Journal of High Energy Physics, Gravitation and Cosmology, 2026, 12(2), 984-1009
https://www.scirp.org/journal/jhepgc
ISSN Online: 2380-4335
ISSN Print: 2380-4327

A Vortex-Based Geometric Interpretation of Atomic Shells and Periodicity

Nader Butto

Independent Researcher, Petah Tikva, Israel
Email: nader.butto@gmail.com

How to cite this paper: Butto, N. (2026) A Vortex-Based Geometric Interpretation of Atomic Shells and Periodicity. *Journal of High Energy Physics, Gravitation and Cosmology*, **12**, 984-1009.
https://doi.org/10.4236/jhepgc.2026.122053

Received: January 18, 2026
Accepted: April 5, 2026
Published: April 8, 2026

Abstract

The structure of atoms and the ordering of electronic states are traditionally described within quantum mechanics through abstract wavefunctions and empirical filling rules. While this formalism successfully predicts atomic spectra and chemical behavior, it provides limited physical intuition regarding the spatial organization of shells, subshells, and orbitals, as well as the origin of well-known filling anomalies observed in transition elements. In this work, we propose a geometric reinterpretation of atomic structure based on a vortex framework, in which electrons are described as localized vortex excitations embedded in a structured vacuum. Atomic shells are modeled as concentric vortex structures, subshells as internal vortex layers, and orbitals as stable localization regions arising from vortex topology. Within this framework, the principal, azimuthal, magnetic, and spin quantum numbers acquire a coherent geometric meaning without modifying their standard quantum-mechanical definitions. The model naturally reproduces fundamental results such as the shell capacity rule $N = 2n^2$ and the subshell orbital multiplicity $2\ell + 1$, while providing a physical explanation for the Aufbau principle, the $n + \ell$ (Madelung) rule, and their apparent exceptions. In particular, the configurations of chromium and copper, as well as the distinction between orbital filling order and ionization order in transition metals, emerge as consequences of subshell overlap, radial scaling, and vortex stability. Recent high-resolution experiments demonstrating spatially extended atom-light interaction regions are consistent with the physical plausibility of structured atomic interaction zones. The analysis further shows that the same geometric counting laws governing electron capacity within atomic shells also determine the recurrence of elements across periods, indicating that atomic structure and chemical periodicity arise from a unified organizing principle. Taken together, these results suggest that atomic structure and periodicity may be understood as manifestations of a hierarchical vortex organization, offering a physically intuitive complement to the conventional quantum-mechanical description.

Keywords

Atomic Shell Structure, Quantum Numbers, Vortex Geometry, Orbital Topology, Electron Configuration, Pauli Exclusion Principle, Subshell Overlap and Contraction, Periodic System Unification

1. Introduction

The atomic structure of matter represents one of the foundational achievements of modern physics and chemistry. Early experimental investigations by Rutherford established the nuclear model of the atom [1], while Bohr introduced the concept of discrete electronic energy levels to explain atomic spectra [2]. Sommerfeld subsequently extended this picture by incorporating angular momentum quantization and relativistic corrections [3]. These developments paved the way for the formulation of quantum mechanics, which replaced classical electron orbits with a wave-based description of electronic states.

The quantum-mechanical framework, formalized through the work of Schrödinger, Pauli, Born, and Dirac, provides a mathematically rigorous and highly successful description of atomic behavior [4]-[7]. Within this formalism, electrons are described by wavefunctions whose squared modulus gives the probability density of finding an electron in a given region of space. Atomic orbitals arise as solutions of the Schrödinger equation and are characterized by a set of quantum numbers that determine their energy, angular momentum, spatial orientation, and spin. This approach accurately predicts atomic spectra, selection rules, and chemical periodicity.

Despite its predictive success, the standard quantum-mechanical description of atomic structure remains largely abstract. Orbitals are mathematical constructs rather than physical objects, and the spatial organization of shells and subshells is encoded algebraically rather than explained geometrically. As a result, several central features of atomic structure are introduced as empirical rules. These include the shell capacity rule $N = 2n^2$, the subshell orbital multiplicity $2\ell + 1$, the Aufbau principle governing orbital filling, and the $n + \ell$ (Madelung) ordering rule [8]. While these rules are remarkably effective, they offer limited physical intuition, and their well-known deviations—such as the electron configurations of chromium and copper or the distinction between orbital filling order and ionization order in transition metals—are typically treated as exceptions rather than as predictable outcomes [9].

In parallel with the development of the standard formalism, alternative interpretative approaches have been explored that seek to provide a more physically intuitive picture of quantum phenomena. Early ideas by de Broglie and later developments by Bohm introduced wave-based and hydrodynamic interpretations in which quantum behavior emerges from structured motion rather than from purely abstract postulates [10] [11]. More recent work has shown that vortex dy-

namics and circulation in a continuous medium can reproduce key quantum features without altering the formal predictions of quantum mechanics [12].

Motivated by these ideas, the present work proposes a vortex-based reinterpretation of atomic structure. In this framework, electrons are modeled as localized vortex excitations embedded in a structured vacuum. Atomic shells are interpreted as concentric vortex structures, subshells as internal vortex layers, and orbitals as stable localization regions arising from vortex topology and overlap. The standard quantum numbers n, ℓ, m_ℓ, and s are retained without modification, but are given a coherent geometric interpretation within this hierarchical vortex organization [13]-[15].

Within this picture, fundamental results of atomic theory emerge naturally. The shell capacity rule and subshell orbital multiplicity follow from the cumulative construction of vortex layers, while the Aufbau principle and the $n+\ell$ rule arise from the three-dimensional geometry and radial overlap of subshell vortices. Apparent anomalies in electron configurations, such as those observed in chromium and copper, are interpreted as stability extrema of subshell vortex structures rather than as violations of filling rules. Likewise, the preferential removal of 4s electrons during ionization of transition metals is explained by the radial exposure of orbital localization regions after subshell formation.

Importantly, this reinterpretation remains compatible with experimental observations. Recent high-resolution optical experiments have demonstrated that even a single atom interacts with light over a spatially extended and structured region, rather than behaving as an idealized point scatterer [16]. Such findings support the physical plausibility of models in which atomic and subatomic entities possess internal spatial organization, while remaining consistent with conventional quantum electrodynamics.

The aim of this article is therefore not to replace quantum mechanics, but to complement it by providing a geometric and physically intuitive interpretation of atomic structure. By unifying shells, subshells, orbitals, quantum numbers, and electron configurations within a single vortex-based framework, the present work seeks to clarify the physical origin of well-established rules and to offer a coherent explanation for their scope and limitations.

2. Atomic Orbitals and Shell Structure

2.1. Atomic Orbitals: Wave-Particle Duality and Physical Meaning

With the advent of quantum mechanics, it became evident that electrons bound to an atomic nucleus cannot be described as classical particles moving along well-defined trajectories. Instead, electrons exhibit wave-particle duality, behaving as localized entities in measurement processes while simultaneously displaying wave-like properties such as interference and standing-wave formation [4] [6] [7].

In the quantum-mechanical framework, the state of an electron in an atom is described by a wavefunction $\psi(\boldsymbol{r})$, whose squared modulus $|\psi|^2$ represents the probability density of finding the electron at a given location. Atomic orbitals

arise as stationary solutions of the Schrödinger equation in the effective electrostatic potential created by the nucleus and the other electrons. These orbitals are therefore not physical trajectories, but probability distributions that encode the spatial structure of allowed electronic states [4].

Several experimental observations, however, demonstrate that electrons also possess well-defined particle-like properties. The number of electrons bound to a nucleus is always an integer, electronic transitions between states occur discretely, and interactions with electromagnetic radiation involve individual electrons absorbing or emitting single photons. Moreover, each electronic state carries a fixed electric charge and a quantized intrinsic angular momentum (spin) [5] [7]. These facts require a description that incorporates both wave-like and particle-like aspects.

Atomic orbitals are thus best understood as physically meaningful but nonclassical entities: they represent regions of space in which electrons can exist in stable quantum states, rather than literal paths of motion. In single-electron systems such as hydrogen, the orbital probability distribution provides a direct description of the electron's spatial organization. In multi-electron atoms, electron-electron interactions lead to an effective averaging of motion, resulting in the familiar concept of an electron cloud surrounding the nucleus [9].

While quantum mechanics precisely specifies the mathematical form of orbitals and their associated energies, it does not provide a direct physical explanation for *why* orbitals possess their characteristic spatial organization, nor how shells and subshells arise as a structured hierarchy. The concepts of shells, subshells, and orbitals are introduced through quantum numbers and boundary conditions, rather than through an explicit geometric mechanism.

In the present work, atomic orbitals are reinterpreted within a vortex-based framework, in which orbitals correspond to stable localization regions of an underlying structured flow. This reinterpretation does not modify the quantum-mechanical definition of orbitals or their associated quantum numbers. Instead, it seeks to provide a geometric and physical intuition for the spatial organization implied by the formalism. In the following sections, this approach is developed by first examining the structure of atomic shells and their relation to the principal quantum number.

2.2. Shell Geometry and the Principal Quantum Number

In quantum mechanics, the large-scale organization of atomic structure is described by the principal quantum number n, which defines the main energy levels, or shells, of an atom. All orbitals sharing the same value of n belong to the same shell. The principal quantum number determines the overall energy scale of electronic states as well as the characteristic radial extent of the electron probability distribution [4]-[7].

Experimentally and theoretically, shells with higher values of n are associated with electrons that are, on average, located farther from the nucleus and are more

weakly bound. As n increases, the energy spacing between successive shells decreases, and when sufficient energy is supplied, an electron may escape the atomic potential entirely, leading to ionization. These features are well established within the quantum-mechanical formalism and are reflected in atomic spectra and ionization energies.

Within the vortex-based interpretation proposed here, the principal quantum number n acquires a geometric meaning. Rather than representing only an abstract energy index, n is interpreted as the number of global shell evolutions of an underlying atomic vortex structure. Each shell corresponds to a concentric vortex layer surrounding the nucleus, and successive values of n represent progressively expanded vortex configurations.

For $n=1$, the shell consists of a single, compact vortex layer localized closest to the nucleus. As n increases, additional vortex layers form outward, producing a nested system of concentric shells. This geometric picture naturally mirrors the quantum-mechanical result that shells with higher n values have larger radial extent and reduced binding strength.

The systematic expansion of shell geometry also provides an intuitive basis for the shell capacity rule. In quantum mechanics, the total number of orbitals within a shell of principal quantum number n is n^2, and since each orbital can accommodate two electrons of opposite spin, the maximum number of electrons per shell is $N=2n^2$ [8]. In the vortex framework, this result emerges from the cumulative construction of internal subshell structures within each global shell, as will be discussed in the following section.

By starting from the shell as the highest level of organization, the vortex-based interpretation establishes a clear structural foundation for atomic architecture. Sublevels, subshells, and orbitals then arise naturally as internal features of this global geometry, rather than as independent abstract entities. This shell-first perspective sets the stage for a detailed examination of subshell structure and orbital multiplicity in the next section.

2.3. Shells, Sublevels, Subshells, and Orbitals

In quantum mechanics, the internal organization of each atomic shell is described by a hierarchy of quantum numbers. While the principal quantum number n defines the shell as a whole, the azimuthal quantum number ℓ determines the allowed sublevels within that shell. For a given value of n, the azimuthal quantum number takes integer values from $\ell=0$ to $\ell=n-1$, giving rise to n distinct sublevels within each shell [4]-[7].

These sublevels are conventionally labeled by capital letters S, P, D, F, G, H, I, corresponding to $\ell=0,1,2,3,4,5,6$, respectively. Each sublevel contains a cumulative set of subshells, denoted by the lowercase letters s, p, d, f, g, h, i.

Importantly, a sublevel does not correspond to a single subshell. Rather, each sublevel includes all subshells from s up to the subshell associated with that sublevel. Thus, the P sublevel contains the subshells s and p; the D sublevel con-

tains s, p, d; the F sublevel contains s, p, d, f; and so on. This cumulative structure is essential for understanding orbital multiplicity and electron capacity.

Within each subshell, the magnetic quantum number m_ℓ specifies the number of distinct orbitals. For a subshell characterized by azimuthal quantum number ℓ, the magnetic quantum number takes values from $m_\ell = -\ell$ to $+\ell$, yielding a total of $2\ell+1$ orbitals. Each orbital can accommodate two electrons of opposite spin, in accordance with the Pauli exclusion principle [5] [7].

As a result, the total number of orbitals associated with a given sublevel increases in the sequence 1, 3, 5, 7, …, corresponding to the subshells s, p, d, f, …. When these contributions are summed cumulatively within a shell, they yield the well-known result that a shell with principal quantum number n contains n^2 orbitals and can accommodate up to $2n^2$ electrons [8].

Table 1 summarizes the relationship between the principal quantum number, sublevel designation, subshell content, orbital multiplicity, and maximum electron capacity. It provides a compact overview of the hierarchical structure defined above.

Table 1. Relation between principal quantum number, sublevel, subshells, orbitals, and maximum electron capacity.

Principal quantum number *n*	Sublevel (capital)	Subshells present	Number of orbitals	Maximum electrons
1	S	s	1	2
2	P	s, p	3	6
3	D	s, p, d	5	10
4	F	s, p, d, f	7	14
5	G	s, p, d, f, g	9	18
6	H	s, p, d, f, g, h	11	22
7	I	s, p, d, f, g, h, i	13	26

While this hierarchy is rigorously defined within the quantum-mechanical formalism, it is introduced algebraically rather than geometrically. In the vortex-based interpretation developed in this work, shells, sublevels, subshells, and orbitals acquire a spatial organization, in which sublevels and subshells emerge as internal structures within a global shell geometry.

2.4. Principal Quantum Number *n* and Shell Structure

In quantum mechanics, the principal quantum number n defines the global energy level and radial scale of an atomic shell. All orbitals sharing the same value of n belong to the same shell, and shells with increasing n are characterized by larger spatial extent and lower binding energy [4]-[7]. As n increases, the spacing between adjacent energy levels decreases, and sufficiently energetic electrons may escape the atomic potential, leading to ionization.

Within the vortex-based framework, the principal quantum number n acquires a direct geometric interpretation. Each shell corresponds to a global vortex structure surrounding the nucleus, and the value of n represents the number of successive shell evolutions of this vortex. In this picture, shells are not abstract energy labels but concentric vortex layers that expand outward as n increases.

For $n=1$, the atomic structure consists of a single, compact vortex layer localized closest to the nucleus. As n increases, additional vortex layers are formed, producing a nested sequence of shells with progressively larger radial extent. This geometric expansion mirrors the quantum-mechanical result that electrons in higher shells are, on average, located farther from the nucleus and are more weakly bound.

The shell capacity rule follows naturally from this construction. In quantum mechanics, a shell with principal quantum number n contains n^2 orbitals and can therefore accommodate up to $2n^2$ electrons [8]. In the vortex framework, this result arises from the cumulative inclusion of all subshell vortex layers permitted within a given shell. Each additional shell evolution introduces new internal structures that increase the number of available orbital localization regions in a systematic manner.

Importantly, the expansion of shell geometry does not imply uniform radial growth of all subshells. As shells and subshells develop, internal vortex layers may overlap and reorganize, leading to nontrivial radial embedding of orbitals. This feature becomes essential for understanding the order of orbital filling and ionization, particularly in multi-electron atoms and transition elements.

Thus, the principal quantum number n governs both the energetic and geometric scale of atomic shells. While its formal definition remains unchanged, the vortex-based interpretation provides a physical picture in which shell structure emerges as a hierarchical, spatially organized vortex system. This perspective sets the foundation for analyzing subshell geometry, orbital localization, and electron configurations in the sections that follow.

2.5. Three-Dimensional Visualization of Shell Structure

The shell structure of the atom can be represented schematically as a three-dimensional vortex geometry, in which each shell corresponds to a distinct concentric vortex layer surrounding the nucleus. As an illustrative example, the element boron ($Z = 5$) possesses electrons distributed across three shells corresponding to the principal quantum numbers $n=1,2$ and 3. This configuration can be visualized as three nested vortex layers with progressively increasing spatial extent.

Figure 1 shows a schematic three-dimensional representation of this shell structure, highlighting the concentric organization of vortex layers associated with increasing values of the principal quantum number. The figure provides a geometric visualization of shell hierarchy rather than an experimental image.

By starting from the shell and the principal quantum number, the vortex-based interpretation establishes a clear geometric foundation for atomic structure. Sub-

shells and orbitals then arise naturally as internal features of this global shell geometry, rather than being introduced as abstract entities. This shell-first perspective provides the structural basis for understanding subshell organization, orbital multiplicity, and electron localization developed in the subsequent sections.

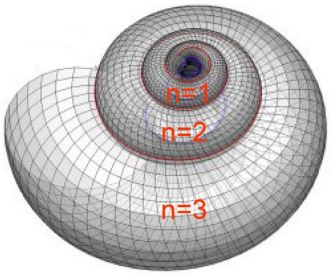


Figure 1. Schematic three-dimensional representation of atomic shell structure, illustrating concentric vortex layers corresponding to increasing values of the principal quantum number n. Boron ($Z = 5$) is shown as an example with three shells.

2.6. Experimental Evidence of Structured Atomic Interaction Regions

While atomic orbitals are defined in quantum mechanics as probability distributions rather than directly observable objects, recent experimental advances have demonstrated that atoms interact with electromagnetic radiation over spatially extended and structured regions, rather than as idealized point scatterers. These observations provide important empirical support for interpretations in which atomic structure possesses intrinsic spatial organization.

A particularly relevant example is the ultra-high-resolution absorption imaging of a single trapped atom performed by Streed *et al.* [16]. In this experiment, individual ytterbium ions were confined in free space using electromagnetic trapping techniques and illuminated with resonant light. By employing an optical system operating near the diffraction limit, the spatial distribution of absorbed light was recorded directly, producing a measurable absorption "shadow" cast by a single atom.

The experimentally observed absorption profile, shown in **Figure 2**, is not point-like but exhibits a finite spatial extent and internal structure determined by the atom-light interaction. While standard quantum electrodynamics accurately predicts the total absorption strength and contrast of such images, it treats the atom as a localized scattering center and does not attribute a specific internal spatial organization to the atomic entity beyond probabilistic interpretation.

In this experiment, individual ytterbium ions were confined in free space using electromagnetic trapping techniques and illuminated with resonant light. By employing an ultra-high-numerical-aperture optical system, the spatial distribution of absorbed light was recorded on a detector, producing a visible shadow cast by a single atom. The results demonstrated that even an isolated atom produces a

measurable and structured interaction profile with incident light when the optical system approaches the fundamental limits imposed by diffraction and atomic absorption cross sections.

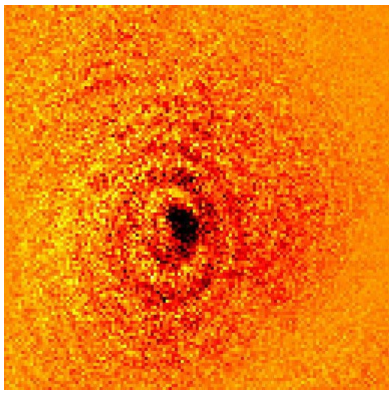


Figure 2. Absorption image of a single trapped ytterbium atom, showing a finite and structured atom-light interaction region (after Streed *et al.* [16]).

The recorded shadow is not point-like, but exhibits a finite spatial extent determined by the atom-light interaction and the electromagnetic response of the atom (**Figure 2**). While standard quantum mechanics and quantum electrodynamics accurately predict the total absorption strength and contrast of such images, they do not assign a physical spatial structure to the atom itself beyond a probabilistic scattering center.

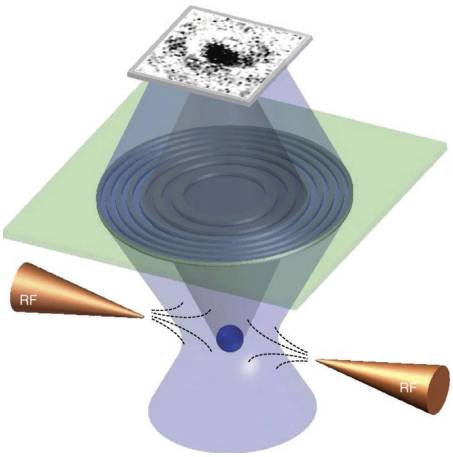


Figure 3. Schematic vortex-based representation illustrating a possible geometric origin of the spatially extended atomic interaction region observed experimentally. The figure is a conceptual visualization and does not represent experimental data.

Within the vortex-based quantum framework, this observation acquires a natural geometric interpretation. An atom is not treated as a structureless point, but as a localized, organized vortex system embedded in a structured vacuum. The

finite spatial extent and symmetry of the observed absorption profile are consistent with the presence of a stable interaction region surrounding the atom, arising from coherent rotational and oscillatory dynamics rather than from a purely abstract point interaction (Figure 3).

It is important to emphasize that this experiment does not directly image internal atomic motion or vortex circulation. However, it provides empirical confirmation that a single atom interacts with light over a spatially extended and structured region, rather than behaving as an idealized mathematical point. Such observations support the physical plausibility of models in which atomic and subatomic entities possess internal organization and spatially coherent interaction zones, as proposed in vortex-based approaches to quantum structure [9]-[12].

Taken together, the experimental observation and its schematic interpretation provide an important experimental anchor for the geometric reinterpretation of atomic structure developed in this work. While remaining fully compatible with conventional quantum mechanics and quantum electrodynamics, these results motivate the exploration of physically intuitive models in which atomic and subatomic entities possess organized spatial structure.

2.7. Subshells and Orbitals as Vortex Structures

Within the vortex-based interpretation, an atomic shell is described as a three-dimensional spiral vortex tube whose global geometry develops progressively with increasing principal quantum number. A shell is not a homogeneous structure, but a composite object formed by the coupling of **internal vortex layers**, each corresponding to an allowed subshell within that shell.

As electrons are added to an atom, additional subshells become available and the shell acquires increasing internal complexity. Geometrically, the shell may be viewed as a **spiral conical structure**, formed by the cumulative stacking of subshell vortex layers extending outward from the nuclear core. The number of subshells contained within a shell increases with the principal quantum number, giving rise to a progressively richer internal vortex topology.

2.7.1. The s Subshell as the Fundamental Building Block

The most fundamental element of this structure is the **s subshell**, which occupies the apex of the shell cone and is present in every shell. As illustrated in Figure 4, the s subshell is represented by a vortex structure composed of **two symmetric spiral arms** winding inward toward the shell interior. At the wider end of each spiral arm, a localized secondary vortex is formed, which acts as a **stable electron localization region**.

These two localized vortices correspond to the two allowed electron states of a single s orbital. Their opposite circulation provides a geometric interpretation of paired electron occupancy, consistent with the Pauli exclusion principle and the existence of two spin states. The s subshell therefore constitutes the **core geometry** upon which all higher subshell structures are built.

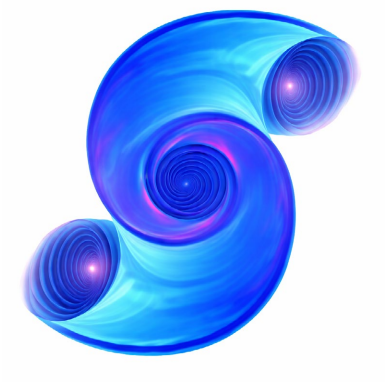


Figure 4. Schematic vortex-based representation of an s subshell located at the apex of the shell cone. The structure consists of two oppositely oriented spiral vortex arms terminating in localized vortex regions that act as electron capture sites, corresponding to the two-electron capacity of a single s orbital. The figure is a conceptual visualization.

2.7.2. Cumulative Construction of Higher Sublevels

Each successive sublevel (p, d, f, …) is constructed by adding **one additional vortex layer** to the complete structure of the preceding sublevel. Each added vortex layer consists of two spiral arms and introduces **two additional orbital localization regions**, while preserving the entire geometry of all lower subshells.

The **p sublevel** therefore contains the complete s subshell together with one additional vortex layer corresponding to the p subshell. As shown in Figure 5, this structure gives rise to **three orbitals** in total: one associated with the inner s subshell and two associated with the p subshell.

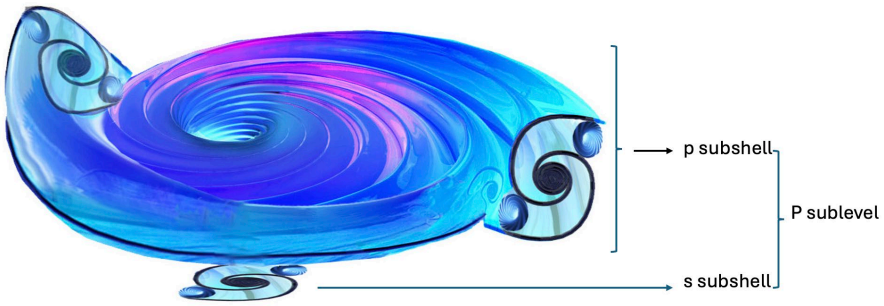


Figure 5. Schematic vortex-based representation of the p sublevel, composed of an inner s subshell and an outer p subshell formed by two spiral arms, together giving three orbitals in total. The figure is a conceptual visualization. In this way, each new sublevel introduces two additional orbitals, while preserving the complete structure of all lower subshells.

The same construction principle applies to higher sublevels. The **d sublevel** contains five orbitals, and the **f sublevel** contains seven orbitals, each obtained by adding one vortex layer to the previous structure. In general, each increase in sublevel introduces two additional orbitals, yielding the standard quantum-mechanical result that a sublevel characterized by azimuthal quantum number ℓ

contains $2\ell+1$ orbitals.

2.7.3. Radial Scaling and Shell Geometry

Within this framework, each newly added subshell vortex layer occupies a **larger radial domain** in order to accommodate the expanding geometry of the shell cone. As the principal quantum number increases, the global shell structure expands outward, and the radii of the subshell vortex layers increase accordingly. Consequently, orbitals are distinguished not only by their angular orientation but also by their **radial extension**, in agreement with the quantum-mechanical increase of the electron-nucleus distance for higher shells.

This behavior arises naturally from geometric constraints rather than from an imposed radial quantization rule. Although all subshell vortex layers share the same fundamental topology—paired spiral arms with localized capture regions—their relative spatial orientation and radial embedding differ, giving rise to the full diversity of orbital structures.

Example: The G Sublevel ($n = 5$)

Elements with principal quantum number $n = 5$ are characterized by a shell whose highest allowed sublevel corresponds to the **G sublevel**. In this case, the atomic structure comprises **five concentric shells**, each containing a cumulative set of subshells:

s; s, p; s, p, d; s, p, d, f; and s, p, d, f, g.

As illustrated in Figure 6, this cumulative arrangement forms a hierarchical three-dimensional structure composed of nested vortex layers with progressively increasing internal complexity. The figure provides a static geometric visualization of how subshell vortex layers are organized within a shell.

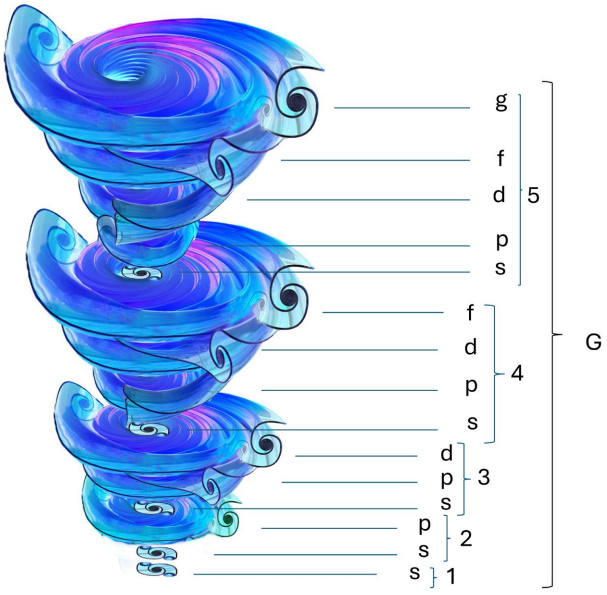


Figure 6. Three-dimensional schematic representation of the electron orbital structure of the G sublevel ($n = 5$), showing five concentric shells with a cumulative increase in subshells from s to g.

3. Quantum Numbers: Geometric Interpretation in the Vortex Framework

In standard quantum mechanics, the electronic structure of atoms is described by four quantum numbers: the principal quantum number n, the azimuthal quantum number ℓ, the magnetic quantum number m_ℓ, and the spin quantum number s. These quantum numbers arise from the mathematical structure of the Schrödinger equation and provide a complete classification of allowed electronic states.

In the present work, these quantum numbers are **not redefined**. Instead, they are given a **coherent geometric interpretation** within the vortex-based framework developed in the preceding sections. This approach preserves the orthodox quantum-mechanical formalism while providing a physically intuitive picture of atomic organization.

3.1. Principal Quantum Number (n)

The principal quantum number n defines the global energy level and spatial scale of an atomic shell. All orbitals sharing the same value of n belong to the same shell.

Within the vortex-based interpretation, n corresponds to the number of **global vortex shell evolutions** surrounding the nucleus. Each increase in n introduces a new concentric vortex layer, leading to a systematic radial expansion of the atomic structure. This geometric interpretation is consistent with the quantum-mechanical result that shells with higher n values exhibit larger spatial extent and lower binding energy.

3.2. Azimuthal Quantum Number (ℓ)

For a given principal quantum number n, the azimuthal quantum number ℓ takes integer values from 0 to $n-1$ and defines the allowed **subshells** within a shell. In standard quantum mechanics, ℓ determines the magnitude of the orbital angular momentum and is associated with the characteristic spatial form of the subshell.

The subshells are conventionally denoted by s, p, d, f, g, h, i, corresponding to $\ell = 0,1,2,3,4,5,6$, respectively.

Within the vortex-based framework, ℓ is interpreted as the number of **internal subshell vortex layers** embedded within the global shell vortex. Each subshell is constructed cumulatively by adding one additional vortex layer to the complete structure of the preceding subshell. Each added layer consists of two spiral arms and introduces **two new orbital localization regions**.

The simplest case is the s subshell ($\ell = 0$), which forms the core structure present in every shell. Its basic vortex topology is illustrated in Figure 7.

The cumulative construction of higher subshells from s through g is shown schematically in Figure 8.

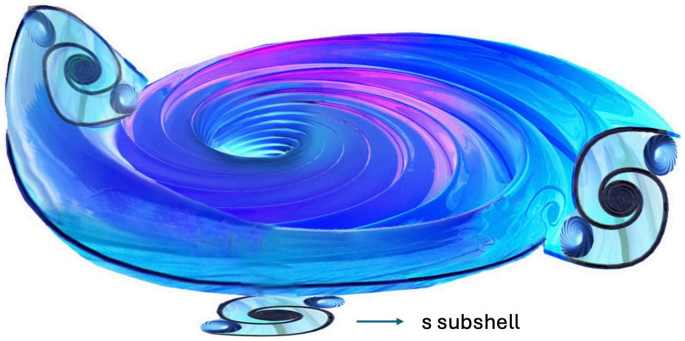


Figure 7. Schematic vortex-based representation of the s subshell ($\ell = 0$), illustrating the fundamental vortex topology that forms the core structure of every shell. The subshell consists of a single internal vortex layer with two spiral arms terminating in localized vortex regions corresponding to an orbital.

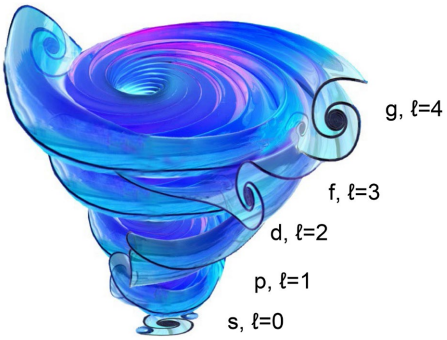


Figure 8. Cumulative vortex-based representation of subshells s-g ($\ell = 0$ - 4), showing successive internal vortex layers within a shell.

This construction yields a total number of orbitals given by

$$2\ell + 1,$$

in exact agreement with standard quantum mechanics. Higher subshells therefore always contain the complete geometric structure of all lower subshells.

3.3. Magnetic Quantum Number (m_ℓ)

The magnetic quantum number m_ℓ specifies the allowed **spatial orientations** of orbitals within a given subshell. For a subshell characterized by azimuthal quantum number ℓ, m_ℓ takes integer values from $-\ell$ to $+\ell$, yielding $2\ell+1$ distinct orbital orientations.

The physical origin of m_ℓ was first revealed experimentally through the Zeeman effect, in which atomic spectral lines split in the presence of an external magnetic field. This splitting reflects the existence of multiple angular momentum projections associated with a given subshell.

Within the vortex-based interpretation, m_ℓ corresponds to the **relative angular orientation of orbital localization regions** generated by the subshell vortex structure. While all orbitals within a subshell share the same fundamental topology, they differ in their spatial orientation with respect to the global shell geometry.

For the s subshell ($\ell = 0$), only a single orbital exists, corresponding to $m_\ell = 0$. This centrally symmetric configuration is illustrated schematically in Figure 9.

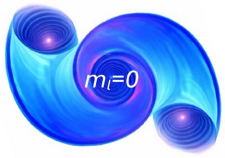


Figure 9. Schematic vortex-based representation of the s subshell ($\ell = 0$), showing a single orbital orientation corresponding to $m_\ell = 0$.

For the p subshell ($\ell = 1$), three orbitals are present, corresponding to $m_\ell = -1, 0, +1$. These are conventionally labeled p_x, p_y, and p_z, reflecting their alignment along orthogonal spatial axes. Their vortex-based representation is shown in Figure 10.

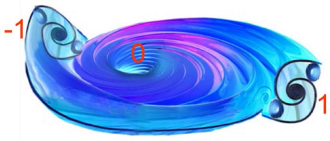
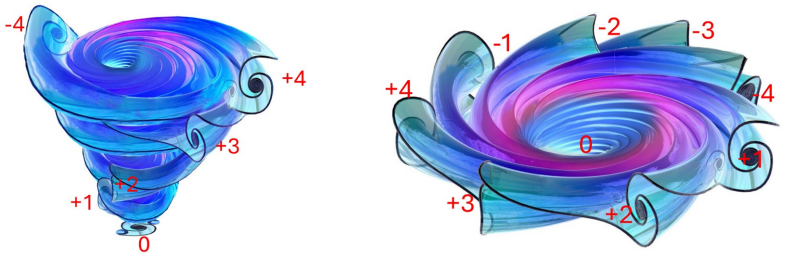


Figure 10. Vortex-based schematic of the three p orbitals ($\ell = 1$), corresponding to $m_\ell = -1, 0, +1$ and labeled p_x, p_y, and p_z.

For higher subshells, the same rule applies. For example, the g subshell ($\ell = 4$) contains nine orbitals with magnetic quantum numbers ranging from $m_\ell = -4$ to +4. A schematic three-dimensional representation of these orientations is shown in Figure 11(a), while alternative projections highlighting symmetry relationships are shown in Figure 11(b).



(a) (b)

Figure 11. (a) Schematic vortex-based representation of orbital orientations within the G sublevel (n = 5), illustrating the nine magnetic quantum number values ($m_\ell = -4$ to +4) associated with the g subshell. The figure shows the full three-dimensional arrangement, in which some orientations may overlap in projection; (b) Alternative schematic projection of the G-sublevel orbitals shown in Figure 10a, displayed in selected planes (e.g., g_x - g_z) to make the relative orientations and opposite pairing of orbitals corresponding to different magnetic quantum numbers more clearly visible. The figures are conceptual visualizations.

3.4. Spin Quantum Number (s) and the Pauli Exclusion Principle

The spin quantum number s characterizes the intrinsic angular momentum of the electron and takes two allowed values, $+\frac{1}{2}$ and $-\frac{1}{2}$. In standard quantum mechanics, spin does not correspond to literal mechanical rotation, a statement that remains valid in the vortex-based framework.

In this interpretation, the electron is modeled as a **continuous vortex excitation** with non-rigid, differential circulation. The rotational velocity is highest near the vortex core and decreases continuously toward the periphery, avoiding any requirement for superluminal tangential velocities.

Within the vortex-based framework, electron spin corresponds to a **quantized circulation state (chirality)** of the electron vortex. The two allowed spin states represent two stable circulation orientations of the vortex structure. This interpretation is illustrated schematically in Figure 12.

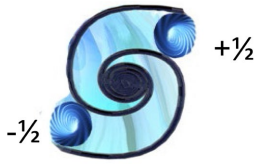


Figure 12. Schematic vortex-based representation of the two allowed spin states of the electron ($s=+\frac{1}{2}$ and $s=-\frac{1}{2}$), illustrated as opposite circulation orientations (chirality states) of an electron vortex. The figure is a conceptual visualization.

For two electrons occupying the same orbital localization region, the **Pauli Exclusion Principle** requires that their spin quantum numbers be opposite. In the vortex-based interpretation, this requirement corresponds to a **dynamical stability condition**: two electron vortices sharing the same orbital must adopt opposite circulation states, ensuring cancellation of net circulation and magnetic moment. As a result, orbitals may accommodate at most two electrons with opposite spin. Atoms with all electrons spin-paired are **diamagnetic**, whereas atoms containing one or more unpaired electrons are **paramagnetic**.

4. Electron Configurations and Orbital Filling Order

Each atom consists of a positively charged nucleus surrounded by electrons distributed in discrete quantum states. In quantum mechanics, electrons do not move along classical trajectories but occupy **orbitals**, which are regions of space where the probability of finding an electron is maximal. The arrangement of electrons among these orbitals in the ground state of an atom is referred to as its **electron configuration**.

Electron configurations determine many fundamental physical and chemical properties of elements, including periodic trends, bonding behavior, magnetic

properties, and ionization processes. In the conventional quantum-mechanical description, electrons are treated as moving independently in an average field created by the nucleus and the other electrons, and configurations are specified using shells, subshells, and orbitals defined by the quantum numbers n, ℓ, m_ℓ, s [4]-[7].

Within the vortex-based framework developed in this work, electron configurations acquire a **geometric interpretation**. Each electron occupies a dynamically stable orbital localization region within a structured vortex system. The **ground state** corresponds to the configuration in which electrons occupy the most stable localization regions available during shell formation, while any departure from this arrangement represents an **excited state**.

4.1. Aufbau Principle and the $n+\ell$ (Madelung) Rule

The empirical rules governing electron configurations are commonly summarized by the **Aufbau principle**, which states that electrons are added sequentially to the lowest available energy orbitals, subject to the Pauli exclusion principle and Hund's rules. The approximate ordering of orbital energies in neutral atoms is described by the $n+\ell$ **rule** (Madelung rule): orbitals with lower values of $n+\ell$ are filled first, and for equal $n+\ell$, the orbital with the lower value of n is filled first [8].

Although widely successful, this rule is empirical and is known to exhibit deviations, particularly among transition metals and heavier elements. These deviations have been systematically analyzed in the literature and are often described as "configuration irregularities" [9].

4.2. Geometric Origin of Orbital Overlap and Filling Order

4.2.1. Subshell Geometry without Overlap

In the lowest atomic shells, subshell vortex layers remain largely **radially separated**. The s subshell forms the innermost vortex layer, while higher subshells (p, d, …) appear progressively farther from the nucleus without significant interpenetration. In this regime, orbital localization regions are clearly ordered by radial distance, and the filling sequence follows the intuitive subshell order.

This situation is illustrated schematically in **Figure 13(a)**, which shows subshell vortex layers arranged without overlap. Under these conditions, the Aufbau principle follows directly from radial proximity to the nucleus.

4.2.2. Onset of Subshell Overlap and Orbital Inversion

As the principal quantum number increases, subshell vortex layers acquire greater curvature and radial extent. Beyond the first few shells, the apical region of a higher-ℓ subshell vortex may **penetrate into the conical structure of a lower-ℓ subshell** belonging to the preceding shell. This geometric overlap alters the effective radial ordering of orbital localization regions.

This regime is illustrated schematically in **Figure 13(b)**, which shows penetration of a d-subshell vortex into the cone of an s-subshell vortex from the previous shell.

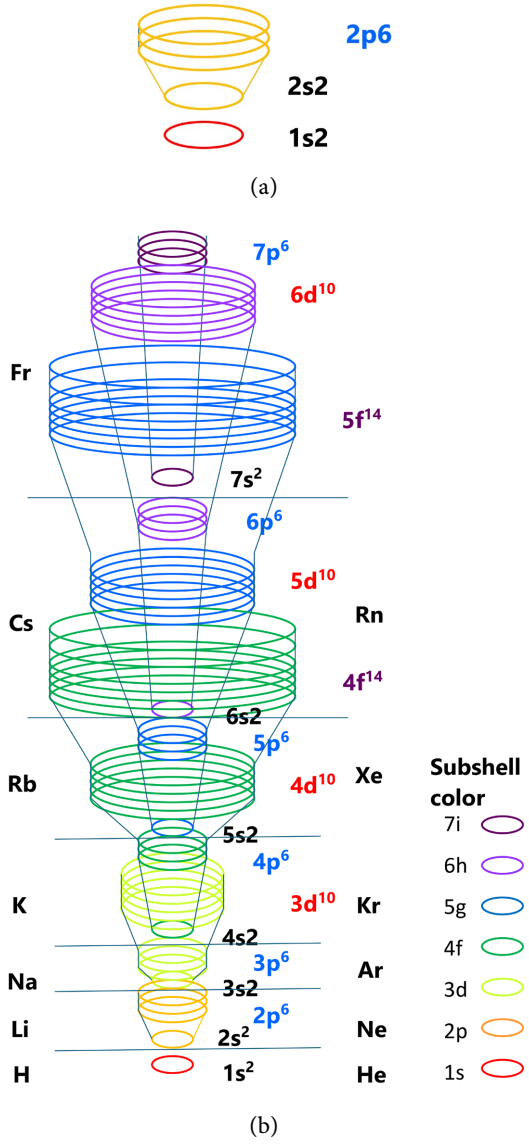


Figure 13. (a) Schematic vortex-based representation of subshell geometry without radial overlap, illustrating early atomic shells in which subshell vortex layers remain radially separated. In this regime, orbital filling follows simple radial ordering, and the Aufbau principle applies directly. Individual subshells are represented by circles of different diameters, where each circle corresponds to a single orbital with a maximum capacity of two electrons. The number of circles indicates the number of orbitals within each subshell. The filling sequence follows the geometric order in which orbital localization regions appear in the diagram. (b) Schematic vortex-based representation of subshell overlap, showing penetration of a higher-ℓ subshell vortex into the cone of a lower-ℓ subshell from a preceding shell. This geometric overlap alters radial ordering and leads naturally to orbital filling inversion (e.g., 4s before 3d).

As a result, although the s orbital belongs to a higher principal quantum number, its localization region lies closer to the nucleus during filling. This explains why orbitals such as 4s are filled before 3d, despite having a higher value of n. The inversion arises naturally from vortex topology and does not require modifi-

cation of the quantum-mechanical formalism.

4.3. Filling Order versus Ionization Order

In multi-electron atoms, the order in which orbitals are filled during electron addition does not necessarily coincide with the order in which electrons are removed during ionization. This distinction is particularly evident in transition metals and is often introduced as an empirical rule.

Within the vortex-based framework, this behavior follows from **geometric reorganization and radial exposure** of orbital localization regions once the atom is fully formed. During electron addition, orbitals are filled according to the availability of stable localization regions within the evolving shell geometry. After subshell vortices are established, subshell contraction and overlap may alter the relative radial positioning of orbitals.

As a result, orbitals that become more radially exposed after shell stabilization are preferentially ionized, even if they were filled earlier. This explains the preferential removal of 4s electrons before 3d electrons in transition metals.

4.4. Radial Contraction within a Shell and Periodic Decrease of Atomic Radius

A well-established empirical fact of atomic physics is that, **within a given principal quantum number**, the atomic radius decreases progressively as the atomic number increases. For example, across a period from left to right, elements become systematically smaller despite the addition of electrons to the same shell. In the conventional quantum-mechanical description, this behavior is attributed to increasing effective nuclear charge and partial screening by inner electrons.

Within the vortex-based framework, this phenomenon acquires a **direct geometric interpretation**.

As electrons are added to orbitals belonging to the same shell, they populate subshell vortex layers that progressively increase in coherence and symmetry. Each additional electron contributes to the stabilization of the subshell vortex structure, strengthening its coupling to the nuclear core. As a result, the subshell vortex undergoes **radial contraction**, drawing its orbital localization regions closer to the nucleus.

This contraction is not driven by direct electron-electron repulsion, but by enhanced nuclear attraction acting on an increasingly coherent vortex configuration. In particular, half-filled and fully filled subshells represent extrema of geometric symmetry and dynamical stability, and therefore contract more strongly than partially filled subshells.

Consequently, even though electrons are added to the same principal shell, the effective radial extent of the shell decreases across a period. The observed reduction of atomic radius with increasing electron number thus emerges naturally from the progressive stabilization and inward contraction of subshell vortex layers.

This same geometric mechanism underlies more pronounced effects observed

in transition elements, where subshell contraction not only modifies atomic size but also alters orbital energy ordering, filling sequences, and ionization behavior, as discussed below for chromium and copper.

4.4.1. Chromium: Half-Filled Subshell Stability

This geometric mechanism resolves the well-known case of chromium, which is traditionally presented as an exception to the Aufbau rule. The observed ground-state configuration of chromium is

$$\mathrm{Cr}:[\mathrm{Ar}]4s^1 3d^5,$$

rather than the naïvely expected $[\mathrm{Ar}]4s^2 3d^4$.

As illustrated schematically in Figure 14(a), prior to electron redistribution the partially filled $3d^4$ subshell forms an asymmetric and weakly coherent vortex layer. When one electron transfers from the 4s subshell into the 3d subshell, the resulting half-filled $3d^5$ configuration becomes highly symmetric and dynamically stable. This increased stability leads to **inward contraction of the 3d vortex layer**, as shown in Figure 14(b), lowering the total energy of the system and making this configuration energetically favored.

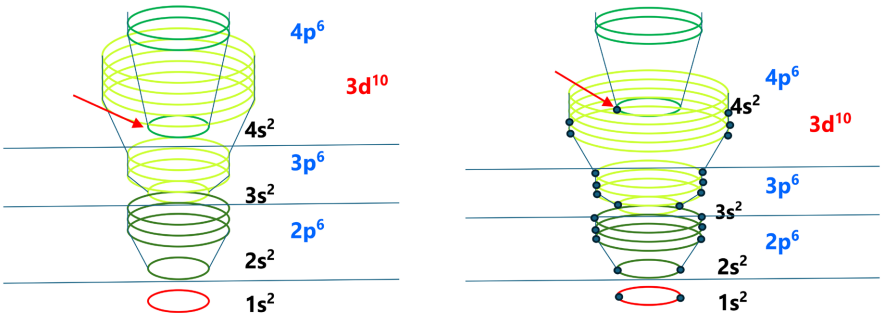


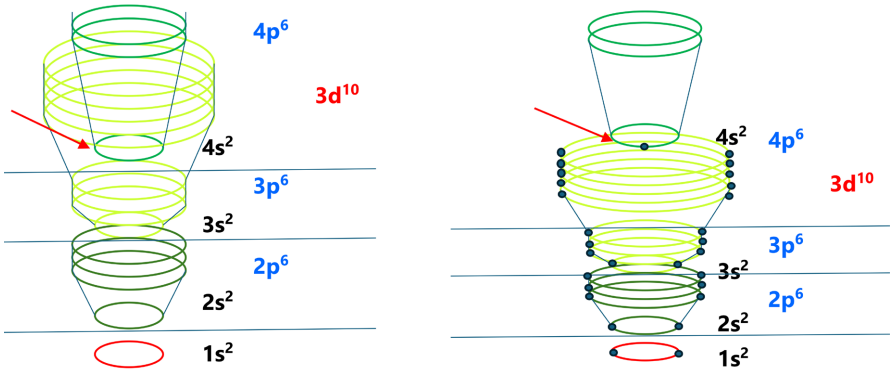
Figure 14. (a) Schematic vortex-based representation of chromium prior to electron redistribution, showing $4s^2$ closer to the nucleus than 3d. (b) After redistribution to the $3d^5$ configuration, the half-filled d subshell forms a symmetric and dynamically stable vortex layer that contracts inward toward the nucleus.

4.4.2. Copper: Fully Filled Subshell Stability

A similar stabilization mechanism applies to copper. The observed ground-state configuration,

$$\mathrm{Cu}:[\mathrm{Ar}]4s^1 3d^{10},$$

arises because a fully filled $3d^{10}$ subshell constitutes a **maximally coherent vortex layer.** Retaining the configuration $[\mathrm{Ar}]4s^2 3d^9$ would leave the d-subshell vortex incomplete and less stable. The system therefore reorganizes to achieve full d-subshell closure, accompanied by inward contraction of the 3d vortex and outward displacement of the 4s localization region, as illustrated in Figure 15(a) and Figure 15(b).



(a) (b)

Figure 15. (a) Schematic illustration of ionization behavior in copper, showing inward contraction of the d-subshell vortex and outward displacement of the 4s localization region. (b) Electron removal proceeds from the most radially exposed orbitals, independent of filling order.

4.4.3. Ionization of Transition Metals

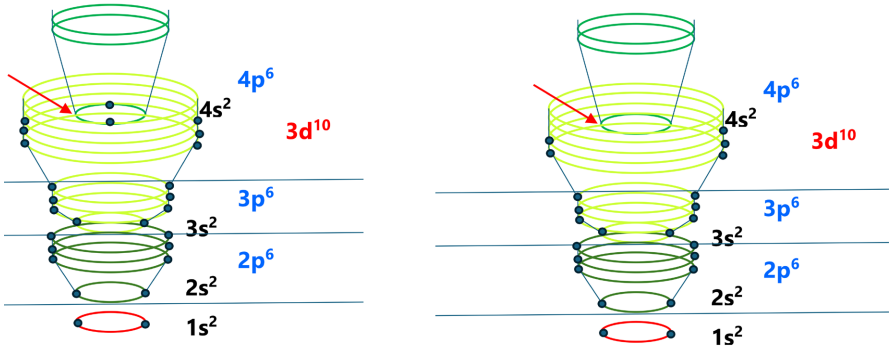
Upon ionization, electrons are removed preferentially from the most radially exposed localization regions, rather than from the subshell that filled last. For iron, the neutral atom has the configuration

$$\mathrm{Fe}:[\mathrm{Ar}]3\mathrm{d}^6\,4\mathrm{s}^2,$$

yet ionization proceeds by removal of the 4s electrons first, yielding

$$\mathrm{Fe}^{3+}:[\mathrm{Ar}]3\mathrm{d}^5.$$

This behavior is illustrated schematically in **Figure 16**. Within the vortex-based interpretation, the progressive filling of the d subshell causes its vortex structure to contract inward and become more deeply embedded in the nuclear region.



(a) $\mathrm{Fe}:[\mathrm{Ar}]3\mathrm{d}^6\,4\mathrm{s}^2$, (b) $\mathrm{Fe}^{3+}:[\mathrm{Ar}]3\mathrm{d}^5$.

Figure 16. (a) Schematic illustration of ionization behavior in Fe, showing inward contraction of the d-subshell vortex and outward displacement of the 4s localization region. (b) Electron removal proceeds from the most radially exposed orbitals, independent of filling order.

As the d vortex claims the inner domain, the 4s localization region is displaced

outward and becomes less tightly bound. Electron removal therefore follows **radial exposure**, not the original filling sequence.

The same mechanism applies across the transition series from scandium to zinc, as well as to chromium and copper. What appear as empirical irregularities in standard quantum mechanics emerge here as **predictable outcomes of subshell contraction, overlap, and vortex stability**.

4.5. Summary of Electron Configuration Mechanisms

Electron configurations within the vortex-based framework emerge from the interplay of **shell expansion, subshell overlap, radial scaling, and dynamical stability**. The Aufbau principle and the $n+\ell$ rule remain valid as effective descriptions of filling behavior, while their apparent deviations arise naturally from geometric transitions within the shell vortex.

Filling order, ionization order, and subshell stability are unified within a single geometric mechanism. Classical "exceptions" such as chromium and copper correspond to stability extrema of half-filled or fully filled subshell vortices rather than to anomalies in quantum mechanics.

5. Unified Counting Law from Atomic Shells to the 3D Gastropod Periodic System

The vortex-based atomic framework developed in this work yields a central quantitative result: a shell with principal quantum number n contains n^2 orbitals and therefore accommodates up to

$$N_e(n) = 2n^2$$

electrons. This counting law is not introduced as an empirical rule, but follows from the hierarchical construction of subshell vortex layers, where each increase in ℓ contributes two additional orbital localization regions and the total orbital count per shell becomes $1+3+5+\cdots+(2n-1)=n^2$.

A key claim of the unified vortex approach is that the same organizing principle that governs orbital capacity within atoms also governs the recurrence of elements across the periodic system. In the previously published 3D gastropod-shell periodic model, the periodic system is divided into seven levels (corresponding to the seven principal shells), and each level contains a specific number of "periods" (defined as 360˚ spiral turns) that determines how many elements appear in that level [14].

5.1. Period Lengths as Shell-Capacity Counting

In a previously published work, a three-dimensional periodic system based on a gastropod-shell geometry was proposed, in which elements are arranged along spiral and vortex structures rather than in a planar table [17]. In the gastropod model, the number of elements in each level reproduces the same numerical sequence familiar from atomic shell capacities:

$$2,8,8,18,18,32,32.$$

These numbers can be computed from the number of spiral turns ("periods") assigned to each level. For the first five levels, the number of elements N_e is given by

$$N_e = 2(l+1)^2,$$

and for levels 6 and 7 by

$$N_e = 2l^2,$$

where l denotes the number of periods (spiral turns) assigned to the level.

Using the assignments described in that paper—Level 1 has no period (2 elements), Levels 2 - 3 have one period (8 elements each), Levels 4 - 5 have two periods (18 elements each), and Levels 6 - 7 have four periods (32 elements each)—one recovers the standard period lengths.

5.2. Why This Indicates a Single Organizing Principle

The central unifying point is that the "period length" of the chemical system is not an independent law. It reflects the number of available stable electron-localization states produced by the same vortex hierarchy that organizes shells and subshells within atoms. In the atomic description, increasing n expands the global vortex shell and increases the number of available orbital localization regions. In the gastropod periodic system, increasing the level and/or the number of spiral turns increases the number of available element configurations expressed in that level—recovering exactly the same counting sequence.

Thus, the periodic recurrence of chemical properties across levels is interpreted as the macroscopic manifestation of the same vortex geometry that governs electron capacity within shells. In this sense, the 3D gastropod periodic system is not merely a graphical rearrangement of the table, but a scale-up of the same hierarchical vortex construction described in the present atomic model.

5.3. Placement of the Gastropod Model in the Present Paper

The present article establishes the microscopic foundation: shells, subshells, orbital multiplicity, overlap, and stability are derived geometrically within a vortex-based description. The earlier gastropod-shell periodic system provides the macroscopic counterpart: a 3D spiral arrangement in which levels and turns reproduce the same electron-capacity/period-length sequence and resolve long-standing placement discontinuities in the conventional periodic table.

Together, these results support the hypothesis that atomic structure and chemical periodicity are governed by a unified vortex principle operating across scales.

The mechanisms developed in the preceding sections establish a coherent geometric description of atomic structure and electron configuration, from shell formation and subshell overlap to orbital filling and ionization behavior. These results naturally raise a broader question: whether the same organizing principles that govern the internal structure of atoms also determine the large-scale recur-

rence of chemical properties across the periodic system. In the following section, this question is addressed by showing that the counting laws and geometric constraints derived at the atomic level extend directly to the three-dimensional organization of elements in the gastropod-shell periodic system, indicating that atomic structure and chemical periodicity are governed by a unified vortex-based principle.

6. Conclusions

In this work, a vortex-based geometric interpretation of atomic structure has been presented, offering a physically intuitive framework for understanding shells, subshells, orbitals, quantum numbers, and electron configurations without altering the formal structure of quantum mechanics. Rather than introducing new postulates or replacing established theory, the approach assigns a coherent spatial meaning to existing quantum numbers and empirical rules by interpreting them as manifestations of a hierarchical vortex organization embedded in a structured vacuum.

At the atomic scale, shells emerge as global vortex structures whose expansion is governed by the principal quantum number. Subshells arise as internal vortex layers within these shells, and orbitals correspond to dynamically stable localization regions associated with vortex topology. This construction naturally reproduces fundamental results of quantum mechanics, including the shell capacity rule $2n^2$, subshell orbital multiplicity $2\ell+1$, and the allowed orientations specified by the magnetic quantum number. Electron spin is interpreted as a quantized circulation state of the electron vortex, providing a natural geometric basis for the Pauli exclusion principle as a dynamical stability condition.

Electron configurations and orbital filling behavior are shown to follow from the geometric evolution of subshell vortex layers. In early shells, subshells remain radially separated and filling follows simple radial ordering. As shells grow, subshell overlap and radial reorganization modify the effective ordering of orbital localization regions, leading to the observed $n+\ell$ filling sequence and its well-known deviations. Classical "exceptions" such as chromium and copper emerge as stability extrema associated with half-filled and fully filled subshell vortices, while the distinction between filling order and ionization order arises from radial exposure in the fully formed atomic vortex.

Beyond individual atoms, the analysis reveals a unifying principle operating across scales. The same counting laws and geometric constraints that determine electron capacity within atomic shells also govern the number of elements in each period of the periodic system. This correspondence supports the view that chemical periodicity is not an independent empirical pattern, but a macroscopic expression of the same vortex-based organization that governs atomic structure.

The proposed framework remains fully compatible with orthodox quantum mechanics and available experimental observations, including evidence that atoms interact with light over spatially extended and structured regions. While the

present work does not claim direct experimental visualization of vortex structures, it provides a consistent geometric interpretation that unifies atomic structure, electron configurations, and periodicity within a single organizing principle.

In this sense, the vortex-based interpretation complements the abstract formalism of quantum mechanics by restoring physical intuition to its foundational rules, and suggests that atomic and chemical order may be understood as manifestations of a unified geometric structure acting across multiple scales.

7. Limitations and Outlook

The vortex-based framework presented in this work is intended as a **geometric and interpretative model** that complements, rather than replaces, the formalism of quantum mechanics. While it provides a coherent physical interpretation of quantum numbers, shell structure, and electron configurations, the present analysis remains qualitative in nature and does not attempt to derive quantitative energy spectra or wavefunctions from first principles. As such, it should be understood as a conceptual framework aimed at clarifying structure and organization, rather than as a complete dynamical theory.

The schematic vortex representations introduced throughout the article are **conceptual visualizations** rather than direct experimental images. Although recent high-resolution experiments demonstrate that atomic interactions occur over spatially extended regions, the internal vortex structures proposed here have not been directly observed. Further theoretical work will be required to establish whether specific experimental signatures—such as characteristic spectroscopic patterns, scattering profiles, or dynamical responses—can be uniquely associated with the predicted vortex geometry.

Future developments of this framework may include quantitative modeling of vortex dynamics in structured vacuum systems, numerical simulations of subshell overlap and contraction, and investigation of how vortex geometry influences measurable atomic properties such as ionization energies, transition probabilities, and magnetic moments. Extending the model to multi-atom systems and molecular bonding may also provide new insights into chemical structure and reactivity.

At a broader scale, the demonstrated correspondence between atomic shell capacity and periodic organization suggests that the vortex-based approach may serve as a unifying principle across different levels of physical organization. Further exploration of this scale-invariant geometry—both theoretically and experimentally—may help clarify the relationship between atomic structure, chemical periodicity, and emergent material properties.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Rutherford, E. (1911) LXXIX. The Scattering of α and β Particles by Matter and the

Structure of the Atom. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, **21**, 669-688. https://doi.org/10.1080/14786440508637080

[2] Bohr, N. (1913) I. On the Constitution of Atoms and Molecules. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, **26**, 1-25. https://doi.org/10.1080/14786441308634955

[3] Sommerfeld, A. (1916) Zur Quantentheorie der Spektrallinien. *Annalen der Physik*, **356**, 1-94. https://doi.org/10.1002/andp.19163561702

[4] Schrödinger, E. (1926) Quantisierung als Eigenwertproblem. *Annalen der Physik*, **384**, 361-376. https://doi.org/10.1002/andp.19263840404

[5] Pauli, W. (1925) Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren. *Zeitschrift für Physik*, **31**, 765-783. https://doi.org/10.1007/bf02980631

[6] Born, M. (1926) Zur Quantenmechanik der Stoßvorgänge. *Zeitschrift für Physik*, **37**, 863-867. https://doi.org/10.1007/bf01397477

[7] Dirac, P.A.M. (1928) The Quantum Theory of the Electron. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, **117**, 610-624. https://doi.org/10.1098/rspa.1928.0023

[8] Madelung, E. (1936) Die mathematischen Hilfsmittel des Physikers. Springer.

[9] Meek, T.L. and Allen, L.C. (2002) Configuration Irregularities: Deviations from the Madelung Rule and Inversion of Orbital Energy Levels. *Chemical Physics Letters*, **362**, 362-364. https://doi.org/10.1016/s0009-2614(02)00919-3

[10] de Broglie, L. (1927) La mécanique ondulatoire et la structure atomique de la matière et du rayonnement. *Journal de Physique et le Radium*, **8**, 225-241. https://doi.org/10.1051/jphysrad:0192700805022500

[11] Bohm, D. (1952) A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I. *Physical Review*, **85**, 166-179. https://doi.org/10.1103/physrev.85.166

[12] Sbitnev, V.I. (2016) Hydrodynamics of the Physical Vacuum: I. Scalar Quantum Sector. *Foundations of Physics*, **46**, 606-619. https://doi.org/10.1007/s10701-015-9980-8

[13] Butto, N. (2020) Electron Shape and Structure: A New Vortex Theory. *Journal of High Energy Physics, Gravitation and Cosmology*, **6**, 340-352. https://doi.org/10.4236/jhepgc.2020.63027

[14] Butto, N. (2020) A New Theory on Electron Wave-Particle Duality. *Journal of High Energy Physics, Gravitation and Cosmology*, **6**, 567-578. https://doi.org/10.4236/jhepgc.2020.64038

[15] Butto, N. (2021) A New Theory for the Essence and Origin of Electron Spin. *Journal of High Energy Physics, Gravitation and Cosmology*, **7**, 1459-1471. https://doi.org/10.4236/jhepgc.2021.74088

[16] Streed, E.W., Jechow, A., Norton, B.G. and Kielpinski, D. (2012) Absorption Imaging of a Single Atom. *Nature Communications*, **3**, Article No. 933. https://doi.org/10.1038/ncomms1944

[17] Butto, N. (2023) The Gastropod Shell Structure as a Blueprint for a Periodic System: A New Theory for Element Configurations. *Journal of Biophysical Chemistry*, **14**, 31-58. https://doi.org/10.4236/jbpc.2023.142002