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# Cosmological Gravitational Redshift, Spectral Shift and Time in the Taub-NUT Universe

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## Abstract

We demonstrate that: 1) The Taub-NUT universe is finite. 2) The Taub-NUT universe is much larger than the maximum observable distance according to the standard theory of cosmology. 3) At large distances the spectral shift turns into a blueshift. 4) At large distances time dilation turns into time contraction.

## Keywords

Cosmology, General Relativity

## 1. Introduction

Recently McGruder [1] developed a theory of cosmology based on two basic cosmological observations: the redshift-distance relationship and time dilation without any additional assumptions. He showed how the Taub-NUT solutions to the Einstein field equations of general relativity, which are generalizations of the Schwarzschild metric, lead to a model of the universe, which is not expanding. Additional references to those listed in [1] are: [2]-[4].

In the Taub-NUT universe the redshift-distance relationship is due to gravitational redshift, whereby time dilation has the exact same dependency on redshift that in the standard theory of cosmology is seen as proof that space is expanding. The Taub-NUT model resolves fundamental issues of the standard theory of cosmology: dark energy, Hubble tension, lack of antiparticles in the universe, etc.

In this work we investigate how the basic quantities of the Taub-NUT universe: cosmological gravitational redshift, $(z+1)^2$, time dilation, $(z+1)$, and spectral shift, $z$, change as a function of distance.

## 2. Cosmological Gravitational Redshift

In [1], it was demonstrated that four mathematically different Taub-NUT solutions to Einstein field equations lead to the same cosmological gravitational redshift, $(z+1)^2$ vs. distance relationship. The figure in [1] shows that the cosmological gravitational redshift agrees with the latest supernova observations [5].

These latest observations of Type Ia Supernova are in the range of $r < 8.3$ Gpc. We now predict via extrapolation the cosmological gravitational redshift out to distances much greater than current observational data. In a separate section, 5, we discuss the pitfall of extrapolation.

In order to accomplish this task we need to incorporate two equations from [1]. Equation (8):

$$(z+1)^2 = g_{00}(r) \tag{1}$$

and Equation (13) [6],

$$g_{00}(r) = -\frac{r^2 - 2\alpha r - n^2}{r^2 + n^2} \tag{2}$$

where $n$ is called the NUT parameter. In [1] we showed that curve fitting leads to numerical values of the constants: $\alpha = 219.822$ and $n = 31.204$.

Figure 1 depicts the cosmological gravitational redshift as a function of distance out to distances far beyond the supernova data used to compute the constants in Equation (2). The black vertical line in the figure is the maximum observable distance according to the standard theory of cosmology. Specifically, the age of the universe, 13.78 billion years, multiplied by the speed of light. Figure 1 was constructed via extrapolation. In section 5 we discuss the pitfall of extrapolation.

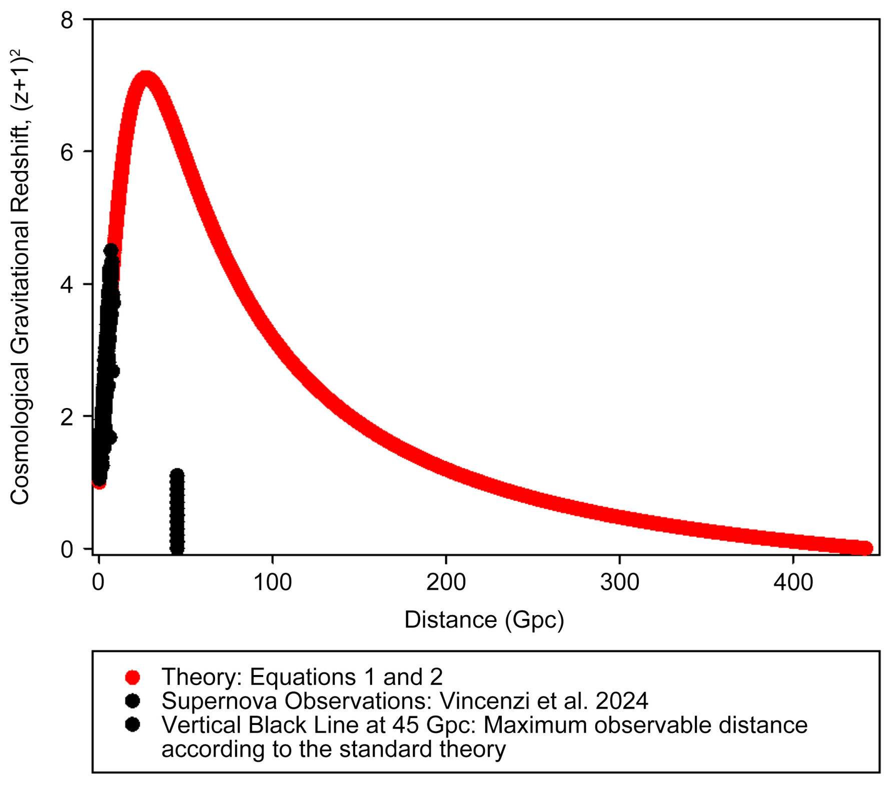


Figure 1. Cosmological gravitational redshift vs. distance.

The cosmological gravitational redshift, Equation (1), must fulfil the condition:

$$(z+1)^2 = g_{00}(r) \geq 0 \tag{3}$$

At what distance does the cosmological gravitational redshift become 0? We obtain this distance by setting Equation (2) equal to 0.

$$g_{00}(r) = -\frac{r^2 - 2\alpha r - n^2}{r^2 + n^2} = 0 \tag{4}$$

The non-negative solution is:

$$r = \alpha + \sqrt{n^2 + \alpha^2} \tag{5}$$

This equation yields: $r = 441.847$ Gpc, which is the size of the Taub-NUT universe. Consequently, we conclude the Taub-NUT universe is finite. The distance of the maximum value of the cosmological gravitational redshift is obtained by differentiating Equation (2) and setting it equal to 0, which is:

$$-\frac{2r - 2\alpha}{n^2 + r^2} + 2r\frac{r^2 - n^2 - 2r\alpha}{\left(n^2 + r^2\right)^2} = 0 \tag{6}$$

This yields the distance of maximum redshift, which is 27.08734 Gpc. Inserting this value into Equation (1) gives the maximum of the cosmological gravitational redshift of 7.115293. These values are the same for all four Taub-NUT models in [1]. The number of digits after the decimal point is determined by the agreement of all four Taub-NUT models.

In this section we have come to two major conclusions. First, by employing expression 3 we were led to the conclusion that the Taub-NUT universe is finite. Secondly, we see that the Taub-NUT universe is much larger than the maximum observable distance according to the standard theory of cosmology as **Figure 1** makes clear.

## 3. Time

In [1] we showed that time dilation is proportional to $(z + 1)$. From equation 1 it follows that:

$$z + 1 = \sqrt{g_{00}(r)} \tag{7}$$

**Figure 2** shows that the theoretical time dilation (from Equations (2) and (7)) matches the observations of [5]. Next we extrapolate this theoretical solution to distances much larger than the [5] data. **Figure 3** depicts this extrapolation.

The distance at which the time dilation is 0 is obtained by setting the above equation equal to 0, which leads to the same distance that the cosmological gravitational redshift becomes 0, which is: $r = 441.847$ Gpc.

The maximum value of the time dilation is obtained by differentiating Equation (7) and setting it equal to 0, which yields the same distance at which the cosmological gravitational redshift obtains its maximum value: 27.08734 Gpc. The maximum value of the time dilation is 2.6674507, whereby all four Taub-NUT models in [1] agree to the digits after the decimal point. **Figure 3** shows that time dilation for all four model universes.

Striking in Figure 3 is the fact that at $r > 27.08734$ Gpc the time dilation decreases instead of constantly increasing as the standard theory of cosmology maintains. Thus, cosmic events in this range as observed by terrestrial observers take less time to unfold then the time predicted by the standard theory.

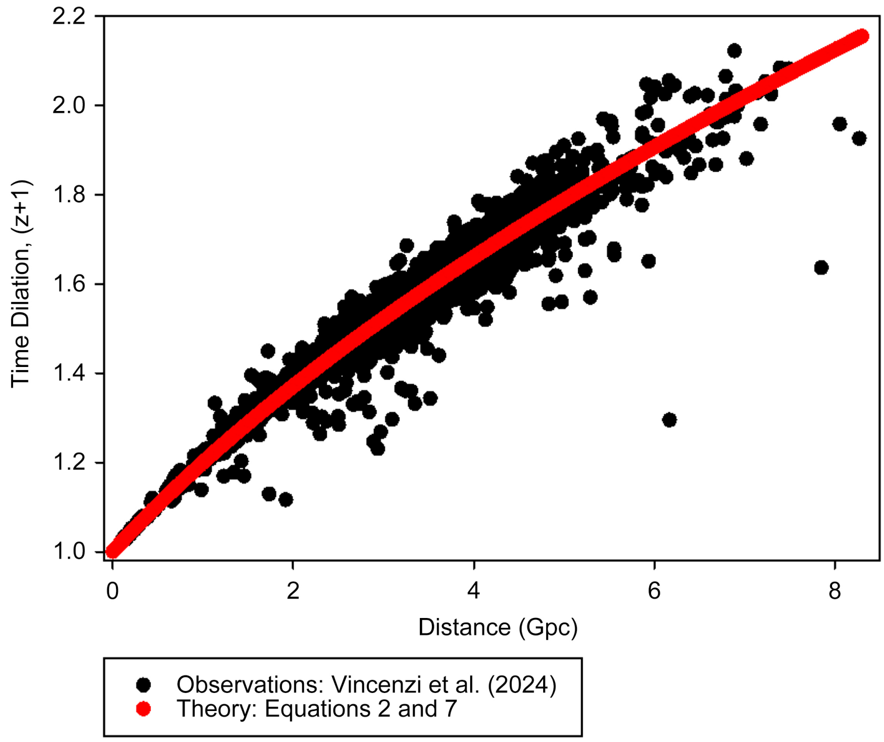


Figure 2. Time dilation vs. distance.

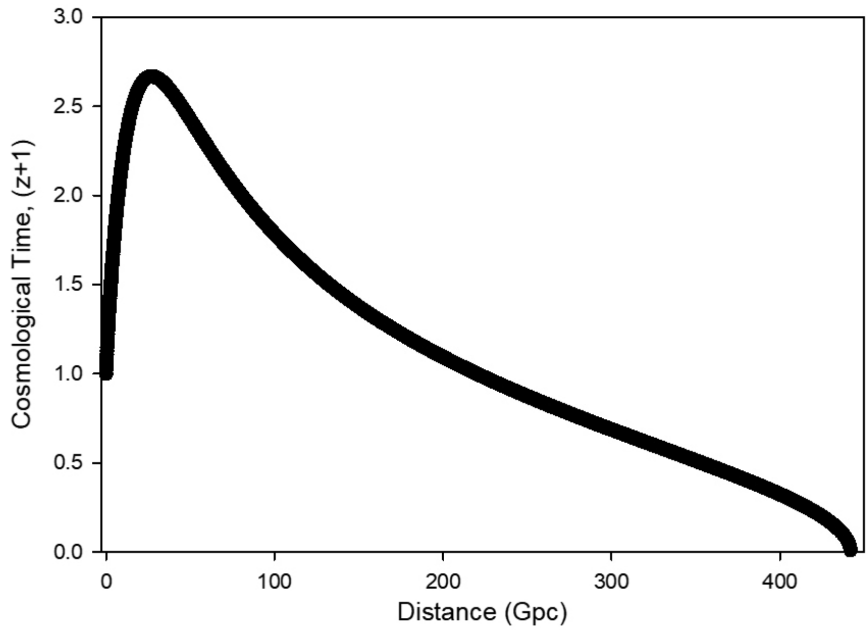


Figure 3. Cosmological time vs. distance.

Terrestrial observers experience no time dilation that is the time dilation factor is 1. But, setting Equation (7) equal to 1 for all four models leads to the conclusion that events at $r = 219.82176$ Gpc also have no time dilation as seen by terrestrial observers.

However, the most striking feature in Figure 3 is the following: for $r > 219.82176$ Gpc the time factor is less than 1 meaning the terrestrial observers see events taking place in less time than they would near earth. This is time contraction. There is no such feature either in Einstein's special relativity [7] or in the standard theory of cosmology.

## 4. Spectral Shift, *z*

In physical cosmology the most fundamental observational relationship is depicted in the redshift-distance diagram. In the standard theory of cosmology it is assumed that it is caused by the expansion of space. To the contrary in the Taub-NUT universe it is a manifestation of the gravitational redshift, which was first predicted by Einstein in 1907 [8].

From Equation (1) it follows that the spectral shift, $z$ is:

$$z = \sqrt{g_{00}(r)} - 1 \tag{8}$$

Figure 4 shows that Equation (8) along with the Equation (2) (red line) leads to the observed redshift-distance relationship (black data points [5]).

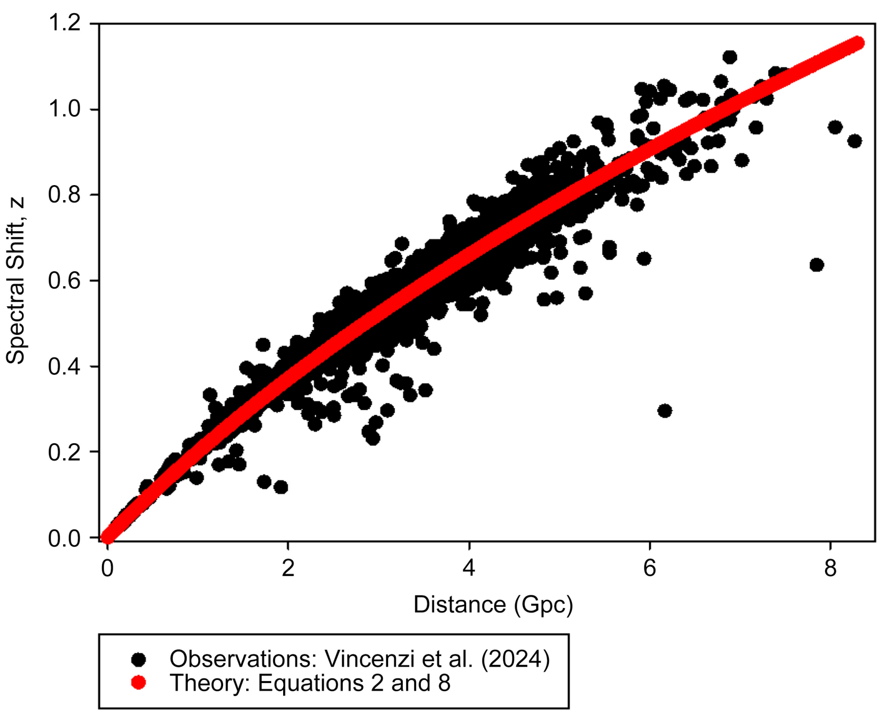


Figure 4. Spectral shift vs. distance.

We now extrapolate Equation (8) (Figure 5) out to the end of the universe derived from expression 3. Following the procedure described in section 2 we obtain

$r = 27.0873$ Gpc for the distance where the redshift reaches a maximum. Thereafter it declines reaching 0 at $r = \alpha = 219.822$ Gpc. At distances greater than this value, $z < 0$, which means blueshift.

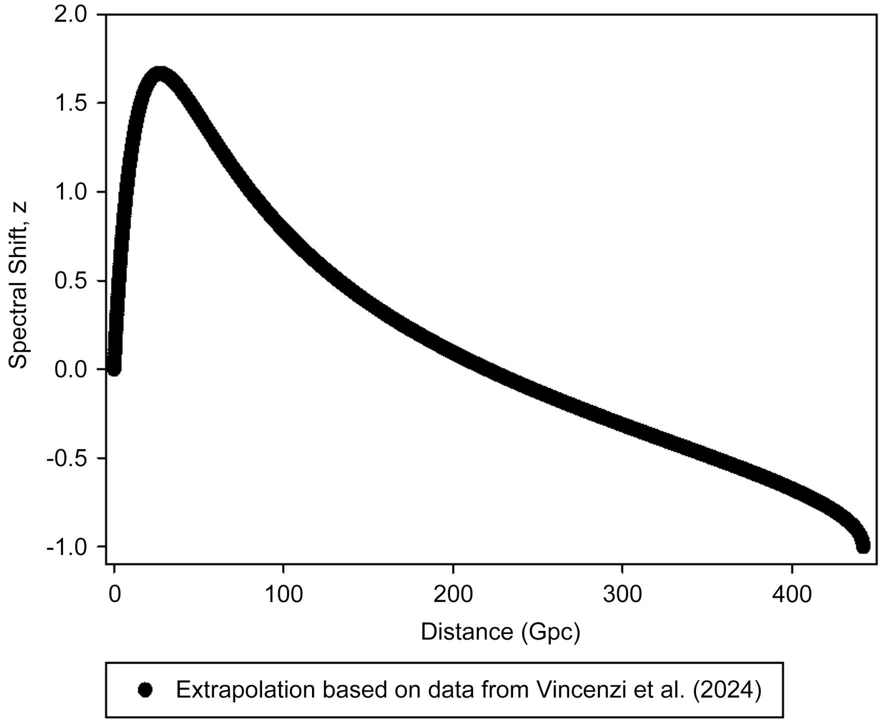


**Figure 5.** Spectral shift vs. distance.

## 5. Extrapolation Revisited

Extrapolation is a risky procedure and often leads to erroneous results. We have employed this method to obtain the above properties of the Taub-NUT universe. We will now show that the numerical values derived are in fact inaccurate, although the conclusions remain intact.

Following the procedure described in section 2 we calculate the maximum redshift in **Figure 5** to be: $z = 1.667$. This value is clearly inaccurate because to date the maximum redshifted object is the galaxy JADES-GS-z13-0 with $z = 13.2$ [9]. Our value is erroneous because the data we extrapolated from [5] does not extend far enough.

Brout *et al.* [10] published data that contains 14 supernova that are farther out than the maximum data in [5]. So we create a new data set by adding these sources to the [5] data. We follow the same procedure as described in [1] to obtain the constants in Equation (2): $\alpha = 83766.033$ and $n = 617.951$. These values are much larger than the above values derived from just the [5] data even though they differ by only 14 supernova. It shows how sensitive the calculations are to distant supernova. **Figure 6**, **Figure 7** and **Figure 8** confirm that these values lead to the observed supernova data from [5] augmented by 14 points from distant supernova from [10].

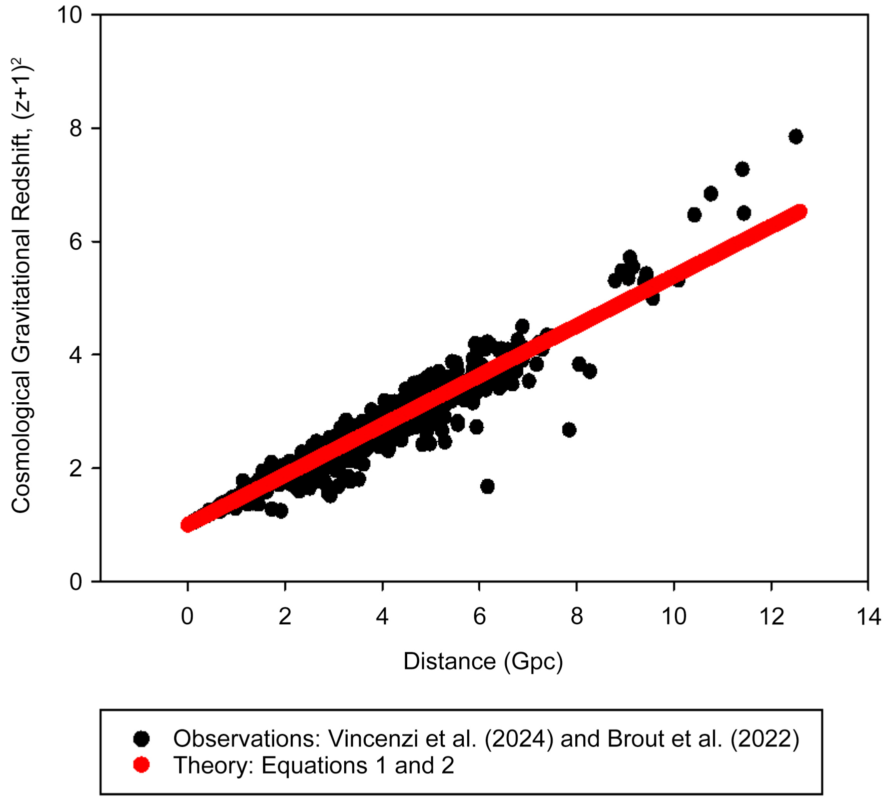


Figure 6. Cosmological gravitational redshift vs. distance.

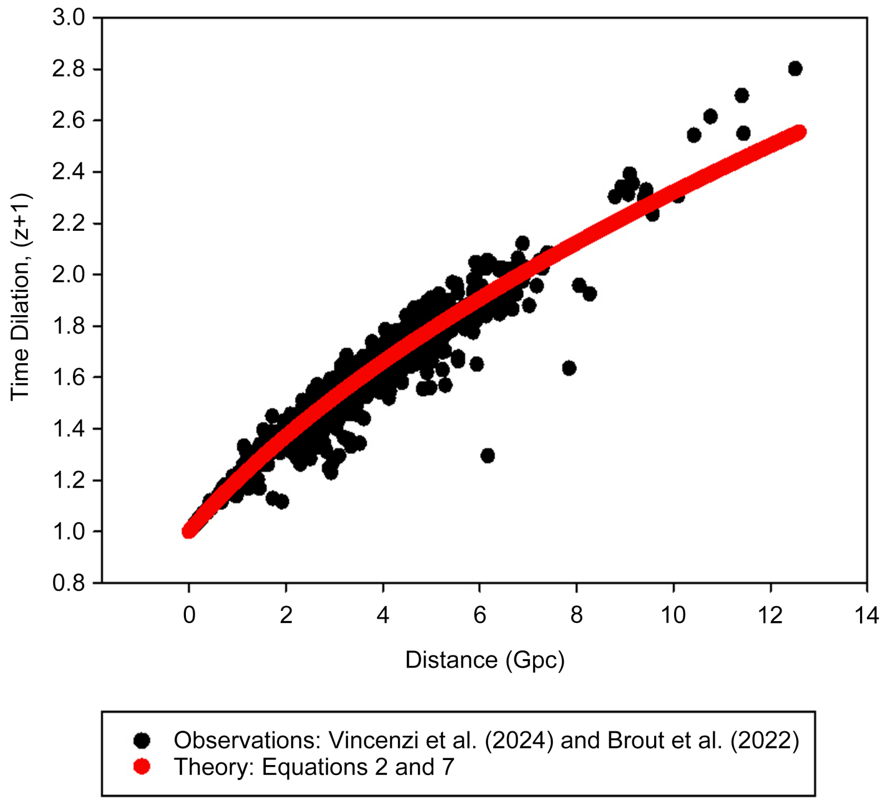


Figure 7. Time dilation vs. distance.

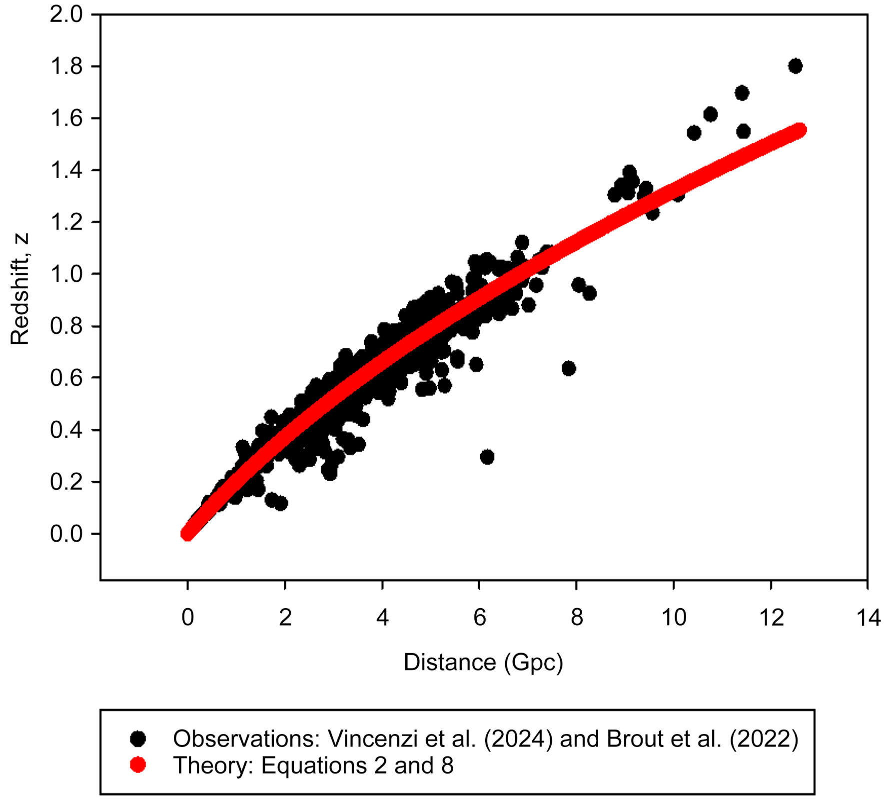


Figure 8. Redshift vs. distance.

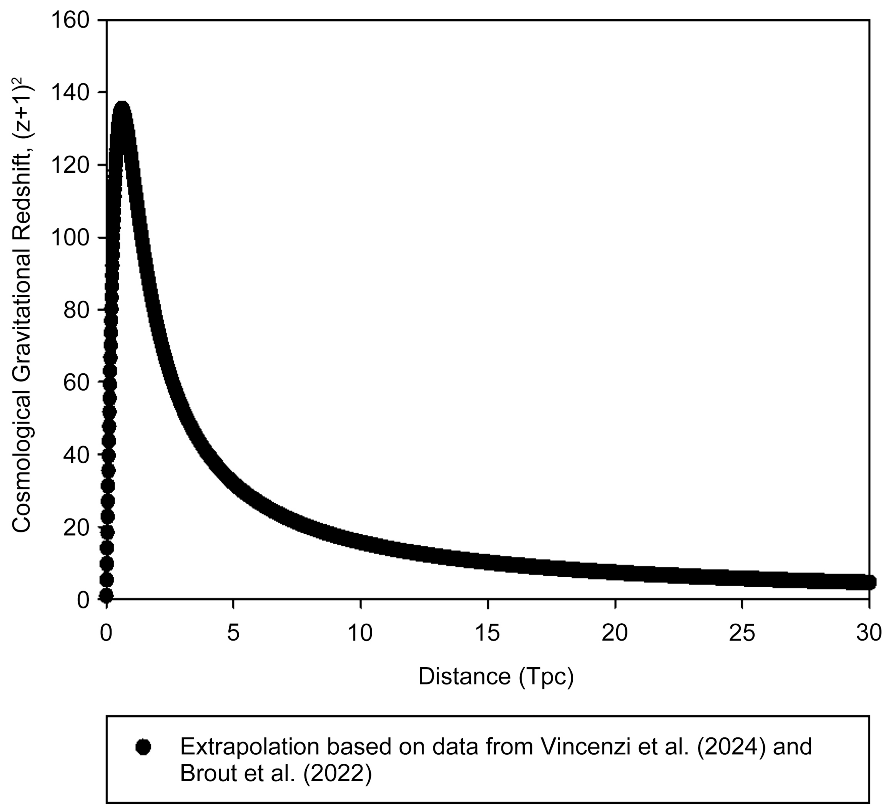


Figure 9. Cosmological gravitational redsift vs. distance.

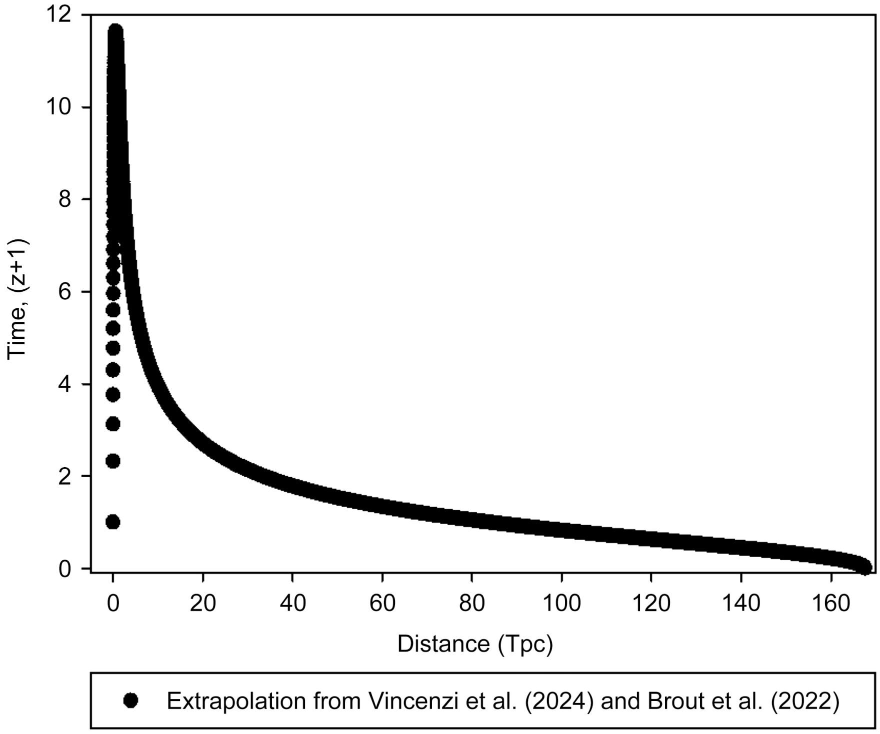


**Figure 10.** Time vs. distance.

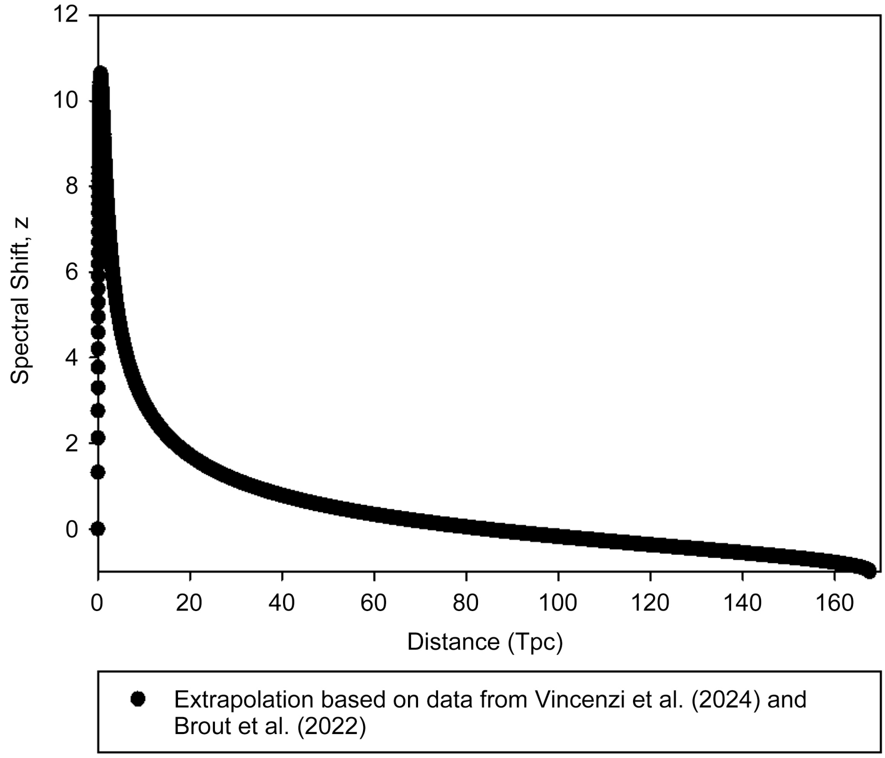


**Figure 11.** Spectral shift vs. distance.

Now we extrapolate this new data set to distances far larger than those observed. Figure 9 shows the cosmological gravitational redshift, $(z+1)^2$, out to 30 Tpc.

Following the procedure in Section 2 we computed the distance at which the maximum value of this variable is reached: $r$ = 613.409 Gpc, which corresponds to a maximum: $(z+1)^2 = 135.558$. The size of the universe is: 167,534 Gpc or 167.534 Tpc.

**Figure 10** depicts time in the Taub-NUT universe extrapolated far beyond the distance range in the new data set. The distance that the maximum value is obtain is $r$ = 613.409 Gpc with a value of $z+1=11.643$. The transition from time dilation to time contraction occurs at: $r=\alpha=83.766$ Tpc.

**Figure 11** is a plot of spectral shift, $z$ vs. distance in Tpc. The distance that the maximum value is obtain is $r$ = 613.409 Gpc with a value of $z$ = 10.643. The transition from redshift to blueshift occurs at: $r=\alpha=83.766$ Tpc.

## 6. Summary and Conclusion

The Taub-NUT universe represents a theory of cosmology, which is not laden with the fundamental problems associated with the standard theory of cosmology: dark energy, Hubble tension, lack of antimatter, etc [1]. Assuming the validity of general relativity, the Taub-NUT model was derived following the approach pioneered by Einstein in his theory of special relativity [7], formulating a theory by deriving the consequences of observations or experiments without additional assumptions. In contrast, the standard theory of cosmology assumes the validity of both the cosmological principle, which maintains that the universe is homogeneous and isotropic and the Copernican principle, which maintains that the earth relative to cosmological observations is not in anyway in a special location in the universe.

In addition to proving in [1] that the cosmological gravitational redshift, $(z+1)^2$, in the Taub-NUT universe agrees with the observations of Type Ia Supernova [5], in this work we showed that the redshift-distance and the time dilation relationships also agree with observations of supernova.

In order to ascertain how these three quantities—cosmological gravitational redshift, $(z+1)^2$, time dilation, $(z+1)$, and spectral shift, $z$, change with distance, we first extrapolated using the solution obtained by comparing the theoretical expressions of the Taub-NUT model with the supernova observations from [5]. This extrapolation leads us to conclude that the Taub-NUT universe is finite and that the Taub-NUT universe is much larger than the maximum observable distance according to the standard theory of cosmology.

We demonstrated that the spectral shift is a non-linear function of distance meaning that a given value of z can be at anyone of two different distances. Similarly we showed that any given cosmological gravitational redshift or time dilation corresponds to two different distances. Thus, determining the values of these quantities does not give us uniquely the distance of the cosmic source.

By invoking the mathematical necessity that the cosmological gravitational redshift, $(z+1)^2 > 0$, we concluded that the Taub-NUT universe must be finite. At small distances we observe redshifts. We noted, however, that at large distances in the Taub-NUT universe the spectral shift turns into a blueshift. Finally, time dilation turns into time contraction at large distances, a new phenomena, which is not a part of special relativity or the standard theory of cosmology.

Extrapolation is at best uncertain and at worse leads to completely erroneous results. So we compared the prediction of our extrapolation of the maximum redshift that can be observed with observations and noted that the prediction was smaller than the maximum observed redshift by the James Webb Space Telescope [9]. Presumably this mismatch was caused by the supernova data from [5] not reaching far enough. So we added the supernova data from [10] whose distances were greater than the Vincenzi *et al.* [5] maximum distance.

First, we showed that the Taub-NUT model lead to values of the three cosmological parameters that agreed with observations of [5] extended by data from [10]. Then we extrapolated this solution to much greater distances than those of the observations. This new extrapolation lead to values of all numbers much greater than those of the first extrapolation but the conclusions described above where the same.

It is clear that we are only going to get reliable estimates of quantities of the Taub-NUT universe, when we have highly accurate data on the relationship between spectral shift and distance that extend to large distances.

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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