ \text{or}\ d\tilde{x} = \rho(x)dx;\rho(x) = \sqrt{g_{11}}. \tag{9.7}$$

Here, $g_{11}(x)$ is a 1D component of the metric tensor, and $\rho(x)$ have a meaning of spacetime "density" or unitless deformation $\rho = \tilde{x}'(x)$. Decomposing spacetime into ASF functions (§8) leads to the formula (8.4) for curved spacetime $\tilde{x}$ expansion (**Figure 9**)

$$\tilde{x}(x) = \sum_k d_k AString\left(\frac{x - ab_k}{a}\right) \tag{9.8}$$

where due to (4.8) deformation $\rho(x)$ becomes the superposition of pulses $up(x/a)$ with half-width a:

$$\rho(x) = \tilde{x}'(x) = \sum_k \rho_k up\left(\frac{x - ab_k}{a}\right);up(x) = AString'(x). \tag{9.9}$$

Now, we can see the emergence of a "common block"—atomic function $up(x)$—between quantum $\varepsilon(x)$ (9.5) and spacetime $\rho(x)$ (9.9) fields:

$$\varepsilon(x) = \sum_k \varepsilon_k up\left(\frac{x - b_k s}{s}\right);\rho(x) = \sum_k \rho_k up\left(\frac{x - ab_k}{a}\right) \tag{9.10}$$

Atomic pulses $cup\left((x-b)/a\right)$ describe the "finite regions of space" in "atomic theory" envisaged by A. Einstein in 1933 [25]. Energy density $\varepsilon(x)$ describes a field, for example Higgs or an electron field, while $\rho(x)$ describes the spacetime deformation distributions. Those fields are not independent but related by GR and QFT Equations (9.1), (9.2)

$$\varepsilon(x) \sim \rho(x). \quad (9.11)$$

The goal is to examine this relationship and understand how mathematical blocks $\varepsilon_k up\left(\frac{x-b_k s}{s}\right)$ and $\rho_k up\left(\frac{x-ab_k}{a}\right)$ can be linked.

9.3.2. Step 2—Matching the Fields to General Relativity

Let's now explore how the atomized field and spacetime (9.10) can be related by GR equations (9.1) [33]

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = kT_{\mu\nu}; k = \frac{8\pi G}{c^4}. \quad (9.12)$$

For a 1D manifold, $\rho(x)$ (9.9) is the square root of a metric function (9.7)

$$\sqrt{g_{11}(x)} = \rho(x) = \sum_k \rho_k up\left(\frac{x-ab_k}{a}\right). \quad (9.13)$$

Next, field energy density $\varepsilon(x)$ (9.10) has the same units as stress-energy tensor $T_{\mu\nu}$, hinting that in 1D case $\varepsilon(x)$ can be associated with T_{11} component:

$$\varepsilon(x) = T_{11}(x) = \sum_k \varepsilon_k up\left(\frac{x-b_k s}{s}\right), \varepsilon_k = T_{11k}. \quad (9.14)$$

One atomic quantum would have energy dependent on the size s of a field quantum (6.10):

$$\int \varepsilon up\left(\frac{x}{s}\right) dx = \varepsilon s = T_{11} s. \quad (9.15)$$

For a 1D manifold, the generic GR (9.12) can be simplified into the dilaton gravity equation for the metric function $g_{11}(x)$ [42]

$$\frac{8\pi G}{c^4} T_{11} = -\nabla_1 \nabla_1 \phi + g_{11} \nabla^2 \phi + \frac{1}{2} W g_{11} (\nabla \phi)^2 - W(\phi')^2 - \frac{1}{2} g_{11} V = L(\phi, g_{11}). \quad (9.16)$$

Here, the energy density function $T_{11}(x) = \varepsilon$ is expressed in terms of the dilaton function ϕ, its first ∇_1 and second derivatives ∇^2, and the model functions $W(\phi)$, $V(\phi)$ [42]. With $T_{11} = \varepsilon$ and $g_{11} = \rho^2$ from (9.13), (9.14), this equation can be reformulated in terms of energy densities $\varepsilon(x)$ and spacetime deformations $\rho(x)$

$$\frac{8\pi G}{c^4} \varepsilon = -\nabla_1 \nabla_1 \phi + \rho^2 \nabla^2 \phi + \frac{1}{2} W \rho^2 (\nabla \phi)^2 - W(\phi')^2 - \frac{1}{2} \rho^2 V = L(\phi, \rho). \quad (9.17)$$

This equation provides the desired expression for a generic 1D relation (9.11).

9.3.3. Step 3—Linking Atomized Spacetime and Fields

By itself, atomic theory, being the theory of finite representations of continuous functions, cannot establish the relationships between fields, and for that, we need the field equations like (9.17). Injecting series (9.10)

$$\varepsilon(x) = \sum_k \varepsilon_k up\left(\frac{x-b_k s}{s}\right); \rho(x) = \sum_k \rho_k up\left(\frac{x-ab_k}{a}\right) \quad (9.18)$$

into (9.17) allows extracting some link function/matrix/operator $Link_{kl}$ which links energy density ε_k at lattice node k to the spacetime deformations on another lattice of nodes l:

$$\varepsilon(x) \sim \rho(x), \quad \varepsilon_k = Link_{kl}(\rho_l). \tag{9.19}$$

In some particular cases, like Friedmann solutions for homogeneous spacetime expansions (8.2), the link function is especially simple and can be matched for every node $\varepsilon_k = \alpha\rho_k$, but for generic curved manifolds, it is not the case, and we have to use a more generic linkage (9.19).

9.3.4. Step 4—Analyzing Atomized Spacetime and Fields

Analyzing the atomic representations (9.18), (9.19), we need to acknowledge that those fields operate on different scales; the width parameter s has a scale of fields quanta (6.10) (like Compton length [21]), while spacetime (and graviton) fields (8.7) operate on the scales of Planck lengths:

$$\varepsilon(x) = \sum\nolimits_k \varepsilon_k up\left(\frac{x - b_k s}{s}\right); \rho(x) = \sum\nolimits_k \rho_k up\left(\frac{x - l_p b_k}{l_p}\right), \tag{9.20}$$

$$s = 1.58 \times 10^{-18}\ \text{m}, \quad a = l_p = 1.616 \times 10^{-35}\ \text{m}. \tag{9.21}$$

Because Planck lengths are much granular, the fields lattice size s can always be subdivided and expressed via some large number N of Planck lengths $s = Na$, ($N \approx 10^{18}$ for Higgs (9.21)), and it is always possible to "upscale" the spacetime lattice to the lattice of fields, so spacetime can "accommodate" any macroscopic field. Furthermore, due to AF finiteness (4.13), only a limited number of spacetime quanta would be required around a given point.

Relating atomized QFT fields and spacetime can be intuitively simply expressed for energies, which are the integrals of AF quanta (9.16), (6.10)

$$\int \varepsilon up\left(\frac{x}{s}\right) \mathrm{d}x = \varepsilon s, \int \rho up\left(\frac{x}{l_p}\right) \mathrm{d}x = \rho l_p. \tag{9.22}$$

Because spacetime is much granular, the field energy is representable by a sufficient number of energies of spacetime quanta.

9.4. How Macroscopic Atomic Fields Curve Spacetime

This atomic model suggests the following process of how the field curves the spacetime on the level of quanta. Appearance of a Higgs-boson or electron as a ripple of quantum field $\varepsilon up\left(\frac{x}{s}\right)$ with energy εs creates "pressure" on spacetime to curve, or, as J. Wheeler famously said [33], "tells the spacetime how to curve". Like for elastic fabric with Einsteinian proportionality coefficient $\frac{8\pi G}{c^4}$ (9.1), the curvature/deformation is spread around the local area defined by the Link function (9.19) derived from the Einsteinian GR field Equations (9.1)-(9.17). The spacetime deformation spreads with atomic pulses $\rho_k up\left(\frac{x - l_p b_k}{l_p}\right)$ causes the displacement of curved spacetime

$$\tilde{x}(x)=\int\rho(x)\mathrm{d}x=\sum_k d_k AString\left(\frac{x-ab_k}{a}\right). \tag{9.23}$$

This model provides a hint of how the nature that knows nothing about functions works on the fundamental level (§12). It leads to simple, fast, and effective rules of shifting a few neighboring spacetime atoms/strings/loops to accommodate the field configurations, for example, 9 atoms (4.13) to reproduce a parabola at every point x

$$\sum_{k=-4}^{k=+4}\left(\frac{k^2}{64}-\frac{1}{36}\right)up\left(x-\frac{k}{4}\right)\equiv x^2. \tag{9.24}$$

9.5. Common Mathematical Block between Quantum and Spacetime Fields

A concise linkage between spacetime and quantum fields can be formulated by denoting the atomic functions in (9.20) as

$$up_{field}\left(\frac{x}{s}\right)=up\left(\frac{x}{s}\right);up_{spacetime}\left(\frac{x}{l_p}\right)=up\left(\frac{x}{l_p}\right). \tag{9.25}$$

Fields operate on a lattice with unique size s, while spacetime is granulated by the Planck length l_p. Both functions are derived from the same core atomic function $up(x)$ (4.1), so we can derive the desired expression

$$\boxed{up_{field}(sx)=up_{spacetime}(l_p x)}. \tag{9.26}$$

AF $up(x)$ in turn is made of two AStrings (4.8)

$$up(x)=AString'(x)=AString(2x+1)-AString(2x-1) \tag{9.27}$$

which describe flat and curved spacetime expansion (9.23), (4.9) as a translation of AString kinks resembling "open strings" from string theory (§11):

$$x\equiv\cdots+AString(x-1)+AString(x)+AString(x+1)+\cdots$$

$$\tilde{x}(x)=\sum_k c_k Astring((x-b_k)/a_k). \tag{9.28}$$

These formulae introduce common mathematical blocks—atomic functions $up(x/s)$—linking together fields and spacetime. In traditional QFT mathematics, operating with featureless "points" obtaining this relation is hardly possible because there is no easily derivable "common block" between "dots". As A. Einstein mentioned [25], operating with infinite-spread functions and point "dots" can cause the "stumbling blocks" proven hard to overcome. In the same lecture [25], he proposed a perspective "atomic theory" based on "finite regions of space", and atomic functions make this theory a reality. Physicists M. Kaku [31] [43] and C. Rovelli [26] [27] also mentioned the difficulties of finding "common blocks" between "dots" replaced by finite objects in LQG and string theories [26]-[31] [43].

Let's preempt the key question, whether it is possible to use alternative functions, for example, traditionally used in QFT and string theories Fourier series based on the core trigonometric function $c\sin(ax+b)$, or Gaussians $ce^{-b(x-a)^2}$ as the alternative "common blocks" between GR and Quantum Theories. The problem with those infinitely-spread functions is that with a limited number of harmonics, they cannot represent even the simplest polynomial— $f(x)=1$. Discretization techniques based on splines, like ax^2+bx+c, are universally applicable to *approximate* any fields, but due to limited smoothness, they are not capable of *exactly* representing a polynomial of *any* order (§4.4). Atomic functions are unique.

Atomic common blocks (9.25) allow enriching major "point-space" QFT and GR theories with new kinds of atomized solutions which naturally uphold quantization and finiteness of nature and avoid controversial singularities, QM wavefunctions collapse into point, infinitesimal dots, and other Einsteinian "stumbling blocks" (§1.2). Novel interpretations of atomized spacetime and QM fields are discussed in [1]-[9] [20], and in §8-12.

9.6. AString Quantum Gravity for Reconciling Quantum Theories with General Relativity

Atomic String Functions provide desired "common blocks" between theories—QFT theories include Atomic Pulses $up(x)$ while spacetime GR theory leads to $AString(x)$, the integrals of Atomic Functions, and because they are related $up(x)=AString'(x)=AString(2x+1)-AString(2x-1)$, the quanta of fields can be treated as the deformations/distortions of spacetime (§9.3). In string terminology (§11), spacetime is "made of" open strings ($AStrings$) while fields are made of "closed strings" $up(x)$ (solitonic atoms (§4.5), and closed strings are just made of two open strings connecting "matter" and "spacetime". This leads to the new theory of AString Quantum Gravity (AQG).

9.7. AString Quantum Gravity and QMGR Theory

In the simple, described version, AQG does not alter either GR nor QFT equations, focusing on "common finite blocks" by extending "points" into finite "regions of space", like in string and LQG theories [25]-[31]. As described in [20], for strong gravitational fields near black holes or small scales, the QM Schrödinger Equation (5.2) needs to account for curved coordinates while GR spacetime needs to incorporate QM wavefunction ψ, leading to QMGR equations [20] including cross-terms linking QM and GR with a common Lagrangian:

$$-\frac{\hbar^2}{2m}\nabla^{\mathrm{T}}\nabla\psi+V\psi+SQE\left(\psi,g_{\mu\nu}\right)=E\psi, \tag{9.29}$$

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=k\left(T_{\mu\nu}+QSE_{\mu\nu}\left(g_{\mu\nu},\psi\right)\right). \tag{9.30}$$

$$L\left(\psi,g_{\mu\nu}\right)=\int\sqrt{-g}\,\mathrm{d}^4x\left(\mathcal{L}_{GR}\left(g_{\mu\nu}\right)+\mathcal{L}_{QM}\left(\psi,g_{\mu\nu}\right)\right);\delta L=0. \tag{9.31}$$

ASF functions can be applied here as well because the "preservation of polynomiality" (§5,8) guarantees that if the equations are in principle resolvable in polynomials, they can be "atomized" with the Atomic Series. In the 1D case, it is interesting the appearance of $g\psi$ cross-function [20] linking 1D metric function (9.7) with QM wavefunctions in one conglomerate:

$$g\psi(x,t) = g(x)\psi(x,t). \quad (9.32)$$

It hints that wavefunction ψ acts as a "quantization factor" on top of spacetime density, somewhat similar to what ASF functions try to do—represent a field as a superposition of quantum-like pulses. Moreover, strong gravity should force QM wavefunctions to collapse [20] so the quantum fields would be mostly defined by peaks in spacetime densities described by ASF functions (9.20)-(9.26), which also prevent singularities. Further integration of AString Quantum Gravity with QMGR theory [20] is another interesting research direction (§13).

9.8. "Atomization" Instead of "Quantization" of Gravity

"Quantization of gravity" [41] has always been a main obstacle towards the unified theory of fields. With electromagnetic, strong, and weak forces united in QFT [21]-[24], complex but successful GR [33] was a distinct roadblock [26]-[28] [41]. Traditional mathematics based on infinitely spread functions and space-points has led to the "renormalization of infinities" and other problems [8] [25]. AString Quantum Gravity suggests the way of overcoming these "stumbling blocks" by using finite atomic functions where both QFT and GR can be "atomized" ("atomically quantized" §4.3) rather than "quantized". It leads to the common atomic function blocks (9.26), which provide the missing bridge between the theories (§9.3). This atomic theory can describe quantum fields (§5-7), spacetime (§8) [1]-[8], quantum mechanics [9] (§10), quantum gravity (§9), with the prospects of string and unified theory (§11). Interestingly, none of these successful major theories requires major modifications—we just need to acknowledge the existence of new previously unknown atomic solutions for QFT (§6), QM, and GR fields (§8), which introduce desired quantized finiteness and common unification blocks. For example, gravity is already "quantized" (§8.2) with AStrings (§8.4) and gravitons (§8.5) if we replace continuous functions with their finite representations accounting for quantized Planck lengths (§8.4) missing in classical GR [33]. The hardest unification part was not the altering theories but discovering that atomic functions (1971) and AStrings (2018) can exactly represent the polynomials of any order (1975) appearing in analytic solutions of all major theories (2022).

In Quantum Gravity discussions [26]-[31] [41], one frequently asked question is "which theory is "wrong"—quantum or GR, and needs to be "fixed" for unification". To answer, we need to understand that traditional continuous analysis based on infinitely spread functions and infinitesimal "dots" used in both Quantum and GR theories is the good *approximation* of finite quantum reality (§1.2), but dealing with approximations often leads to "stumbling blocks" [25] and missing links. Both Quantum and GR theories are very successful, but to unite them, we need to replace the approximations with finite functions better suited to describe finite "building blocks" like strings, loops, and excitations (§1.2). Using the right series not only simplifies the mathematical analysis but also allows uncovering hidden commonalities between theories [8]. So, none of the theories is wrong, but they seem "incomplete" in the sense of using approximated infinitely spread functions absent in localized nature. Traditional continuous GR, QFT, and QM theories still can be improved to include finiteness, and right finite functions can help with that.

It is tempting to acknowledge again the genius of A. Einstein, who, apart from his major revolutionary theories, also predicted "atomic theory" in a relatively unnoticed 1933 lecture [25]. Building the theory took 90 years, from Atomic Functions (1971) to AStrings (2018), Atomic Spacetime (2022), Atomic Quantum Mechanics (2024), and now AString Quantum Gravity and AString theories.

9.9. Questions from "Unresolved Problems in Physics"

For decades, Quantum Gravity was included in the list of unresolved problems with the following sequence of questions [41]. Let's provide the short answers based on the AString Quantum Gravity theory (§9), Atomic Quantum Fields (§5-7), Atomic Spacetime (§8, [1]-[8]), Quantum Gravity [20], and Atomic Quantum Mechanics [9] theories evolving for 8 years.

1) *Can quantum mechanics and general relativity be realized as a fully consistent theory* (*perhaps as a quantum field theory*)?

Yes. Theorem-based reformulation of those theories with Atomic String Functions [1]-[20] introduces quantum finiteness and allows composition of continuous fields from finite smooth "solitonic atoms". Atomic Quantum field theory (§5-7) matches "solitonic atoms" to quanta of fundamental QFT fields with known energies and treats them as atomic spacetime distortions (§9). "Atomic theory" was envisaged by A. Einstein in 1933 [25].

2) *Is spacetime fundamentally continuous or discrete*?

Both. Spacetime is continuous but composed of finite (but not discrete) smooth overlapping atoms described by ASF functions (§8). Discretization involves unwanted approximation, which does not provide smoothness, while "atomization" provides a "hybrid" model uniting continuity with finiteness (§5-9).

3) *Would a consistent theory involve a force mediated by a hypothetical graviton, or be a product of a discrete structure of spacetime itself* (*as in loop quantum gravity*)?

Both. Atomic theory provides "advanced lattice discretization"—smooth "atomization"—of fields composed from finite atoms which can be matched to spacetime gravitons (§8.5) [1] [3]-[8]. LQG [26] [27] is also based on the discretization and quantization of spacetime, but without ASF functions, with the interesting integration opportunity. AQG theory can be a hybrid between string and LQG theories (§11, 13).

4) *Are there deviations from the predictions of general relativity at very small or very large scales or in other extreme circumstances that flow from a quantum gravity mechanism*?

Yes. Atomic Spacetime theory, based on atomic functions, introduces the Planck length constant absent in classical GR (§8, 9). GR is based on infinite-spread functions and point-in-space concepts, which A. Einstein objected in 1933 as "stumbling blocks" in lecture [25] where he proposed the solution—"atomic theory". Introducing finiteness (§8,9) should eliminate the GR infinities due to the quantum mechanism at very small scales [20], and should alter the quantum mechanical wavefunctions near black holes [20].

Whether AString Quantum Gravity fully resolves the quest of Quantum Gravity is the open question to physicists and string theorists. It certainly describes the Standard Model particles quanta (§6, 7) and provides the "common mathematical blocks" for unification of "atomized fields" with "atomized spacetime" (§8, 9, 10) and linkages to string and LQG theories (§11). Coming from a mathematical school with 55 years of AFs history (§2, 3), we hope that the theory would attract the attention of theoretical physicists for further validation and collaboration.

Extending the questions, it is tempting to answer the following question. *Why has the atomic quantum gravity theory not been developed earlier*? The prospective "atomic theory" was envisaged by A. Einstein in 1933 [25], but in the absence of proper mathematical apparatus, the opportunity seems to have gone unnoticed. While the theory of atomic functions [1]-[20] has been evolving since the 1970s with a hundred publications, it was relatively unknown due to language and other barriers (§3). Systematic expansion of atomic functions into fundamental theories only started in 2017. Hope the theory would attract the attention of physicists and string theorists, with opportunities to contribute to the new variants of QFT (§8), quantum gravity (§9, 10), and string theory (§11).

10. Atomic Quantum Theory and Quantum Mechanics

AString Quantum Gravity (§9) shows the way how QFT and GR spacetime theories can be united based on common blocks of atomic functions. Now, let's elaborate on how the theory is related to Quantum Mechanics (QM). For this, we need to reevaluate the QM foundations in the 1920s.

10.1. The History of Quantization Rules

QM originated in the 1920s from M. Planck's idea (1900) that field energies should be quantized—come in discrete integer m levels of increments ΔE:

$$E = E_0 + m\Delta E. \tag{10.1}$$

The first Bohr-Sommerfeld model (1920) postulated that the energy of a rotating electron with momentum p and position x in a hydrogen atom should be quantized with the levels $m = 0, 1, 2, \cdots$ of the Planck constant:

$$E = \int p\mathrm{d}x = m\hbar. \tag{10.2}$$

Einstein won the Nobel prize (1921) for the explanation of the photoelectric effect when an atom emits the light of frequency ν when an electron jumps from one orbit to another with energy increment $\Delta E = \hbar\nu$. W. Heisenberg (1925) proposed a more advanced model [21] by representing momentum p and position x via a Fourier series, which yielded not only the quantization rule $m\Delta E$ but also the ground-state energy E_0 in (10.1). In 1926, E. Schrödinger used the generic wave representation $\mathrm{e}^{i(px-\omega t)}$ and published his Equation (5.2) [21] [24] where the discrete energy levels appear as eigenvalues of the boundary value problem [24] for the probabilistic wavefunction ψ. Finally, those ideas have been generalised into the concepts of the quantum harmonic oscillator [24] where the discrete energy levels are "equally spaced":

$$E_m = \left(m + \frac{1}{2}\right)\hbar\omega = E = E_0 + m\Delta E . \tag{10.3}$$

10.2. Einsteinian Perspective "Atomic Theory"

Interestingly, in 1933 lecture [25], A. Einstein, being deeply unsettled in his famous debates with N. Bohr about the probabilistic nature of QM ("*The god does not play dice*"), proposed the perspective "*perfectly thinkable*" "*atomic theory*" where "*finite regions of space where the charge vanishes everywhere*" have discrete integer energy levels. This A. Einstein's note resonates well with the theory of Atomic Functions, solidified 50 years later, but started being applied to Spacetime, QM, and QFT theories only since 2018 [1]-[9]. Really, "finite regions of space" is strikingly similar to a finite Atomic Quantum pulse model (6.10) with energy:

$$E = \int \varepsilon up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x = \varepsilon s^n . \tag{10.4}$$

The Einstein-inspired goal is to quantize this energy with discrete levels $E_m = E_0 + m\Delta E$, which can be achieved in two ways described below—in the standard QM way, or in the "atomic theory" way.

In the traditional QM and QFT way [21] [24], the finite quantum region of space (10.4) collapses into the "point", the quantum size parameter s is ignored, atomic function $up_n\left(\frac{x_n}{s}\right)$ collapses into a delta-function, and energy density $\varepsilon(x_n)$ function becomes energy $E(x_n)$. It locks the problem in the domains of mathematics with infinitely spread functions like $e^{i(px-\omega t)}$ where the main quantization opportunity is to use Schrödinger (5.2), or generally Dirac QFT equations (5.4). In this case, we need to operate with traditional wavefunction formulation (§6.3) with Atomic Wavefunction (6.4) $\psi up\left(\frac{x-b}{s}\right)$ ([9], Eremenko, 2024). It works for QM and generally QFT, but hides the problem of integrating the quantum theories with General Relativity.

10.3. Quantization Based on Einsteinian "Atomic Theory"

Let's now elaborate on the second Einsteinian-inspired "atomic way" of quantization for Atomic Quantum (10.9). Here, we should note that using finite functions and objects like strings introduces a new parameter—the size/volume $V = s^n$ of a quantum (like Planck length or Compton length). In string theories [29]-[31], this parameter would be the size of a vibrating string for a given field, while in LQG [26]-[28], it is the size of a fundamental loop. The energy $E = s^n \varepsilon$ becomes the product of two parameters—volume $V = s^n$ and energy density ε. It creates an interesting new option for the quantization of energy E with some discrete levels $E_1, \cdots, E_n$ which can be attributed to the quantization of volumes V_i, densities ε_i, or both

$$E_i = V_i \varepsilon, \quad E_i = V \varepsilon_i, \quad E_i = V_i \varepsilon_i. \tag{10.5}$$

The quantization of sizes/volumes V_i is much easier to understand than the quantization of energy $E_i = V_i \varepsilon$, because the volumes follow the obvious law of addition $V = V_1 + V_2 + \cdots$. We just need to hypothesize that nature's fields and spacetime are built from the finite "building blocks" like strings or loops, and this underlying finiteness inevitably causes the discrete levels of energy quanta. It looks like some exclusion principle (like the Pauli exclusion principle [21] [24]) prevents quanta (photons, bosons) from stacking up on top of each other, so spacetime at a given point can host only one quantum packet. A second photon trying to occupy the same space would force the first photon to move out, creating movements of matter. Having an extra size parameter in the energy definition $E = \varepsilon s^n$ allows us to *shift* the problem of energy quantization to a much easier problem of size/geometry/spacetime quantization, like in Loop Quantum Gravity [26]-[28]. This simplifies the QM concepts built upon point functions, where this opportunity is much harder to realise. Let's demonstrate how atomic quantum theory upholds the quantization idea.

10.4. Examples of Atomic Energy Quantization

Let us consider a uniform, for example, Higgs field (§5.6), with energy level $E =$ 126 GeV/c^2 and quantum size $s = 1.58 \times 10^{-18}$ m. With atomic functions (§6.4), it can be decomposed into the sequence of m identical Atomic Quanta (6.10) with the same energy density ε located at points $x = \cdots, -2s, -1s, 0, 1s, 2s, \cdots$ (**Figure 2**, **Figure 10**):

$$\varepsilon \equiv \cdots + \varepsilon up\left(\frac{x-2s}{s}\right) + \varepsilon up\left(\frac{x-s}{s}\right) + \varepsilon up\left(\frac{x}{s}\right) + \varepsilon up\left(\frac{x+s}{s}\right) + \varepsilon up\left(\frac{x+2s}{s}\right) + \cdots. \tag{10.6}$$

For 1D Atomic Quanta (6.10) and properties (4.5) of AF, the total energy (integral) E of this field would be

$$E = m\Delta E = ms\varepsilon; \Delta E = \varepsilon s, \tag{10.7}$$

and coming in discrete portions, exactly like for Bohr-Heisenberg quantization rules (10.1)-(10.3). This energy can be "quantized" independently of energy density ε and attributed to the quantization of size $ms = s + s + \cdots + s$.

A similar "geometric" idea was postulated by A. Einstein for quantized energy of photons $E = \hbar \nu$ [21] emitted by electrons jumping from one equidistant atom orbit to another, shifting the problem to the geometrical radial quantization of orbits.

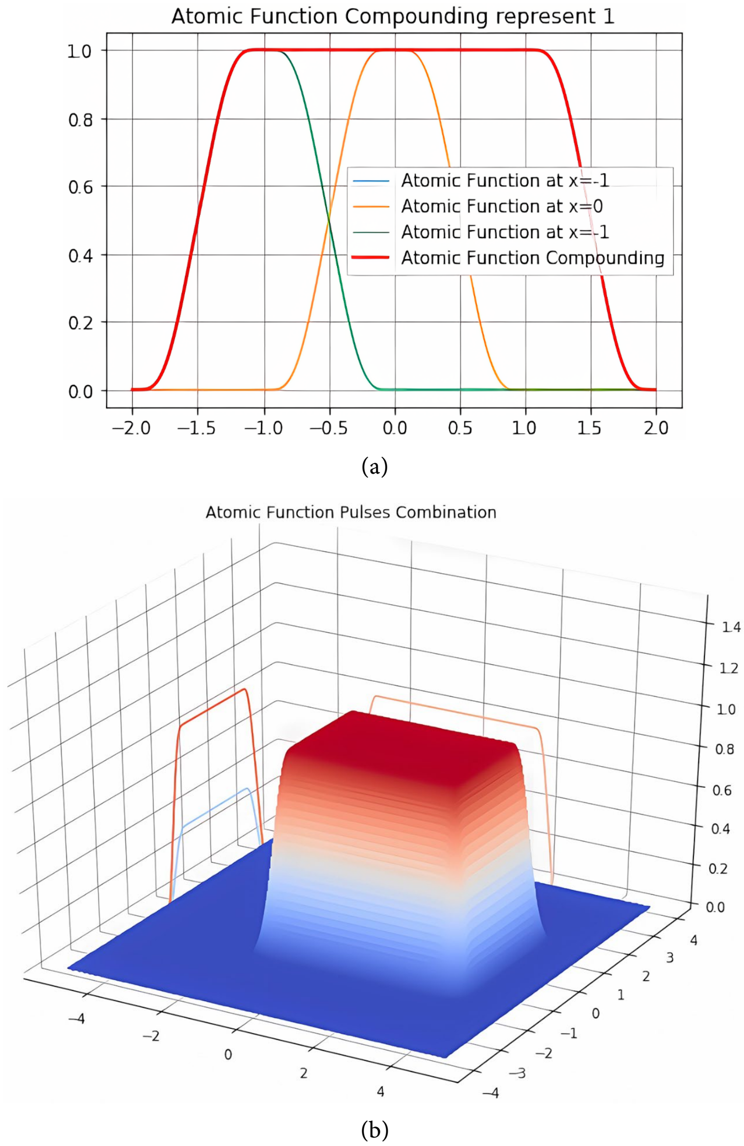

Figure 10. (a) Partition of a constant field with Atomic Functions; (b) Representation of flat surface via summation of 2D Atomic Functions

While introducing quantum wave-particle dualities, L. de Broglie [24] also assumed that many wavelengths should exactly "fit" the electron orbits radius (§10.4), so "size" information was explicitly accounted for in classical QM.

Another example is the classical quantum model of the hydrogen atom [21], [24] leading to the Rydberg formula for atomic orbits, where the approximate expectation value of the electron radius of n orbital is

$$r_n = a_0 n^2, \tag{10.8}$$

where a_0 = 0.529A is a Bohr radius. The orbital radii sequence is $1a_0$, $4a_0$, $9a_0$, $16a_0$ exactly matches the 2D Atomic Quanta model (6.10) with simple discrete rule $1s^2, 2s^2, 3s^2, 4s^2$ for sizes/areas. It looks like many, if not all, QM energy quantization cases are related to the quantization of some sizes/areas/volumes. LQG [26] [27] also relates quantization of fields to the discretization of spacetime.

In summary, Atomic QFT and ASring Quantum Gravity theories, with explicit inclusion of finite sizes in Atomic Series and Atomic Quanta definitions (10.4) not only uphold, but also simplify some QM concepts. No wonder that A. Einstein in 1933 [25] proposed "atomic theory" as the way of resolving the "quantum-riddle".

10.5. Atomic Quantum Mechanics (2024)

Atomic Quantum Mechanics based on atomic functions was described in detail in [9]. It represents QM probability fields as a series of interacting, overlapped Atomic Wavefunctions (6.4)

$$\psi(x)=\sum_k \psi_k up\left(\frac{x-b_k}{s}\right). \tag{10.9}$$

The theory offers the resolution of the "boundary wavefunction discontinuity" problem [40] and introduces desired quantum finiteness to integrate with Quantum Gravity theories [20]. In summary, we can see that the Atomic QFT theory (§6) not only upholds the quantization origins of QM, but also makes some interpretations easier. In QM and QFT theories based on continuous mathematics with infinitely-spread functions, the useful concept of size s is abstracted out to the space point $s=0$, so the only option to quantize energy was to invent Schrödinger-Dirac-QFT equations [21] [24]. It does not mean those theories are incorrect, but they introduced another level of complexity, sometimes leading to Einsteinian "stumbling blocks" [25] (§1) for resolving problems of Quantum Gravity (§9) (infinite series, boundary wave-functions discontinuity problem [9] [40], "renormalisation of gravity" [21]). QM Schrödinger equation expressing the energy conservation law $E=K+V$ is physically correct, but is based on the infinitely spread function $\psi(x,t)=\mathrm{e}^{i(px-\omega t)}$ which not only creates "boundary problem" [9] [40] but also enforces the linearity of differential equations, making it hard to integrate with nonlinear GR for the unified problem of Quantum Gravity. Atomic theory offers the way to remove some of these "stumbling blocks" with the approach consistent with QM, where the complex problem of energy quantization is shifted to an easier problem of quantization of sizes/geometry/spacetime.

11. The Prospects of Atomic String (AString) Theory

Atomic Quantum Field Theory (§6), Atomic Spacetime (§8), and AString Quantum Gravity (§9) theories are built upon the universality of Atomic String Functions (ASF) to compose different fields from the same "building blocks". It naturally leads to the prospects of a new variant of string theory—AString theory first envisaged in 2020 [1]-[5], with more details in [8]. The AString theory is based on the following foundations.

11.1. The Mathematical Foundations

First, it is the universality of polynomials and Taylor power series to represent any analytic functions, vectors, and tensors in many dimensions:

$$y(x_i) = \sum_{n=0}^{\infty} \frac{y^{(n)}(0)}{n!} x_i^n = \sum_{n=0}^{\infty} B_n x_i^n \quad (11.1)$$

Second, V.L. Rvachev and V.A. Rvachev (1967-1975) have discovered that a finite atomic function (AF) $up(x)$ pulse (4.1)

$$up'(x) = 2up(2x+1) - 2up(2x-1) \quad (11.2)$$

can *exactly* represent the polynomials of *any* order n with *limited* N number of finite pulses around point x (4.13):

$$P_n(x) \equiv \sum_{k=-N}^{k=N} C_k up\left(\frac{x - ab_k}{a}\right); \quad x^n \equiv \sum_{k=-N}^{k=+N} C_k up\left(x - k2^{-n}\right). \quad (11.3)$$

So, what we (and nature) know as polynomials are just the overlapping superposition of some finite functions resembling quanta (§6).

Third, it was discovered ([1]-[3], Eremenko, 2020) that differential operators in major physical theories, including GR, QM, QFT, preserve polynomiality (§5,8) and, being applied to polynomial operands, produce other sets of polynomials allowing "in-principle" to resolve all equations in polynomials or their sums (analytic functions [32]) representable by finite ASF functions (11.3). So, polynomials are just made of "mathematical atoms", as atomic functions (11.2) were often called in the 1970s.

Fourth, it was discovered [1]-[9] (Eremenko, 2018) that AF $up(x)$ is a "solitonic atom" (Atomic Soliton [4]) made of two $AString$ kink functions (§4.2), which are also the integrals of $up(x)$:

$$up(x) = AString(2x+1) - AString(2x-1) = AString'(x). \quad (11.4)$$

In translation, AStrings compose what we know as a dimension x in flat (4.9)

$$x \equiv \sum_k sAString(x - ks) \quad (11.5)$$

or curved spacetime (§8)

$$\tilde{x} \equiv \sum_k C_k AString\left(\frac{x - b_k}{a}\right). \quad (11.6)$$

In combination, the theory led to the Atomic Series (§4.3, §5) [1]-[6] (2022) as universal as Taylor and Fourier series, but based on finite functions resembling quanta and strings when continuous analytic functions can be decomposed into a superposition of ASF functions on some lattice:

$$y(x) \equiv \sum_l up(x, a_l, b_l, c_l) = \sum_k AString(x, a_k, b_k, c_k). \quad (11.7)$$

The Atomic Series can be universally applied to linear and nonlinear physical theories, including GR, QFT, and QM, leading to the AString theory.

11.2. From "Mathematical Atoms" to Atomic Quanta and Vibrating Strings

In application to QFT theories (§5-7), the Atomic Series allows decomposing continuous energy density fields (6.8)

$$\varepsilon(x_1,\cdots,x_n)=\sum_k \varepsilon_k up_{kn}(x_n/s) \tag{11.8}$$

into Atomic Quanta of energy (6.10)

$$E=\int AQuant(x)\,\mathrm{d}^n x=\int \varepsilon up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x=\varepsilon s^n \tag{11.9}$$

which can be matched to the experimental values of particles from the Standard Model (**Table 1**, §6) (eq $s=1.58\times10^{-18}$ m, $E=125$ GeV, $\varepsilon=5.1\times10^{45}\ \mathrm{J/m^3}$ for the Higgs boson).

Using the energy-frequency relationship $E=\hbar\nu$, it is possible (§6.9) to reformulate these energy density distributions in terms of frequencies of "vibrating strings"

$$E=\int \varepsilon up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x=\varepsilon s^n=\hbar\nu;\nu=\varepsilon s^n/\hbar. \tag{11.10}$$

This establishes the direct link to string theories [29]-[31].

11.3. Fields as Atomic Distortions of Spacetime

Atomic Quanta (11.9) as "building blocks" of fields made of finite AF pulses $\varepsilon up_n\left(\frac{x_n}{s}\right)$ can be linked to spacetime deformations using GR field equations (§9), the fundamental ASF theory relation (11.4)

$$up(x)=AString(2x+1)-AString(2x-1)=AString'(x), \tag{11.11}$$

and the ability of AStrings to compose curved spacetime (11.6)

$$\tilde{x}\equiv\sum_k c_k AString\left(\frac{x-ab_k}{a}\right). \tag{11.12}$$

This allows formulating the AString Quantum Gravity (§9), establishing the "common finite blocks" between GR with Quantum Theories in line with the foundations of string theories [29]-[31] in the 1980s.

11.4. From Atomic Quanta to Open and Closed Solitonic Strings

Atomic Quanta (11.6) described by AF pulses look like enclosed "atoms" resembling "closed strings" from string theory [29]-[31]. Really, pulse $up(x)$ (11.4) is a "solitonic atom" (§4.2) (**Figure 11(a)**, **Figure 11(b)**) made of two *opposite* AStrings $up(x)=AString(2x+1)-AString(2x-1)$. But if AStrings are pointing in *one* direction, they compose the spacetime expansion (**Figure 11(c)**)

$$\begin{aligned}x\equiv\cdots&+AString(x-2s)+AString(x-s)+AString(x)\\&+AString(x+s)+AString(x+2s)+\cdots.\end{aligned} \tag{11.13}$$

So, the same AString function can create either an enclosed solitonic object (*closed string*) or expand into spacetime as the *open string*, stinkingly similar to string theories [28]-[31].

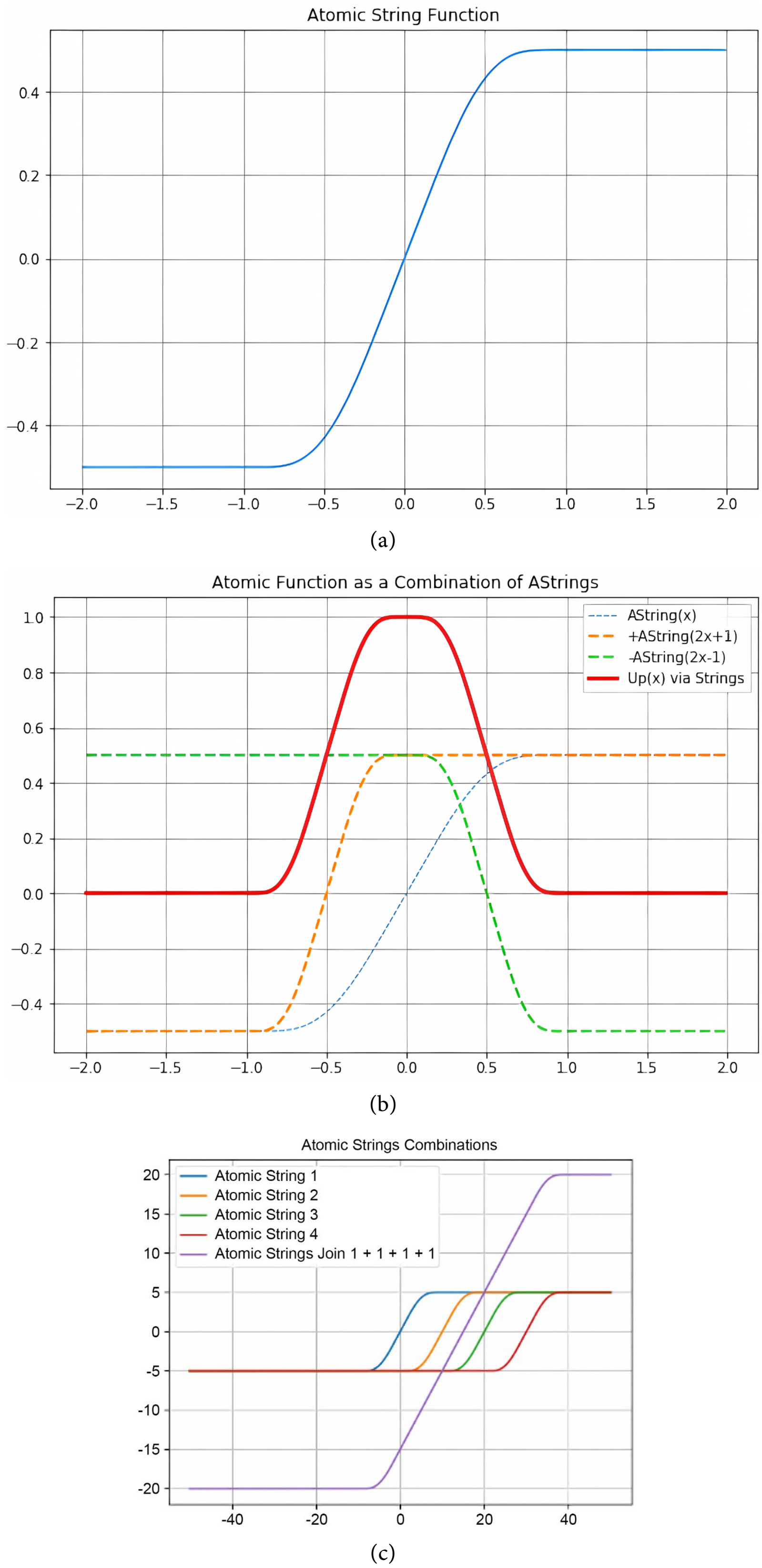

Figure 11. (a) AString function as "open string"; (b) Atomic function ("closed string") as a combination of two open AStrings; (c) Spacetime as an expansion of AStrings.

11.5. AStrings and Loop Quantum Gravity

Atomic Functions can also describe loops (§7.6) presumed to be the building blocks of spacetime in LQG theory [26]-[28]. Radial AF (4.6) (**Figure 1**, **Figure 6**) is defined on the spherical area resembling a quantized loop, while 1D AF (**Figure 11(b)**) describes the 1D cross-section of loops with size s assumed to be the Planck length in LQG:

$$\begin{aligned} up(r,c,s) &= cup\left(\sqrt{x^2+y^2+z^2}/s\right), \\ up(x,c,s) &= cup(x/s). \end{aligned} \tag{11.14}$$

In this interpretation, atomic distributions (11.8) describe the overlapping interconnections between loops. Apart from the new model of "atomic loops", AString theory may suggest the link between string and LQG theories as major contenders for Quantum Gravity [41]. Really, the enclosed $up(x)$ "loops" (11.4) also describing the fundamental particles-excitations of fields and spacetime (§6,7,8) can be interpreted as "solitonic atoms" composed of two open AStrings (§11.4) creating the research opportunity for converging finite strings and loops into one "atomic theory" envisaged by A. Einstein [25] in 1933 (§1, 8, 9).

11.6. AStrings Described by Solitonic Atomic Functions

In this prospective AString theory, the strings are described by a non-analytic core functions $up(x), AString(x)$ while in string theories [29]-[31] known for their complexity, it seems like the sinusoidal vibrations $c\sin(ax+b)$ in up to 11 dimensions [28]-[31]. This raises the key question for future research, which core function is "better" and "used by Nature" [8]. With a limited number of pulses, Atomic Functions can exactly and simply compose polynomials of any order (11.2), while a few sine harmonics cannot compose even the simplest constant. The AString theory points to some solitonic "building block" of nature consisting of two opposite AString kinks (11.4) held by some forces (perhaps, entanglement). The principle of smooth least action described in [8] may be the guiding principle leading to the formation of an infinitely smooth local distribution of fields leading to atomic functions. While the expansion of the theory towards the modern string theories [29]-[31] requires further collaborative research, let's note that AString theory leads to simple models of GR, QFT, and QM fields, with the hope that Atomic Functions can help to simplify complex string theories.

In the 1933 paper [25], where A. Einstein envisaged the "perfectly thinkable" "atomic theory", he wrote. "*The important point for us to observe is that all these constructions and the laws connecting them can be arrived at by the principle of looking for the mathematically simplest concepts and the link between them. In the limited number of the mathematically existent simple field types, and the simple equations possible between them, lies the theorist's hope of grasping the real in all its depth.*" The proposed atomic theory seems to follow A. Einstein dreams of simplicity and universality.

12. The Simple Model of Nature's Fields

Atomic functions, as the mathematical foundations of atomic theories described in this work, provide an interesting insight into the simple rules of how nature can operate on the fundamental level of quanta/strings/qubits/atoms [1] [8]. For example, from the atomized representation of a parabola

$$\sum_{k=-N}^{k=+N}\left(\frac{k^2}{64}-\frac{1}{36}\right)up\left(x-\frac{k}{4}\right)\equiv x^2, \tag{12.1}$$

we can see that the shape of a parabola is defined in *interaction zones* between equally spaced *overlapping* atoms (**Figures 1-4**), and incoming energy is *distributed* between neighboring atoms. It looks like all smooth fields, trajectories, shapes, orbits, and movements in nature can be envisaged as simple interactions of a few atoms/quanta/strings within a local finite atomic network, pointing to the elegant simplicity of nature on all micro- and macroscopic levels, as described in detail in [1] [8] [34]. LQG also leads to similar spinfoam networks [26] [27]. It also correlates with G. 't Hooft [38] "cellular automata" rules, with opportunities for further research on the common theory.

13. Summary and Future Research Directions

13.1. Summary of Findings

1) Finite Atomic (1971) and AString (2018) Functions (ASF) (§4) *exactly* compose the polynomials of *any* order (1975) from a *limited* number of finite "solitonic atoms" (2018). It leads to Atomization Theorems and Atomic Series (2022) as an extension of universal Taylor series to represent a wide range of analytic functions and solutions of differential equations via superpositions of finite functions (§4, 5).

2) In application to fundamental GR, QM, and QFT theories (2018-2026), ASF theory allows decomposing continuous fields via superposition of finite objects resembling quanta, strings, loops, solitons, excitations, and ripples.

3) In 1933, A. Einstein [25] envisaged "*perfectly thinkable*" "*atomic theory*" with "*finite regions of space*" similar to Atomic Functions (2022) to overcome "*stumbling blocks*" for theories' unification (§1, 6, 9).

4) In application to GR (2018-2024), Atomic Spacetime theory offers a new way of quantizing/atomizing the spacetime field, leading to the "atomic graviton" model and spacetime composed of open AStrings (§8, 9, 11).

5) For QFT, finite Atomic Functions can be deduced from QFT gauge theories (§5), which allows introducing new Atomic Quanta of the Standard Model of fields (§6,7) with energies matching the experiments with the core formula

$$E=\int AQuant\left(x_1,\cdots,x_n\right)\mathrm{d}^n x=\int \varepsilon up_n\left(\frac{x_n}{s}\right)\mathrm{d}^n x=\varepsilon s^n. \tag{13.1}$$

which introduces ASF functions into QFT theories. Fields differ by the atomic quanta sizes included in the definitions of energy. ASF can also describe QFT wavefunctions.

6) Unified description of quantum and spacetime fields allows uncovering the "common blocks"—atomic functions—contributing to the long-standing problem of reconciliation of GR with Quantum Theories based on AString Quantum Gravity (§9). Quantum fields are the distortions of spacetime described by the same underlying finite functions.

7) Fields atomization with ASF functions upholds the quantisation rules of QM (§10), leading to Atomic Quantum Mechanics ([9], 2024).

8) Universality of Atomic Series and unified fields composition from finite functions open the prospects of a new variant of string theory—AString theory—with spacetime made of "open strings" while "matter" fields are composed of "closed strings" (§11). Radial atomic functions can also describe loops from LQG theory, with the prospects of some convergence of LQG and AString theory.

9) On the quantum level, nature follows the simple rules of a local atomic network (2021), for example, the interaction of 9 atoms composes a parabola (§12).

13.2. Future Research Direction

Coming from a mathematical school, we seek the validation of the proposed atomic theories by physicists and string theorists. Atomic Functions may not only simplify but also unify the fundamental theories that require the collaboration of many scientists.

The main open question is to what degree the uncovered common Atomic Functions blocks between "atomized" GR and QFT theories can contribute to the resolution of the long-standing problem of Quantum Gravity.

The Atomic Series has the potential to become as universal as Taylor and Fourier Series. They operate with finite functions naturally linked to quantization and finite string-like theories.

Relationships with string and quantum gravity theories are the most important directions for future collaborative research. Especially interesting whether AStrings would provide a new description of strings, simplifying the complex string theories and possibly avoiding many controversial dimensions.

Describing loops with atomic functions can not only contribute to Loop Quantum Gravity but also provide the linkage to string theories.

Integrating AString Quantum Gravity with QMGR theory [20] is an interesting research direction that may resolve the GR singularity problem.

Universality of finite atomic functions to describe different fields may be the foundation of unified "theories of everything".

13.3. Afterword

In final words, it all comes down to appreciating the beauty and elegant simplicity of nature that knows nothing about functions and Lagrangians, yet operates through simple and efficient rules of local Atomic Networks. Every move, shape, configuration, orbit, and trajectory we observe is just an interaction of a few solitonic atoms, which reflects the beautiful simplicity of nature. It correlates perfectly with A. Einstein [25] "*I have deep faith that the principle of the universe will be beautiful and simple*". Uncovering this beauty with atomic functions was a challenging but happy moment in life.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Eremenko, S.Y. (2024) Theory and Properties of Atomic Spacetime. *Journal of Applied Mathematics and Physics*, **12**, 1967-2006. https://doi.org/10.4236/jamp.2024.126121

[2] Eremenko, S.Y. (2023) Atomization Theorems in Mathematical Physics and General Relativity. *Journal of Applied Mathematics and Physics*, **11**, 158-191. https://doi.org/10.4236/jamp.2023.111012

[3] Eremenko, S.Y. (2022) Atomic Spacetime Model Based on Atomic Astring Functions. *Journal of Applied Mathematics and Physics*, **10**, 2604-2631. https://doi.org/10.4236/jamp.2022.109176

[4] Eremenko, S.Y. (2018) Atomic Solitons as a New Class of Solitons. *Journal Nonlinear World*, **16**, 39-63. https://www.researchgate.net/publication/329455498

[5] Eremenko, S.Y. (2018) Atomic Strings and Fabric of Spacetime. *Achievements of Modern Radioelectronics*, No. 6, 45-61. https://www.researchgate.net/publication/320457739

[6] Eremenko, S.Y. (2020) Soliton Nature. Amazon Kindle Publishing. https://www.amazon.com/dp/1951630777

[7] Eremenko, S.Y. (2022) AString Functions in Theoretical Physics. *International Conference on Atomic and R-Functions (ICAR), Virtual Workshop*, Split. https://www.researchgate.net/publication/358264308

[8] Eremenko, S.Y. (2025) Prospects of Reconciling General Relativity and Quantum Theories based on AString Quantum Gravity. https://www.researchgate.net/publication/397291642

[9] Eremenko, S.Y. (2024) Atomic Quantum Mechanics Based on Atomic Functions.

Journal of Applied Mathematics and Physics, **12**, 3941-3963.
https://doi.org/10.4236/jamp.2024.1211240

[10] Rvachev, V.L. and Rvachev, V.A. (1971) About One Finite Function. *DAN URSR*, No. 6, 705-707.

[11] Rvachev, V.L. (1982) Theory of R-Functions and Their Applications. Naukova Dumka.

[12] Rvachev, V.L. and Rvachev, V.A. (1982) Non-Classical Methods in the Theory of Approximations in Boundary Value Problems. Naukova Dumka.

[13] Kravchenko, V.F. and Rvachev, V.A. (1996) Application of Atomic Functions for Solutions of Boundary Value Problems in Mathematical Physics. *Foreign Radioelectronics: Achievements of Modern Radioelectronics*, **8**, 6-22.

[14] Kravchenko, V.F., Kravchenko, O.V., Pustovoit, V.I., Churikov, D.V., Volosyuk, V.K. and Pavlikov, V.V. (2016) Atomic Functions Theory: 45 Years Behind. 2016 9*th International Kharkiv Symposium on Physics and Engineering of Microwaves, Millimeter and Submillimeter Waves* (*MSMW*), Kharkiv, 20-24 June 2016, 1-4.
https://doi.org/10.1109/msmw.2016.7538216

[15] Kravchenko, V.F., Kravchenko, O.V. and Safin, A.R. (2010) A New Construction of Solution of Schrodinger Equation on the Basis of Atomic Functions. 2010 *International Kharkov Symposium on Physics and Engineering of Microwaves, Millimeter and Submillimeter Waves*, Kharkiv, 21-26 June 2010, 1-3.
https://doi.org/10.1109/msmw.2010.5546001

[16] Gotovac, B. and Kozulic, V. (1999) On a Selection of Basis Functions in Numerical Analyses of Engineering Problems. *International Journal for Engineering Modelling*, **12**, 25-41.

[17] Eremenko, S.Y. (1991) Finite Element Methods in Solid Mechanics. Osnova.
https://www.researchgate.net/publication/321171685,
https://books.google.com.au/books?id=dyTLDwAAQBAJ

[18] Eremenko, S.Y. (2022) Novel Model of Atomic Spacetime Based on Atomic AString Metriant Functions. *International Journal of Innovative Research in Sciences and Engineering Studies*, **2**, 1-15.

[19] Eremenko, S.Y. (2022) Atomic Function and AString Function (Python Code).

[20] Eremenko, S.Y. (2023) Quantum Gravity Based on Generalized Thermodynamics. *Journal of Applied Mathematics and Physics*, **11**, 988-1028.
https://doi.org/10.4236/jamp.2023.114066

[21] Griffithz, D. (2010) Introduction to Elementary Particles. 2nd Edition, Wiley, 454 p.

[22] Peskin, M.E. and Schroeder, D.V. (1995) An Introduction to Quantum Field Theory. CRS Press, 864 p.

[23] Wald, R.M. (1994) Quantum Field Theory on Curved Spacetime and Black Hole Thermodynamics. University of Chicago Press.

[24] Susskind, L. and Friedman, A. (2014) Quantum Mechanics: The Theoretical Minimum. Basic Books.

[25] Einstein, A. (1933) On the Method of Theoretical Physics. Oxford University Press.
https://openlibrary.org/books/OL6292654M

[26] Rovelli, C. and Vidotto, F. (2014) Covariant Loop Quantum Gravity. Cambridge University Press. https://doi.org/10.1017/cbo9781107706910

[27] Rovelli, C. (2007) Quantum Gravity. Cambridge University Press, 486 p.

[28] Smolin, L. (2000) The Cubic Matrix Model and a Duality between Strings and Loops.

arXiv: hep-th/0006137.

[29] Polchinski, J. (1998) String Theory. Vol. I: An Introduction to the Bosonic String. Cambridge University Press.

[30] Becker, K., Becker, M. and Schwarz, J.H. (2007) String Theory and M-Theory: A Modern Introduction. Cambridge University Press.

[31] Kaku, M. (1999) Introduction to Superstrings and M-Theory. Springer.

[32] Analytic Function. https://en.wikipedia.org/wiki/Analytic_function

[33] Carroll, S. (2003) Spacetime and Geometry: An Introduction to General Relativity. Pearson Education (US).

[34] Eremenko, S.Y. (2018) Atomic Machine Learning. *Neurocomputers*, **3**, 1-21. https://www.researchgate.net/publication/322520539

[35] Rothe, H.J. (2012) Lattice Gauge Theories: An Introduction. 4th Edition, World Scientific Publishing.

[36] DeGrand, T. and DeTar, C. (2006) Lattice Methods for Quantum Chromodynamics. World Scientific Publishing Co. Pte. Ltd. https://doi.org/10.1142/9789812773982

[37] Hawking, S.W. and Penrose, R. (1996) The Nature of Space and Time. Princeton University Press.

[38] 't Hooft, G. (2016) The Cellular Automaton Interpretation of Quantum Mechanics. Springer International Publishing. https://doi.org/10.1007/978-3-319-41285-6

[39] Veinik, A.I. (1991) Thermodynamics of Real Processes. Nauka i Technika.

[40] Particle in a Box. https://en.wikipedia.org/wiki/Particle_in_a_box

[41] List of Unsolved Problems in Physics. https://en.wikipedia.org/wiki/List_of_unsolved_problems_in_physics

[42] Grumiller, D., Kummer, W. and Vassilevich, D.V. (2002) Dilaton Gravity in Two Dimensions. arXiv: hep-th/0204253.

[43] Kaku, M. (2021) The God Equation: The Quest for a Theory of Everything. Doubleday.

[44] Fabius Function. https://en.wikipedia.org/wiki/Fabius_function

[45] Kamber, G., Gotovac, H., Kozulić, V., Malenica, L. and Gotovac, B. (2020) Adaptive Numerical Modeling Using the Hierarchical Fup Basis Functions and Control Volume Isogeometric Analysis. *International Journal for Numerical Methods in Fluids*, **92**, 1437-1461. https://doi.org/10.1002/fld.4830

[46] Faridi, W.A. and AlQahtani, S.A. (2024) The Formation of Invariant Exact Optical Soliton Solutions of Landau-Ginzburg-Higgs Equation via Khater Analytical Approach. *International Journal of Theoretical Physics*, **63**, Article No. 31. https://doi.org/10.1007/s10773-024-05559-1