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Examining Toroidal Geometry in Terms of a 3-Dimensional "Tokamak", and the Use of a Toroidal Universe

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Abstract

We begin with a review of Friedman geometry in order to get a relationship between vacuum energy density and Hubble expansion parameter. Furthermore, in line with Utpal Sarkar, we can write the Hubble parameter as proportional to the square of background temperature, which is important in our derivational work. Furthermore, in line with work presented by Guth and Vilikin, we can obtain $g_{00}=1$ for a time component in the Toroidal universe by setting a three dimensional line element as embedded in the square of a scale factor $a(t)$ times the "line element" given by R. Murdzek (which has a 3 dimensional presentation of a Ricci scale factor). Incorporating the Guth and Vilikin "line element" trick, for a Toroidal universe allows $g_{00}=1$ for obtaining a non-zero stress energy tensor we can write as T_{00} as a non-zero value even if the Ricci component R_{00} in our construction is zero. We also close with a model of how all this is proportional to low entropy conditions in the early universe, citing a paper done by the author and Lousto *et al.* who modeled early universe conditions on black hole physics.

Keywords

Toroidal Universe, Friedman Geometry, Hubble Parameter, Vacuum Energy Density, Early Universe Entropy

1. Describing the Dynamics of Space-Time with Time Not Included, as Given in Reference [1]

Our task is to include in time EXPLICITLY in a working representation of Tokamak geometry, so we can perform graviton production rate calculations. Unfortunately, if time is not written in directly, it is extremely hard to make the necessary connections to engineering physics relevant to experimental tasks we wish to perform. So we first review a time INDEPENDENT geometry, with respect to Tokamaks, and then proceed to put in time by adjustments of the line element arguments used to parameterize our problem.

Note that [1], [2] and [3] as well as in reference [4] in a radius of the universe argument do refer to repeating universe arguments. However, [1] does NOT include TIME directly as given in **Figure 1**, **Figure 2**, and **Figure 3** which we illuminate below, whereas reference [4] again refers to a repeating universe but without time included.

We wish to refer to [1] which has the time component we want but which does NOT include in the geometry of a Tokamak, in terms of a space-time embedding of a 3-dimensional Tokamak in terms of a generalized line element which has a time component in it.

Afterwards, we describe the way we embed in the 3-dimensional Tokamak as an approximation of a torus, in a line element which includes in time explicitly.

Below are the basics of [1] which does include in a time component in terms of a Toroidal geometry, but which does NOT include in Tokamak geometry directly. This is from [1]in terms of Branes, using references [2] and [3] in terms of Brane geometry.

Not that we could include in the 3 dimensional toroidal version of a Tokamak as an approximation of a torus explicitly embedded in the Ekpyrotic model in a time dependent sense, but [1] as well as [4] does NOT include in a 3 dimensional TOROID explicitly in terms of time components.

Figure 1 below does NOT explicitly refer to time. This means our construction will have difficulty including in particle production in the regime of a Tokamak used as an approximation of a Toroidal Cosmology. **Figure 2**, is also NOT dependent upon time explicitly. Also **Figure 3** gives the basic idea of toroidal geometry embedded in Brane cosmology but ALSO does not include in time explicitly either.

Having said that, let us examine how Murdzek in [4] comes up with a Toroidal Universe as embedded in Brane construction as given in [1], [2] and [3].

2. Preliminaries for a 3-Dimensional Torus, along the Lines of Murdzek as in [4]

In doing this, we recognize that this 3-dimensional Torus will have $g_{00} = 0$, and so to begin, we look at a donut geometry which can be ascribed in **Figure 1** below, and **Figure 2** as well as in **Figure 3**. This creates a problem, in terms of time because having $g_{00} = 0$ also would leave us to have $T_{00} = 0$. We wish to eventually have a space-time cosmological linkage so time would have to be considered, especially if we want a Toroidal geometry which will have a rate of particle production to work with.

In order to have a particle production, *i.e.* massive graviton production from our Tokamak device which we reference toward the end, we will want to have

$T_{00} \neq 0$, and $g_{00} \neq 0$, but we will still access the Toroidal geometry as given in [4].

Having said that, let us initiate a discussion of [4] recognizing that [4] while highly innovative still will not allow one to have particle creation in it, and to understand how to reinsert the time dimension is a way to make an argument for embedding the line element given in [4] and discussed in detail in our section II, in the next several pages.

Begin first with two diagrams as of the Toroid. **Figure 2** is from [4].

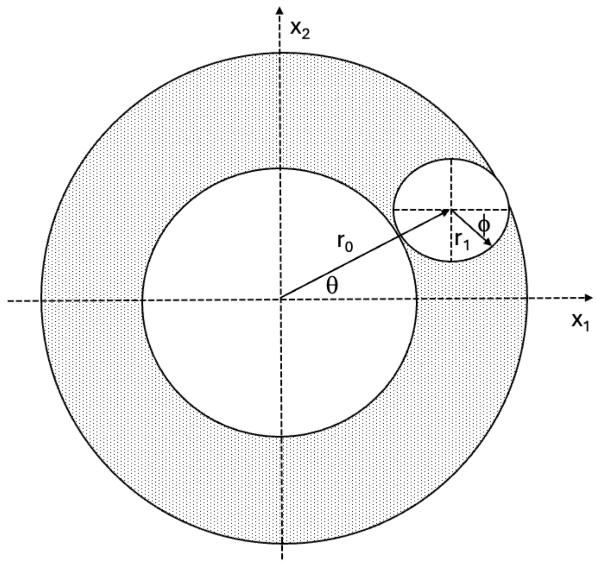

Figure 1. Simple visualization of a torus, in 3-dimensional geometry.

The z axis is perpendicular to the donut figure as given in **Figure 1**. *I.e.* along the lines given by Murdzek [1], and [4] and this is made super explicit in **Figure 2** below.

I.e. see **Figure 2**.

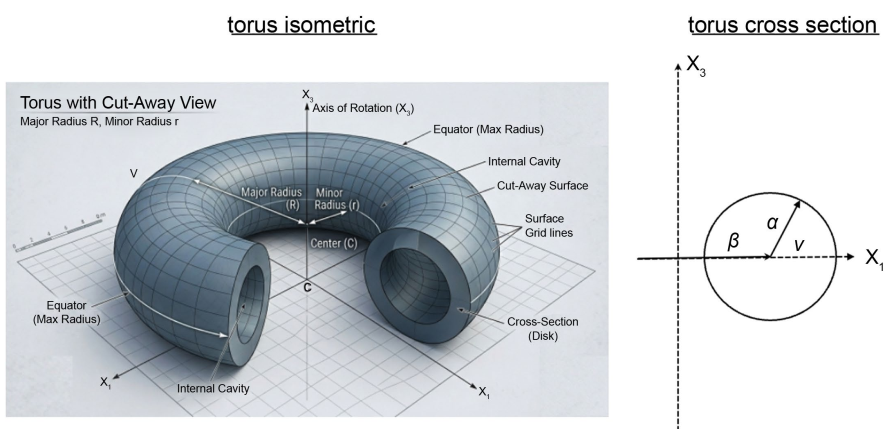

Figure 2. Coordinates used in the torus. The position of a point on the torus cross section is in coordinates (α, β, and γ).

This is from [1] and [4] and is the working geometry which well be assuming for Tokamak physics.

This has the following pertinent geometry: Go to **Figure 2**.

Now, more on the geometry of the Murzdzek coordinates used for the Toroid as given in references [1] and [4]. We find that the geometry coordinates are given in the following way as set in this **Figure 2**. See the formulation as set in this **Figure 2**.

Here, we do this in Cartesian coordinates

$$\begin{aligned} x_1 &= (\beta + \alpha \cos\nu)\cdot\cos\phi \\ x_2 &= (\beta + \alpha \cos\nu)\cdot\sin\phi \\ x_3 &= \alpha \sin\nu \end{aligned} \tag{1}$$

Here, this is for the following surface equation in Cartesian co-ordinates [1] and [4]

$$\left(\beta - \sqrt{x_1^2 + x_2^2}\right)^2 + x_3^2 = \alpha^2 \tag{2}$$

Also we will set from **Figure 3**, as well as

$$\begin{aligned} \Re &= R \\ \theta &= \vartheta \end{aligned} \tag{3}$$

Furthermore, this will if we specify a "Toroidal universe radii" as given [4] by set

$$\Re \sin\Theta = x_1 \tag{4}$$

Note that for making our notation consistent, $\Re = R$ in **Figure 2**, and we do this so our R which we relabeled is NOT confused with the Ricci scalar.

This assumes x_1 has ϕ set to zero, and Θ is the angle of a straight line $\Re$ with respect to the drawn axis from x_1 equals zero in figure 2 to the surface of the donut in **Figure 2**. Which we call $\Re$.

So, then we have the line element as given by

$$\mathrm{d}s^2 = -(\beta + \alpha\cos\nu)^2\,\mathrm{d}\phi^2 + \alpha^2\mathrm{d}\nu^2 \tag{5}$$

In doing this, using [3] again we refer to the only non-zero Riemannian tensor given as

$$R_{1212} = \alpha\beta\cos\nu + \alpha^2\cos^2\nu \tag{6}$$

Then the only non-zero Ricci tensor components of the toroid are specified as [4]

$$\begin{aligned} R_{11} &= -\frac{(\beta + \alpha\cos\phi)\cdot\cos\phi}{\alpha} \\ R_{22} &= -\frac{\alpha\cos\phi}{(\beta + \alpha\cos\phi)} \end{aligned} \tag{7}$$

This is extremely important to what we do next, because it means

$$R_{00} = 0 \tag{8}$$

In doing so, effectively in the line element as given in Equation (4) we have that

$$g_{00} = 0 \tag{9}$$

This Equation (8) means then that we would have $T_{00}=0$ which makes connection to the Space-time geometry next to impossible, this way.

We will be still observing Equation (8) but we wish to embed the Equation (5) line element in a setting where we have $g_{00}=1$, which is crucial to using $T_{00}\neq 0$. *I.e.* a non-zero energy density term in our physics comparison between two Ricci scalar terms, for getting $T_{00}\neq 0$, And in doing so, we are looking in the [1] and [4] Toroidal case as given by line element Equation (5) as by [1] and [4]

$$R_s=-\frac{2\cos\nu}{\alpha\left(\beta+\alpha\cos\nu\right)} \tag{10}$$

This is part and parcel of **Figure 3** which we put in below. And this is also in [3] [4] as well.

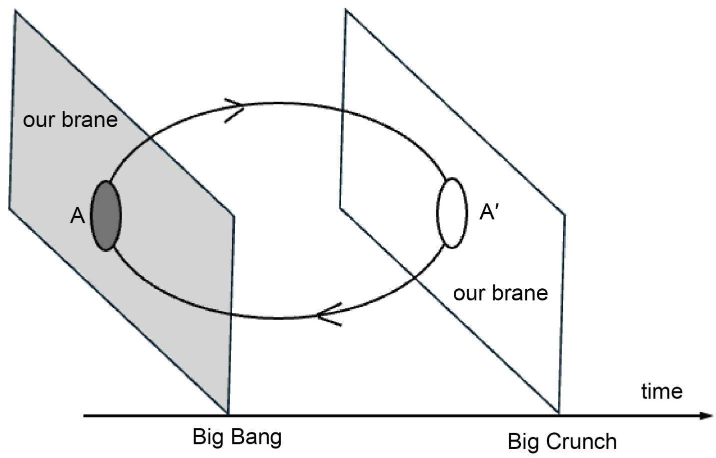

Figure 3. Our brane in the Ekpyrotic model. By identifying A with A′ we obtain a torus.

Here below is the basics of [1] put in which does NOT include in a time component in terms of a Toroidal geometry. It does NOT include in Tokamak geometry directly. This is from [1] in terms of Branes, using references [1] [2] and [3] in terms of Brane geometry. See page 3 of [1] for where this came from.

Note that there are very abstract definitions of Toroidal geometry and quantization in 2 + 1 geometry as seen in [5] in page 54 which refer to Toroid Moduli, in 2 + 1 geometry in what is called Teichmuller space. Which leads to a very strange metric given in Page 55, Formula (3.74).

The motivation of the Teichmuller space is to come up with a constant Hamiltonian which appears good if you want invariance laws. The down side is that in the Teichmuller space, when the Hamiltonian is CONSTANT, we have zero MOMENTUM.

So strange topological constructions do not rescue us. *I.e.* if we wish for a linkage to space-time geometry which MAY be proportional to Tokamaks, plus a time component, we need to do better. That is what our article is about.

So let us dive into this situation.

We refer to the abstract treatment of toroidal geometry brought up in reference [1] which is included in Figure 3. It is included in time but does not configure to the Tokamak's inherent geometry. This leaves us with the geometry of the Tokamak mapped right back in later after this discussion.

The Brane embedding so given does NOT have a direct linkage to 3 DIMENSIONAL TOKAMAKS.

Figure 3, taken from [1], is almost complete except that we lack the three-dimensional Tokamak as an approximation of a torus, which is explicitly embedded in the Ekpyrotic model in a time dependent sense.

3. Plan of Action Is to Use Guth and Villikin in Order to Have a Non-Zero $T_{00} \neq 0$ and Also $g_{00} = 1$, *i.e.* to Rework Our Problem in Terms of a Space-Time Cosmology Which Can Be Spatially Approximated by a 3 Dimensional Tokamak. First We Will Outline [4] in Its Description of Universe Dynamics

I sincerely wish to praise [4] for, without an explicit time dependence, worked in, giving a description of a Toroidal Universe. So I will briefly outline their program and yes it is excellent However, it DOES NOT allow for incorporating in particle production And to have Gravitons produced, one needs a TIME element worked in.

First let us go back to what was brought up in [4], and it is ALMOST complete, except they keep people away from a time component.

What is done, also by re writing Equation (1) in spherical co-ordinate is that [1] re-images the radii of the Toroidal geometry to obey the following quadratic Equation for the purported radii $\Re$ of the mini-Universe, and this is in fidelity with Figure 2 above.

$$\Re^2 - 2\Re\beta\sin\Theta + \beta^2 - \alpha^2 = 0 \tag{11}$$

The most interesting solution for this is when $\Re = 2\beta\sin\Theta$ as in [4] which corresponds to an oscillating Universe. However, in all of this, there will still not be any explicit time dependence and at best we will have say this situation.

4. How to Get $g_{00} = 1$ Re-Inserted Back into Space-Time Geometry of the Torus So $T_{00} \neq 0$

What we will be doing is to reference [6] Alan H. Guth and Alexander Vilenkin in which a Toroidal Universe has

$$\mathrm{d}S^2 = \mathrm{d}t^2 - a^2(t)\,\mathrm{d}X^2 \tag{12}$$

In doing so the scale factor, by [6]is written up as part of a cosmology with "Riemannian Metric line element" methodology leading to

$$a(t) = a_0 \exp(H_v t) \tag{13}$$

Here what we call the square of d*X*, is in actuality in our local geometry, Equation (5)

We will return to reference [6] later, but we will use it again in concluding remarks. But we will start off by assuming a term κ is set equal to Zero, which is necessary for Equation (13) to be chosen

5. Now How to Formulate $T_{00} \neq 0$ First by Using I and II, for the Toroid, and Also Comparing That with $T_{00} \neq 0$ in a Friedman Universe, by First Calculating H_{vol}

As asked by the referee, why do we then go straight to the Friedmann Equations?

Simply put because if we wish to calculate the time dynamics for Tokamaks later we need to make a connection to Friedmann geometry which has time explicitly.

To do this we will isolate what we can do with $g_{00} = 1$ always true, by first referring to the Friedman Universe case given below.

Starting off, we look at [7] which has the simple representation of a Friedmann space-time evolution equation given as

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{8\pi}{3} \cdot \rho_{vac} \tag{14}$$

Here we examine $\kappa = 0$ as the simplest case, *i.e.* here we have by [6] and [7] as well as the innovative treatment of the Hubble parameter given by Utpal Sarkar [8]

$$H_{vac} = \sqrt{\frac{8\pi}{3} \cdot \rho_{vac}} = \frac{1.66\sqrt{g_*} \cdot T_{vac}^2}{M_{Pl}} \tag{15}$$

ρ_{vac} is the energy density, and g_* is the initial degrees of freedom frequently set at 100.

Then let's go to calculating for both the Toroidal geometry and the Friedman Universe *i.e.* if

$T_{00} \neq 0$ **then by Hooper, [8] as well as [9] we have then that**

$$\begin{aligned} T_{00} &= \frac{M_{Pl}^2}{8\pi} \cdot \left(R_{00} - \frac{R_s \cdot g_{00}}{2}\right) \\ \text{which} &= \frac{M_{Pl}^2}{8\pi} \cdot \left(-\frac{R_s}{2}\right) \\ &= (\text{Toroid}) = \frac{M_{Pl}^2}{8\pi}\left(\frac{\cos\phi}{\alpha(\beta + \alpha\cos\phi)}\right) \\ &\approx (\text{Friedman}) = \frac{3M_{Pl}^2}{8\pi}\left(\frac{1.66\sqrt{g_*} \cdot T_{vac}^2}{M_{Pl}}\right)^2 \end{aligned} \tag{16}$$

6. Examining What Equation (16) Is Telling Us

To do this, consider how much energy may be pumped into a Tokamak, for T_{00}

To best estimate, [10], we can write Tokamak temperature of about 13 keV (150 million kelvin) and 13 KeV is such that we can consider this phenomenology as follows.

This is for an optimal regime of about 25 KeV, which comes out to 2.5 times 10^−5 GeV, and this would be for the Equation (16) inputs.

Here, we have that Planck Mass in GeV is by [11]

$$M_{Pl} \approx 1.22 \times 10^{19}\ \text{GeV}/c^2 \xrightarrow[c \to 1]{} 1.22 \times 10^{19}\ \text{GeV} \tag{17}$$

Then the ratio, for the Tokamak would be

$$T(\text{tokamak})^2 / M_{Pl} \approx 5.1229 \times 10^{-39}\ \text{GeV} \tag{18}$$

I.e. the rest mass of a Graviton is, say [12]

$$m(\text{graviton}) \approx 10^{-65}\ \text{grams} \approx 5.61 \times 10^{-41}\ \text{GeV} \tag{19}$$

This means, then, that if we take the following ratio

$$\frac{T(\text{tokamak})^2 / M_{Pl}}{m(\text{graviton})} \approx \frac{5.1229 \times 10^{-39}\ \text{GeV}}{5.61 \times 10^{-41}\ \text{GeV}} \approx 10^2 \tag{20}$$

This means in a cubic centimeter say of space, per second we would have approximately 100 gravitons per second in a cubic centimeter of space.

Keeping in mind naturalized units, we would have to consider the geometry of space which would be evaluated, *i.e.* we would by a Killing vector argument approximate the energy density of the Toroid as being a constant in the rotation of the "donut" [12] given in **Figure 2**, *i.e.* for the Toroid itself we would have [13] if $R = \beta$, and $r = \alpha$ and that this is a constant, as given in the Toroid

$$T_{00} \doteq \frac{E(\text{Energy Toroid})}{(2\pi R)\cdot(\pi r^2)} \cong \frac{E(\text{Energy Toroid})}{(2\pi\beta)\cdot(\pi\alpha^2)} \tag{21}$$

We will elaborate upon the consequences of all this in our follow up publication w.r.t. actual Engineering applications of this geometry as far as our search for Gravitons and GW radiation. *i.e.* in particular if this is accepted as legitimate to use data as to ITER, as a baseline calculation.

We wish to say that the explicit quantization methods of [9] will be compared as a baseline as to nucleation of Gravitons in Toroidal geometry, in both cosmology and instrumentation follow ups to this document.

7. Final Frontier of Investigation, Why Early Universe Conditions Are Low Entropy

To get to this, the author, myself cites work done in an earlier paper, which is for the purpose of showing entropy modeling from early universe initial conditions.

To do this, look at [14] and its treatment of early universe conditions which will lead to LOW entropy.

Quote as given directly from reference [14].

To begin with. Look at how to construct entropy for black holes and the early universe.

Note that for gravity one has, if k is Boltzmann's constant, and N the number of Microstates. Note that formula 1 turns to formula 2 if N is large

$$S = k \ln N \tag{22}$$

Now, by Muller and Luosto [14] [15] as well as Crowell [14] [16] one can write for the early universe:

$$S = kA/4l_P^2 \tag{23}$$

B1. What if one looks at a treatment of black holes?

The area A is such, that by Crowell [2] [14] [16] we can write this area as, for a black hole of mass M

$$A = 16\pi M^2 \tag{24}$$

For a string theory treatment of black holes we will write [2] [14] [16]

$$A = 16\pi\alpha \sum_{i=1}^{N} n_i \tag{25}$$

We also ask the readers to investigate what is in [14] [17] before going to the remainder of this argument.

So what is α ?

If what Ng writes for Quantum infinite statistics [14] [18] [19] is true, then

$$E = \alpha E_P \sqrt{n} \Leftrightarrow \alpha = \frac{1}{2}\frac{\sqrt{\ln 2}}{\pi} \tag{26}$$

Partition function treatment of black holes [14] [16].

Crowell wrote having a partition function for Black holes defined by

$$Z = \sum_n \exp\left[4\pi\omega_n\right] \cdot \exp\left[-\beta\alpha\sqrt{n}\right] \tag{27}$$

This was achieved by normal modes for black holes, of mass M which was of the form [14]-[16]

$$\omega_n = \alpha^2 = \frac{\ln 3}{8\pi M} + \frac{i}{4M} \cdot \left(n + \frac{1}{2}\right) \tag{28}$$

The imaginary component to (28) above is what is not used if one uses the (26) result, which will lead to a bridge to early universe results. We will differentiate between the early universe result and (28) above by keeping fidelity with respect to the early universe, if one is looking at the real component of (28) above, while not looking at the imaginary results. This is in tandem with looking at the full expression of (28) for black holes, with real and imaginary results, while speculating that by way of contrast, if we have only the real part of (28), we are looking at a re do of the Ng entropy result, which would be in tandem with having (27) having no appreciative imaginary component.

How we wish to interpret how to interpret the rise of entropy from a black hole and entropy of the early universe. Note that [14] [15] has an alternative expression for the early universe which can be written as, if a is the scale factor, of radii r_H

for a horizon radius, with

$$S = \frac{0.3 r_H^2}{a^2} \tag{29}$$

And [14] [15]

$$r_H = \sqrt{\frac{3}{\Lambda}} \tag{30}$$

Here, the cosmological constant as given by [14] [17] by Park *et al.* is of the form with T the background temperature, as given by

$$\Lambda \propto T^{\tilde{\beta}} \sqrt{\frac{3}{\Lambda}} \Rightarrow r_H = \sqrt{\frac{3}{\Lambda}} \cong \sqrt{3} T^{-\tilde{\beta}/2} \Rightarrow S \approx 0.3 \cdot 3 T^{-\tilde{\beta}} / a^2 \tag{31}$$

Above almost scales exactly as having the universe with entropy proportional to one over the temperature to the minus beta power times one over the square of the scale factor for early universe conditions.

To make it more revealing, note from [14] [15] that one can write

$$S_{\text{Early Universe}} \sim 16\pi\alpha^2 n \tag{32}$$

Note that this is very similar to work done by Ng, in [18].

Here also, from [14] [15] we have an energy expression from (26) above, as well as employing the string theory result of

$$\begin{aligned} & S_{\text{Early Universe}} \sim 16\pi\alpha^2 n \sim T^{-\tilde{\beta}} / a^2 \Rightarrow T^{-\tilde{\beta}} \propto 16\pi\alpha^2 n a^2 \\ & \Rightarrow T \approx \frac{1}{\left(16\pi\alpha^2 n a^2\right)^{\beta}} \end{aligned} \tag{33}$$

Assuming we have a condition for which α is in a short period of time a constant in the early universe and that we have for H the initial Hubble expansion parameter, and the time, then if what is below, is

$$a \sim a_0 \exp(H \cdot t) \sim a_0 (\text{Plank time}) \tag{34}$$

Then in the regime of Planck time we are looking at [14] [15]

$$T \approx \frac{1}{\left(16\pi\alpha^2 n a^2\right)^{\beta}} \sim \left[\frac{(1 - H \cdot t)^{\tilde{\beta}}}{a_0^{\tilde{\beta}}}\right] \cdot \frac{1}{n^{\tilde{\beta}}} \propto \frac{1}{n^{\tilde{\beta}}} \tag{35}$$

The proportionality of temperature, T, in the Planck time regime is saying that as n is "nucleated" or created, that the temperature scales down. Note that beyond the Planck interval of time, one will be beginning to look at a time dependence, according to the coefficient $\left[\frac{(1 - H \cdot t)^{\tilde{\beta}}}{a_0^{\tilde{\beta}}}\right]$ with H a constant. Before then the dominant effect of scaling down will be on the creation of $\frac{1}{n^{\tilde{\beta}}}$ contributions to dropping of the temperature. [14] [15].

End of quote

This definitely using black hole physics, as well as entropy, in terms of quantum number n as stated in this document as a way to scale entropy, providing for conditions in which if we start of with very high quantum number n, we will have vanishingly small entropy contributions to our early universe.

This question as to early universe entropy, and why it was so low initially was asked by a reviewer, and my answer is an extensive explainer as to what is actually seen.

This is also, with investigation also partly interlocking with Stoica, C, in [19] as well.

8. Conclusions

What we wish to do is outline a procedure for modeling the situation in the Early Universe using toroidal geometry. Applied to the early universe (as well as Tokamak geometry), it will be in complete fidelity with Section 7 results.

Recall the geometry cited for a radius for the "universe" as brought up in Equation (4), *i.e.* this is a geometry worked out by [1].

We wish to make this geometry with its potential for entropy production as well as for the joint symmetries of space-time inherent in both a repeating universe, as well as the symmetries inherent in entropy, and temperature, for the early universe, as well as the Tokamak, in sync with Section 7 above.

That will be the subject of a next paper.

We wish to add that references [20] and [21] are pertinent to the development of spacetime that will be joined to deliver Tokamak geometry similar to the developments in these papers.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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