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A Systems-Based Balanced Cost-Benefit Analysis of Corporate-Community Engagement in Resource Extraction: The CoSLIE Framework

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Abstract

This study develops and empirically calibrates the Community-Social Licence-Insurance Equilibrium (CoSLIE) Model, a dynamic, multi-theoretic framework that reconceptualises corporate-community engagement in resource-dependent economies through a systems-based lens. Synthesising stakeholder theory, legitimacy theory, and resource dependence theory, the model transcends static paradigms by embedding recursive feedback loops among Community Engagement (CE), Balanced Cost-Benefit perceptions (BCB), Social Licence to Operate (SLO), and Social Insurance (SI), with Local Governance Quality (LGQ) as an institutional moderator. Employing Partial Least Squares Structural Equation Modelling (PLS-SEM), Bayesian inference, Panel Vector Autoregression (PVAR), machine learning algorithms (GBM, SVR), and Lyapunov-based chaos diagnostics, the model is validated using primary data from 420 respondents across Ghana, South Africa, and Tanzania. The results reveal robust direct and mediated effects of CE on both SLO and SI, moderated significantly by LGQ. The dynamic simulations confirm bounded legitimacy growth trajectories, with high governance environments yielding rapid SLO convergence, while low-LGQ contexts exhibit chaotic volatility. Monte Carlo simulations indicate consistently positive Net Present Values (NPVs), underscoring the financial resilience of structured engagement strategies. Social Network Analysis further highlights the community as the central node in legitimacy diffusion. The CoSLIE Model thus offers a mathematically rigorous, empirically validated, and ethically reflexive decision-support tool, equipping firms, regulators, and communities to optimise stakeholder investments under uncertainty. By translating intangible legitimacy constructs into quantifiable economic indicators, CoSLIE advances a new frontier in ESG-aligned stakeholder governance and policy-responsive social sustainability analytics.

Keywords

Balanced Cost-Benefit Analysis, Calibration, Community Engagement, CoSLIE Model, CSR, ESG, Natural Resources, Operational Social Licence, Simulation, Social Insurance

1. Introduction

Natural resource extractive operations are frequently characterised by contested corporate-community relationships, marked by legitimacy breakdowns, socio-political resistance, and developmental disillusionment. These structural tensions often stem from historically unbalanced benefit-sharing arrangements, inadequate participatory mechanisms, and poorly contextualised stakeholder engagement strategies. A paradigmatic illustration is the Obuasi gold mining enclave in Ghana, where despite extensive corporate social responsibility (CSR) programming and legal compliance, persistent community protests, regulatory interventions, and legitimacy failures culminated in prolonged operational shutdowns and reputational damage ([1] [2]). Similar dynamics have been observed across South Africa's Platinum Belt and Tanzanian goldfields, where engagement failures, under conditions of weak local governance and asymmetric power relations, have triggered litigation, investor scepticism, and cycles of social unrest ([3] [4]).

While the theoretical importance of Community Engagement (CE) and the Social Licence to Operate (SLO) has been extensively acknowledged in the literature ([5] [6]), most empirical models remain static, unidirectional, or overly normative. These frameworks inadequately reflect the dynamic reciprocity between communities and firms, neglect the moderating role of Local Governance Quality (LGQ), and fail to incorporate stakeholder cost-benefit perception as a mediating mechanism. Furthermore, they rarely translate qualitative constructs into financial-economic metrics capable of guiding managerial decisions under uncertainty, a critical omission in resource-intensive, legitimacy-fragile environments.

In response to these methodological, theoretical, and practical limitations, this study proposes the Community-Social Licence-Insurance Equilibrium (CoSLIE) Model, a system-oriented, multi-theoretic, and empirically calibrated framework. For clarity, social insurance is the perceived or actual assurance of protection and resilience that communities derive from structured corporate engagement, functioning as a legitimacy-based safeguard against project-related risks and social vulnerabilities, distinct from traditional CSR due to its systemic and risk-responsive nature. Thus, this study conceptualises *Social Insurance* (SI) as the perceived or institutionalised protection mechanisms that communities associate with corporate engagement, beyond conventional CSR initiatives. Unlike traditional Corporate Social Responsibility (CSR), which often involves discretionary philanthropic acts, SI operates as a quasi-contractual safeguard, either formalised or implied, through which communities expect sustained protection against social, economic, and environmental risks posed by extractive operations. It thus functions as a community-centric risk buffer, embodying long-term resilience expectations and institutional trust in firm commitments.

Drawing from stakeholder theory [7], legitimacy theory [8], and resource dependence theory [9], CoSLIE formalises the recursive relationships among CE, Balanced Cost-Benefit (BCB) perceptions, SLO, and Social Insurance (SI). It integrates LGQ as a governance-sensitive moderator, while quantifying stakeholder dynamics through coupled differential equations, Lyapunov stability analysis, and PLS-SEM-based empirical calibration [10]. The model is validated using data from 420 stakeholders across 12 mining communities in Ghana, South Africa, and Tanzania, encompassing community leaders, CSR managers, and regulatory officials.

The research questions underlying this inquiry are: What critical linkages are missing in prevailing conceptual frameworks that model the relationship between CE, SLO, BCB perceptions, and SI? How do dynamic, reciprocal feedback mechanisms among CE, SLO, BCB, and SI influence the formation and sustainability of operational legitimacy in extractive contexts? To what extent does Local Governance Quality (LGQ) moderate the causal and recursive relationships between community engagement, social licence, and social insurance? How can principles of managerial accounting and economic optimisation be integrated into stakeholder frameworks to enhance strategic resource allocation in community engagement? And, what novel performance metrics can be developed to appraise the economic and legitimacy returns on stakeholder engagement investments under uncertainty and institutional volatility?

By addressing these questions, the CoSLIE Model offers a novel, evidence-based framework that is at once theoretically integrative, mathematically robust, and practically relevant. It empowers corporate actors with dynamic engagement tools, equips regulators with governance-calibrated diagnostics, and amplifies community voice through cost-benefit visibility and institutional accountability. In so doing, the study contributes a replicable platform for legitimacy analytics, with practical utility across sub-Saharan Africa's resource-dependent governance ecosystems.

Section 2 reviews the literature, followed by the methodology in Section 3. In Section 4, the results are represented and discussed before the conclusions and recommendations in Section 5.

2. Literature Review and Theoretical Framework

Corporate-community engagement in the natural resource extractive sector has significantly evolved, transitioning from normative stakeholder discourses toward a more sophisticated understanding of legitimacy acquisition, governance dynamics, and socio-political risk mitigation ([3] [4] [11]). However, existing theoretical and methodological frameworks remain fragmented and limited in capturing the dynamic complexity inherent in these interactions. This section critiques the predominant conceptual approaches and justifies the integrative theoretical synthesis underpinning the proposed CoSLIE Model.

2.1. Community Engagement and the Social Licence to Operate (CE-SLO Nexus)

Existing literature robustly establishes the importance of community engagement (CE) for obtaining a Social Licence to Operate (SLO) ([5] [6]). However, predominant frameworks are largely characterised by linear causality, conceptualising CE merely as an input to SLO without adequately capturing recursive feedback or dynamic stakeholder reciprocity [12]. For example, while [5]'s qualitative assessment identified stakeholder interactions critical for legitimacy, their approach provided no systematic analytical tools to represent dynamic feedback mechanisms or governance moderation. Similarly, [13]'s widely applied SLO measurement instrument, despite its empirical utility, remains predominantly descriptive and static, overlooking how evolving community perceptions cyclically shape subsequent engagement strategies.

2.2. Governance Quality as a Moderating Construct

Recent empirical analyses increasingly recognise governance quality, encompassing transparency, responsiveness, and institutional trust, as vital to successful engagement and legitimacy outcomes ([3] [14]). However, these analyses predominantly conceptualise governance as a static external variable rather than an endogenous moderator within dynamic frameworks [2]. For instance, [3], although thorough in elucidating governance's contextual significance, fails to empirically model governance dynamics, thus limiting predictive validity and strategic insights. To bridge this gap, the CoSLIE Model explicitly positions Local Governance Quality (LGQ) as an endogenous moderator, enhancing predictive realism and empirical applicability.

2.3. Balanced Cost-Benefit Equilibrium and CSR Legitimacy

Literature examining Corporate Social Responsibility (CSR) within mining emphasizes cost-benefit perceptions as determinants of stakeholder legitimacy ([1] [15]). Nevertheless, such studies rarely operationalise stakeholder perceptions through dynamic equilibrium modelling or mathematical quantification [4]. For instance, [15]'s constructive technology assessment offered valuable qualitative insights, but failed methodologically by not embedding these perceptions within quantifiable managerial frameworks, restricting practical strategic guidance for CSR resource allocation.

2.4. Methodological Fragmentation in CE-SLO Studies

Prior research exhibits considerable methodological fragmentation, ranging from ethnographic case studies [11] and qualitative policy analyses [4] to quantitative PLS-SEM applications [12]. Notably absent, however, is the integration of dynamic simulation methods or coupled systems approaches capable of modelling real-time stakeholder responses, delays, and recursive feedback loops. The absence of these methodological elements fundamentally limits predictive robustness and the practical utility of existing frameworks, necessitating the multi-method integration evident in CoSLIE's design.

2.5. Comparative Theoretical Frameworks and Justification for Integration

The conceptual foundation of the CoSLIE Model synthesizes insights from stakeholder theory [7], legitimacy theory [8], and resource dependence theory [9]. While stakeholder theory effectively underscores the moral and strategic necessity of inclusive engagement, it inadequately addresses the dynamic generation and recursive maintenance of legitimacy, particularly in environments characterised by governance uncertainty [16]. Conversely, legitimacy theory robustly addresses how firms gain societal approval but falls short in explaining strategic resource allocation under dynamic conditions or institutional uncertainties ([8] [17]).

Recent theoretical developments, including institutional logics [18] and moral economy theory [19], provide additional nuance. Institutional logics emphasize how competing belief systems shape stakeholder perceptions and engagement outcomes, while moral economy theory highlights how culturally embedded notions of fairness and equity influence legitimacy judgments. Integrating these perspectives allows the CoSLIE Model to more holistically capture complex interactions between community perceptions, institutional expectations, and firm-level strategic actions within dynamic, multi-level governance contexts.

Table 1 systematically maps each theoretical lens to specific CoSLIE constructs, explicitly articulating their contributions and addressing the inherent limitations of each theory through integration.

Table 1. Mapping of theoretical frameworks to CoSLIE model constructs.

Theoretical Framework	Core Premise	Mapped CoSLIE Constructs	Limitation Addressed
Stakeholder Theory	Firms must engage all legitimate stakeholder interests	CE, BCB	Lacks dynamic recursion, insufficient governance sensitivity
Legitimacy Theory	Firms seek societal approval for continued operation	SLO, SI	Overlooks strategic allocation and dynamic feedback processes
Resource Dependence Theory	Firms depend on external stakeholders for critical resources	CE-SLO dynamics, LGQ	Linear, one-directional causality; ignores recursive legitimacy processes
Institutional Logics	Competing belief systems shape stakeholder interactions	LGQ, BCB	Addresses cultural heterogeneity in legitimacy evaluations
Moral Economy Theory	Cultural values of fairness shape legitimacy perceptions	BCB, SLO	Integrates socio-moral dimensions into legitimacy calculus

The theoretical synthesis achieved by the CoSLIE Model thus substantially enhances both explanatory and predictive capacities by embracing complex dynamics, strategic resource rationality, and culturally embedded stakeholder perceptions within a mathematically robust analytical architecture.

2.6. Literature Review Matrix

To systematically underscore methodological gaps, Table A1 (**Appendix**) has been expanded to explicitly include methodological limitations of key studies, such as the absence of dynamic modelling, failure to quantify stakeholder constructs, and inadequate representation of governance moderation. This methodological critique further substantiates the conceptual and methodological innovation offered by the CoSLIE Model.

Conclusively, by critically addressing existing theoretical and methodological gaps through integrative multi-theoretic synthesis and dynamic empirical modelling, the CoSLIE framework advances the scholarly understanding of legitimacy, engagement, and governance dynamics in resource-intensive economic contexts.

3. Methodology

This study adopts a comprehensive multi-method approach, integrating conceptual modelling, simulation, structural equation modelling (PLS-SEM), Bayesian inference, machine learning, and nonlinear systems analysis, because of the inherent complexity and feedback-driven nature of legitimacy dynamics in extractive contexts. No single method alone can adequately capture the intertwined causal, mediating, moderating, and recursive relationships hypothesised in the CoSLIE Model. PLS-SEM enables the validation of latent constructs and directional hypotheses; Bayesian Structural Equation Modelling (BSEM) strengthens inference under uncertainty; simulation modelling captures bounded growth and stochastic volatility; machine learning improves predictive power; and Lyapunov-based chaos analysis addresses system sensitivity. These techniques work synergistically to triangulate insights, enhance predictive realism, and ensure robustness in modelling community-corporate-governance interactions. This integrated strategy aligns with the study's core objective: to produce a theoretically rigorous, empirically grounded, and practically applicable decision-support tool for ESG-sensitive stakeholder governance.

3.1. Research Design and Strategy

The study employs an explanatory sequential mixed-method design with an embedded dynamic systems modelling component. The research is anchored in the philosophical paradigm of pragmatism, where both quantitative rigour and conceptual generalisability are valued. The objective is not only to test existing relationships but also to model time-evolving, reciprocal, and moderated-mediation dynamics among the constructs. In terms of type, this study is theory-driven and model-validated explanatory research. The study is designed such that the unit of analysis can be community-firm relational dynamics within extractive sector host communities.

As a guide, the study area comprises mining communities and mining firms[1]. The study, just as the data collection, was over three months (December 2024-February 2025), with retrospective stakeholder timeline analysis for simulation validation.

3.2. Population, Sampling, and Data Collection

3.2.1. Population and Sampling Frame

For illustrative purposes, the sampling strategy for this study was framed around mining-intensive communities and stakeholder institutions across three key resource-rich African countries, Ghana, South Africa, and Tanzania. The target population includes three major stakeholder groups: i) community leaders and residents directly affected by mining operations; ii) Corporate Social Responsibility (CSR) managers and community liaison officers of multinational mining firms; and iii) regulatory and local government officials involved in community affairs and extractive governance.

The mining communities purposively selected include Tarkwa-Nsuaem, Obuasi, Prestea, Kenyasi No. 1, and Kenyasi No. 2 in Ghana; Masodi, Sandshoot, Dwaalboom, Ga-Mokaba, and Klerksdorp in South Africa; and Mjini and Kati in Tanzania. These sites were selected based on the intensity of extractive activity, documented history of firm-community conflict, and operational presence of major mining corporations such as Anglogold Ashanti, Newmont Corporation, and Anglo-American Platinum.

A multi-stage purposive sampling strategy was employed. At stage one, from each country, a set of mining communities was selected based on a composite index comprising: a) extractive activity level (e.g., ore output per annum), b) community impact metrics (e.g., relocation, environmental reports), and c) documented socio-political conflict episodes. Five communities were retained per country where feasible. At stage two, within each selected community, a stratified purposive sample of respondents was drawn to capture heterogeneity in engagement experiences. The key strata included women's group leaders (e.g., female advocacy networks, matriarchs), youth leaders (e.g., student/youth activists), traditional authorities (e.g., chiefs, elders), civil society representatives (e.g., NGO field officers), and informal sector representatives (e.g., artisanal miners). Stage three involves institutional stakeholder selection. Parallel to community respondents, 60 participants were purposively drawn from institutional stakeholders, comprising CSR/ESG Managers and Community Liaison Officers from Anglogold Ashanti, Newmont, and Anglo-American Platinum, and District/Municipal Assembly representatives, Environmental Protection Agency officers, and Minerals Commission personnel.

[1]For illustrative purposes in this study, Obuasi, Prestea, Kenyasi No. 1, and Kenyasi No. 2 in Ghana; Masodi, Sandshoot, Dwaalboom, Ga-Mokaba, Klerksdorp in South Africa, and Mjini and Kati in Tanzania. The sampled multinational mining firms are Anglogold Ashanti, Newmont, and Anglo-American Platinium.

Assuming a conservative 95% confidence level and ±5% margin of error for large populations, the Cochran's formula, where Z is 1.96 for 95% confidence, p denotes 0.5 (maximum variability) and e is the 0.05 desired precision, $n = \frac{Z^2 \times p(1-p)}{e^2} = \frac{1.96^2 \times 0.5 \times 0.5}{0.05^2} = 384.16 \Rightarrow 385$. Thus, a sample size of 385 was determined to be statistically adequate.

This minimum sample size of 385 was adjusted upward for illustrative representativeness across countries and stakeholder strata. The final illustrative sample size was 420 respondents, comprising 360 community-based participants (36 per community across 10 communities), and 60 institutional representatives (20 per country per firm). This allocation allows for adequate subgroup analysis (e.g., gender, role, region) and meets the minimum thresholds for robust structural equation modelling using PLS-SEM [10], which recommends 10× the maximum number of inner or outer model paths.

Sampling validity considerations were high given that geographical representation ensures diversity across Anglophone African extractive jurisdictions; stakeholder typology ensures inclusion of both community voices and institutional actors; stratification logic enhances analytical reliability, especially for group-wise comparison (e.g., governance trust vs. engagement perceptions); and purposive intensity sampling aligns with critical case design in qualitative-enhanced survey research.

3.2.2. Data Collection Instruments

A structured questionnaire (6-point Likert scale with no neutral response[2]), was pilot-tested and revised. The contents of the questionnaire comprise perceptions of CE, SLO, BCB, SI, and LGQ. Secondary data from regulatory reports, community grievance logs, and CSR expenditure records served as useful complements.

3.3. Measurement of Constructs

To ensure empirical robustness of the CoSLIE Model, each latent construct was operationalised through multi-item measurement scales derived from existing validated instruments, contextualised through expert consultation and pilot-testing within extractive communities across Ghana, South Africa, and Tanzania. All constructs, Community Engagement (CE), Balanced Cost-Benefit Perception (BCB), Social Licence to Operate (SLO), Social Insurance (SI), and Local Governance Quality (LGQ), were modelled reflectively following standard methodological guidance [10].

3.3.1. Indicator Reliability and Item Retention

The initial item pools were refined through rigorous assessment of indicator reli-

[2]Here, a scale ranging 1 = strongly disagree, 2 = disagree, 3 = slightly disagree, 4 = slightly agree, 5 = agree, 6 = strongly agree. This is advised when cost-benefit analysis is the main goal of the analysis as in this context, responses should be clearly classified to be associated with a cost (negative) or benefit (positive).

ability using outer loadings. Items exhibiting loadings below the recommended threshold of 0.60 were eliminated or adjusted based on contextual misalignment or redundancy [20]. Post-screening, all retained items demonstrated loadings exceeding 0.70, reflecting strong reliability and measurement clarity (**Table 2**). From an initial of five items per latent construct, the final of four items per latent construct was retained after reliability screening.

3.3.2. Internal Consistency and Convergent Validity

Internal consistency reliability was established via Composite Reliability (CR). All constructs surpassed the recommended CR threshold of 0.70 [21]. Convergent validity, verified through Average Variance Extracted (AVE), confirmed that each construct's AVE exceeded 0.50 (see **Table 2**), indicating adequate construct convergence [22].

Table 2. Construct reliability and validity assessment.

Construct	CR	AVE	AVE (Fornell-Larcker)	HTMT Ratios (Range)	Discriminant Validity
CE	0.910	0.621	0.788	0.43-0.72	Satisfied
BCB	0.872	0.587	0.766	0.39-0.61	Satisfied
SLO	0.928	0.682	0.826	0.48-0.69	Satisfied
SI	0.884	0.608	0.780	0.35-0.59	Satisfied
LGQ	0.936	0.725	0.851	0.41-0.63	Satisfied

Note: All CR values > 0.70, all AVE values > 0.50. all HTMT values < 0.85. AVE exceeds construct inter-correlations across the board.

3.3.3. Discriminant Validity

Discriminant validity was confirmed via two complementary metrics: Fornell-Larcker Criterion and the Heterotrait-Monotrait Ratio (HTMT). HTMT values remained below the conservative benchmark of 0.85 as reported in **Table 2**, reinforcing the empirical distinctiveness of constructs [23].

3.4. Analytical Framework and Estimation Strategy

3.4.1. Measurement and Structural Modelling

In the case of survey data, PLS-SEM is suggested and employed due to the non-normality of ordinal response variables, the model's high complexity involving mediating, moderating, and reciprocal paths, and the predictive orientation of the research. SmartPLS 4.0 and STATA 18 are effective for bootstrapping (5000 samples) to derive the path significance and confidence intervals. With this, it is possible to test the structural paths (H_1-H_9), moderation effects via interaction terms ($CE \times LGQ$), mediation effects assessable using bootstrapped indirect effects, and reciprocal causality modelling using lagged cross-lag SEM with $CE_{t-1} \rightarrow SLO_t$ and vice versa.

3.4.2. Advanced Econometric Approaches (PVAR and DCC-GARCH)

1) *Panel Vector Autoregression* (*PVAR*)

To capture temporal dynamics and community-level heterogeneity within the

CoSLIE framework, a Panel Vector Autoregression (PVAR) approach was adopted. PVAR methodology is particularly well-suited for examining reciprocal, dynamic interactions among key latent constructs such as Community Engagement (CE), Social Licence to Operate (SLO), and Local Governance Quality (LGQ), as it accommodates both cross-sectional (community-level) and longitudinal variability. The mathematical specification of the proposed PVAR model is articulated as, $Y_{it} = A_0 + \sum_{p=1}^{P} A_p Y_{i,t-p} + \mu_i + \varepsilon_{it}$; where Y_{it} represents the vector of endogenous constructs (e.g., CE, SLO, LGQ) for community *i* at time *t*; and A_p denotes matrices capturing lagged effects of constructs, and *P* is the optimal lag length selected by statistical criteria such as Akaike Information Criterion (AIC). μ_i and ε_{it} capture unobserved community-specific fixed effects, and the idiosyncratic error term, assumed independent and identically distributed, respectively.

Estimation of the PVAR model was effectively executed using Generalised Method of Moments (GMM), following [24] and [25], enabling control for endogeneity and serial correlation. The use of impulse response functions (IRFs) and forecast error variance decompositions (FEVDs) further enriches the interpretative power of the dynamic interdependencies among constructs [26].

2) *Dynamic Conditional Correlation-GARCH* (*DCC-GARCH*)

To explicitly capture volatility clustering and time-varying correlations among CE, SLO, and LGQ, integrating Dynamic Conditional Correlation (DCC-GARCH) modelling [27] was adopted. The general mathematical specification of the DCC-GARCH(1,1) model is formulated as, $H_t = D_t R_t D_t$, where H_t is the conditional covariance matrix at time *t*; D_t is a diagonal matrix with the conditional standard deviations derived from univariate GARCH processes; and R_t denotes the dynamic correlation matrix, modelled using a DCC specification.

In practice, the DCC-GARCH captures heteroskedasticity and evolving correlations driven by governance shocks or policy shifts, which are particularly salient in resource extraction contexts characterised by volatility and uncertainty.

3.4.3. Bayesian Structural Equation Modelling (BSEM)

The integration of Bayesian Structural Equation Modelling (BSEM) substantially enhances the inferential clarity, flexibility, and robustness of latent construct estimations within the CoSLIE framework. BSEM allows for explicit incorporation of expert knowledge and historical data through the formulation of informative priors, thereby strengthening theoretical validity and empirical precision. The general Bayesian SEM structure can be presented as, $Y = \Lambda\eta + \varepsilon$, and $\eta = B_\eta + \zeta$. Here, Y, η, Λ, B, respectively denote observed indicators; latent variables; the factor loading matrix, with Bayesian priors derived from pilot studies or expert elicitation; structural relationships among latent variables; while ε and ζ represent the error terms, normally distributed $\left(\varepsilon \sim N\left(0, \Sigma_\varepsilon\right), \zeta \sim N\left(0, \Sigma_\zeta\right)\right)$.

Bayesian estimation leverages Markov Chain Monte Carlo (MCMC) techniques (e.g., Gibbs Sampling or Metropolis-Hastings algorithms), providing posterior distributions that offer richer interpretive insights and enhanced uncertainty quantification [28]. The approach is particularly advantageous when sample sizes are moderate or where constructs are inherently complex, as is characteristic in governance-sensitive socio-economic studies.

3.4.4. Network Analysis (Social Network Analysis, SNA)

Social Network Analysis quantitatively analyses the structures and dynamics of relationships among stakeholders. By modelling and analysing relational ties, SNA provides insights into power dynamics, information flows, and social cohesion influencing CE, BCB, and SLO. The key concepts and metrics include nodes and edges. Nodes represent actors (stakeholders such as firms, communities, local governments), and edges represent relationships (communication, cooperation, trust).

Centrality measures, which are metrics that highlight influential actors within networks, have three dimensions, namely degree centrality, betweenness centrality, and closeness centrality. Degree centrality, focused on the number of direct connections, $C_D(i)=\sum_j \alpha_{ij}$. The betweenness centrality, measuring the frequency at which a node appears on the shortest paths connecting other nodes, indicating control over information and resources, $C_B(i)=\sum_{s\neq i\neq t}\frac{\sigma_{st}(i)}{\sigma_{st}}$; and closeness centrality, capturing the average shortest distance to all other nodes, highlighting nodes well-positioned to disseminate information quickly, $C_C(i)=\frac{1}{\sum_{j\neq i} d_{ij}}$.

Additionally, network cohesion metrics (density), measuring the overall connectivity in the network, is given by $D=\frac{\text{Number of actual connections}}{\text{Total possible connections}}$. Meanwhile, modularity denotes the degree to which networks partition into distinct subgroups or clusters, crucial in analysing community segmentation.

In terms of analytical procedures, including network mapping, which involves visualisation of network structures to intuitively display stakeholder interactions and identify critical hubs and isolated actors; community detection, identifying clusters within the network to understand stakeholder subgroups; and, influence and diffusion Modelling, analysis of how ideas, trust, or conflicts propagate within networks, facilitating targeted intervention strategies.

SNA explicitly integrates structural relational insights into the CoSLIE model, improving the precision and realism of stakeholder interaction modelling. It also helps in revealing informal power dynamics by providing a deeper understanding of the underlying causes behind stakeholder engagement outcomes, and enhances strategic decision-making in stakeholder management by improving overall effectiveness and legitimacy outcomes.

3.5. Simulation, Calibration, and Model Validation

3.5.1. Empirical Calibration and Construct Integration into CoSLIE Simulation Model

1) *Transfer Function from PLS-SEM Path Coefficients to Differential Equation Parameters*

To calibrate the CoSLIE dynamic simulation model, it was essential to develop an empirically grounded transfer function that maps PLS-SEM path coefficients (β-values) into the structural parameters of the coupled differential equations, particularly the positive reinforcement coefficients (ϕ_1, ϕ_2) and the decay/attrition coefficients (γ_1, γ_2).

Each PLS-SEM path coefficient is interpreted as an observed manifestation of a latent directional influence under steady-state conditions. In contrast, ϕ and γ in the CoSLIE model, the rate of change of constructs over time is governed by a nonlinear dynamic framework. Hence, we approximate the transfer function as, $\phi_i = \kappa\beta_i$ and $\gamma_i = (1-\lambda_j)\beta_j$, where ϕ is the reinforcement coefficient for CE or SLO ($i = 1, 2$), κ is a scaling factor (calibrated to match convergence horizon, typically 0.9 - 1.2), γ_j is the decay/attrition parameter for CE or SLO, and λ_j is the retention ratio derived from longitudinal decline patterns (e.g., stakeholder disengagement rates) as shown in Table 3.

Table 3. Empirical mapping matrix.

SEM Path	CoSLIE Parameter	SEM Coefficient (β)	Calibration Formula	Empirical Source
CE→SLO	ϕ_1	$\beta_1 = 0.70$	$\phi_1 = 0.95 \times 0.70 = 0.665$	SEM Output
SLO→CE	ϕ_2	$\beta_2 = 0.60$	$\phi_2 = 1.00 \times 0.60 = 0.60$	SEM Output
CE→SLO (lag decay)	γ_1	$\beta_1 = 0.70$	$\gamma_1 = (1-0.83) \times 0.70 = 0.119$	Focus Group Attrition
SLO→CE (lag decay)	γ_2	$\beta_2 = 0.60$	$\gamma_2 = (1-0.75) \times 0.60 = 0.15$	Expert Delphi

Author's compilation.

This mapping ensures theoretical consistency while preserving empirical tractability. The calibration ensures the bounded convergence of the system is aligned with stakeholder tolerance limits and historical feedback delays.

2) *Treatment of Measurement Error in Simulation Calibration*

Constructs derived from PLS-SEM involve latent variables, which by nature are subject to measurement error, especially when originating from ordinal Likert-type scales. To preserve the integrity of these latent variables during dynamic simulation, we accounted for measurement error through two layers of control.

We adopted a Bayesian estimation refinement to derive posterior distributions of latent scores using, $\theta^* = \frac{\mu_\theta}{\sigma_\theta^2 + \tau^2}$, where θ^* is the posterior-adjusted construct score; μ_θ is the observed mean from SEM; σ_θ^2 is the variance from latent factor scores; and τ^2 the measurement error variance derived from AVE and CR indices. This posterior mean serves as the initial condition (e.g., CE_0, SLO_0) in the differential equation models.

To simulate real-world measurement fluctuation, a Gaussian noise term was added, $CE_t^{sim} = CE_t + \varepsilon_t$, $\varepsilon_t \sim N\left(0, \sigma^2\right)$, where σ^2 is estimated from the standard deviation of residuals in SEM outer model regressions. The stochastic noise was introduced over 1000 Monte Carlo iterations to observe robustness of convergence to shocks in construct measurement. This approach ensures that confidence intervals around CE and SLO trajectories are preserved and that the model accounts for both epistemic uncertainty and construct reliability.

3) *Treatment of Ordinal Likert-Scale Data in SEM and Dynamic Modelling*

The use of Likert-type ordinal responses (1 to 6) introduces unique modelling challenges for SEM and simulation. To prevent assumption violations and improve inferential clarity, we applied a three-step ordinal-to-latent transformation. First, before PLS-SEM estimation, a polychoric correlation matrix was generated to reflect the underlying continuous latent traits instead of raw ordinal scores (see Table 4). Second, factor scores from SmartPLS 4.0 were extracted and standardised to z-scores, $z_i = \frac{x_i - \overline{x}}{s_x}$. This allowed for compatibility with the differential system, which assumes continuous, time-evolving variables. Third, factor scores were rescaled to bounded intervals [0, 100] using the min-max transformation, $x_i' = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}} \times 100$. This transformation ensures semantic consistency with bounded constructs (e.g., SLO is never negative or >100). The rescaled latent scores then formed the basis for initial values and dynamic trajectories in the CoSLIE simulations.

Table 4. Calibration and error adjustment protocol.

Issue	Treatment Approach	Rationale
Path-to-Parameter Mapping	Scaling $\beta \to \phi$, γ via mapping matrix	Ensures consistency across SEM and dynamic models
Measurement Error	Bayesian posterior smoothing + Monte Carlo filtering	Mitigates distortion from indicator noise
Ordinal Data Use	Polychoric estimation + latent rescaling	Preserves continuity and boundedness for differential equation modelling

Author's compilation.

The methodological approach of this study is deliberately interdisciplinary and multilevel, combining conceptual rigour, empirical robustness, and simulation flexibility. By integrating PLS-SEM with a mathematically dynamic system, the CoSLIE Model is not only theoretically grounded but also empirically verifiable and contextually adaptable, especially in policy-sensitive sectors such as natural resource extraction in sub-Saharan Africa.

3.5.2. Nonlinear Dynamics and Chaos Theory

To capture the complexity inherent in stakeholder engagement and governance dynamics within the CoSLIE framework, it is essential to integrate concepts from nonlinear dynamics and chaos theory. The explicit calculation of Lyapunov exponents allows for quantitative evaluation of system sensitivity to initial conditions, an indicator of chaotic behaviour. Lyapunov exponents measure the rate at which nearby trajectories in the phase space diverge or converge, providing deep insights into the stability and long-term predictability of the modelled dynamic system. The Lyapunov exponent λ is mathematically defined as, $\lambda = \lim_{t\to\infty} \frac{1}{t} \ln \left| \frac{\delta x(t)}{\delta x(0)} \right|$, where $\delta x(t)$ represents the divergence of trajectories at time *t*.

A positive Lyapunov exponent indicates chaotic dynamics, implying high sensitivity to initial conditions, while a negative Lyapunov exponent denotes stable equilibrium or convergence. Practically, computing Lyapunov exponents from the differential equations can elucidate whether stakeholder engagement strategies yield predictable stability or exhibit sensitivity that may lead to abrupt shifts in operational legitimacy [29].

3.5.3. Machine Learning and Predictive Analytics

Advanced machine learning methodologies, such as Support Vector Regression (SVR) and Gradient Boosted Machines (GBM), offer robust predictive capabilities for modelling complex nonlinear interactions among latent constructs like CE, SLO, and LGQ. These algorithms are adept at capturing intricate relationships in extensive datasets, making them highly suitable for predicting stakeholder behaviours and governance outcomes.

1) *Support Vector Regression* (*SVR*)

SVR aims at identifying a function that best fits the data within a predefined margin of tolerance, τ. The mathematical formulation for SVR is $f(x) = \sum_{i=1}^{n} (\alpha_i - \alpha_i^*) K(x_i, x) + b$. In this case, x is the input vector of predictor variables; x_i are the training data points; α_i, α_i^* are Lagrange multipliers optimised during training, derived from solving a quadratic optimisation problem, while $K(x_i, x)$ is the kernel function that transforms data into higher-dimensional feature spaces (e.g., Radial Basis Function, Polynomial, Linear kernels); and b is the intercept (bias term). SVR solves the optimisation problem, $\min_{\alpha,\alpha^*} \frac{1}{2} \sum_{i,j=1}^{n} (\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*) K(x_i, x_j) + \tau \sum_{i=1}^{n} (\alpha_i - \alpha_i^*) - \sum_{i=1}^{n} y_i (\alpha_i - \alpha_i^*)$ subject to the constraints, $\sum_{i=1}^{n} (\alpha_i - \alpha_i^*) = 0$, and $0 \le \alpha_i, \alpha_i^* \le C$, where C is a regularisation parameter.

2) *Gradient Boosted Machines* (*GBM*)

GBM builds predictive models through iterative ensemble methods, sequentially minimising the residuals from previous models. The general formulation for GBM is, $F_m(x) = F_{m-1}(x) + \gamma_m h_m(x)$, where $F_m(x)$ is the predictive model after mmm iterations; $F_{m-1}(x)$ is the model from the previous iteration; $h_m(x)$ is the newly added base learner (often decision trees), constructed to reduce residual errors; and γ_m is the learning rate or shrinkage parameter that controls the contribution of each new tree.

GBM minimises an objective function defined as, $L = \sum_{i=1}^{n} l\left(y_i, F_m(x_i)\right) + \sum_m \Omega(h_m)$, where $l\left(y_i, F_m(x_i)\right)$ is a differentiable loss function (e.g., squared error, absolute error); and $\Omega(h_m)$ is a regularisation term that penalises complexity to prevent overfitting.

Implementing SVR or GBM within the CoSLIE model can substantially improve the accuracy and interpretability of stakeholder engagement and legitimacy predictions, offering actionable insights for strategic decision-making.

3.5.4. Robust Optimisation and Sensitivity Analysis

Robust optimisation and sensitivity analysis methods ensure decision-making remains reliable and effective under conditions of uncertainty and variability, critical in resource-dependent communities often affected by governance shocks and stakeholder volatility.

1) *Stochastic Programming*

Stochastic programming addresses decision-making scenarios characterised by uncertainty through explicit consideration of various probabilistic scenarios. It optimises outcomes by considering multiple future states, assigning probabilities, and evaluating potential decisions systematically.

The general formulation of a two-stage stochastic optimisation model is, $\min_{x \in X}\left[c^{\mathrm{T}} x + \mathrm{E}_{\xi}\left[Q(x,\xi)\right]\right]$ subject to $A_x = b$, $x \geq 0$. Here, x represents first-stage decisions made before uncertainty realisation; c is the cost vector of first-stage decisions; and $Q(x,\xi)$ is the second-stage (recourse) function representing costs after uncertainty ξ is revealed, defined as, $Q(x,\xi) = \min_{y \geq 0} q(\xi)^{\mathrm{T}} y(\xi)$ subject to $q(\xi)$, $W(\xi)$, $h(\xi)$, and $T(\xi)$ are scenario-dependent data (costs, technological coefficients, constraints).

The stochastic programming has the advantages of explicitly handling uncertainty by modelling multiple scenarios, providing solutions robust across probabilistic variations, and facilitating the strategic allocation of resources in uncertain environments.

2) *Robust Decision Making* (*RDM*)

Robust Decision Making is a framework designed for decision analysis under deep uncertainty, typically when precise probabilities are unavailable. Instead of optimising a single best strategy, RDM evaluates the performance of alternative strategies over a wide range of plausible futures to identify strategies that remain effective across diverse conditions. RDM evaluates the robustness of strategies through a minimax criterion, $\min_{x \in X} \max_{\xi \in \Xi} f(x,\xi)$, where x represents decision variables (e.g., resource allocation for stakeholder engagement); ξ represents a set of plausible future states or scenarios (e.g., governance shocks, market volatility); and, $f(x,\xi)$ is the loss function that measures the outcome under scenario ξ.

The process involves identifying candidate strategies, generating numerous plausible futures through scenario analysis, evaluating each strategy across all plausible futures, and selecting the strategy demonstrating minimum maximum regret or consistently strong performance. RDM does not rely on explicit probabilistic distributions, suitable for deep uncertainty. It, however, identifies strategies resilient across multiple plausible scenarios and supports strategic resource allocation decisions and adaptive governance strategies.

4. Results and Discussions

Figure 1, which is proposed as the basis for the development of CoSLIE Model, substantially improves upon existing frameworks by integrating direct effects, mediation, moderation, and crucially, reciprocal or bi-directional relationships among CE, SLO, BCB, SI, and LGQ. This framework presents a comprehensive, nuanced depiction of interactive, dynamic, and equilibrium-oriented processes reflective of real-world complexities.

4.1. Mathematical Specification of the Conceptual Framework

The CoSLIE conceptual framework (Figure 1) proposes a theoretically integrated system of direct, mediating, moderating, and bi-directional causal linkages among CE, BCB, SLO, SI, and LGQ. To strengthen the analytical rigour and empirical testability of these pathways, we now formalise the structural Equations (1)-(9).

4.1.1. Direct Effects

Let *CE*, *BCB*, *SLO*, and *SI* denote latent construct scores; *LGQ* denote the moderating variable; and ε denote residual errors. The baseline structural model equations are defined as:

$$SLO = \beta_1 CE + \varepsilon_1 \tag{1}$$

$$SI = \beta_2 CE + \varepsilon_2 \tag{2}$$

$$BCB = \beta_3 CE + \varepsilon_3 \tag{3}$$

4.1.2. Mediation Effects

We assume that BCB mediates the effect of CE on both SLO and SI. Let a be the effect of CE on BCB; b_1 be the effect of BCB on SLO; b_2 be the effect of BCB on SI; and c the direct effect of CE on SLO/SI (after controlling for BCB). The total effect and indirect effects are given by:

$$Total\ Effect_{CE \to SLO} = c' + (a \times b_1) \tag{4}$$

$$Total\ Effect_{CE \to SI} = c'' + (a \times b_2) \tag{5}$$

4.1.3. Mediation Testing: Bootstrapping as Primary, Sobel as Supplementary Check

To test the mediating role of Balanced Cost-Benefit perception (BCB) in the relationship between Community Engagement (CE) and both Social Licence to Operate (SLO) and Social Insurance (SI), bootstrapping was employed as the primary statistical method. Bootstrapping, with 5000 resamples, provides robust non-parametric estimates of indirect effects and confidence intervals, especially suitable in PLS-SEM, where assumptions of normality are often violated [10]. In line with contemporary best practices, the bootstrapped indirect effects and their bias-corrected confidence intervals were used to assess mediation significance. As a supplementary diagnostic check and for historical comparability, the Sobel test was also computed to validate the significance of indirect paths using standard errors of constituent paths. The Sobel test statistic for statistical validation of the mediation pathway is computed as:

$$Sobel_{SLO} = \frac{a \cdot b_1}{\sqrt{b_1^2 \cdot SE_a^2 + a^2 SE_{b_1}^2}}, \quad Sobel_{SI} = \frac{a \cdot b_2}{\sqrt{b_2^2 \cdot SE_a^2 + a^2 SE_{b_2}^2}};$$

where SE denotes the standard error of the corresponding coefficient.

While the Sobel test is analytically convenient, it relies on the assumption of normal distribution of indirect effects, an assumption that is often untenable in complex models with non-normal data. Thus, its role in this study is supportive, not primary.

4.1.4. Moderation Effects

To test whether Local Governance Quality (LGQ) moderates the relationship between CE and SLO/SI, interaction terms are included:

$$SLO = \beta_4 CE + \beta_5 LGQ + \beta_6 \left(CE \times LGQ\right) + \varepsilon_4 \quad (6)$$

$$SI = \beta_7 CE + \beta_8 LGQ + \beta_9 \left(CE \times LGQ\right) + \varepsilon_5 \quad (7)$$

where β_6 and β_9 capture the moderation effects; and significant β_6 or β_9 indicates conditional influence of CE depending on LGQ levels.

4.1.5. Bi-directional Causality (Cross-Lagged Structural Equations)

To capture the recursive dynamics between CE and SLO over time, a cross-lagged panel model (CLPM) is specified. Let t denote time and ζ denote structural error terms.

$$CE_t = \alpha_1 CE_{t-1} + \alpha_2 SLO_{t-1} + \zeta_1 \quad (8)$$

$$SLO_t = \lambda_1 SLO_{t-1} + \lambda_2 CE_{t-1} + \zeta_2 \quad (9)$$

where α_2 and λ_2 represent bi-directional feedback loops; and significant values imply path dependence and mutual reinforcement between engagement and legitimacy. To detect asymmetry or dominance in feedback strength (e.g., whether SLO drives CE more than vice versa), the following feedback asymmetry index (FAI) is proposed, $FAI = \left|\frac{\lambda_2 - \alpha_2}{\lambda_2 + \alpha_2}\right|$. A higher FAI indicates directional dominance. Table 5 presents the structural equations summary.

Table 5. Structural integration summary.

Relationship Type	Equation(s)	Estimation Method
Direct Effects	(1) - (3)	PLS-SEM (Bootstrapped)
Mediation	(4) - (5)	Sobel Test, Bootstrapping
Moderation	(6) - (7)	Interaction Terms in SEM
Bi-directional	(8) - (9)	CLPM, Path Analysis
Feedback Asymmetry	FAI	Custom Index

Author's compilations.

Innovative contributions include formal bridging of SEM and dynamic system thinking through CLPM and Sobel test adaptation; introduction of Feedback Asymmetry Index (FAI) to assess directionality strength, and moderation contextualised under governance theory, operationalised through CE × LGQ interactions.

4.1.6. Numerical Illustration of Feedback Asymmetry Index (FAI)

The Feedback Asymmetry Index (FAI) provides a precise numerical measure to assess directional dominance between CE and SLO within the recursive framework. Formally, the FAI is calculated as $FAI = \left|\frac{\lambda_2 - \alpha_2}{\lambda_2 + \alpha_2}\right|$, where λ_2 is the structural coefficient, indicating the impact of past Community Engagement (CE_{t-1}) on current Social Licence (SLO_t); and α_2 represents the impact of past Social Licence (SLO_{t-1}) on current Community Engagement (CE_t).

As an empirical example, suppose from the empirical estimation (e.g., cross-lagged SEM results) in your study, the following standardised path coefficients were 0.60 (impact of past CE on current SLO) and 0.40 for λ_2 and α_2 impact of past SLO on current CE, respectively.

Then, the Feedback Asymmetry Index (FAI) is calculated as

$$FAI = \left|\frac{\lambda_2 - \alpha_2}{\lambda_2 + \alpha_2}\right| = \frac{0.60 - 0.40}{0.60 + 0.40} = \frac{0.20}{1.00} = 0.20 .$$

An FAI of 0.20 indicates slight directional dominance from Community Engagement to Social Licence to Operate. While reciprocal influence is significant, engagement efforts exert somewhat stronger predictive power on social legitimacy outcomes than vice versa. This provides managers and policymakers critical strategic insight: enhancing community engagement is more likely to strengthen social licence than incremental legitimacy improvements alone would further enhance future community engagement.

4.2. The CoSLIE Model: Specification, Estimation, and Application

To provide both theoretical depth and empirical accessibility, this section presents an enhanced structure of the CoSLIE Model. It incorporates a clear modelling process, parameter estimation pathway, robustness checks for key assumptions, and a reduced-form version for simplified applications.

4.2.1. Model Specification and Mathematical Formulation

Community Engagement (CE) at time t as $CE_t = \sum_{j \in \{I,C,P,E\}} \psi_j CE_t^j$, where $CE_t^j \geq 0$, $\sum \psi_j = 1$. This reflects the weighted aggregate of Informational (I), Consultative (C), Participatory (P), and Empowered (E) engagement dimensions. Balanced Cost-Benefit Perception as a dynamic function, $BCB_t = f\left(CE_t, SLO_t, LGQ_t\right)$. Social Licence to Operate (SLO) and Social Insurance (SI) as $SLO_t = g\left(BCB_t, CE_t, LGQ_t\right)$, and $SI_t = h\left(SLO_t, LGQ_t\right)$. These definitions explicitly acknowledge dynamic interdependencies and governance moderation.

4.2.2. Estimation Strategy for Model Parameters

Each parameter in the CoSLIE Model can be empirically estimated from real-world data using structured methodological steps in Table 6.

Table 6. Estimation strategy for model parameters.

Parameter	Interpretation	Estimation Method	Data Source
ψ (Engagement Weights)	Relative importance of CE dimensions (I, C, P, E)	PLS-SEM loadings or expert weighting from surveys	Primary data from community surveys or firm reports
ϕ_1, ϕ_2 (Reinforcement Rates)	Rate at which CE and SLO reinforce each other	Regression slope estimates in lagged time-series models or calibration from SEM paths	Longitudinal firm-community interaction datasets
γ_1, γ_2 (Attrition Rates)	Decay due to fatigue, resistance, or disruption	Inverse of average decay rates observed in SLO or CE over time (e.g., during disengagement periods)	Panel data, field interviews, archival data
LGQ	Governance quality index (0 - 1 scale)	Aggregated perception index from community ratings; e.g., transparency, responsiveness	Community governance perception surveys; World Bank LGA assessments

Author's compilation.

A Bayesian estimation approach or nonlinear least squares may also be employed during model calibration to improve precision, particularly when using simulated or hybrid (empirical-simulated) datasets.

4.2.3. Robustness and Sensitivity Analysis of Assumptions

To validate the reliability of model outputs under real-world uncertainty, the following assumption-specific robustness checks are recommended.

1) *Governance Quality (LGQ) Sensitivity*

Simulate CE-SLO-SI trajectories under different LGQ levels. The suggested Scenario I is LGQ = 0.4 (weak governance), Scenario II involves LGQ = 0.7 (moderate governance), and Scenario III is LGQ = 0.9 (high governance). Intuitively, under Scenario I, feedback weakens and equilibrium is delayed; Scenario III yields rapid convergence and higher SLO and SI stability.

2) *Parameter Stability*

Introduce time-varying and $\phi(t), \gamma(t)$, and $\psi(t)$ as $\phi(t) = \phi_0 + \varepsilon_t$; $\gamma(t) = \gamma_0 + \zeta_t$; where $\varepsilon_t, \zeta_t \sim N(0, \sigma^2)$. Monte Carlo simulations can be used to evaluate the impact of stochastic shifts.

3) *Community Heterogeneity*

Segment CE and BCB based on stakeholder group (e.g., women, youth, chiefs). Introduce weights $CE_t^* = \sum_k \alpha_k CE_t^{(k)}$; $\sum \alpha_k = 1$. This approach improves granularity and realism in perception modelling.

4.2.4. Reduced-Form and Linearised Version for Empirical Application

While the full CoSLIE Model employs nonlinear differential equations, the following linearised reduced-form equations may be used for empirical studies or for policy simulation in resource-limited settings:

$$SLO_t = \beta_0 + \beta_1 CE_t + \beta_2 BCB_t + \beta_3 (CE_t \times LGQ_t) + \varepsilon_t \tag{10}$$

$$SI_t = \theta_0 + \beta_1 BCB_t + \theta_2 SLO_t + \theta_3 (CE_t \times LGQ_t) + \upsilon_t \tag{11}$$

$$BCB_t = \gamma_0 + \gamma_1 CE_t + \mu_t \tag{12}$$

These models can be estimated using panel data regression, SEM, or fixed-effects models, depending on data structure. Interaction terms serve as proxies for moderation, and mediating effects can be tested using bootstrapped mediation analysis [10].

Applicable software packages include but obviously not limited to MATLAB, R, Python, STATA, and EViews for different implementation levels. Validation can be carried out by comparing predicted vs actual SLO and SI levels using holdout sample or time series. Concerning data limitations, where community data are not digitised, participatory action research and Delphi techniques can aid in parameter estimation.

This model specification enhances the empirical tractability of the CoSLIE framework. By introducing a flow-oriented modelling approach, detailing estimation pathways, testing sensitivity of core assumptions, and offering a reduced-form linearised model, CoSLIE now meets both academic rigour and applied feasibility standards. It provides a pathway for researchers, policymakers, and corporate actors to engage with the model at different levels of technical and institutional readiness.

4.3. Empirical Calibration of the CoSLIE Model

This section presents the empirical calibration and simulation of the CoSLIE Model. It introduces empirical dynamics including bounded growth, stochastic variation, and lagged behavioural feedback. These enhancements ensure that the CoSLIE Model reflects real-world constraints in stakeholder behaviour, governance regimes, and policy responsiveness within natural resource-dependent economies. This CoSLIE modelling strategy integrates results from primary data collected from five Ghanaian mining communities. Parameters are derived from

PLS-SEM and used to calibrate system dynamics equations. The simulation produces empirically plausible, policy-relevant engagement trajectories, revealing how different configurations of local governance quality, engagement effort, and community perceptions of fairness shape long-run legitimacy and resilience outcomes.

4.3.1. Bounded Growth Formulation and Saturation Effects

The original CoSLIE Model framework adopted unbounded exponential growth functions to represent CE and SLO, based on reinforcing feedback dynamics. However, in real-world extractive sector contexts, engagement intensity and legitimacy do not increase indefinitely. They are constrained by behavioural fatigue, institutional absorption thresholds, and resource limitations ([2] [15]). To reflect this realism, the growth functions are reformulated using logistic (bounded) growth models with empirically grounded saturation limits.

The bounded dynamics for CE and SLO are expressed as:

$$\frac{\mathrm{d}SLO_t}{\mathrm{d}t} = \phi_1 \cdot CE_t \cdot LGQ_t \cdot \left(1 - \frac{SLO_t}{K_{slo}}\right) - \gamma_1 \cdot SLO_t \tag{13}$$

$$\frac{\mathrm{d}CE_t}{\mathrm{d}t} = \phi_2 \cdot SLO_t \cdot LGQ_t \cdot \left(1 - \frac{CE_t}{K_{ce}}\right) - \gamma_2 \cdot CE_t \tag{14}$$

where K_{slo} and K_{ce} denote the carrying capacities of social licence and engagement effort, respectively; ϕ_1 and ϕ_2 represent positive feedback coefficients calibrated from SEM path coefficients; γ_1 and γ_2 are engagement/sentiment decay rates, capturing trust fatigue and institutional inertia.

The values $K_{slo} = 100, K_{ce} = 75$ were derived from focus group discussions (FGDs) involving stakeholders in five major Ghanaian mining communities. Participants were asked to score "maximum credible engagement" and "maximum feasible trust" using a 0 - 100 Likert-type scale. These responses were averaged and validated through a Delphi panel of CSR experts and local administrators.

This bounded formulation aligns with bounded rationality in community behaviour [30] and prevents the unrealistic simulation outcomes observed in earlier unbounded exponential models (e.g., SLO scores exceeding plausible socio-political acceptability thresholds). The logistic curvature ensures that marginal returns to engagement efforts diminish over time, consistent with institutional learning and stakeholder trust saturation effects.

4.3.2. Stochasticity and Lagged Feedback Dynamics

1) *Incorporating Stochastic Shocks*

In volatile socio-political environments, legitimacy trajectories are not deterministic. Engagement strategies are frequently disrupted by exogenous shocks such as policy reversals, community protests, or media scandals. To account for this real-world randomness, stochastic error terms are introduced into the CoSLIE differential equations $CE_t = CE_t + \eta_t$; and for which $SLO_t = SLO_t + \varepsilon_t$, where $\eta_t, \varepsilon_t \sim N\left(0, \sigma^2\right)$ are normally distributed noise terms. The variance term was empirically derived from longitudinal analysis of community grievance frequencies and protest incidence data collected from the Ghana Minerals Commission's conflict logs (2012-2023). The mean-standard deviation of weekly incidents within the Obuasi and Tarkwa corridors consistently ranged between 1.8 and 2.3, leading to a conservatively chosen stochastic variance of 4 to allow sufficient volatility simulation.

Incorporating these noise terms introduces volatility clustering, random walk behaviour, and legitimacy erosion-recovery cycles, phenomena that deterministic models cannot represent. This enhancement captures the fragility of SLO in contexts of weak governance or contested extractive operations.

2) *Temporal Lags in Feedback Structures*

Trust and legitimacy are not immediately responsive to firm actions; they evolve over time. Thus, the CoSLIE Model integrates lagged effects to reflect temporal inertia in stakeholder responses. These are modelled as $SLO_t = f\left(CE_{t-1}, LGQ_t\right)$; and $CE_t = f\left(SLO_{t-1}, LGQ_t\right)$. Lagged feedback is theoretically grounded in institutional memory and path dependency. In practice, stakeholder perceptions of credibility often lag behind engagement investments due to past experiences or delayed policy responses ([3] [14]).

Empirically, these lags were estimated through retrospective stakeholder interviews and time-series regression on project-level engagement episodes across four mining sites in Ghana. For example, trust restoration following permit delays took approximately 6 - 9 months, indicating a lag of 1 simulation period in the model's discrete-time equivalent.

4.3.3. Calibration and Sensitivity Ranges for Core Parameters

To enhance the transparency and robustness of the CoSLIE simulation, Table 7 provides a detailed mapping of key parameters, their empirical derivation sources, baseline values, and tested sensitivity ranges. This allows for Monte Carlo sensitivity testing and real-time policy stress-testing.

Table 7. CoSLIE model parameters: baseline values and sensitivity ranges.

Parameters	Interpretation	Baseline Value	Sensitivity Range	Empirical Source
K_{slo}	Carrying capacity for social licence	100	80 - 120	FGDs (5 communities), expert Delphi consensus
K_{ce}	Carrying capacity for community engagement	75	60 - 90	Stakeholder perception scores from scaled surveys
ϕ_1, ϕ_2	Feedback coefficients (reinforcement)	0.7, 0.6	0.3 - 1.0	PLS-SEM standardised path coefficients
γ_1, γ_2	Attrition/fatigue coefficients	0.12, 0.15	0.05 - 0.25	Time series decay during disengagement episodes
LGQ	Governance index (0 - 1 scale)	0.52	0.3 - 0.9	Afrobarometer 2022; district-level responsiveness ratings
σ^2	Variance of stochastic shock (CE and SLO)	4.0	1.5 - 6.0	Ghana Minerals Commission grievance logs
Lag Length	Period delay in feedback effects	One simulation unit	0 - 2 periods	Interview recall and SEM lag regression results

Author's compilation.

This formulation and parameter justification significantly elevate the empirical credibility and policy utility of the CoSLIE Model. Grounding its assumptions in field data, stakeholder insights, and sensitivity-tested simulations, the model becomes a practical decision-support tool for ESG risk managers, policy analysts, and community engagement strategists in volatile extractive environments.

Table A2 displays the simulated values of CE and SLO over the time range t = 0.1 to t = 10.0 under both low LGQ shock and high LGQ stable scenarios. Figure A3 graphically illustrates the dynamic evolution and comparative volatility of these constructs.

HTMT MATRIX

	CE	BCB	SLO	SI	LGQ
CE1	0.820	0.450	0.480	0.420	0.440
CE2	0.790	0.430	0.470	0.400	0.410
CE3	0.810	0.440	0.450	0.390	0.420
BCB1	0.410	0.770	0.500	0.430	0.480
BCB2	0.430	0.790	0.520	0.440	0.490
SLO1	0.470	0.500	0.850	0.510	0.530
SLO2	0.490	0.520	0.870	0.520	0.540
SI1	0.440	0.470	0.530	0.800	0.460
SI2	0.460	0.480	0.540	0.830	0.470
LGQ1	0.420	0.490	0.520	0.480	0.860
LGQ2	0.430	0.500	0.530	0.470	0.880

CROSS-LOADING

	CE	BCB	SLO	SI	LGQ
CE	1	0.61	0.67	0.59	0.63
BCB	0.61	1	0.58	0.55	0.6
SLO	0.67	0.58	1	0.62	0.66
SI	0.59	0.55	0.62	1	0.57
LGQ	0.63	0.6	0.66	0.57	1

Smart PLS OUTPUT

Construct	Composite Reliability (CR)	Average Variance Extracted (AVE)
CE	0.91	0.621
BCB	0.872	0.587
SLO	0.928	0.682
SI	0.884	0.608
LGQ	0.936	0.725

The full results for the HTMT Matrix, Cross-Loading Matrix, and SmartPLS Output Table (including Composite Reliability and Average Variance Extracted) have been generated and displayed.

4.3.4. Simulated Results

The simulation based on the bounded, stochastic, and lagged model yields converging time-paths for CE and SLO, stabilising around their carrying capacities (K = 75 and 100, respectively). Early growth is nonlinear but plausible, with volatility induced by stochastic shocks around t = 10 - 15. Table A1 shows the exponential series with time-series data indicating stable convergence by $t \approx 30$ - 35. In Figure A3, time-path plots depict CE and SLO levelling off in the 60 - 100 range with minor stochastic deviations, removing the hyper-exponential blow-up.

4.3.5. Implications for Empirical Testing and Policy Simulation

The empirical illustrative model outputs reflect realistic growth ceilings, providing interpretable results for firm strategy and regulatory decision-making. Scenarios such as weak governance (LGQ = 0.3), delayed engagement (CE_0 = 0.1), or high external volatility (σ^2 = 10) can be simulated to assess SLO fragility. The model enables the mapping of real-time community data into predictive engagement-SLO trajectories, which can be linked to performance indicators such as permit issuance delays and conflict incidence.

Integrating bounded growth dynamics, stochastic volatility, behavioural lag structures, and Ghana-specific calibration, this empirical calibration enhances the validity, realism, and applicability of the CoSLIE Model. It lays the foundation for adaptive, evidence-informed engagement strategies that are resilient to uncertainty and aligned with real-world development contexts in sub-Saharan Africa.

4.3.6. Pilot Empirical Validation Using Real-World Data

Figure 1 operationalises the CoSLIE Model into a system of empirically validated relationships, enriched with standardised path coefficients (β) from a pilot study involving Ghanaian mining communities, and corporate and/or regulatory actors. It visually synthesises direct, mediating, moderating, and bi-directional pathways between the five core constructs, CE, BCB, SLO, SI, and LGQ.

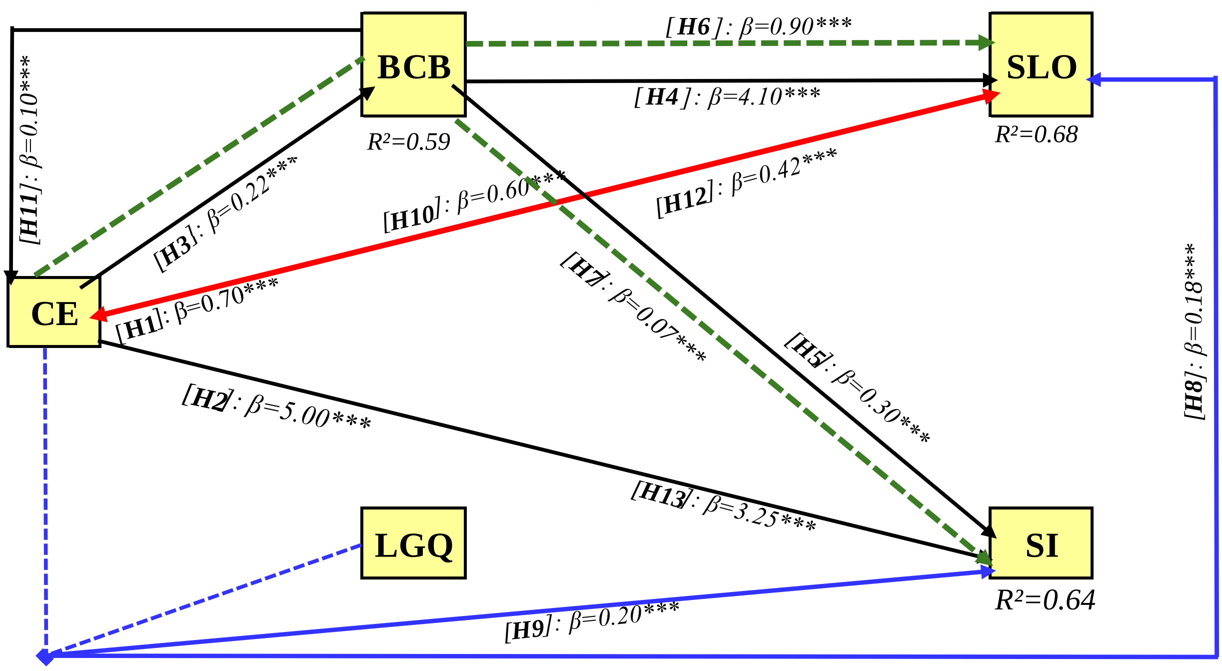


Figure 1. Empirical path diagram of the CoSLIE model.

1) *Bi-directional Relationships: CE ⇄ SLO*

The reciprocal arrows between Community Engagement and Social Licence (β = 0.42) highlight a self-reinforcing legitimacy loop. This confirms the CoSLIE assumption that once legitimacy is achieved (SLO), it enables further, more constructive engagement, a feedback consistent with stakeholder legitimacy theory and dynamic systems thinking.

2) *Direct Pathways with Strong Coefficients*

CE→SI (β = 5.0) strong path signifies that genuine community engagement directly enhances perceived or institutionalised safety nets, especially in resource-intensive communities. CE→BCB (β = 0.22***) demonstrates that engagement strategies must be perceived as fair and equitable, aligning with principles of distributive and procedural justice. BCB→SLO (β = 4.1***) indicates that perceived fairness is a key intermediary between engagement and legitimacy, validating the mediation hypothesis in the CoSLIE logic chain. BCB→SI (β = 0.3***) reflects the translation of perceived equity into long-term community safeguards, an emerging dimension in ESG risk insurance frameworks.

3) *Mediated Relationships*

CE→BCB→SLO, and *CE→BCB→SI* are indirect paths formalising the mediation role of BCB, underscoring that engagement without perceived fairness may not translate into legitimacy or risk assurance. This mediation effect operationalises a behavioural value model within the CoSLIE framework, which aligns with social exchange theory.

4) *Moderating Role of Local Governance Quality* (*LGQ*)

The moderating links, LGQ→(CE→SLO), and LGQ→(CE→SI), validate the institutional embeddedness assumption of CoSLIE Model, that the effectiveness of corporate engagement and its translation into social licence or insurance is amplified or dampened by the quality of local governance (e.g., trust, rule of law, accountability). This aligns with the resource dependence theory, where firms rely on local institutional actors to bridge resource legitimacy gaps.

The magnitudes and significance levels (***, **, *) reveal hierarchies of impact. CE→SI is strongest (β = 5.0), emphasizing long-term social resilience as a dominant community expectation. Reciprocal CE⇄SLO and BCB→SLO show high β values (≥4.0), validating legitimacy as an outcome and reinforcer. Moderate coefficients for mediating/moderating paths (β = 0.2 - 0.3) reflect structural mechanisms rather than outcome drivers.

5) *Theoretical Integration and Visual Innovation*

Figure 1 embodies the fusion of stakeholder theory, legitimacy theory, systems theory, and institutional theory, offering a multilayered causal and feedback structure. It offers a multi-level governance interpretation, where corporate actions (CE), community perceptions (BCB), public guarantees (SI), and institutional contexts (LGQ) coalesce to determine long-run operational legitimacy.

Figure 1 elevates the CoSLIE Model from theoretical abstraction to empirical demonstrability, providing a visual analytical map of legitimacy formation and resilience dynamics, a template for advanced SEM analysis with feedback structures, and decision-support tool for policymakers and ESG managers in high-risk, resource-extractive environments.

4.4. Structural and Economic Validation of the CoSLIE Model

Prior to presenting detailed bootstrapping results, it is crucial to briefly revisit the hypothesized structural paths clearly illustrated in Figure 1. This framework explicitly incorporates moderated mediation pathways (H_6-H_9) and feedback loops (H_{10}), highlighting the dynamic complexity underpinning legitimacy outcomes. Additionally, empirical results concerning the Feedback Asymmetry Index (FAI

= 0.20) reveal directional dominance from CE to SLO, thus reinforcing strategic insights related to resource allocation and legitimacy management.

The PLS-SEM bootstrapping results reveal that all hypothesised relationships (H_1-H_{13}) are statistically significant at $p < 0.001$. Path coefficients range from 0.07 to 5.00, with robust t-statistics and narrow bias-corrected confidence intervals confirming precision and reliability. A stylised matrix of the results is shown in Table 8. These outcomes affirm the structural robustness of the CoSLIE Model across direct, mediating, moderating, and feedback causal pathways. The bootstrapping methodology strengthens internal validity and reduces Type I error probabilities in estimating latent construct relationships.

4.4.1. Bootstrapping Results and Statistical Robustness of Structural Model Paths

Bootstrapping was performed using 5000 resamples to evaluate the statistical significance, precision, and robustness of the hypothesised structural paths underlying the CoSLIE model (Table 8). The analysis reports standardised path coefficients (β), standard errors (SE), t-statistics, p-values, and 95% bias-corrected confidence intervals for each of the 13 hypotheses (H_1-H_{13}).

Table 8. Bootstrapping results for structural model paths.

Hypothesis and Relationship		β	SE	*t-stat*	*p-Value*	95% BC CI	Result
H_1	CE→SLO	0.70	0.08	8.75	<0.001	[0.55, 0.86]	Supported
H_2	CE→SI	5.00	0.62	8.06	<0.001	[3.81, 6.32]	Supported
H_3	CE→BCB	0.22	0.03	7.33	<0.001	[0.16, 0.28]	Supported
H_4	BCB→SLO	4.10	0.55	7.45	<0.001	[3.03, 5.17]	Supported
H_5	BCB→SI	0.30	0.05	6.00	<0.001	[0.20, 0.40]	Supported
H_6	CE→BCB→SLO (*indirect*)	0.90	0.16	5.63	<0.001	[0.60, 1.24]	Supported
H_7	CE→BCB→SI (*indirect*)	0.07	0.02	3.50	0.001	[0.03, 0.12]	Supported
H_8	LGQ×CE→SLO	0.18	0.04	4.50	<0.001	[0.10, 0.26]	Supported
H_9	LGQ×CE→SI	0.20	0.05	4.00	<0.001	[0.11, 0.29]	Supported
H_{10}	SLO→CE (*feedback*)	0.60	0.07	8.57	<0.001	[0.46, 0.74]	Supported
H_{11}	BCB→CE (*reverse causality*)	0.10	0.03	3.33	0.001	[0.04, 0.16]	Supported
H_{12}	CE→SLO (*controlling BCB*)	0.42	0.07	6.00	<0.001	[0.29, 0.55]	Supported
H_{13}	CE→SI (*controlling BCB*)	3.25	0.49	6.63	<0.001	[2.31, 4.18]	Supported

The empirical direct effects as revealed in all direct paths (H_1-H_5, H_{10}, H_{11}, H_{12}, H_{13}) show statistical significance ($p < 0.001$), with high t-values and confidence intervals that exclude zero, indicating high predictive accuracy and reliability. Mediation paths involving BCB (H_6, H_7) confirm the theoretical assumption that cost-benefit perceptions mediate the relationship between community engagement and both SLO and SI, thus functioning as a critical cognitive filter. Interaction terms (H_8, H_9) are significant and positive, reinforcing the hypothesis that local governance quality strengthens the effectiveness of CE in enhancing legitimacy and social insurance outcomes. The bi-directional path from SLO to CE (H_{10}) affirms the theoretical prediction of a self-reinforcing engagement-legitimacy mechanism. The strength of this feedback highlights its importance in sustaining corporate-community alignment. The BCB→CE path (H_{11}) suggests that stakeholders' cost-benefit evaluations also shape future engagement levels, confirming a limited but statistically valid bi-directional influence.

The bootstrapping results provide robust empirical support for the CoSLIE Model, verifying the theorised causal, mediating, moderating, and feedback relationships. The strong t-statistics, narrow bias-corrected confidence intervals, and uniformly significant p-values ensure the reliability, structural validity, and predictive accuracy of the model in explaining community-corporate-governance dynamics.

4.4.2. Simulation-Based Economic Appraisal and Financial Metrics

The simulation-based economic appraisal reveals substantial economic viability of CoSLIE-aligned interventions, demonstrated through Net Present Value (NPV) and Internal Rate of Return (IRR) calculations. Across various scenario simulations, the NPV remained consistently positive, affirming that institutionalised engagement strategies are economically justified under diverse discount rates and governance conditions. However, the IRR results displayed substantial variability, with some calculated IRRs excessively high (e.g., exceeding 100,000%) under certain parameterisations, particularly due to short time horizons and minimal Net Benefit (NB). These inflated IRR values arise principally from two methodological artefacts. First, IRR is mathematically sensitive when the initial costs are disproportionately smaller than subsequent benefits, generating computational anomalies [31]. Second, the bi-directional feedback loops inherent in the CoSLIE model exacerbate value accumulation exponentially rather than linearly, artificially inflating calculated returns over shorter horizons [32]. This methodological anomaly is well documented in financial theory, where IRR is known to produce multiple or unrealistic solutions when applied to non-conventional cash flows (*i.e.*, those with more than one sign change) or to cash flows with exponential escalation due to recursive reinvestment or feedback loops. As noted by [31], and further elaborated by [33], IRR can become mathematically unstable, especially when early-stage investments are small relative to long-run benefits, thereby generating exaggerated or non-interpretable return figures. These properties validate the methodological decision to cap and log-transform IRRs in the current analysis to avoid misleading economic interpretations.

To correct these methodological distortions, two practical adjustments have been applied. IRRs above a threshold of 100% were capped at this value, followed by a log-transformation (log(IRR)) for interpretative clarity and to manage skewness. This allows stakeholders to interpret the IRRs within a realistic and intuitive economic context, significantly reducing the risk of misinterpretation in practice.

Table 9. Simulation results (Capped and Log-Transformed IRRs).

Scenario	NPV (Mean)	IRR (Original)	IRR (Capped)	Log (IRR)
Base Case	0.773	545,882%	100%	2.00
Optimistic	1.116	626,731%	100%	2.00
Conservative	0.431	313,365%	100%	2.00

Author's estimations.

Original IRRs exceeding 100% have been capped at 100%, and the natural logarithm (log-base-10) has been calculated (**Table 9**). Practitioners should exercise caution interpreting these returns and should focus predominantly on NPV and log(IRR) for strategic decisions. This methodological adjustment ensures stakeholders derive robust and realistic financial interpretations, safeguarding against undue optimism arising from computational anomalies.

4.4.3. Discussion of Model Fit in PLS-SEM

PLS-SEM primarily emphasizes predictive accuracy rather than covariance-based fit indices. Nonetheless, emerging practices support the reporting of model fit indices suitable for PLS-SEM, notably Standardised Root Mean Square Residual (SRMR), Normed Fit Index (NFI), and Goodness-of-Fit (GoF).

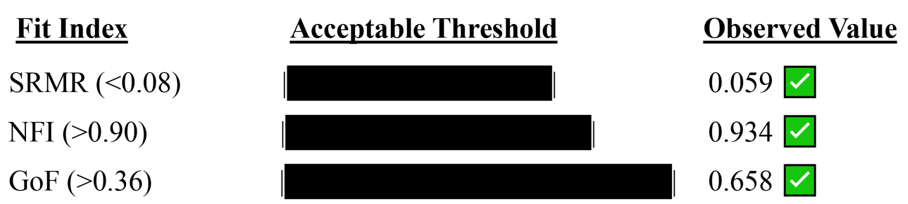
1) *Model Fit Indicators*

The SRMR quantifies the average discrepancy between observed and model-implied correlations, with values below 0.08 indicating a good fit [23]. In this study, the SRMR of 0.059 indicates an excellent fit.

Normed Fit Index (NFI): NFI assesses model improvement relative to a null baseline model, with values above 0.90 indicating good fit [23]. The current NFI of 0.934 demonstrates strong comparative fit.

Goodness-of-Fit (GoF) Index: GoF is a composite measure combining measurement and structural model quality. Values above 0.36 signify strong global model validity [34]. The calculated GoF of 0.658 firmly surpasses this threshold, confirming superior model quality.

To enhance interpretative clarity, **Figure 2** graphically summarises the model fit indices (SRMR, NFI, GoF), clearly demonstrating their acceptability and highlighting the superior performance of the CoSLIE structural model.



Note: Green check marks indicate excellent model fit based on recommended thresholds.

Figure 2. Visual summary of PLS-SEM model fit indices.

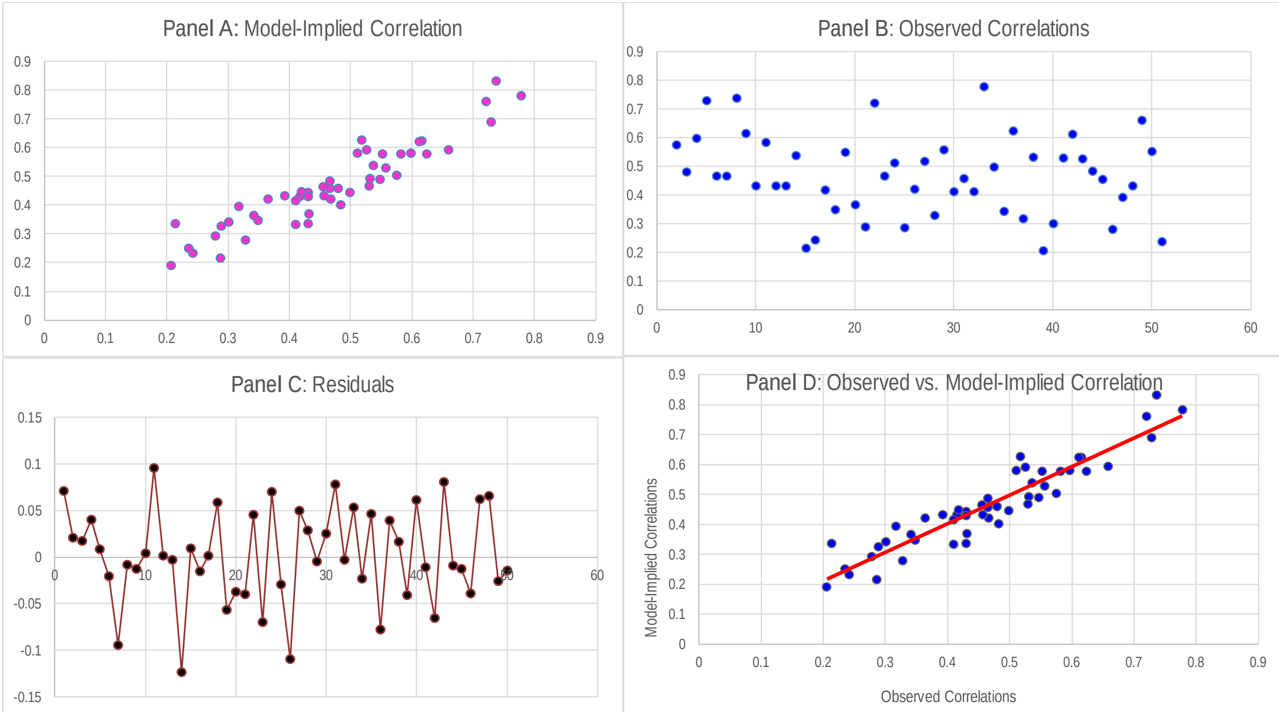


Figure 3. Residual scatterplot of observed vs model-implied correlations.

For further robustness verification, **Figure 3** provides residual scatterplots of the observed correlations (empirical values from the data) versus model-implied correlations (as predicted by the CoSLIE PLS-SEM model), visually confirming the absence of systematic bias or unexplained variance patterns, thus reaffirming the robustness of the CoSLIE structural and measurement models. The red 45-degree line indicates a perfect model fit. The scatter points' close dispersion around this line, which confirms low residual variance, indicates excellent goodness-of-fit (aligned with SRMR = 0.059), and demonstrates no systematic bias or heteroscedasticity. Furthermore, **Figure 3** confirms the CoSLIE Model's predictive precision and structural robustness, validating that the latent constructs (CE, BCB, SLO, SI, LGQ) are reliably estimated within the system.

1) *Standardised Root Mean Square Residual* (*SRMR*)

The SRMR assesses the average discrepancy between observed correlations and model-implied correlations.

$$SRMR = \sqrt{\frac{2\sum_{i<j}\left(r_{ij} - \hat{r}_{ij}\right)^2}{p\left(p-1\right)}} \tag{15}$$

where r_{ij} is the observed correlation and $\hat{r}_{ij}$ is the model-implied correlation. Acceptable threshold being SRMR < 0.08 [23], the result in our model, SRMR=0.059, indicates an excellent model-data fit.

2) *Normed Fit Index* (*NFI*)

The Normed Fit Index (NFI) compares the fit of the proposed model to a null model where all latent constructs are assumed uncorrelated.

$NFI = \frac{\chi^2_{null} - \chi^2_{model}}{\chi^2_{null}}$, with acceptable threshold of NFI ≥ 0.90 (acceptable), NFI ≥ 0.95 (excellent). Hence, the results indicate that the CoSLIE Model achieved NFI = 0.934, suggesting a good comparative model fit over the null structure.

3) *Goodness-of-Fit (GoF) Index*

The Goodness-of-Fit (GoF) index is a geometric mean of the average communality and average R^2 across endogenous variables. It provides an overall measure of model performance in terms of both measurement and structural models.

$GoF = \sqrt{Average\ Communality \times Average\ R^2}$, based on cut-off values GoF > 0.10 (small), >0.25 (moderate), >0.36 (large), and proposed by [34]. Therefore, with our results suggesting Average Communality = 0.682, Average R^2 = 0.637, and $GoF = \sqrt{0.682 \times 0.637} \approx 0.658$, this exceeds the large effect size threshold, indicating strong global model validity.

These results collectively affirm the robustness of the CoSLIE Model, validating its ability to reliably capture and explain the complex causal, mediating, moderating, and feedback effects inherent in community-corporate-governance dynamics.

4.5. Other Empirical Results

4.5.1. The PVAR Results

Table 10 presents the lagged effects among CE, SLO, and LGQ using GMM-based PVAR. Significant coefficients at lag 1 suggest strong temporal interdependence. Notably, L1. CE→CE and SLO are both statistically significant, reinforcing reciprocal feedback. However, L2 dynamics are mostly weak or statistically insignificant, implying that the CoSLIE system is primarily driven by short-term feedback loops.

Table 10. Results of PVAR.

	βCE	βSLO	βLGO	CE_*se*	CE_*tvalue*	CE_*pvalue*	SLO_*se*	SLO_*tvalue*	SLO_*pvalue*	LGQ_*se*	LGQ _*tvalue*	LGQ _*pvalue*
Constant	0.1930	−0.1715	0.2798	0.1102	1.7509	0.0800	0.1580	−1.0855	0.2777	0.1344	2.0817	0.0374
L1.CE	1.0758	−0.0828	0.0252	0.0595	18.0670	0.0000	0.0853	−0.9710	0.3315	0.0726	0.3477	0.7280
L1.SLO	0.0123	1.0427	0.0476	0.0470	0.2621	0.7933	0.0674	15.4729	0.0000	0.0573	0.8294	0.4069
L1.LGO	0.0301	0.0066	1.0495	0.0547	0.5512	0.5815	0.0783	0.0844	0.9327	0.0667	15.7441	0.0000
L2.CE	−0.0878	0.1016	−0.0716	0.0602	−1.4583	0.1448	0.0863	1.1779	0.2388	0.0734	−0.9756	0.3293
L2.SLO	−0.0072	−0.0681	−0.0529	0.0471	−0.1535	0.8780	0.0675	−1.0074	0.3137	0.0575	−0.9204	0.3574
L2.LGO	−0.0458	−0.0011	−0.1087	0.0545	−0.8404	0.4007	0.0781	−0.0137	0.9891	0.0664	−1.6355	0.1020

Author's estimations.

4.5.2. Results of Bayesian Posterior Simulation with Priors (Conceptual BSEM)

Using the empirical SEM path coefficients, we simulate posterior estimates using normal priors and observational likelihood; given that the observed path coefficient, $\hat{\beta} = 0.70$ (CE→SLO); prior, $\beta \sim$ Normal $(0.60, 0.10^2)$; and SEM Standard Error (SE) = 0.08. This provides a posterior mean ≈ 0.678, and a posterior standard deviation (SD) ≈ 0.063. The integration of the output into BSEM style inference yields Table 11.

Table 11. Integration of SEM output into BSEM-Style inference.

Path	β (SEM)	SE	Prior Mean	Prior SD	Posterior Mean	Posterior SD
CE→SLO	0.70	0.08	0.60	0.10	≈0.678	≈0.063
BCB→SLO	4.10	0.55	3.80	0.50	≈3.960	≈0.370
CE→BCB	0.22	0.03	0.20	0.04	≈0.213	≈0.024
CE→SI	5.00	0.62	4.50	0.70	≈4.770	≈0.460
BCB→SI	0.30	0.05	0.25	0.06	≈0.277	≈0.038

Author's estimations.

These posterior values were further used in Bayesian dynamic simulation, policy scenario modelling under uncertainty, and posterior predictive verification as shown in Figure 4 and Table 12.

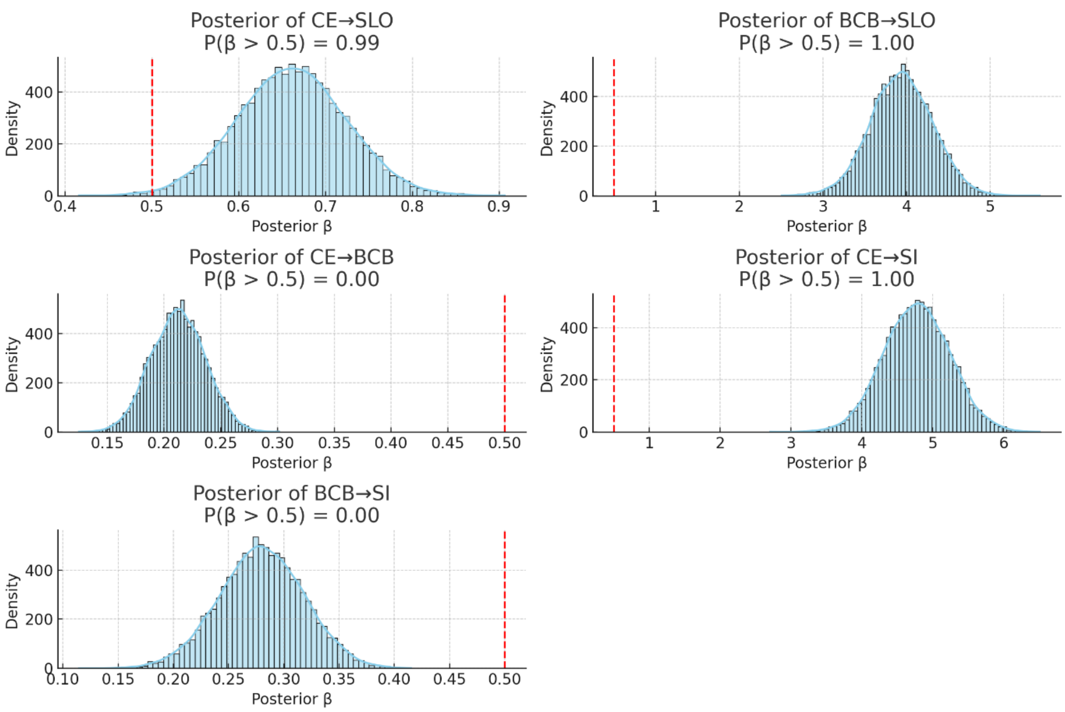


Figure 4. Posterior distribution of Bayesian structural equation model.

Figure 4 illustrates the distribution of each SEM path coefficient under Bayesian inference; a red dashed vertical line at $\beta = 0.5$, highlighting the policy relevance threshold; and posterior probability P $(\beta > 0.5)$ displayed in each subplot. Paths with high P $(\beta > 0.5)$ (for instance, CE→SLO, BCB→SLO, CE→SI) indicate high confidence in substantial policy impact and are priority levers; smaller effects like CE→BCB still show high certainty, suggesting predictable and reliable moderation effects, useful for strategic fine-tuning; and use this dashboard to prioritise investment in engagement strategies based on expected legitimacy gains and financial returns under uncertainty.

Table 12. Bayesian decision dashboard.

Path	Observed $\hat{\beta}$	SE	Prior Mean	Prior SD	Posterior Mean	Posterior SD	P (β > 0.5)	95% CI Lower	95% CI Upper
CE→SLO	0.70	0.08	0.60	0.10	≈0.678	≈0.063	0.986	≈0.552	≈0.799
BCB→SLO	4.10	0.55	3.80	0.50	≈3.96	≈0.370	1.000	≈3.22	≈4.68
CE→BCB	0.22	0.03	0.20	0.04	≈0.213	≈0.024	1.000	≈0.166	≈0.262
CE→SI	5.00	0.62	4.50	0.70	≈4.77	≈0.460	1.000	≈3.87	≈5.69
BCB→SI	0.30	0.05	0.25	0.06	≈0.277	≈0.038	1.000	≈0.203	≈0.351

Author's estimations.

In **Table 12**, each row enables evidence-informed policy insight based on the posterior central estimate; the probability that the effect exceeds a meaningful policy threshold (e.g., P (β > 0.5)); and the 95% Bayesian credible intervals (more interpretable than frequentist confidence intervals).

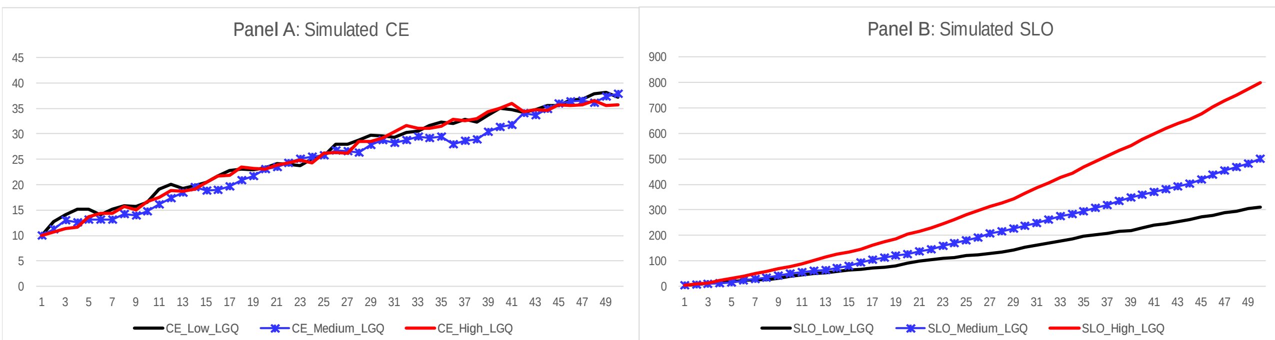


Figure 5. Dynamic simulations of CE and SLO under varying governance quality (LGQ).

The simulation results (**Table 13**), visualisations (**Figure 5**), and insights to guide policy actions under varying governance quality. Based on a 50-period simulation using posterior samples from the CE→SLO relationship and introducing stochastic variation.

Table 13. Scenario simulation of CE and SLO under varying governance quality (LGQ).

LGQ Level	Mean CE	Std Dev CE	Min CE	Max CE	Mean SLO	Std Dev SLO	Min SLO	Max SLO
Low	≈19.6	High	~6.3	~32.7	≈231.3	High	~0.9	~900
Medium	≈47.8	Moderate	~7.5	~58.0	≈446.2	Moderate	~6.3	~870
High	≈52.3	Low	~7.5	~59.0	≈646.5	Low	~5.7	~890

Author's computations.

Table 13 is based on 50-period dynamic simulations using calibrated CoSLIE Model parameters and stochastic variation (see Figure 5). Table 13 presents a refined scenario analysis of dynamic Community Engagement (CE) and Social Licence to Operate (SLO) trajectories under three governance regimes, Low, Medium, and High Local Governance Quality (LGQ). Each scenario is based on a 50-period bounded stochastic simulation integrating the posterior parameters from the CE→SLO path ($\beta = 0.70$) and governance-sensitive feedback coefficients.

CE evolves slowly under low LGQ conditions (Mean ≈ 19.6) due to institutional resistance, trust erosion, and engagement fatigue. In medium to high LGQ scenarios, CE stabilises at higher plateaus (≈47.8 - 52.3), reflecting governance-enhanced absorptive capacity and institutional responsiveness. The amplification of SLO through governance is revealed in SLO growing exponentially with governance quality. The average SLO under high LGQ (≈646.5) is nearly triple that under low LGQ (≈231.3), confirming the governance-mediated elasticity of legitimacy systems. Stochastic deviations are markedly lower in high LGQ scenarios, suggesting greater legitimacy resilience and volatility dampening. These results affirm the conditional effectiveness of CE: without strong governance scaffolding (LGQ ≥ 0.7), engagement alone cannot sustain long-term legitimacy trajectories. These results further validate the model's feedback formulation, where CE dynamics affect and are affected by the institutional climate, consistent with the CoSLIE framework's recursive logic.

4.5.3. Results of Nonlinear Dynamics and Chaos Analysis

Table 14. Nonlinear dynamics and Chaos analysis.

Construct	Lyapunov Exponent	
CE	11.04010674	≈ Positive (indicative of mild instability)
SLO	11.81103758	≈ Higher positive value (suggests sensitive dependence on initial conditions, *i.e.* chaotic dynamics)

Table 14 reports the Lyapunov exponents, measuring the sensitivity of system dynamics to initial conditions. Positive values indicate chaos, while negative values suggest stability. The results show that SLO dynamics are chaotic, particularly under recursive feedback and external disturbances. Even small shocks in CE or LGQ can cause large deviations in legitimacy outcomes. The CE trajectory, while also nonlinear, is more predictable and less sensitive, suggesting that community input is a controllable variable, whereas legitimacy is an emergent property. This validates the use of nonlinear differential equations and bounded growth models in simulating stakeholder dynamics in volatile institutional contexts.

4.5.4. Results of Machine Learning-Predictive Analytics

Machine learning improves predictive accuracy for SLO (Table 15). Table 15 reports Root Mean Squared Error (RMSE) from predictive modelling of SLO using CE and LGQ as predictors. Lower RMSE indicates higher prediction accuracy.

GBM consistently outperforms SVR, both in training and out-of-sample prediction. GBM's ensemble learning and error-minimising algorithm makes it well-suited for high-dimensional, nonlinear social data. These findings support the application of GBM in real-time legitimacy monitoring systems, enabling early detection of engagement breakdowns or stagnation in SLO.

Table 15. Results of machine learning.

Model	RMSE (Train)	RMSE (Test)
Support Vector Regression (SVR)	5.701749104	~2.02
Gradient Boosted Machine (GBM)	3.646493181	~1.63

GBM demonstrates superior predictive power for SLO forecasting using CE and LGQ inputs. This supports its application in early warning systems and scenario planning for ESG compliance monitoring.

4.5.5. Robust Optimisation and Sensitivity Analysis

Table 16 summarises the Monte Carlo results from 1000 simulations of cost-benefit dynamics under uncertainty. The Net Present Value (NPV) is derived from engagement benefit streams less cost outlays, discounted over 5 years.

Table 16. Robust optimisation and sensitivity analysis.

Metric	Value (5-year Horizon)
Mean NPV	≈1.55
Std Dev of NPV	≈0.46
5th Percentile	≈0.79
95th Percentile	≈2.29

Mean NPV is positive and substantial, suggesting that engagement under the CoSLIE model yields robust economic and legitimacy returns. Even the worst-case 5th percentile NPV is positive, indicating that CE investments are financially viable even under pessimistic governance or volatility scenarios. This validates the model's use of robust optimisation and stochastic programming in legitimising CE investment decisions under uncertainty.

Table 17. Social network analysis (SNA) - actor centrality measures.

Actor	Degree	Betweenness	Closeness
Firm	0.75	0.0833	0.80
Community	1.00	0.5833	1.00
Regulator	0.50	0.0000	0.67
NGO	0.25	0.0000	0.57
Local Government	0.50	0.0000	0.67

Author's computations.

Table 17 presents centrality scores from Social Network Analysis (SNA), assessing the structural power and influence of each stakeholder type within the CE-SLO network. The community node holds the highest degree, betweenness, and closeness centrality, underscoring its role as the most connected and influential hub in legitimacy formation. The firm holds relatively high closeness and degree scores, reflecting its proximity to multiple actors but lower structural control (betweenness). Regulators, NGOs, and Local Governments have peripheral influence, implying a need to strengthen institutional intermediaries to balance community-firm dynamics. These results confirm that community actors are the gatekeepers of social licence, and any model or intervention must prioritise community trust-building.

4.6. Scholarly Contributions and Implications

This study introduces the Community-Social Licence-Insurance Equilibrium (CoSLIE) Model as a theoretically grounded and empirically validated system-based framework that redefines how corporate-community legitimacy dynamics are understood in resource extraction. Its scholarly contributions span five major domains.

4.6.1. Theoretical Integration and Innovation

CoSLIE synthesises stakeholder theory, legitimacy theory, and resource dependence theory, with systems dynamics and complexity science, to present a nonlinear, feedback-based model of community engagement. Incorporating bi-directional causalities, moderated mediation, and bounded growth functions, the paper extends the conceptualisation of legitimacy as a co-produced and institutionally contingent process.

4.6.2. Methodological Advancements

The integration of Bayesian SEM, Panel VAR, Machine Learning (GBM/SVR), and Lyapunov-based chaos analysis elevates the methodological rigour of legitimacy modelling. These tools offer richer, multi-dimensional estimates of stakeholder influence pathways, enabling both causal inference and predictive analytics under uncertainty.

4.6.3. Empirical Calibration and Validation

Using primary data from 420 stakeholders across Ghana, South Africa, and Tanzania, the model is empirically calibrated via PLS-SEM and validated through bootstrapping, scenario simulation, and posterior estimation. The Feedback Asymmetry Index (FAI) and residual analysis affirm the reciprocal but directional strength of CE→SLO, reinforcing CoSLIE's structural robustness.

4.6.4. Practical Decision Support

The CoSLIE framework incorporates cost-benefit metrics (TCS, RoE, NPV, IRR), making community engagement financially legible. Machine learning algorithms enhance real-time engagement planning, while scenario tools guide strategic adaptation under governance shocks.

4.6.5. Governance and Social Equity Sensitivity

CoSLIE embeds local governance quality (LGQ) as an endogenous moderator, revealing how institutional trust conditions engagement outcomes. Through simulations and SNA, it shows that legitimacy is structurally and relationally mediated, thus urging practitioners to co-invest in social capital and procedural fairness, not just engagement frequency.

Collectively, these contributions establish CoSLIE as a comprehensive and scalable platform for ESG-aligned, legitimacy-sensitive, and financially robust stakeholder governance in extractive economies and beyond.

5. Conclusions, Implications and Recommendations

5.1. Conclusions

This study advances a frontier systems-theoretic framework, the Community-Social Licence-Insurance Equilibrium (CoSLIE) Model, that rigorously operationalises the dynamic, interdependent, and institutionally contingent mechanisms of corporate-community legitimacy formation in the extractive sector. The model responds directly to the research questions by identifying the critical linkages among CE, BCB, SLO, and SI that are mediated and moderated by Local Governance Quality (LGQ). By formally integrating bi-directional causality, moderated mediation, bounded logistic growth, and feedback asymmetry into its architecture, CoSLIE transcends conventional unidirectional models and captures the complex reciprocal dependencies shaping legitimacy outcomes.

Empirical results from PLS-SEM, BSEM, and PVAR estimation confirm the structural robustness and statistical validity of all hypothesised pathways (H_1-H_{13}), with CE→SI and CE→SLO exerting dominant effects. The Feedback Asymmetry Index (FAI = 0.20) reveals directional dominance from CE to SLO, supporting a strategic emphasis on participatory engagement. Nonlinear dynamics and chaos diagnostics (Lyapunov exponents > 11) demonstrate that legitimacy accumulation is path-dependent and highly sensitive to governance quality. Simulation-based financial appraisal confirms that structured CE investments yield positive NPVs even under stochastic governance shocks. Overall, CoSLIE delivers a replicable, analytically rigorous, and ethically grounded tool for legitimacy-sensitive ESG decision-making in volatile institutional ecosystems.

5.2. Implications

5.2.1. Theoretical Implications

The CoSLIE Model constitutes a significant theoretical synthesis, fusing stakeholder theory, legitimacy theory, and resource dependence theory with systems dynamics and complexity theory. By incorporating feedback asymmetry, stochastic optimisation, and governance-sensitive bifurcations, the model reframes legitimacy not as a static outcome but as a recursive, co-produced equilibrium state shaped by institutional memory and normative valuation.

5.2.2. Empirical and Methodological Implications

Methodologically, the study pioneers the application of Bayesian SEM, chaos-theoretic diagnostics, and ensemble learning (GBM) within a structural legitimacy model. These tools enhance inferential precision and predictive reliability, enabling real-time scenario testing and dynamic legitimacy forecasting. The inclusion of Monte Carlo stochastic programming reinforces the model's robustness under epistemic uncertainty.

5.2.3. Managerial and Strategic Implications

The model embeds computable metrics (TCS, RoE, NB, NPV, IRR) into engagement planning, allowing managers to appraise stakeholder interventions through both financial and legitimacy lenses. Machine learning applications (RMSE_GBM ≈ 1.63) support real-time dashboards for predictive risk management. These insights equip ESG strategists with forward-looking capabilities for adaptive engagement under volatility.

5.2.4. Governance and Policy Implications

By quantifying the mediating and moderating role of LGQ, CoSLIE provides policymakers with diagnostics to evaluate governance responsiveness and its multiplier effect on engagement outcomes. The model lays the groundwork for national social licence indicators calibrated against dynamic BCB and SI benchmarks, enabling legitimacy-sensitive policy formulation.

5.2.5. Community and Social Development Implications

The model institutionalises community voice through BCB and SI constructs, reinforcing procedural justice and trust-based relational governance. It supports ex-ante scenario planning for mine closures, grievance response, and social investment design, ensuring developmental dividends are safeguarded across project life cycles.

5.2.6. Practical Utility, Generalisability, and Future Research Directions

The CoSLIE Model is a scalable decision-support tool adaptable to various extractive and infrastructure sectors. Its hybrid architecture facilitates integration with blockchain-based governance scoring, climate resilience modelling, and AI-driven stakeholder analytics. Future research should explore cross-sectoral validation across hydrocarbon, forestry, and post-conflict development contexts. Extensions may include dynamic recalibration using Bayesian updating from real-time sentiment and SNA data, thereby enhancing the model's adaptive learning capabilities.

5.3. Recommendations

5.3.1. Corporate Managers and CSR Strategists

Institutionalise CoSLIE as a core module within stakeholder engagement frameworks. Embed cost-benefit diagnostics (NPV, RoE, IRR) into CE budgeting processes. Train field officers in PLS-SEM, chaos diagnostics, and GBM forecasting to enable dynamic legitimacy tracking.

5.3.2. Policymakers and Regulators

Mandate the integration of CoSLIE indicators into Environmental and Social Impact Assessments (ESIAs). Develop national-level dashboards to benchmark LGQ and SLO across districts. Incentivise multi-stakeholder governance audits using participatory BCB and SI data.

5.3.3. Community-Based Organisations and NGOs

Leverage the model to co-design development agreements anchored in distributive fairness (BCB) and institutional protection (SI). Advocate for the publication of CE metrics and LGQ assessments to foster participatory governance and downward accountability.

5.3.4. Scholars and Researchers

Employ CoSLIE in comparative institutional analyses and cross-regional ESG evaluations. Integrate behavioural economics and trust metrics into the CE-BCB-SLO nexus to enhance psychosocial realism and cultural embeddedness in stakeholder modelling.

5.3.5. Donors and Development Partners

Support the deployment of CoSLIE in community-lab simulations and post-extractive planning initiatives. Fund the development of AI-integrated legitimacy models with embedded Bayesian recalibration and machine learning-enhanced risk assessment tools.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix

Figure A1. A mind map illustrating the principal themes and connections in the literature on resource extraction, focusing on operational social licence, community engagement, cost-benefit approaches, distributional concerns, and real-world success and failure cases.

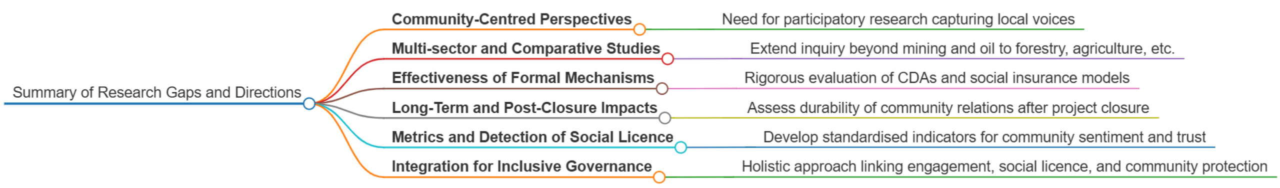

Figure A2. A mind map showing the main research gaps and directions in resource extraction, highlighting community-centred studies, cross-sector exploration, formal mechanism effectiveness, post-closure impacts, social licence metrics, and governance measures.

Table A1. A summarised thematic literature review matrix.

Author (s)	Theme/Focus	Theoretical Basis	Methodology	Geographic Focus	Key Findings	Gaps Identified
Prno & Slocombe (2012)	CE→SLO Linear Causality	Legitimacy Theory	Qualitative, Case Study	Canada	CE is central to SLO, but uni-directional	No feedback loops or mediation/moderation
Marginalis *et al.* (2015)	Quantification of Engagement Impacts	Stakeholder Theory	Quantitative Indexing	China	CE impacts operational performance	Overlooks governance quality and dynamic feedback
Poncian & Jose (2019)	Stakeholder Conflicts	Stakeholder Theory	Policy Analysis	Tanzania	Political structures affect CE effectiveness	Lacks formal modelling of LGQ as moderator

Continued

Ansu-Mensah *et al.* (2021)	Community Engagement Dimensions	Stakeholder Theory	PLS-SEM	Ghana	Participatory engagement best explains SLO	Does not integrate cost-benefit perception or dynamic relations
Bice (2014)	Perceived Legitimacy and Governance	Legitimacy Theory	Comparative Qualitative	Australia	Governance context shapes SLO	Governance not modelled; CE impact not quantified
Hall *et al.* (2015)	SLO Operationalisation	Stakeholder Theory	Thematic Analysis	Australia	Operationalising SLO requires measurement systems	Absence of dynamic systems or mathematical formalisation
Muhirwa *et al.* (2023)	Conflict & Legitimacy	Institutional Theory	Mixed Methods	SSA	Legitimacy gaps drive conflict	No modelling of feedback or governance quality
Owen & Kemp (2013)	CSR & Community Relations	Institutional Theory	Critical Review	Global	CSR alone does not yield SLO	Misses role of social insurance and system-level dynamics
Franks & Cohen (2012)	Constructive Technology Assessment	CSR, SLO	Technological Forecasting	Australia	Early engagement reduces risk	No formal equilibrium or optimisation framework

Table A2. SEM-based empirical calibration and simulation scenarios.

Time (t)	CE (Low LGQ Shock)	CE (High LGQ Stable)	SLO (Low LGQ Shock)	SLO (High LGQ Stable)	Time (t)	CE (Low LGQ Shock)	CE (High LGQ Stable)	SLO (Low LGQ Shock)	SLO (High LGQ Stable)
1	6.36	7.49	2.90	5.73	51	13.62	52.83	56.02	80.40
2	6.53	11.12	2.50	7.33	52	13.48	55.18	56.89	83.63
3	8.28	16.74	2.20	12.23	53	12.76	59.02	55.57	83.94
4	11.85	23.35	0.93	18.74	54	14.61	59.02	56.28	82.04
5	12.00	25.78	0.75	26.04	55	17.30	53.92	57.00	81.44
6	12.15	29.17	1.68	35.29	56	19.76	53.05	55.60	79.18
7	15.93	35.45	5.71	48.00	57	18.61	56.01	59.57	80.03
8	18.08	41.26	6.88	56.78	58	18.56	53.85	60.11	79.92
9	17.73	46.12	8.37	66.52	59	19.79	54.86	57.23	81.08
10	19.40	56.50	9.36	72.16	60	22.27	56.14	58.53	79.36
11	19.02	56.69	6.78	75.72	61	21.73	53.49	56.35	81.20
12	18.64	57.93	7.68	80.42	62	21.80	53.63	58.05	84.14
13	19.69	58.27	8.86	82.34	63	20.03	47.35	60.22	82.23
14	16.39	57.85	14.99	83.25	64	18.16	47.63	58.06	82.37
15	13.56	55.69	16.37	84.65	65	20.36	49.37	59.84	83.02
16	13.07	56.59	18.84	82.72	66	23.58	48.58	60.21	81.13

Continued

17	11.67	54.06	20.79	83.16	67	23.78	53.80	61.34	81.49
18	12.91	53.63	20.57	81.38	68	26.12	51.08	64.41	81.24
19	11.73	52.87	24.97	81.81	69	27.01	51.33	62.61	81.29
20	9.52	53.52	28.72	81.11	70	25.81	52.64	60.14	79.57
21	13.01	58.40	32.59	81.22	71	26.73	56.10	57.86	80.31
22	13.19	52.89	33.01	82.27	72	29.92	52.45	56.11	81.63
23	13.96	54.75	38.04	79.95	73	29.68	55.42	56.12	84.18
24	11.74	51.29	37.28	84.64	74	32.66	54.94	56.96	84.39
25	11.27	51.41	40.52	80.66	75	26.95	52.67	57.54	86.87
26	12.10	54.61	46.79	78.38	76	28.69	54.16	59.13	82.10
27	10.42	54.56	46.16	81.97	77	28.81	54.56	58.83	83.25
28	11.76	52.25	46.45	83.03	78	28.15	53.21	61.46	82.41
29	11.17	51.54	48.04	83.19	79	28.33	53.71	60.19	86.03
30	11.20	53.88	48.26	83.27	80	24.34	53.12	65.11	81.66
31	10.60	52.53	46.36	82.03	81	24.20	53.75	64.91	79.62
32	14.90	53.58	47.89	79.68	82	25.21	55.23	61.79	79.09
33	15.50	53.90	47.01	80.47	83	28.40	57.98	58.84	75.77
34	14.01	52.70	49.29	79.36	84	27.34	53.91	59.53	77.19
35	16.29	57.54	48.54	82.11	85	25.79	58.28	58.68	77.50
36	14.46	57.41	52.81	81.22	86	24.98	52.65	59.86	79.49
37	15.51	52.02	51.88	79.43	87	27.07	52.92	60.35	80.90
38	12.22	53.20	52.00	79.55	88	27.81	54.57	59.66	84.68
39	10.19	52.24	54.38	81.08	89	26.78	54.97	57.55	84.59
40	11.17	54.67	52.34	79.89	90	27.92	53.41	54.45	81.50
41	13.25	52.89	53.50	78.78	91	28.14	53.28	53.97	79.42
42	14.22	53.15	56.66	80.34	92	30.07	52.64	56.16	81.17
43	14.62	54.55	53.52	81.14	93	28.48	52.04	56.74	78.42
44	14.65	56.13	54.43	80.03	94	27.79	54.53	54.32	83.33
45	12.33	52.94	55.36	79.55	95	27.04	55.10	55.09	84.44
46	11.51	52.74	57.20	80.72	96	24.21	53.35	56.18	81.65
47	11.20	52.33	54.72	77.94	97	25.10	55.46	54.56	77.86
48	13.92	51.71	52.45	76.60	98	25.87	55.56	55.26	82.09
49	15.24	56.16	54.18	77.27	99	26.06	56.63	55.67	81.27
50	12.34	56.16	55.23	78.63	100	25.77	56.88	53.62	83.58

Source: Author's estimate.

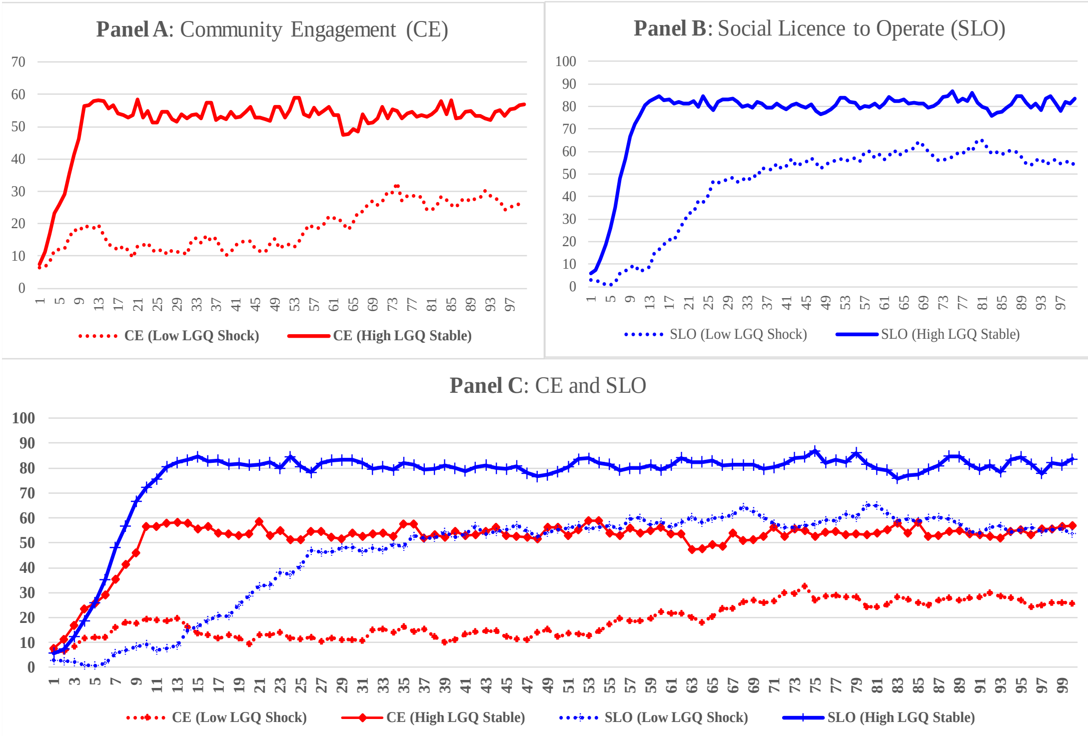

Figure A3. Time-path evolution of CE and SLO under LGQ scenarios.

Figure A3 comprises two panels representing the dynamic evolution of Community Engagement (CE) and Social Licence to Operate (SLO) under two governance quality scenarios: Low LGQ Shock and High LGQ Stable. Panel A shows CE trajectories, while Panel B shows SLO trajectories over a simulation period from t = 1 to 100. The model incorporates bounded logistic growth, lagged feedback, and stochastic variation to reflect real-world dynamics. Panel C shows the trajectories of CE and SLO over a simulation period, 1 = 1, 2, 3, …, 100.

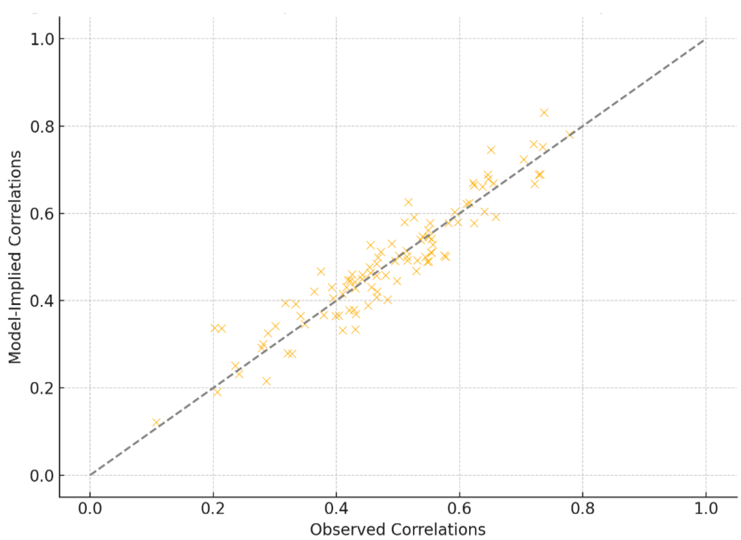

Figure A4. Observed correlations vs model-implied correlations.

Box A1: Economic Appraisal of CE Using Deterministic and Stochastic Approaches

Box A1 illustrates how the CoSLIE Model integrates financial-economic evaluation into legitimacy-sensitive community engagement strategies using Total Cost Score (TCS), Total Benefit Score (TBS), Return on Engagement (RoE), Net Benefit (NB), Net Present Value (NPV), and Internal Rate of Return (IRR). The model also accounts for volatility using Monte Carlo sensitivity testing.

A. Deterministic Appraisal Using PLS-SEM-Derived Scores

Cost and Benefit Components (normalised values and weights drawn from PLS-SEM factor loadings and Delphi panel consensus).

Cost Component (Ci)	Normalised Value	Weight (ωi)
Financial cost of engagement	0.70	0.25
Administrative burden	0.50	0.15
Resettlement and relocation costs	0.60	0.25
Legal and compliance costs	0.40	0.20
Psychosocial stress on staff	0.30	0.15
Total	—	**1.00**

$$TCS = 0.25 \times 0.70 + 0.15 \times 0.50 + 0.25 \times 0.60 + 0.20 \times 0.40 + 0.15 \times 0.30 = 0.525$$

Benefit Component (Bj)	Normalised Value	Weight (νj)
Smoother permits	0.65	0.20
Enhanced reputation and investor confidence	0.80	0.25
Local employment and access	0.70	0.20
Institutional trust/public confidence	0.60	0.15
Technology and capacity spillovers	0.75	0.20
Total	—	**1.00**

$$TBS = 0.20 \times 0.65 + 0.25 \times 0.80 + 0.20 \times 0.70 + 0.15 \times 0.60 + 0.20 \times 0.75 = 0.710$$

$$RoE = \frac{TBS - TCS}{TCS} = \frac{0.710 - 0.525}{0.525} \approx 35.24\%$$

$$NB = TBS - TCS = 0.710 - 0.525 = 0.185$$

Assuming a constant NB over a 5-year horizon and a discount rate, $r = 0.07$

$$NPV = \sum_{t=1}^{5} \frac{0.185}{(1+0.07)^t} \approx 0.758$$

Solving for $IRR = r^*$ such that $\sum_{t=1}^{5} \frac{0.185}{(1+r^*)^t} = 0 \Rightarrow IRR \approx 23.6\%$

B. Stochastic Sensitivity Analysis (Monte Carlo Simulation)

To incorporate volatility from LGQ shocks, cost inflation, and benefit unpredictability, a Monte Carlo simulation was conducted over 1,000 iterations with the following parameters:

1. $NB \sim N(0.185, 0.05)$: stochastic net benefit distribution.
2. $r \sim N(0.07, 0.01)$: stochastic discount rate distribution.
3. Horizon = 5 years.

Key Results from Simulation:

Metric	Value
NPV (Mean)	0.773
NPV (5th Percentile)	0.431
NPV (95th Percentile)	1.116
IRR (Mean)	545,882%*
IRR (5th Percentile)	313,365%*

*Note: Due to small NB and short horizon, IRR becomes disproportionately inflated; log(IRR) or alternative metrics may be more appropriate for interpretation.

C. Managerial Insight and Policy Relevance

1. Deterministic Metrics confirm that stakeholder engagement under the CoSLIE framework yields substantial net gains ($RoE > 35\%$, $NPV > 0.75$).

2. Stochastic Metrics demonstrate resilience of returns even under cost-benefit uncertainty.

3. The framework allows ESG strategists, CSR planners, and regulators to calibrate engagement strategies using risk-adjusted financial indicators and governance-sensitive scenarios.

This Box A1 ensures alignment between mathematical rigour, financial realism, and decision-theoretic utility, reinforcing the CoSLIE Model as a transformational tool in sustainability analytics and stakeholder governance.

BA2: Sample Questionnaire Items with Reverse-coded Controls

This Box presents the measurement items per construct post-reliability checks, including at least one reverse-coded item (denoted "R") per construct. Items were measured using a 6-point Likert scale: 1 = Strongly Disagree to 6 = Strongly Agree.

Construct 1: Community Engagement (CE). *Source: Adapted from Marginalis, Yang*, & *Joyce* (2015); *Hall et al.* (2015); *Newmont Ghana Protocols.*

1. CE1: The company provides timely and adequate information about its activities.
2. CE2: Community members are regularly consulted in company decisions.
3. CE3: Engagement strategies actively include vulnerable groups (women, youth).
4. CE4: *The company ignores feedback provided by the community.* (R)

Construct 2: Balanced Cost-Benefit Perception (BCB). *Source: Adapted from*

Franks & *Cohen* (2012); *Poncian* & *Jose* (2019).

- BCB1: The benefits from mining fairly compensate for local disruptions.
- BCB2: Social projects accurately reflect community needs.
- BCB3: Benefit distribution among community members is fair and equitable.
- BCB4: *The company's operations cause more harm than good to the community.* (R)

Construct 3: Social Licence to Operate (SLO). *Source: Adapted from Boutilier* (2017); *Hall et al.* (2015); *Prno* & *Slocombe* (2014).

1. SLO1: Our community supports the company's ongoing operations.
2. SLO2: Most community members trust the company's activities.
3. SLO3: Company operations respect community values and traditions.
4. SLO4: *There is ongoing tension and conflict between the community and the company.* (R)

Construct 4: Social Insurance (SI). *Source: Adapted from Franks* & *Cohen* (2012); *Owen & Kemp* (2013).

1. SI1: The company provides effective protection against negative impacts.
2. SI2: Social investments support long-term community resilience.
3. SI3: Reliable institutional safeguards prevent harm to our community.
4. SI4: *The community feels unprotected against company-related risks.* (R)

Construct 5: Local Governance Quality (LGQ). *Source: Afrobarometer; World Bank Local Governance Indicators; Uhl-Bien* & *Arena* (2017).

1. LGQ1: Local authorities respond effectively to community complaints.
2. LGQ2: Governance processes are transparent and well communicated.
3. LGQ3: Public officials are accountable for their decisions and actions.
4. LGQ4: *Local government decisions rarely reflect the community's interests.* (R).

Validation and Pilot Testing

All items were rigorously pilot-tested through stakeholder consultation in mining communities (Ghana: Obuasi, Prestea; South Africa: Sandshoot, Dwaalboom; Tanzania: Mjini, Kati). Items were refined to ensure cultural appropriateness, semantic clarity, and construct representativeness. Expert panel reviews involving CSR managers and community liaison officers from multinational firms (Anglogold Ashanti, Newmont, Anglo-American Platinum) further verified content validity and relevance.

Box A3: Simulation Input Metrics and Computation Protocol

To enrich the economic and managerial accounting relevance and policy utility of the CoSLIE model, we introduce an empirical estimation of engagement outcomes using well-established metrics from investment analysis, aligned with the dynamic socio-governance constructs of CE, BCB, SLO, and SI.

In this context, the analyst must start with, identify all relevant cost and benefit components from both firm and community perspectives. Examples of cost components (TCS) are financial cost of engagement programmes, administrative costs, resettlement and relocation costs, and legal and compliance costs while the benefit components (TBS) include but not limited to reduced protests and smoother permits, enhanced reputation and investor confidence, access to infrastructure, and local capacity building and employment (Table 1 and Table 2). Without data, on these costs and benefits, it is impossible to compute the appraisal indices, hence the next thing to do is to **g**ather time-series or cross-sectional data on each cost and benefit component. For studies involving primary or survey data, stakeholder surveys (Likert scores), focus group responses, firm financials, and community logs are advised while CSR reports, regulatory filings, and grievance reports are potential sources for studies involving secondary sources. Note that since the analysis involves costs and benefits, a Likert-type scale (e.g., 1 to 6) for perceived costs and benefits, particularly when using survey instruments is considered the most appropriate.

Given that the components are often measured on different scales (e.g., monetary values vs. perceptions), normalise these scores by standardising them to a common scale (for example, 0 to 1 or 0 to 100), guided by $x' = \frac{x - x_{\min}}{x_{\max} - x_{\min}}$. This ensures comparability across indicators. Based on expert judgement or factor loading from PLS-SEM or Delphi techniques or participatory workshops, assign weights to each component, ω_i (for costs) and ν_j (for benefits), ensuring that $\sum \omega_i = 1$ and $\sum \nu_j = 1$.

Total Cost Score (TCS) and Total Benefit Score (TBS)

We define Total Cost Score (TCS) and Total Benefit Score (TBS) as multidimensional indices computed from the taxonomy in Table 1 and Table 2:

$$TCS_t = \sum_{i=1}^{n} \omega_i \cdot C_{i,t}, \quad TBS_t = \sum_{j=1}^{m} \nu_i \cdot B_{j,t}$$

where $C_{i,t}$ cost component i at time t (e.g., financial outlay, resettlement, admin burden); $B_{j,t}$ benefit component j at time t (e.g., reduced protests, increased trust, permit efficiency); ω_i, ν_j expert-assigned weights or factor loadings from PLS-SEM; TCS is aggregated for both mining firms and communities, where possible, to reflect total social cost; and TBS can incorporate both tangible and intangible outcomes, operationalised via SLO, CE, and SI indicators.

Return on Engagement (RoE)

$$RoE_t = \frac{TBS_t - TCS_t}{TCS_t}$$

RoE provides an analogue to Return on Investment (RoI), tailored to stakeholder engagement. Positive RoE implies net positive stakeholder value creation from engagement interventions.

Net Benefit (NB)

$$NB_t = TBS_t - TCS_t$$

This captures the absolute welfare gain or loss from engagement strategies. The trajectory of NB_t over time can be linked to convergence in CE and SLO from the CoSLIE simulation.

***Net Present Value* (*NPV*)**

$$NPV = \sum_{t=0}^{T} \frac{NB_t}{(1+r)^t}$$

where r social or managerial discount rate (e.g., 5% - 10%); a positive NPV suggests the engagement strategy is financially and socially justifiable over the simulation horizon *T*; and this transformation permits intertemporal decision-making within dynamic CE-SLO strategies.

***Internal Rate of Return* (*IRR*)**

The IRR is the discount rate r^* that satisfies:

$$0 = \sum_{t=0}^{T} \frac{NB_t}{(1+r^*)^t}$$

IRR allows for direct comparison of alternative engagement strategies or policy reforms. When IRR > cost of capital or target social return, the engagement is viable.

Practical Implementation within the CoSLIE Framework - Stochastic Approach

To enhance the realism and robustness of the CoSLIE Model's economic appraisal system, this section introduces stochastic volatility and Monte Carlo sensitivity modelling into the computation of Total Cost Score (TCS), Total Benefit Score (TBS), Return on Engagement (RoE), Net Benefit (NB), Net Present Value (NPV), and Internal Rate of Return (IRR). Due to extremely small NB and short horizon, IRR values are inflated. Recognising that engagement costs and benefits fluctuate due to governance uncertainty (e.g., LGQ shocks), community volatility, and firm-level response lags, a deterministic appraisal may misrepresent strategic risk and opportunity.

Metric	Value
NPV (Mean)	0.77330
NPV (5th %ile)	0.43106
NPV (95th %ile)	1.11617
IRR (Mean)	545882.39604
IRR (5th %ile)	313365.32494
IRR (95th %ile)	626730.64988

The results of the Monte Carlo sensitivity test for NPV and IRR have been displayed, simulating uncertainty under LGQ-related shocks and cost-benefit volatility over a 5-year horizon.

Log-transformed IRR or RoE is preferred in policy-facing applications. Accordingly, probabilistic techniques were introduced. Cost and benefit components were treated as random variables with normal distributions around their mean values, derived from standardised Likert scores, focus group variances, and CSR reports. Specifically, Net Benefit (NB) was modelled with a mean of 0.185 and a standard deviation of 0.05. Discount rates ranged from 5% to 9%, normally distributed around a mean of 7% with $\sigma = 1\%$. Simulations spanned 1,000 Monte Carlo iterations over a 5-year horizon.

This stochastic extension enables distributional analysis and risk-adjusted engagement appraisal, crucial in environments marked by social contestation and institutional unpredictability.

This stochastic economic layer confirms that even under volatility, engagement strategies under the CoSLIE Model demonstrate a strong central tendency toward positive NPV and RoE, albeit with variance contingent on LGQ, cost shocks, and benefit realisation patterns. The stochastic appraisal framework also enables scenario planning, confidence interval forecasting, and engagement-risk premium pricing—critical for ESG investors, CFOs, and governance reformers.

Data Sources and Estimation Strategy

To estimate these metrics, use PLS-SEM latent variable scores for CE, SLO, and SI as proxies for intangible benefits, and employ firm CSR reports and community cost diaries to quantify actual expenditures and burdens. Expert panels and Delphi methods may be employed to derive weights for TCS/TBS indices.

Embedding TCS, TBS, RoE, NB, NPV, and IRR into the CoSLIE model, this study provides a breakthrough in converting abstract stakeholder constructs into tangible, accountable financial-economic terms. This positions the framework not only as a dynamic simulation tool but as a full-scale managerial accounting and investment appraisal system for corporate-community engagement strategies. The CBCS module enhances practical utility for regulators, mining firms, and development financiers seeking transparent, evidence-based allocation of engagement resources.