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A First Order Wave Equation Composed of a Curved Spacetime Metric

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Abstract

The ultimate quest of field theory is to pursue a fundamental law that can address both micro and macro issues, namely quantum entanglement and the inflationary universe, respectively. Here in this paper, a wave equation revealing the causality of a de-Sitter Space explains the so-called spooky action at a distance and the expansion of the universe within a single theory, quantum field theory on curved spacetime. An Astrophysical aspect of the theory is pursued at the end. Historic Background of Quantum Field Theory in Curved Spacetime: A century ago, there were two theories in physics that split the universe into aspects: the quantum theory dealing with microscopic world and the theory of general relativity dealing with macroscopic world. From the development of physics, we started with global phenomena like Newton's gravitation law and came down to local phenomenon like particle wave duality in early twentieth century. Gravity was a well-known phenomenon since Newton's time and reformulated by Einstein as general theory of relativity. There is no doubt that gravity expressed by the general theory of relativity coincides with the observational astronomy like gravitational wave, gravitational lensing and blackhole dynamics. At the same time, it is well tested by high energy physics that quantum mechanics is the fundamental theory for particle physics. It would be wrong to favor one over the other. Therefore I propose that the dual aspect of universe respecting both quantum and classical physics is simply expressed by one fundamental law. The appropriate candidate is Quantum Field Theory in Curved Spacetime. Working towards a unified theory for four fundamental forces, namely gravitation, electromagnetic force, strong and weak. Three out of the four, except for gravitation, can be explained by quantum mechanics. People are challenged to quantize gravity based on their mathematical skills, like gauge theory or string theory but still cannot escape the consequence of mathematical remedial technique—renormalization of the loops. They may be successful in one-loop renormalization but keep on renormalizing the subsequent loops without ending. If the theory does not make mathematical sense, it is a sign we should reconsider all the other possible ones. Here quantum field theory in curved space is studied [1]-[3].

Keywords

Field Theory, Curved Spacetime, De-Sitter Space

1. Introduction

Quantum Field Theory in curved space brings the dynamics to a quantum ensemble by introducing the classical spacetime manifold, a 4 × 4 metric to the wave equation [4]. The 4 × 4 metric is a signature of the curved spacetime manifold determined by Einstein Field Equation [5], $G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$.

Originally, the cosmological term, $\Lambda g_{\mu\nu}$ didn't exist in the equation until the realization of the expansion of the universe was confirmed after Hubble with his constant $H = \alpha\sqrt{\Lambda} = \frac{1}{R}$.

Where Λ is the cosmological constant, α is a proportionality constant and R is the radius of curvature of the observable universe. The cosmological constant arises due to the hypothesis of dark energy that exists to explain the expansion of the universe. The values of the cosmological constant will not be discussed here since they are irrelevant to the quantum mechanical part of the theory. In the quantum part of the theory, gravity is expressed as the geometric (four dimensional) manifold from the theory of general relativity and it cannot be treated as a quantum field without running into the catastrophe of ultraviolet divergence [6]. The only way to avoid the endless cycle of renormalization is to rely on the quantum theory of field in curved space in which the electromagnetic, strong and weak forces are regarded as Klein-Gordon fields and gravity is regarded as classical field. Thus, string theory as a fundamental theory for unifying four fundamental forces is not essential since the quantum dynamics of fields in curved spacetime can serve the purpose of unification. That will be explored in sections 4, 5, 6 and 7.

In my previous work, there is a mathematical concept stating that a Klein-Godon field can gain a scalar potential under a unitary transformation, *i.e.* time evolution on de-Sitter space [7]. First, Klein Gordon equation is invariant under a de-Sitter transformation, otherwise it is not a unitary transformation or Klein Gordon field is not a quantum field. Since, Klein Gordon equation yields quantum dynamics in curved spacetime, de-Sitter universe is regarded as the best approximation of the cosmological space [7]. Now we are ready to use the mathematics developed by Friedlander [4] to the content of this section. The divergence of a vector field on a spacetime manifold, M is a scalar field, a function on M see (1.1.24) in local coordinates [4],

$$div\, v = |g|^{-\frac{1}{2}} \frac{\partial}{\partial x_i}\left(|g|^{\frac{1}{2}} v_i\right)$$

where $g = \det g_{ij}$. Now, suppose the vector field is replaced by a distribution, or a wavefunction on a manifold (1.1.26) [4] states that the linear differential operator up to second order on the distribution in the manifold equals to a scalar function in the manifold. Considering only the second order and the zeroth order terms with the first order term being omitted in (2.8.19) [4], along with the hypothesis (1.1.26) [4], we have the familiar Klein Gordon equation wave equation on Riemannian metric that has been studied [7]. Considering the first order term only, we have a first order linear differential equation expressing the wave mechanics in curved spacetime as follows in natural units $\hbar = c = 0$

$$-i\partial_{\mu} g^{\mu\nu} \psi_{\nu} = p_{\mu} g^{\mu\nu} \psi_{\nu} \tag{1}$$

where $g_{\mu\nu}$ is a 4×4 metric tensor signifying the curvature of a curved manifold. The RHS of (1) can be understood as a scalar function which is an infinitely differentiable continuous function on the manifold, M or simply said as analytical. After the functional analysis of (2.8.19) [4] and (1.1.26) [4] in addition with the generalization to a vector field, ψ_{μ} from a scalar Klein-Gordon field, ϕ and to a four momentum p_{μ} as an operator, $-i\partial_{\mu}$ in configuration space, from a scalar potential V [7], we can write the wave equation on curved spacetime manifold as an eigenvalue equation (1). It is a wave equation in local coordinates, the Cauchy data defined on a global manifold, M (see Dimock) [8]. Please note that the 4×4 metric satisfies relativistic covariance as shown in the next section so that (1) is consistent with the principle of relativistic invariant.

2. De-Sitter Space

The metrics are given by

$$g^{\mu\nu} = \left(\frac{t}{R}\right)^2 \operatorname{diag}(-1,1,1,1) \tag{2}$$

$$g_{\mu\nu} = \left(\frac{R}{t}\right)^2 \operatorname{diag}(-1,1,1,1) \tag{3}$$

where $\mu = \nu = (0,1,2,3)$ and R is the radius of the universe or the inverse of the Hubble constant.

Embedded in five dimensions, de-Sitter universe is a highly symmetric manifold, a hyperbolic surface, $x_0^2 - x_1^2 - x_2^2 - x_3^2 - x_4^2 = -R^2$

Where $x_0 = t$, the other four components are spatial. It's metric contains no matter and a constant curvature that explains the current state of inflationary universe or accelerating universe due to Euclidean vacuum [9]. It is the geometry of the large scale structure of the universe worth for studying because of the wave equation being analytical as will be seen below. The radius of the curvature R can tell the so-called the age of the universe according to Hubble's constant. Hubble's inference of the big bang theory was true provided time-like and light-like geodesics on the de-Sitter manifold in the intergalactic space or the large scale of the universe. What if the geodesics is spacelike causality upon the rate of expansion of the universe beyond the speed of light according to the steady state constant curvature of de-Sitter space. Then there is no time-like or light-like causality in the de-Sitter universe to connect the past event with the present event. Hubble's idea of the origin of the big scale universe might not sound right. On the mathematical analysis, the hyperbola is intercepted by a flat space plane at 45 degrees under the Lorentzian transformation in 5 dimensions [10] as time goes back to infinite past, namely $R = 0$. Upon the symmetry breaking of the de-Sitter manifold, a wedge of the manifold cut, the manifold intercepted by the 45 degree plane has no origin. The origin of time doesn't exist in de-Sitter hyperbola. No big bang occurred.

3. Wave Equation Solved in de-Sitter Space

The metric (2) and (3) fulfill the principle of relativistic covariance such that

$$g^{\mu\nu} g_{\nu\sigma} = \delta^{\mu}_{\sigma}$$

$$x'^{\mu} = g^{\mu\nu} x_{\nu}$$

Considering

$$\begin{aligned} x'^{\mu} x'_{\mu} &= g^{\mu\nu} x_{\nu} g_{\mu\sigma} x^{\sigma} \\ &= \delta^{\nu}_{\sigma} x_{\nu} x^{\sigma} \\ &= x_{\nu} x^{\nu} \end{aligned}$$

Relativistic covariance was proved.

We are now in the position to solve the equation (1) as follow

$$\left|\det\left(g_{\mu\nu}\right)\right|^{\frac{1}{2}} = \left(\frac{R}{t}\right)^{4}$$

(1) follows $\sum_{\mu,\nu} \left|\det\left(g_{\mu\nu}\right)\right|^{-\frac{1}{2}} \frac{\partial}{\partial x_{\mu}} \left(\left|\det\left(g_{\mu\nu}\right)\right|^{\frac{1}{2}} g^{\mu\nu} \psi_{\nu} \right) = i p_{\mu} g^{\mu\nu} \psi_{\nu}$

$$t^{4} \sum_{\mu,\nu} \frac{\partial}{\partial x_{\mu}} \left(t^{-4} g^{\mu\nu} \psi_{\nu} \right) = i p_{\mu} g^{\mu\nu} \psi_{\nu}$$

$$t^{4} \frac{\partial}{\partial t} \left(-\frac{1}{R^{2} t^{2}} \psi_{0}(t) \right) + \frac{t^{2}}{R^{2}} \sum_{i=1}^{3} \frac{\partial}{\partial x_{i}} \psi_{i}(x_{i}) = \frac{i t^{2}}{R^{2}} \left(\sum_{i=1}^{3} p_{i} \psi_{i}(x_{i}) - E \psi_{0}(t) \right)$$

Solving the temporary component and spatial components separately,

$$t^{4} \frac{\partial}{\partial t} t^{-2} \psi_{0}(t) = i t^{2} E \psi_{0}(t)$$

$$t^{2} \frac{\partial \psi_{0}(t)}{\partial t} - 2t \psi_{0}(t) = i t^{2} E \psi_{0}(t)$$

$$\frac{\partial \psi_{0}(t)}{\partial t} = \left(\frac{2}{t} + iE \right) \psi_{0}(t)$$

$$\ln\left(\psi_{0}(t)\right) = i \int E \mathrm{d}t + 2 \ln(t)$$

After putting back the natural units, we have the solutions

$$\psi_0(t)=\left(\frac{ct}{R}\right)^2\exp\left(i\varphi(t)\right) \tag{4}$$

$$\psi_i(x_i)=\exp\left(i\theta_i(x_i)\right) \tag{5}$$

where $\varphi(t)=\frac{1}{\hbar}\int E\mathrm{d}t$ and $\theta_i(x_i)=\frac{1}{\hbar}\int p_i\mathrm{d}x_i$

After normalization factor was recovered by integrating the wavefunctions (4) and (5) over the entire space and time, we have

$$\psi_0(t)=\frac{1}{\sqrt{2\pi}}\cdot\left(\frac{ct}{R}\right)^2\exp\left(i\varphi(t)\right) \tag{6}$$

$$\psi_i(x_i)=\frac{1}{\sqrt{2\pi}}\exp\left(i\theta_i(x_i)\right) \tag{7}$$

4. Analysis of Solutions of the Wave Equation

The expansion rate of the universe determined after Hubble constant, $H = 1/R$ brings global aspect to the local phenomenon (4) and (6). The wavefunction of (4) / (6) reveals the nature of inflationary universe. There is a correlation relationship between quantum entanglement and inflationary cosmology in [11]. The nature of quantum entanglement manifests since the wavefunction (6) is time dependent with a factor $\left(\frac{ct}{R}\right)^2$ and the phase is a superposition of linearly independent energy eigenstates if the integral is replaced by an infinite sum. If the wavefunction is to remain fixed at an arbitrary time, t that means the distribution of energy $E(t)$ has to change corresponding to the variation of any energy eigenstates in the distribution of energy at that arbitrary time. In other words, an energy eigenstate changes (maybe spin state), all the others change accordingly in order to preserve the phases, $\varphi(t)$ and $\theta_i(x_i)$ and in turn the wavefunction (6) as a global phenomenon. Note that the distribution of energy varies independent with the de-Sitter manifold's causal domains – timelike, spacelike or lightlike. Quantum entanglement can be observed over different cosmological causal domains. Therefore, it is a universal phenomenon over the three stages of cosmological expansions, timelike or lightlike relation $v \le c$ as our current stage or spacelike relation $v > c$ in the future stage. This is a signature behavior of quantum fields in de-Sitter spacetime.

Remark: The integral in the phases can be regarded as a Riemann sum of energy eigenstates in which one eigenstate varies, all the others follow so that the sum remains fixed.

5. De-Sitter Space to Minkowski Space

Revisiting (6), at the limit $ct = R$, back to flat spacetime limit, we recover the plane wave solution of Schrödinger equation. Solution of the wave equation in Minkowski space or Schrödinger equation exists in the light cone of causality as a well known fact. So, quantum entanglement in Minkowski's limit is restricted only valid in local coordinates, not in global manifold, M, like the one shown previously in de-Sitter space in section 3. Thus, it cannot be called spooky actions at a distance or quantum entanglement in global coordinates. In other words, Schrödinger equation, a local wave theory is just a special case (or the Minkowski's limit), the temporary component of the solutions ψ_μ, of the general wave equation on curved spacetime (1).

6. Discussions on Dark Matter/Energy in the de-Sitter Universe

Evidence, like Cosmic Background Radiation shows that our inflationary universe due to a positive value of cosmological constant is due to the existence of dark matter which has an energy called dark energy, comprising more than 90% of the energy of the universe. It is called dark energy because it is not measurable but determined from Einstein Field Equation in the observable universe and its existence was assumed after Hubble observation of the expanding universe. There is an interesting phenomenon, particles creation or annihilation can occur in de-Sitter universe regardless of the stages of evolution of the universe, accelerated or steady state [12]. This result signifies the existence of dark energy in de-Sitter universe as a pair of photons with cosmic background radiation energy 29.5 MeV that in turn gives rise to an electron-positron pair creation or annihilation ($\gamma\gamma \to e^- e^+$) [12]. The missing mass 29.5 MeV of dark matter can be analysed through the conservation of four momentum, $p^\mu p_\mu = m^2$ or simply as the particles on shell in local coordinates. The four momentum p^μ here is the same as the one in (1) so the wavefuctions (6) and (7) both describe the behavior of dark matter as well as ordinary matter. Then integrals of energy and momentum in the phases $\varphi(t)$ and $\theta_i(x_i)$ are the statistical values of energy and momentum for the ensemble. They are supposed to be the ones used in experimental work, roughly m = 29.5 MeV in the Planck scale.

7. Conclusion

As shown up to this point, quantum field theory in flat space metric is only a partial truth of the one in curved space. Quantum entanglement was illustrated in global coordinates, t as well as the expansion of the universe manifested by the metric carrying the factor $R = 1/H$, both in the same wave equation. The wave equation, a local phenomenon, gauged by the global metric shows the transformation of the vector field through the differential operator $p_\mu = -i\partial_\mu$, in configuration space based on the mathematical concepts developed by Friedlander [4]. The philosophy of this theory is that quantum field dynamics is governed by causality, particularly in de-Sitter universe. This proves that the theory of general relativity can bring the role of gravity in the theory of quantum field. In the tradition of quantum field theory development, Klein-Gordon equation, Schrödinger equation, and Dirac equation are all good up to local phenomena, no gravitation is conceived if it is considered as a global field. However, in terms of global geometry, one has to rely upon the theory of quantum field on curved spacetime. The wave solutions (6) and (7) being consistent with the hypothesis of quantum entanglement and inflationary universe, address the skeptical question of public concern over the idea of quantizing gravity through the use of the metric of the space as a function of being turned into an operator. It is evident that quantizing gravity is not mathematically justified through the study of this paper, as discussed in section 1. This paper can also shed light on the hypothesis of dark matter as a further investigation on the existence of dark energy by looking at the solutions of the wave equation (1). It seems dark energy embedded with the background manifold like de-Sitter space and Schwarzschild blackhole satisfies Einstein field equation [13]. Interested researchers may study the field behaviors further, particularly with the metrics of Schwarzschild blackhole [13] or Kerr blackhole [14].

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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