Journal of High Energy Physics, Gravitation and Cosmology, 2026, 12(1), 392-430
https://www.scirp.org/journal/jhepgc
ISSN Online: 2380-4335
ISSN Print: 2380-4327

Standard Cosmological Model vs. Time Variable Light Speed Model

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How to cite this paper: Pipino, G. (2026) Standard Cosmological Model vs. Time Variable Light Speed Model. *Journal of High Energy Physics, Gravitation and Cosmology*, **12**, 392-430.
https://doi.org/10.4236/jhepgc.2026.121024

Received: September 12, 2025
Accepted: January 20, 2026
Published: January 23, 2026

Abstract

The standard cosmological model (ΛCDM = Λ-Cold Dark Matter) is illustrated in its current inflationary version. The main reasons for this model are explained, which are primarily the experimental evidence of the cosmological redshift, the existence of the cosmic microwave background (CMB), and the apparent need for so-called dark matter to explain certain observational phenomena such as anomalies in galaxy velocity curves. Numerous problems related to the standard model are illustrated. From this: 1) The horizon problem; 2) The flatness problem; 3) The Hubble tension; 4) The size and chemical composition of primordial galaxies; 5) The failure to detect WIMPS particles that should constitute dark matter; 6) The fact that the expansion of the universe appears to be accelerating. As an alternative, the TVSL (Time Varying Speed of Light) model is proposed, which assumes that the speed of light in a vacuum is a function of time. The model predicts a minimal variation of c (about 2.2 (cm/s)/year), which is sufficient to explain the cosmological redshift z and the Hubble law, according to which z is proportional to the distance of the galaxies. By attributing the redshift to a variation in the signal coming from galactic sources, we eliminate the need to assume that the universe is expanding. This immediately solves all the problems listed above. The variations of c with time are so modest that Special Relativity, SR, remains substantially correct. SR is therefore used to redefine the physics of Black Holes (BH). This eliminates the singularity and allows the BH to emit matter, as plasma beams. From these plasma beams will arise a new generation of atoms, stars and galaxies. The resulting model of the universe is completely different from that suggested by the Big Bang model. The universe did not originate at a specific point in the past, starting from microscopic dimensions and then expanding exponentially. Rather, it has always existed in its current form. Matter undergoes phase transformations, from plasmatic to atomic and vice versa. The engines of this transformation are supermassive black holes, which absorb energy in the form of atomic material, transform it into plasmatic material, and emit it in the latter form, as relativistic jets. From these jets, new generations of galaxies will be born. The atoms of newborn galaxies emit photons at maximum speed. As they age, the speed of light emitted decreases. This phenomenon causes cosmological redshift, which is therefore related to the age of the galaxy, as well as their distance.

Keywords

Special Relativity, General Relativity, Big Bang, CMB, TVSL, Flatness Problem, Horizon Problem, Black Hole, Klein-Gordon Equation

1. PART ONE: ΛCDM Model

1.1. Origin of the Redshift According to the ΛCDM Model

The experimental observation that led to the development of the ΛCDM (Λ-Cold Dark Matter) cosmological model, also known as the standard model or Big Bang model, is the fact that the spectrum of light from a distant galaxy (with a distance of 100 Mpc or greater) is shifted relative to the spectrum emitted by atoms in our galaxy.

We denote by λ_{obs} the wavelength of light emitted by an atom belonging to a galaxy G at a time t_e, and by λ_{lab} the wavelength emitted, at the present time t_0, by the same type of atom belonging to our galaxy (relative to the same energy transition). The shift

$$z = \frac{\lambda_{obs} - \lambda_{lab}}{\lambda_{lab}} \quad \textit{cosmological redshift}$$

is called the *cosmological redshift*. The ΛCDM model predicts that atoms in the galaxy G emit photons with wavelength λ_{lab} at time t_e (being λ_{lab} the same wavelength measured at the present time), and that these photons increased in wavelength during their journey from the galaxy G to the terrestrial observer O. Therefore, they decreased in frequency, since the product $c = \lambda f$, according to General Relativity on which the model is based, must remain constant.

The mutual removal of galaxies due to the expansion of space can be thought to produce a Doppler effect on the frequency, given by the relativistic relation:

$$f_{obs} = f_{lab}\sqrt{\frac{1-\beta}{1+\beta}}$$

Or:

$$\lambda_{obs} = \lambda_{lab}\sqrt{\frac{1+\beta}{1-\beta}} = \lambda_{lab}\sqrt{\frac{(1+\beta)^2}{1-\beta^2}}$$

If $\beta \ll 1$ we have:

$$\lambda_{obs} \approx \lambda_{lab}(1+\beta).$$

We have defined

$$z = \frac{\lambda_{obs} - \lambda_{lab}}{\lambda_{lab}} = \frac{\lambda_{obs}}{\lambda_{lab}} - 1 .$$

Then

$$z \approx \beta = \frac{v}{c}$$

The Hubble constant H relates the recession velocity v to the galaxy's distance r. That is,

$$v = Hr \quad \text{Hubble's law}$$

Using the previous law, the redshift (for small distances) is:

$$z = \frac{H}{c} r$$

Currently, $H \approx 72$ (km/s)/Mpc is experimentally measured using direct galaxy distance. Instead, $H \approx 67.3$ (km/s)/Mpc is obtained using background radiation to indirectly deduce the value of H (Planck satellite, 2015).

Cosmological redshift concerns distant galaxies, with distances of the order of 100 Mpc or greater. In fact, all galaxies have random proper motions (similar to thermal motions) of the order $v \approx 100$ km/s. These individual motions overlap with the recession motion that instead affects all galaxies without distinction. The cosmological recession velocity V for a galaxy at a distance of one megaparsec (Mpc) is of the same order (72 km/s), and increases proportionally to the distance. Therefore, if one wishes $V \gg v$, the distance must be 100 Mpc or greater.

1.2. Critical Issues of the ΛCDM Model

The Ptolemaic model was capable of making many accurate predictions regarding the positions of the planets and stars, despite being patently false. It predicted that the stars were fixed on a crystal sphere with the Earth at its centre, and that the planets were also fixed on crystal spheres with the same centre but different radii. To explain the sudden reversals in the apparent orbits of the planets, it was imagined that they travelled along circles (epicycles), whose centres in turn moved on circles of larger radii (deferens). This demonstrates that making many accurate predictions is necessary, but not sufficient, to establish a model as correct. Despite many accurate predictions (predicting the existence of the CMB, predicting cosmological redshift, predicting that galaxies with high redshifts are small and depleted of heavy elements), the Big Bang model may be flawed, as was the Ptolemaic model. Below, we will outline some reasons why the Standard Model may be incorrect.

1.3. Fundamental Criticality

The main criticality of the ΛCDM model, also known as the BB (Big Bang) model or the Standard Model, is the following. The BB model predicts that the universe originated from a very specific moment in the past, called the Big Bang and denoted by t_{bb}. The time t_{bb} is linked to the so-called Hubble constant H_0, which expresses the relationship between redshift and distance at the present time. Roughly speaking, the temporal distance t_{bb} with respect to the present time, can be calculated from the relation $t_{bb} \approx 1/H_0 \approx 13.8$ billion years. The BB model predicts that everything, including time and space, originated at instant t_{bb} in the past. Therefore, the temperature and energy density of the universe at that instant should have been infinite (initial singularity due to the null volume). Time also originated at instant t_{bb}. However, without time, neither velocity nor acceleration can exist. Therefore, without time, neither kinetic energy (related to velocity) nor any other type of energy (related to acceleration, or the forces that, by doing work, produce this energy) can exist. Without time, radiation cannot exist either, since an essential characteristic of radiation is its frequency, which is related to time. Therefore, without time, there is only NOTHING. Therefore, according to the BB model, all the energy of the universe would have been created, from NOTHING, at instant t_{bb}.

There is only one sacred principle in physics, never invalidated by any experiment: energy neither originates from nothing nor is lost to nothing.

Not only does the terrifyingly large energy of the entire universe not originate from nothing, but not even a microjoule of energy originates from nothing!

1.4. Other Critical Issues of the ΛCDM Model

Other critical issues of the ΛCDM model are:

- The horizon problem
- The flatness problem
- The Hubble tension
- The size and chemical composition of primordial galaxies, detected by the JWST satellite program, which operates in the infrared range. This program has recently detected the existence of primordial galaxies (JADES GS z11-0) [1] with very high z (greater than 11), therefore close to the time t_{bb} of the Big Bang (a temporal distance of approximately 400 million years) but with very large dimensions and a chemical composition typical of evolved galaxies. The presence of these galaxies is not compatible with an age of the universe equal to $t_{bb} \approx 13.8$ billion years. Therefore, either a way is found to increase the age of the universe, as the authors of the article [2] do, which takes into account the hypothesis that the fundamental constants c, G, h can vary, or it is admitted that the idea underlying the Big Bang, that is, that the universe originated at a well-defined time in the past, is incorrect.
- Problem of the failure to detect WIMPS particles that should constitute dark matter, despite over 50 years of research.
- The fact that the expansion of the universe appears to be accelerating, while an attractive force, such as the gravitational force between galaxies, should cause a deceleration.

To understand the previous problems, it is necessary to review the foundations of relativity.

1.5. Theoretical Foundations of Relativity

Relativistic theory, both Special and General, is based on an unexpressed principle, upon which all others depend:

All laws of physics can be expressed as geometric relations.

Since many relations in physics involve time, and since geometric relations must be relations between lengths, the implicit principle requires that time must be expressible through a measure of length. Therefore, there must exist a factor, which we will denote by *c*, which, when applied to a quantity with the dimensions of time, transforms it into a quantity with the dimension of length. That is, if Δt denotes a time interval between two events, this interval must be transformed into a spatial interval Δx between them, using the relation $\Delta x = c\Delta t$. Obviously, the factor c must have the dimensions of a velocity. In this way, the space in which physical events occur will be composed of four coordinates which, in the case of a Cartesian reference, are indicated by $x^0 = ct$; $x^1 = x$; $x^2 = y$; $x^3 = z$. The three coordinates x^1, x^2, x^3 are called spatial coordinates, while the coordinate ct is called the temporal coordinate. The four-dimensional geometric space in which the reference is defined (x^0, x^1, x^2, x^3) is called spacetime. The word "space" instead indicates three-dimensional space. A point in spacetime is called an event. Any physical law causally relates an event (represented by point P_1 in spacetime) to another event caused by the first (represented by point P_2). The law that expresses how a system evolves can be made to coincide with the line (generally curved) that connects event P_1 to event P_2 (called the worldline). Since physical laws are curves in spacetime $x^\alpha = \left(x^0, x^1, x^2, x^3\right)$, if we want these laws to be independent of time, the factor c must be constant. This implies that some physical phenomenon must exist capable of producing a velocity c that is not only constant, but also independent of the observer's velocity.

1.6. Reference Frames and Inertial Observers

We define *inertial frame* a reference in which galaxies appear to have constant coordinates. A system fixed to the Earth's surface is not inertial because, due to the Earth's rotation, galaxies appear to rotate. Conversely, if, in a terrestrial laboratory, three gyroscopes are taken with their axes oriented in the x, y, z directions, orthogonal to each other, in the (x, y, z) reference frame, any galaxy will always have the same coordinates. Galaxies have proper motions of the order of 100 km/s, but given their enormous distance, their proper displacements are unobservable over observation periods of years. An observer is said to be inertial when his velocity, measured in an inertial frame, is zero or constant.

1.7. First Law of Special Relativity

The first law of Special Relativity is that inertial observers measure the same speed of light, *c*. That is, in an inertial frame, the speed of light does not depend on the observer's velocity. Special Relativity does not address gravity. For it, gravitational fields are zero, and the frames of reference in which physical events occur are inertial. Conversely, General Relativity addresses the effects of gravitational fields.

1.8. First Law Extended to General Relativity

The principle of the constancy of the speed of light is extended to General Relativity:

Observers in any frame of reference (*inertial or otherwise*) *measure the same speed of light*, *c*. *That is*, *in any frame of reference*, *the speed of light is constant and equal to that measured in an inertial frame*.

In relativity, a measurement system is often used that assumes the value 1 for both c and G (gravitational constant). This system is named Geometrized System. We will avoid using this system for four reasons.

In this system, all physical quantities have the dimension of a length, or a power of a length. For example, space, time, mass, energy, and momentum all have the same dimension (length). In physics, quantities with the same physical dimension coincide (up to a dimensionless numerical constant which can be made unitary with an appropriate choice of measurement units).

The second reason is that a measurement system capable of describing both mechanical and electromagnetic phenomena must possess at least two physical quantities.

If mechanical phenomena are described, for example, by the quantity mass, electrical phenomena will be described by a quantity such as charge, independent of mass.

That charge and mass are independent is demonstrated by the existence of the proton, which has the same charge as the electron (with a different sign) but a different mass.

The third reason is practical. If we use the Geometric System GS, then the mass density ρ = mass/volume coincides with the energy density ϵ = energy/volume.

In fact, in the GS system, since mass coincides with energy, we have $\rho = \epsilon$. The equation that provides, for example, $\rho(t)$ in Friedman's model of a homogeneous and isotropic universe is

$$\dot{a}^2 - \frac{8\pi\rho(t)a^2}{3} = -k \quad \textit{Friedman equation in the geometric system}$$

This equation (in GS) is valid whether ρ represents mass density or energy density. In fact, if $c = 1$, then mass and energy coincide ($E = mc^2$). However, in the International System IS, where $c \neq 1$, $G \neq 1$, the two cases are represented by different equations. If $\rho(t)$ is a mass density, that is, $[\rho]$=mass/volum, given that mass in GS has the dimension of a length, we have $[\rho]_{GS} = L^{-2}$.

In the International System IS, however, we have

$[\rho]_{IS} = \frac{\text{mass}}{\text{volum}} = \frac{mc^2}{L^3} = \frac{mc^2}{L^3}\frac{T^2 m}{T^2 m}$. But $[G] = \frac{L^3}{T^2 M}$ Then

$[\rho]_{IS} = \frac{c^2}{G}T^{-2} = \frac{c^2}{G}[\rho]_{GS}$; $[\rho]_{GS} = [\rho]_{IS} * \frac{G}{c^2}$.

To write the Friedman formula in the international system, we must replace ρ

with $\rho G/c^2$. And, with similar reasoning, given that $[t]_{GS} = L$, we must replace *t* with *ct*. The Friedman equation, referred to the mass density *ρ*, then becomes:

$$\frac{\dot{a}^2}{c^2} - \frac{8\pi\rho(t)Ga^2}{3c^2} = -k \quad \textit{FRW equation, IS, } \rho = \textit{matter density.}$$

It is easy to verify that if *ρ* represents the energy density the Friedman equation in the geometric system remains unchanged, while in the international system it becomes:

$$\frac{\dot{a}^2}{c^2} - \frac{8\pi\rho(t)Ga^2}{3c^4} = -k \quad \textit{FRW equation, IS, } \rho = \textit{energy density.}$$

That is, the same equation in the geometric system translates into two different equations in the international system.

The fourth reason is that assuming *c* = 1; *G* = 1, means definitively assuming that both *c* and *G* cannot vary over time. Now, this conclusion is highly questionable. In fact, the precision with which both *c* and *G* were measured does not allow us to rule out the possibility that both could undergo small variations over time.

The last measurement of *c* was made in 1983, when the value 299,792,458.1 m/s was measured (within experimental errors). Since then, it has been conventionally assumed that $c = 299{,}792{,}458\ \text{m·s}^{-1}$, and the experimental value of *c* has not been measured again. Since about 50 years have passed since then, if *c* were measured again now (with the same method and the same precision), it would be found to be $c = 299{,}792{,}457\ \text{m·s}^{-1}$.

In fact, according to the TVSL model, d*c* = −*H c* d*t*. Assuming for *H* (Hubble constant) the value $H = 2.33 \times 10^{-18}\ \text{s}^{-1}$, we obtain d*c* = 2.2 (cm/s)/year. This means that in 50 years, *c* should decrease by approximately one meter/sec.

If this experimental prediction were verified, it is clear that the entire ΛCDM model, based on the constancy of c, would need to be revised.

1.9. Line Element

The distance between two events is denoted by Δ*s*, or by d*s*, if the distance is infinitesimal. Given any surface, its geometry depends on its shape and therefore cannot depend on the observer's velocity. This means that two observers with relative velocity *V* must measure the same line element d*s*. The combination of these two principles—*c* constant, d*s* independent of the observer's velocity—leads to the conclusion that the line element, in the case of Special Relativity (absence of gravitational forces), is:

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}R^2$$

with $\mathrm{d}R^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$ (line element of space in Cartesian coordinates).

1.10. Universe According to the FRW Model

The FRW (Friedman, Robertson, Walker) model, adopted by the Standard Model, assumes that the energy of the universe is spatially distributed homogeneously and isotropically. This assumption is confirmed by the distribution of the cosmic microwave background radiation (CMB). Galaxies are assimilated to a pressureless gas because their random motion (about 100 km/s) gives rise to a thermal energy much lower than the rest energy of the galaxies themselves.

1.11. First Law of Thermodynamics

The first law of thermodynamics states that the total energy change in a volume ΔV containing a fixed number of particles is equal to the work done when ΔV is varied by $d(\Delta V)$, minus the energy flux leaving the volume. The assumption of homogeneity implies that, at a given time t, the temperature of every point must be the same. Therefore, there can be no flow of energy out of ΔV. Then the change in energy due to a change in volume, which we denote by $d(\Delta E)$, is equal to the work done to change the volume by $d(\Delta V)$, that is, it is equal to $-p\, d(\Delta V)$, where p is the pressure. That is, $d(\Delta E) = -p\, d(\Delta V)$.

[The minus sign is due to the fact that if $d(\Delta V) > 0$, or if the volume increases, the work done is negative, that is, it is work done on the system, and not provided by the system.]

The FRW model conceives the universe as a fluid. Each galaxy is thought as a point in this fluid with Cartesian coordinates (x, y, z). To determine the coordinates, a reference system is needed with respect to which each galaxy appears to have a fixed position. The coordinates of any galaxy in this system, called the *comoving system*, are therefore constant over time.

If (x, y, z) are the comoving coordinates of the volume ΔV, the FRW model metric, in Cartesian coordinates, is:

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]$$

In this formula, the factor $a(t)$, called the scale factor, is a dimensionless factor on which the expansion of space depends. Without the factor $a^2(t)$, the metric of space would simply be: $\mathrm{d}R^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$, and since (x, y, z) are comoving coordinates, they would be constant over time, so the distance to any galaxy (obtained by integrating $\mathrm{d}R$) would be constant.

The volume ΔV will then be $\Delta V = a^3(t)\Delta x\Delta y\Delta z = a^3(t)\Delta V_{com}$, where ΔV_{com} denotes the volume calculated in comoving coordinates. If $\rho = \frac{\Delta E}{\Delta V}$ the relation $d(\Delta E) = -p\cdot d(\Delta V)$ gives:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\rho a^3\Delta V_{com}\right) = -p\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\Delta V_{com}\right)$$

Since the volume computed in comoving coordinates is constant, we obtain:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\rho a^3\right) = -p\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\right) \quad \textit{first law, } \rho = \textit{energy density}$$

The previous law is the first thermodynamic law, according to the FRW model for a homogeneous and isotropic universe. In it, $\rho = \rho(t)$ is the total energy density of the universe, p is the pressure, while $a = a(t)$ is the scaling factor related to the expansion of the universe. It contains the pressure, which is zero for the material component of the galactic fluid (galaxies). But it is not zero for the radiation component (positive pressure) or that provided by the vacuum energy (negative pressure).

1.12. Energy Density of Radiation $\rho_r(t)$

If the fluid composing the universe is considered a black body, then the energy density $\rho_r(t)$ due to radiation depends on the absolute temperature of the fluid, and therefore on its pressure $p(t)$, according to the relation

$$p_r(t) = \frac{\rho_r(t)}{3}.$$

In it, $\rho_r(t)$ is the energy per unit volume. From the first law of thermodynamics:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\rho a^3\right) = -\frac{\rho}{3}\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\right) \quad \text{we get:}$$

$$\rho_r(t) = \rho_{r0}\left[\frac{a_0}{a(t)}\right]^4 \quad \textit{energy density of radiation.}$$

1.13. Radiation Temperature

In a black body, the energy density of the emitted radiation depends on the fourth power of the temperature: $\rho_r = kT^4$ (k is a constant function of other fundamental constants).

So, we get:

$$T_r(t) = T_{r0}\frac{a_0}{a(t)} \quad \textit{radiation temperature}$$

The temperature of radiation depends inversely on the scale factor $a(t)$. The BB model assumes that at time t_{bb} the dimensions of the universe were zero. That is, $a(t_{bb}) = 0$. The formula $T_r(t) = T_{r0}\frac{a_0}{a(t)}$ then yields $T_r(t_{bb}) = \infty$. The temperature, near the BB, therefore assumes such high values that elementary particles (electrons, protons, neutrons) have energy greater than that which binds these particles in the atom. Therefore, elementary particles are free to move and constitute a primordial plasma. As time passes, $a(t)$ increases, and therefore $T(a)$ decreases. When the temperature drops below 3000˚K, the energy of elementary particles drops below the atomic binding energy between electrons and protons, which is approximately 0.25 eV (*i.e.*, 3000˚K if measured in ˚K) [3]. Therefore, it is assumed that the cosmic microwave background radiation is the radiation emitted at the instant of recombination t_{ric}, the epoch at which the temperature of the plasma sphere dropped below the 3000˚K required for elementary particles to combine. Starting from t_{ric}, the temperature of the CMB continued to decrease, until reaching its current value of $T(t_0) = 2.725\ \mathrm{K}$. It can be calculated that $t_{ric} \approx 360000$ years after the Big Bang.

1.14. Energy Density of Matter $\rho_m(t)$

Matter, as mentioned, is conceived as a pressureless gas, so from the first law of thermodynamics we find: $\frac{\mathrm{d}}{\mathrm{d}t}\left(\rho_m a^3\right)=0$, where ρ_m is its energy density. From the previous, we deduce $\rho_m a^3 = \text{constant}$, that is, considering a generic instant *t*, and the instant t_0 relative to the current time, we have:
$\rho_m(t)a^3(t)=\rho_m(t_0)a^3(t_0)$. From which:

$$\rho_m(t)=\rho_{m0}\left[\frac{a_0}{a(t)}\right]^3 \quad \textit{energy matter density.}$$

As can be seen, the FRW model predicts a decrease in the energy density of matter inversely proportional to the cube of the scale factor. In the early days of the universe, when $a(t) \approx 0$, the energy density of matter was also very high. However, it was lower than the energy density of radiation. In fact, the latter is proportional to ($1/a^4$), while that of matter is proportional to ($1/a^3$), and, for $a \to 0$, we have $1/a^4 > 1/a^3$.

1.15. Vacuum Energy Density ρ_Λ

Currently, the vacuum energy density ρ_Λ is believed to be constant and positive. The first law of thermodynamics, if $\rho=\rho_\Lambda=\text{constant}$, gives $p=-\rho_\Lambda$. Vacuum pressure is therefore considered negative. That is, its sign leads to the accelerated expansion of the universe. Vacuum energy has no known value. It has become conventional to write ρ_Λ in the form:

$$\rho_\Lambda=\Lambda\frac{c^4}{8\pi G} \quad \text{vacuum energy density}$$

The constant Λ, which has dimensions L^{-2}, (both in the international system and in the geometric system) is called the *cosmological constant*.

1.16. Generalized Metrics for a Homogeneous and Isotropic Space

A homogeneous and isotropic space does not necessarily have a flat metric. The metric must only satisfy the relation:

$$\mathrm{d}s^2=-c^2\mathrm{d}t^2+a^2(t)\mathrm{d}R^2$$

It is shown that a homogeneous and isotropic space with a spherical geometry (finite volume) has a metric given by the relation

$$\mathrm{d}R^2=\left[\mathrm{d}\chi^2+\sin^2\chi\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right)\right]$$

where χ, θ, and ϕ are angular coordinates. The previous one was obtained by assuming that space is the three-dimensional surface of a quadrisphere of unit radius belonging to four-dimensional space. Another shape that space can assume is that of a hyperboloid. The three possible cases, which describe the metric of a flat, spherical, or hyperbolic space, can be summarized with:

$$\mathrm{d}s^2=-c^2\mathrm{d}t^2+a^2(t)*\left[\mathrm{d}\chi^2+F\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right)\right]$$

where F is a dimensionless factor that can be $\chi^2, \sin^2\chi, \sinh^2\chi$ for flat, spherical, hyperbolic space. Setting $r = \sin\chi$, the previous formula can be written:

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)\right]$$

where k, the curvature factor, is 0 for flat space, 1 for spherical space, and −1 for hyperbolic space.

The previous equation refers to space with any curvature. However, numerous satellite measurements of the cosmic microwave background (CMB) show that the geometry of our universe does not deviate from flatness. That is, $k = 0$. Analysing the cases $k \neq 0$ helps us understand the evolution of the universe in the past, or in the future, when the geometry of the universe could not be flat.

1.17. Hubble's Law, Hubble's Constant, and Scale Factor

Hubble's law assumes that the recession velocity of galaxies is proportional to their distance, or:

$$v = \frac{\mathrm{d}r}{\mathrm{d}t} = Hr$$

This relationship immediately leads to the relation $H = \dot{a}/a$ where the dot indicates the time-dependent relationship.

Indeed, in Hubble's law, v is the velocity of a galaxy G moving away from the Earthly observer, due to the expansion of the space between galaxies.

That is, if

$$r = a(t)r_{com}$$

is the distance of the galaxy G, with r_{com} = comoving distance (constant in time), then

$$v = \frac{\mathrm{d}r}{\mathrm{d}t} = r_{com}\frac{\mathrm{d}a}{\mathrm{d}t} = r_{com}\dot{a}\text{, and setting } v = Hr \text{ we find } H = \frac{v}{r} = \frac{r_{com}\dot{a}}{r_{com}a(t)}$$

$$H = \frac{\dot{a}}{a} \quad \textit{Hubble's constant definition}$$

1.18. Friedman's Cosmological Models

The temporal evolution of the universe depends on the scale factor $a(t)$. In addition to satisfying the first law of thermodynamics, $a(t)$ must satisfy Einstein's equation.

Making this imposition, we find:

$$\dot{a}^2 - \frac{8}{3}\pi\rho a^2 = -k \quad \textit{Friedman equation in geometric units.}$$

The previous one is the Friedman equation in geometric units $c = 1$, $G = 1$. In international units, if ρ is the energy density the previous becomes:

$$\frac{\dot{a}^2}{c^2} - \frac{8}{3}\pi\rho\frac{G}{c^4}a^2 = -k \quad \textit{Friedman equation in international units.}$$

In the previous $\dot{a} = \mathrm{d}a/\mathrm{d}t$, $\rho = \rho(t)$ is the total energy density (due to three components: radiation, matter and vacuum) while k is the curvature factor which can be 0, 1, −1.

The previous one is written: $\dot{a}^2 - \frac{8}{3}\pi\rho\frac{G}{c^2}a^2 = -kc^2$.

CMB observations indicate that the universe is flat, so $k = 0$. So dividing by a^2 and setting $\dot{a}/a \equiv H$ we have: $H^2 = \frac{8}{3}\pi\frac{G}{c^2}\rho(t)$

From which $\rho(t) = \frac{3c^2H^2}{8\pi G}$. We set $\rho(t) * \frac{8\pi G}{3c^2H^2} = \Omega(t)$. Then

$$\Omega(t) = 1.$$

It is convenient to divide the energy density, $\Omega(t)$, into three parts relating to radiation, matter, and vacuum. That is, we set: $\Omega(t) = \Omega_r(t) + \Omega_m(t) + \Omega_\Lambda(t)$.

Substituting we have: $\Omega_r(t) + \Omega_m(t) + \Omega_\Lambda(t) = 1$.

The previous relation indicates that, at any instant in the life of the universe, the sum of all energy densities due to matter, radiation, vacuum is constant and equal to 1.

With reference to the current time, we then have:

$$\Omega_r(t_0) + \Omega_m(t_0) + \Omega_\Lambda(t_0) = 1;$$

$$\rho_r(t_0) * \frac{8\pi G}{3c^2H_0^2} + \rho_m(t_0) * \frac{8\pi G}{3c^2H_0^2} + \rho_\Lambda(t_0) * \frac{8\pi G}{3c^2H_0^2} = 1$$

Placing

$$\rho_{crit} = \frac{3c^2H_0^2}{8\pi G}$$

The previous one is written:

$$\frac{\rho_r(t_0)}{\rho_{crit}} + \frac{\rho_m(t_0)}{\rho_{crit}} + \frac{\rho_\Lambda(t_0)}{\rho_{crit}} = 1$$

The previous one was written with reference to the current time t_0, but, as said before, it is valid for any instant of time t (naturally ρ_{crit} must also be referred to the instant t).

1.19. Scale Factor for a Flat, Matter-Dominated Universe

We derive the following relation, relative to a flat ($k = 0$), matter-dominated, universe:

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

The Friedman equation, for a flat universe, is:

$$\dot{a}^2 - \frac{8\pi G a^2 \rho}{3c^2} = 0$$

with $\rho = \rho_m$ = density of energy of matter. This is a homogeneous equation.

Therefore, if $b(t)$ is a solution, then $a(t)=kb(t)$ will also be a solution. The constant k can be chosen so that $a(t_0)=1$, t_0 being the current time.

From the previous equation, we have $H^2-\frac{8\pi G\rho}{3c^2}=0$; $\rho=\frac{3c^2H^2}{8\pi G}$.

At present the Friedman equation gives $\rho_0=\frac{3c^2H_0^2}{8\pi G}=\rho_{crit}$.

That is, *if the universe is flat, its total energy density (due to the three components: matter, radiation, vacuum) at the present time is equal to the critical density.*

Suppose that at the present time the total energy density is due essentially to matter. In this hypothesis, the first law of thermodynamics: $\frac{\mathrm{d}}{\mathrm{d}t}(a^3\rho)=-p\frac{\mathrm{d}}{\mathrm{d}t}(a^3)$ provides $\frac{\mathrm{d}}{\mathrm{d}t}(a^3\rho)=0$. Then $a^3\rho=a_0^3\rho_0$; $\rho=\frac{a_0^3\rho_0}{a^3}$ and, in the hypothesis $a_0=1$ we have: $\rho=\frac{\rho_0}{a^3}$. Substituting in the Friedman equation we have: $\dot{a}^2-\frac{8\pi G\rho_0}{3ac^2}=0$. But in the hypothesis of zero curvature the energy density coincides with the critical density. Then $\rho_0=\frac{3c^2H_0^2}{8\pi G}$. Replacing in the previous one, we have: $\dot{a}^2-\frac{H_0^2}{a}=0$. The previous one has the solution:

$$a(t)=\left(\frac{t}{t_0}\right)^{\frac{2}{3}} \quad \textit{scaling factor in case of matter domination}$$

as is easy to verify by substitution [$H_0=\left(\frac{\dot{a}}{a}\right)_{t_0}=\frac{2}{3t_0}$]

A similar relationship is found in the case of a flat, radiation-dominated universe: $a(t)=\left(\frac{t}{t_0}\right)^{\frac{1}{2}}$. These relationships indicate that as t decreases, the scale factor decreases until it becomes zero at $t = 0$ (the time $t = 0$ corresponds to the Big Bang).

The relationship $a(t)=\left(\frac{t}{t_0}\right)^{\frac{2}{3}}$ relative to a matter-dominated universe, allows us to calculate the time t_0 as a function of the Hubble constant $H=\frac{\dot{a}}{a}=\frac{2}{3}\frac{1}{t}$. Assuming $\frac{1}{H_0}=13.8\times10^9$ years we would have $t_0=9.2\times10^9$ years. The previous value is too small, since there are stars, like our Sun, that are much older than 9.2 billion years.

It follows that *the hypothesis that the history of our universe has been dominated by matter or radiation is incorrect. Currently, and even in the past, the history of the universe is heavily influenced by vacuum energy.*

1.20. Scale Factor for Density Equal to Experimental Values

To calculate the scale factor $a=a(t)$ we must remember that

$$\Omega = \Omega_r + \Omega_m + \Omega_\Lambda + \Omega_k = 1$$

The currently most probable values for the four quantities listed above are (Planck satellite, 2015): $\Omega_{r0} = 8\times10^{-5}$; $\Omega_{m0} = 0.315$; $\Omega_{\Lambda 0} = 0.685$; $\Omega_{k0} = 0$.

From the relation $\dot{a}^2 - \frac{8\pi G\rho a^2}{3c^2} = 0$ we have

$$\dot{a}^2 - \frac{8\pi G\rho a^2 H_0^2}{3c^2 H_0^2} = 0$$

Placing $\rho_{crit} = \frac{3c^2 H_0^2}{8\pi G}$

the previous one is written:

$\dot{a}^2 - \frac{\rho a^2 H_0^2}{\rho_{crit}} = 0$ Placing $\frac{\rho}{\rho_{crit}} = \Omega$ the previous one becomes:

$\dot{a}^2 = \Omega a^2 H_0^2$ Or: $\dot{a} = \sqrt{\Omega} aH_0$; $\frac{\mathrm{d}a}{\mathrm{d}t} = \sqrt{\Omega} aH_0$;

$\mathrm{d}t = \frac{\mathrm{d}a}{H_0\sqrt{\Omega} a}$ $\int_0^{t_0} \mathrm{d}t = \frac{1}{H_0}\int_0^1 \frac{\mathrm{d}a}{\sqrt{\Omega} a}$ (we have placed $a(t_0) = 1$).

But $\Omega = \frac{\Omega_{m0}}{a^3} + \Omega_\Lambda$ (neglecting Ω_r due to its small value).

Replacing in the previous one:

$$\int_0^{t_0} \mathrm{d}t = \frac{1}{H_0}\int_0^1 \frac{\mathrm{d}a}{a\sqrt{\frac{\Omega_{m0}}{a^3} + \Omega_\Lambda}} = \frac{1}{H_0}\int_0^1 \frac{\mathrm{d}a}{\sqrt{\frac{\Omega_{m0}}{a} + a^2\Omega_\Lambda}}$$

If we extend the time between instant 0 and any instant *t*, the integral will be extended between 0 and the value *a*. Then,

$$t = \frac{1}{H_0}\int_0^a \frac{\mathrm{d}x}{\sqrt{\frac{\Omega_{m0}}{x} + x^2\Omega_\Lambda}} = \frac{1}{H_0} 0.805\,\mathrm{ArcSinh}\left[1.474 a^{\frac{3}{2}}\right]$$

Placing $\Omega_{m0} = 0.315$; $\Omega_\Lambda = 0.685$ we have

$$t = \frac{1}{H_0} 0.805\,\mathrm{ArcSinh}\left[1.474 a^{\frac{3}{2}}\right]$$

And, being $a(t_0) = 1$, $t_0 = \frac{1}{H_0} 0.805\,\mathrm{ArcSinh}[1.474] = \frac{0.950}{H_0} = 13.1\times10^9$ years.

Also this value of t_0 *is incompatible with the ages of the oldest galaxies.*

For example, the galaxy GN-Z11, detected by JWST, already existed 13.4 billion years ago [4].

In fact, for this galaxy the value $z = 11.6$ was measured. But $z = \frac{\lambda_e}{\lambda_0} - 1$;

$\frac{\lambda_e}{\lambda_0} = \frac{c(t_e)}{c(t_0)} = \frac{a(t_0)}{a(t_e)} = \frac{1}{a(t)}$. That is to say

$$z = \frac{1}{a} - 1 \quad \textit{redshift-scale factor relationship.}$$

For $z = 11.6$ we have $a = 0.079$. This means that $t \approx 366 \times 10^6$ years. This means that this galaxy should have existed approximately 360 million years after the Big Bang. This value is considered largely insufficient for the development of a mature galaxy such as GN-Z11, since it displays emission lines from nitrogen (atomic number 7), which is created in the cores of stars at the expense of carbon and oxygen and is expelled into space when the star in which it formed explodes as a supernova. This takes several generations of stars and several billion years.

The most distant galaxy currently known is JADES-GS-z14-0, discovered in 2024 by the JADES program of the JWST space telescope. It has a redshift of 14.32, meaning it was emitted 273 million years after the Big Bang. Spectroscopic analysis revealed the presence of oxygen. This means that by the time the galaxy emitted light, several generations of stars had formed within it, whose deaths allowed the galaxy to contain an element heavier than hydrogen and helium. According to the ΛCDM model, this would have occurred in just 273 million years.

1.21. Evolution of the Size of the Universe

We can reverse the relation $t = t(a)$ getting:

$$a(t) = 0.772\,\mathrm{Sinh}\left[1.24\frac{t}{t_0}\right]^{2/3}$$

The formula just obtained, allows us to derive the temporal evolution of the dimensions of the universe. These dimensions depend on scale factor $a(t)$. Placing $t_n = \frac{t}{t_0}$, the graph of $a(t_n)$ is shown in **Figure 1**. The dashed line is what would have been obtained if the universe had been dominated by matter [$a(t_n) = t^{\frac{2}{3}}$].

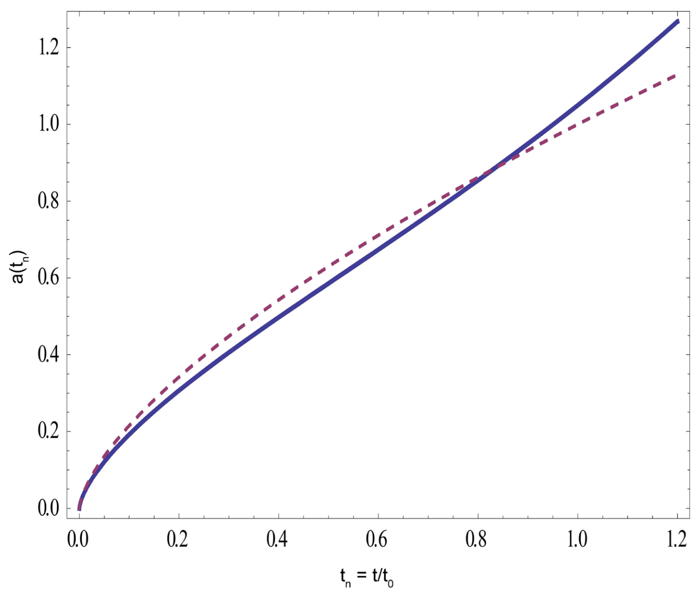

Figure 1. Scale factor as function of normalized time t/t_0.

Double-differentiating $a(t_n)$, we find that $\ddot{a} < 0$ for $t_n < 0.53$; $\ddot{a} > 0$ for $t_n > 0.53$. Then from time t_{bb} (Big Bang) until $t = t_F = 0.53t_0$, the expansion rate of the universe has slowed down, while from time t_F until the present time, the expansion rate of the universe appears to be accelerating. This is expected to continue in the future. Therefore, up until time t_F, matter dominated, while currently, vacuum energy dominates. Regarding the vacuum energy density, the FRW equation is:

$$\dot{a}^2 = \frac{8\pi G a^2}{3c^2}\rho_\Lambda \quad \textit{(flat universe)}.$$

In the previous one $\rho_\Lambda = \frac{\text{energy}}{\text{volum}}$.

Placing $\rho_\Lambda = \frac{c^4}{8\pi G}\Lambda$ the previous becomes $\dot{a}^2 = \frac{a^2}{3}c^2\Lambda$; $\frac{\dot{a}}{a} = c\sqrt{\frac{\Lambda}{3}}$ from which:

$$a(t) = a_0 e^{c\sqrt{\frac{\Lambda}{3}}(t-t_0)} \quad \textit{universe dominated by vacuum energy.}$$

Both Λ and c are assumed to be constant over time. Therefore, the ratio $\frac{\dot{a}^2}{a^2} = \frac{c^2}{3}\Lambda$ is constant and is equal to the Hubble constant squared H_0^2. Then $H_0 = c\sqrt{\frac{\Lambda}{3}}$. If the vacuum energy density is dominant, we get:

$$a(t) = a_0 e^{H_0(t-t_0)} \quad \textit{vacuum energy-dominated universe.}$$

This explains the exponential behaviour of the graph of $a(t)$ for $t_n \geq 0.53$.

The constant H_0 in the previous one represents the Hubble's constant at time t_0.

In fact, it is $H = \frac{\dot{a}}{a} = \frac{a_0 H_0 e^{H_0(t-t_0)}}{a_0 e^{H_0(t-t_0)}} = H_0$.

1.22. The Problem of Dark Energy

In 1998, using Type Ia Supernovae as standard candles, it was observed that galaxies were at a greater distance than they should have been if the expansion coefficient $H = \dot{a}/a$ had been constant.

This latter condition implies that $a(t)$ should follows a linear law in time, while observations indicated that $a(t)$ increased over time according to the law shown in **Figure 1**.

If a term containing the cosmological constant Λ is added to Einstein's equation, the observed behaviour of $a(t)$ is obtained. Clearly, however, remains the problem of explaining the physical reasons that led to adding that term to Einstein's equation.

Providing these reasons is perhaps the biggest problem with the Big Bang model. The most popular explanation is that most of the existing energy in the universe (beyond 70%, given that $\Omega_\Lambda = 0.715$) is dark energy.

The most popular explanation is that the source of dark energy is vacuum energy, or the phenomenon of quantum fluctuations, which predicts that pairs of virtual particles and antiparticles (*i.e.*, those existing for a very short time) can be created in the vacuum, which are at the origin of many physical phenomena such as the Casimir effect or the spontaneous emission of gamma rays.

The energy density that the vacuum would have to have to produce the acceleration of the observed expansion of the universe is $\rho_\Lambda = \Omega_\Lambda * \rho_{crit}$; $\rho_{crit} = \frac{3c^2 H_0^2}{8\pi G} = 8.7\times10^{-10}\ \mathrm{J\cdot m^{-3}}$; $\Omega_\Lambda = 0.685$; $\rho_\Lambda = 5.95\times10^{-10}\ \mathrm{J\cdot m^{-3}}$.
We obtain a value that is about 10^{120} times smaller than the value of the vacuum energy density which is derived from quantum theory.

1.23. Visible Universe Radius

The *visible radius of the universe* refers to the distance of the farthest visible source in space, and therefore also in time. According to the ΛCDM model, the maximum temporal distance is $t_{bb} = 13.8\times10^9$ years. The first light signal was emitted 360,000 years after the Big Bang. Therefore, the source with the maximum temporal distance is 13.79×10^9 years.

If space did not expand, a light signal emitted by such a source would travel a distance of 13.79×10^9 light-years. But, according to ΛCDM, as the signal moves away from the source, space expands. Therefore, the distance travelled by the signal is greater than the previous value. In fact, the FRW metric for a homogeneous and isotropic universe is:

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\mathrm{d}R^2$$

In the previous, $\mathrm{d}R^2$ is the line element (squared) of three-dimensional space. In polar coordinates $\mathrm{d}R^2 = \mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)$. Let us consider a ray of light moving in a radial direction directed towards the terrestrial observer. For it θ = cost; φ = const. So $\mathrm{d}R^2 = \mathrm{d}r^2$. Furthermore for the light we have $\mathrm{d}s = 0$; so $c^2\mathrm{d}t^2 = a^2(t)\mathrm{d}r^2$; $c\mathrm{d}t = a(t)\mathrm{d}r$; $\mathrm{d}r = \frac{c\mathrm{d}t}{a(t)}$. If we integrate the second member between $t = 0$ and $t = t_0$, the first member will be integrated between $r = 0$ and $r = R_0$, where R_0 is the path of a light ray in time t_0, taking into account the fact that in the interval $0 \div t_0$ space has expanded. Then

$$R_0 = c\int_0^{t_0} \frac{\mathrm{d}t}{a(t)}$$

Assigning to $a(t)$ the expression just proved: $a(t) = 0.772\,\mathrm{Sinh}\left[1.24\frac{t}{t_0}\right]^{2/3}$ we find: $R_0 = 3.28ct_0$. Being $t_0 = 13.8\times10^9$ years, we find $R_0 = 45.2\times10^9$ light-years (radius of visible universe).

1.24. Hubble Tension

JWST observations show galaxies with z values around 11 (or higher). With such values, galaxies must have existed at least 13.4 billion years ago. This may be compatible, albeit with difficulty, with an age of the universe of at least 13.8 billion years ago.

This age is obtainable with a value of

$$H_0 = 67.3\left(\text{km/s}\right)/\text{Mpc} = \frac{67.3\times10^3}{10^6\times3.08\times10^{16}} = 2.18\times10^{-18}\ \text{s}^{-1}.$$

With this value $t_H = \frac{1}{H_0} = 14.5\times10^9$ years, and then $t_0 = 0.950 t_H = 13.76\times10^9$ years.

This value of H_0 was actually observed by the Planck satellite in 2015 using the CMB spectrum. However, the problem is that all direct measurements of H_0, which use Supernovae Ia as standard candle method to measure distances, provide values of H_0 around 72 (km/s)/Mpc [5], and the errors provided exclude that the value of H_0 could be 67.4 (km/s)/Mpc.

The problem is therefore very serious because if the value provided by the Planck satellite is correct, then the methods used to obtain cosmic distances are flawed.

And this is a big problem because these measurements are based on the discovery by Saul Perlmutter, Brian P. Schmidt and Adam Riess of the accelerating expansion of the universe, a discovery for which the three scientists received the Nobel Prize in Physics in 2011.

Or the method of measuring distances using standard candles is correct, and then the model associating the CMB with the epoch of recombination is flawed.

1.25. Horizon Problem

The horizon problem is linked to the fact that the CMB background radiation, which according to ΛCDM cosmology is one of the pieces of evidence of the Big Bang, is extremely homogeneous, meaning it has the same temperature (2.725˚K) at all points in the cosmos surrounding us (except for minor differences). This means that the points on the surface of the plasma sphere that emitted these photons at the so-called *epoch of recombination* were in causal contact with each other.

According to Special Relativity, two points in spacetime are in causal contact when the ratio $\Delta r/\Delta t$ (Δr being the spatial distance between the two points and Δt their temporal distance) is less than or equal to the speed of light c, held to be constant and equal to approximately $3\times10^8\ \text{m}\cdot\text{s}^{-1}$.

According to the Big Bang theory, this radiation was emitted at time $t_{ric} \approx 360000$ years after the initial time t_{bb}. At this time, the electrons and protons that constituted the primordial plasma sphere combined (for the first time) to give rise to neutral atoms. Before recombination, photons were unable to leave the plasma sphere because they were continuously reflected by the elementary particles present in the plasma. At time t_{ric}, the sphere is composed of neutral atoms, and therefore photons can leave its surface. The radius of the primordial sphere at the time of recombination is, according to the ΛCDM model, $R \approx 41\times10^6$ light years.

In fact it is $\frac{T_{ric}}{T_0} = \frac{a_0}{a_{ric}}$; But $T_{ric} = 3000\ \text{K}$; $T_0 = 2.725\ \text{K}$;

Then $\frac{a_0}{a_{ric}} = \frac{3000}{2.725} = 1100$.

This means that the radius of the present universe is 1100 times greater than the radius at the time of recombination. The radius of the visible universe would be $R_0 = 3.28c/H_0 = 45.2$ billion light-years. Therefore, the most distant galaxy we can observe is R_0 away, and this would correspond to the size of the universe. Then the radius at the time of recombination was $\left(45.2\times10^9\right)/1100 = 41\times10^6$ light-years. Two relative points A and B on the surface of the plasma sphere, assumed to be antipodal to each other, would require a signal capable of traveling 82×10^6 light-years in 360,000 years to be in random contact, because this is the time it would take for photons to causally bring points A and B into contact. This is impossible if light is assigned a constant speed of 3×10^8 m/s. The solution that the ΛCDM model provides to the previous problem is that of inflation, that is, assuming that space, starting from the instant $t_{infl} = 10^{-35}$ sec, suddenly expands, increasing its dimensions by a factor of 10^{30} in a time equal to 10^{-30} sec. This mechanism would represent a solution to the problem since it is assumed that in the very short time interval t_{infl} that elapses between the instant t_{bb} and the start of inflation, the universe is concentrated into a volume of microscopic dimensions, in which all points are extremely close (such that light can connect them in time t_{bb}) and therefore certainly in causal contact with each other. This connection is preserved even after the start of the inflationary expansion. The inflationary hypothesis solves a major problem, but raises an even bigger one. The question is what the possible causes of such a rapid and incredibly intense expansion of space might be. Furthermore, this hypothesis does not entail any verifiable physical consequences that could confirm or disprove it. It is, in other words, an ad hoc hypothesis made to solve a problem, but it cannot be proven.

1.26. Flatness Problem

For a generic instant t, we have: $\Omega_k(t) = -\frac{kc^2}{H(t)^2 a(t)^2}$. Differentiating Ω_k with respect to time we find:

$$\frac{\mathrm{d}\Omega_k}{\mathrm{d}t} = \Omega_k(t) H(t)\left[2\Omega_r + \Omega_m - 2\Omega_\Lambda\right]$$

(For details, see, for example, [6]). From the previous equation, it is clear that $\Omega_k(t) = 0$ is a solution to the previous equation. In the epoch immediately following the Big Bang, radiation dominated with respect to vacuum energy. Therefore, the term in square brackets is positive. This means that if $\Omega_k(t)$ at the Big Bang epoch had had a positive value even slightly different from zero, that value would have been amplified to enormous values at the present time, contrary to what has been observed. That the curvature could have been positive at the time of the Big

Bang is predictable from the fact that at that epoch the dimensions of the universe were infinitesimal. The physics that must be adopted in such conditions is quantum physics. And this predicts that there were statistical fluctuations that would have led, even for only brief moments, to a positive curvature.

2. Part Two: TVSL Model

2.1. Basics of the Model

The starting hypothesis of the TVSL model is that the speed of light can vary with time, at a minimal rate, so as not to significantly affect Special Relativity, which has proven to be an extraordinarily precise theory. However, sufficient to explain cosmological phenomena that occur on time scales vastly larger than those addressed by SR.

In this hypothesis, we must distinguish between the speed of light at the current time, which we denote by c_0, and the speed at a generic time *t*, which we denote by $c(t)$.

The FRW metric for a homogeneous and isotropic universe must be referred to the current time t_0.

It will then be:

$$\mathrm{d}s^2 = -c_0^2\mathrm{d}t^2 + a^2(t)\mathrm{d}R^2$$

In the previous $\mathrm{d}R^2$ is the line element (squared) of the space which, in polar coordinates is written:

$$\mathrm{d}R^2 = \mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right)$$

Consider a ray of light traveling in the radial direction. For it, θ = constant, φ = constant. Therefore $\mathrm{d}R^2 = \mathrm{d}r^2$. Furthermore, for light, the line element is zero. Then:

$$0 = -c_0^2\mathrm{d}t^2 + a^2(t)\mathrm{d}r^2;\quad \mathrm{d}r\ a(t) = c_0\ \mathrm{d}t$$

In the previous equation, d*r* is the distance travelled by light (in the radial direction) in the time d*t*. Therefore, d*r*/d*t* is precisely the speed of light. That is,

$$c(t) = \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{c_0}{a(t)}$$

Differentiating:

$$\frac{\mathrm{d}c}{\mathrm{d}t} = -\frac{c_0}{a^2}\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{c_0}{a}\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t} = -c\frac{\mathrm{d}a/\mathrm{d}t}{a}$$

Placing:

$$H = \frac{\mathrm{d}a/\mathrm{d}t}{a}$$

We get:

$$\frac{\mathrm{d}c}{\mathrm{d}t} = -cH$$

Which is the fundamental relationship on which the TSVL model is based. At our time, this relationship provides:

$$\left(\frac{\mathrm{d}c}{\mathrm{d}t}\right)_{t=t_0} = -c_0 * H_0 = -2.998\times10^{8}\left[\mathrm{m\cdot s^{-1}}\right]\times 2.33\times10^{-18}\left[\mathrm{s^{-1}}\right]$$
$$= -6.98\times10^{-10}\left[\mathrm{m\cdot s^{-2}}\right]$$

This acceleration value implies that the variation of c in one year is

$$\Delta c = -6.98\times10^{-10}\times365\times24\times3600 = 2.2\times10^{-2}\left[\mathrm{m\cdot s^{-1}}\right]$$

That is, the speed of light, according to the previous model, should decrease by only 2.2×10^{-2} [$\mathrm{m \cdot s^{-1}}$] each year. This value is so small that it has absolutely no impact on laboratory measurements, which occur on much shorter timescales. In any case, even experiments of this duration would produce a relative error $\varepsilon_r \approx 7\times10^{-11}$. However, his value it is capable of explaining many physical phenomena as an alternative to the theory of General Relativity, which, as we have seen, predicts an expanding universe with an accompanying Big Bang and all the problems that this entails, some of which we have already seen. First, we note that the negative acceleration: $a_c = \mathrm{d}c/\mathrm{d}t = -Hc$ can be related to a phenomenon already observed for all the probes (Pioneer 11, Galileo, Ulixes) that have left the solar system to venture into sidereal space. In this space, the gravitational force of the Sun is absolutely negligible. Therefore, the probes should proceed at a constant speed. Instead, a negative acceleration (*i.e.* directed towards the sun) equal to $a_0 \approx -8\times10^{-10}$ [$\mathrm{m \cdot s^{-2}}$] was measured for them.

It has been agreed to attribute this deceleration to a reflection phenomenon of the radiation emitted by the spacecraft's generators and reflected by their antennas. This explanation seems implausible if one considers that the measured deceleration has the same value despite the generators, weights, and shapes of the various spacecraft being very different.

It seems natural, on the contrary, to think that what is being measured is not the deceleration of the spacecraft, but the deceleration of the electromagnetic waves $a_c = -6.98\times10^{-10}$ [$\mathrm{m \cdot s^{-2}}$] that the spacecraft sends toward Earth to signal its position.

Another phenomenon that is easily explained by assuming that light is subject to a deceleration $a_c = -6.98\times10^{-10}$ [$\mathrm{m \cdot s^{-2}}$] concerns the rotation curve of the stars in a galaxy. As far as we know, every galaxy has at its center a supermassive black hole around which the stars of the galaxy rotate according to a law that, as a first approximation, should predict the equality between the gravitational force GMm/r^2 (M = black hole mass, m = star mass), and the centrifugal force: mv^2/r.

That is $v = \sqrt{G\frac{M}{r}}$.

The velocity of the stars should therefore tend to zero as the distance r from the centre of the galaxy increases. In reality, the velocity v, for r greater than a limit value r_{lim}, does not decrease at all, but remains constant, as if for $r > r_{lim}$ the acceleration was not related to the distance *r*, but had a constant value a_0.

The MOND theory explains the rotation curves with an ad hoc hypothesis. That is, it introduces an acceleration a_0 to which it assigns the value $a_0 \approx 2\times10^{-10}$ [m·s^{-2}]. Newton's second law is not $F = ma$, but $F = \mu ma$, with $\mu = 1$ for $a \gg a_0$, while $\mu = a/a_0$ for $a \ll a_0$.

Then, in this last case, we would have: $\mu a = \frac{GM}{r^2}$;

$$\frac{a^2}{a_0} = \frac{GM}{r^2}; \quad \frac{v^4}{r^2 a_0} = \frac{GM}{r^2}; \quad v = \sqrt[4]{G a_0 M}$$

This latter relationship is known as the *Tully-Fisher relation* and links the rotation speed of a galaxy's peripheral stars to the galaxy's normal mass, and not to a phantom dark matter that has so far remained untraced, despite over 50 years of research.

The Tully-Fisher relation has proven valid for elliptical galaxies.

The weakness of the MOND theory is that the acceleration a_0 remains mysterious. Furthermore, the hypothesis that Newton's law varies as $a < a_0$ has been falsified by laboratory experiments.

How then can we explain the validity of the Tully-Fisher law?

The explanation is simple. When the redshift from the stars of a galaxy is measured to find their velocity, this redshift is made up of two parts. The first is actually related to the velocity of the stars. The second is, instead, related to the deceleration a_c that the light undergoes between the instant of emission and the instant of detection.

A comprehensive explanation of the phenomenon is provided in [7].

2.2. TVSL Model and Cosmological Redshift

According to the TVSL, redshift is due to the variation in the speed of light with time. This model assumes that the speed of light in a vacuum, $c(t)$, varies over time proportionally to the value of $c(t)$. More precisely, it decreases with time according to the relationship:

$$\frac{\mathrm{d}c(t)}{\mathrm{d}t} = -Hc(t)$$

This relationship was proposed in [8] and justified in [7] [9]. The dependence of the variation $\mathrm{d}c/\mathrm{d}t$ on velocity suggests that the forces causing this variation are dissipative, *i.e.*, frictional forces with the components of a fluid assumed to be the low-energy neutrino background of the universe. That such a background must exist can be deduced from the fact that neutrinos are produced (at high energy) in the nucleus of stars, where fusion reactions occur that give rise to the energy of stars and heavy elements. Once produced, neutrinos travel at a speed close to *c*, interacting very little with matter. Following countless collisions, they can only lose energy, transforming into low-energy neutrinos, which are currently undetectable. In fact, the current detection threshold for neutrinos is 5 Mev.

The TVSL model also predicts the existence of a cosmological redshift proportional to distance (Hubble's law). However, this is not attributed to the expansion of space, *i.e.*, the separation of galaxies from each other, but rather to the variation of c over time. In fact, since $c = \lambda f$ (λ = wavelength, f = frequency), it is impossible for c to vary without a variation of λ, or f. Since the frequency f is related to energy, if conservation of energy is assumed, the wavelength λ must vary. If c decreases with time, λ will also decrease with time, and therefore a galaxy G observed by us in its distant past will emit light with a larger λ than that emitted by atoms at the present time. This is the explanation of the redshift phenomenon according to the TVSL model. The TVSL model differs from the ΛCDM model, but also from the so-called "tired light" models. In the ΛCDM model, the frequency of the light emitted by a galaxy G varies along the path from G to the observer O, due to the dilation of space. This leads to the conclusion that a photon traveling from G to O loses energy along the way without experiencing any collisions, and this is contrary to the principle of conservation of energy. As for the "tired light" models, the frequency of the emitted photons decreases along the path from the galaxy G to the Earth observer, due to collisions of the photons with interstellar matter. But every collision of a photon with matter causes, due to the Compton effect, a deviation of the photon from the GO path. The photon that suffered the collision would therefore not be seen by an observer on Earth (given the distance, both G and O can be thought of as point-like).

In the case of TVSL, on the contrary, the wavelength during the GO path remains constant. The photons observed by the Earth-based observer are those that have not interacted with interstellar matter or with the neutrino fluid existing in the universe because they are massless. It is instead the electrons in atoms that interact with the neutrino background. In fact, both electrons and neutrinos have mass, albeit minimal. This very weak interaction leads to a decrease in the electronic velocity v, and since this is related to the velocity of the photons emitted during an energy transition, the velocity of the emitted photons varies (decreases over time). Once emitted, the photon travels through space with unchanged velocity (and therefore with λ and f unchanged).

The TVSL model assumes that the speed of light varies with time according to the law:

$$\frac{\mathrm{d}c(t)}{\mathrm{d}t} = -Hc(t).$$

In the previous, H is a constant that, as we will see, coincides with the Hubble constant H_0. Letting t_e indicate the instant at which a ray of light is emitted by a galaxy G, and t_0 the time at which it is observed, we have:

$$\int_{t_e}^{t_0} \frac{\mathrm{d}c(t)}{c(t)} = -H\int_{t_e}^{t_0} \mathrm{d}t;\ \left[\ln\left(c(t)\right)\right]_{t_e}^{t_0} = -H\left(t_0 - t_e\right)$$

$$\ln c(t_0) - \ln c(t_e) = -H\left(t_0 - t_e\right);$$

$$\ln\left[\frac{c(t_0)}{c(t_e)}\right] = -H\left(t_0 - t_e\right);$$

$$c(t_0) = c(t_e)\mathrm{e}^{-H(t_0 - t_e)}$$

2.3. Hubble's Law in the TVSL Model

The cosmological redshift is a parameter that measures the experimental fact that the wavelength λ_{ric} of the signal received from an apparent distant galaxy is longer than the wavelength λ_{emL} of the signal emitted in the laboratory. That is

$$z = \frac{\lambda_{ric} - \lambda_{emG}}{\lambda_{emL}}$$

Regarding the times relative to the previous signals, we have $\lambda_{ric} = \lambda(t_e)$, the received signal was emitted at time of emission t_e, and maintain this value until the present time t_0; while $\lambda_{emL} = \lambda(t_0)$, meaning the laboratory signal is emitted at a time t_0. In the ΛCDM model, atoms are considered immutable objects; therefore, they have emission spectra that remain unchanged over time and, during a given energy transition, they always emit light signals with the same frequency and wavelength. Therefore, $\lambda_{emG} = \lambda_{emL} = \lambda(t_0)$.

The previous equation is therefore written:

$$z = \frac{\lambda_{ric} - \lambda_{emL}}{\lambda_{emL}} = \frac{\lambda(t_e) - \lambda(t_0)}{\lambda(t_0)} = \frac{\lambda_e - \lambda_0}{\lambda_0}$$

In the TVSL model, the difference between λ_{ric} and λ_{emG} has another explanation. This difference is due to the variation in velocity c between the time t_0 of reception and the time t_e of emission of the light signal.

Therefore

$$z \equiv \frac{c(t_e) - c(t_0)}{c(t_0)} = \frac{c(t_e)}{c(t_0)} - 1 \quad \text{def. of redshift in TVSL.}$$

Using the relationship:

$$c(t_e) = c(t_0)\mathrm{e}^{H(t_0 - t_e)}$$

we have

$$z = \mathrm{e}^{H(t_0 - t_e)} - 1$$

For values of $H(t_0 - t_e) \ll 1$ we have $z \approx [1 + H(t_0 - t_e)] - 1$; $z \approx H(t_0 - t_e)$.

But if t_e is the emission time, we have $t_0 - t_e = d/c_e$, where d is the distance traveled by the signal.

$$z = \frac{Hd}{c_e} \quad \text{Hubble's law}$$

The previous is Hubble's law, valid for small distances, when $c_e \approx c_0$. Instead, the relationship between z and d for any distance is:

$$z = \mathrm{e}^{\frac{Hd}{c_e}} - 1 \quad \text{relationship between redshift and distance.}$$

2.4. Resolution to the Dark Energy Problem

The relationship $c(t_0) = c(t_e)\mathrm{e}^{-H(t_0 - t_e)}$, allows us to resolve the puzzle underlying the ΛCDM concept of *dark energy*. Recall that this concept was introduced to explain the behavior of the scaling factor $a(t)$, which exhibits an exponential behaviour starting from time $t/t_0 = 0.53$, indicated in **Figure 1**. The ΛCDM model explains this behaviour by introducing a constant Λ into Einstein's equation (cosmological constant). This means that a third component is added to the two known components that constitute the energy density (radiation and matter), which has the property of creating a repulsive force between galaxies, so that the expansion accelerates, rather than slowing down as would be expected if the only force acting between galaxies were gravitational. This energy component is called *dark energy* and is associated with a negative pressure. The energy density Ω of the universe (relative to the critical energy) then has three components: Ω_r (radiation), Ω_m (matter), and Ω_Λ (vacuum energy). To the component relating to vacuum energy is given the value $\rho_\Lambda = \frac{c^4\Lambda}{8\pi G}$. In this way, in the case of a universe dominated by the vacuum, we find the equation that we have already seen (par. 1.21):

$$a(t) = a_0 e^{H_0(t-t_0)} \quad (\text{with } H_0 = c\sqrt{\frac{\Lambda}{3}})$$

The cosmological constant and dark energy are introduced into the ΛCDM model in order to arrive at the previous formula, which indicates that the expansion of the universe is accelerating and is confirmed by astronomical observations.

But note that the previous formula has a very simple derivation in the TVSL model.

In fact, we have just seen that we have:

$$c(t_0) = c(t_e) e^{-H(t_0-t_e)}$$

From which:

$$c(t) = c(t_e) e^{-H(t-t_e)}$$

But we have seen that $c(t)$ and $a(t)$ are inversely proportional:

$$c(t) = \frac{c_0 a_0}{a(t)}$$

So:

$$\frac{c_0 a_0}{a(t)} = c(t_e) e^{-H(t-t_e)}$$

and placing $t_e = t_0$, we have

$$\frac{c_0 a_0}{a(t)} = c(t_0) e^{-H(t-t_0)}; \quad \frac{a_0}{a(t)} = e^{-H(t-t_0)};$$

Or:

$$a(t) = a_0 e^{H(t-t_0)}$$

Therefore, in the TVSL model, there is no dark energy, and therefore there is *no need to introduce the term Λ into Einstein's equation.*

2.5. Relationship between ΛCDM and TVSL

The measure of distance d of a galaxy, in the TVSL model, depends on the speed of light *c*(*t*), because $z=\frac{\Delta\lambda}{\lambda}=\frac{\Delta c}{c}$. Such measures have an exponential trend because the speed of light varies with time according to an exponential law:

$$c(t)=c(t_e)\mathrm{e}^{-H(t-t_e)}.$$

In the Standard model the constant *H* depends on the scale factor according to the relation $H=\frac{\mathrm{d}a/\mathrm{d}t}{a}$ while, in the TVSL model, *H* depends on *c*(*t*) according to the relation $H=-\frac{\mathrm{d}c/\mathrm{d}t}{c}$.

Therefore, the link between the two models is provided by the relation $a(t)=\frac{c_0 a_0}{c(t)}=\frac{c_0}{c(t)}$ (assuming as usual $a_0=1$).

That is, the two models are different interpretations of the same experimental observation: a galaxy, with a distance greater than 100 *Mpc, exhibits a redshift proportional to its distance.*

This redshift can be interpreted in two alternative ways.

- STANDARD MODEL: The redshift $z=\frac{Hd}{c}=\frac{\Delta\lambda}{\lambda}$ is due to the galaxies moving away from each other (Standard model). The galaxies move away because the distance $d\propto a(t)$ between them increases over time at the rate $H=\frac{\mathrm{d}a/\mathrm{d}t}{a}$.
- TIME-VSL MODEL: The redshift is due to the decrease of the light-speed *c*, over time, at the rate $H=-\frac{\mathrm{d}c/\mathrm{d}t}{c}$ and, because *c* = *λf*, with the rate $H=-\frac{\mathrm{d}\lambda/\mathrm{d}t}{\lambda}$ (the frequency, proportional to energy, is constant).

From a mathematical point of view, we can therefore switch from one model to the other simply by replacing *c* con *c*(*t*) and *a*(*t*) with $\frac{c_0}{c(t)}$ (assuming as usual $a_0=1$). From a conceptual point of view, however, the TVSL model does not present all the critical issues (dark matter that we can't observe, dark energy of which we don't know the nature, cluster of galaxies old as the Big Bang, black holes supermassive old as the Big Bang, and so on…). Note that in the TVSL model, there is no radius of the visible universe, since there is no instant at which light signals were first emitted. The universe is considered stationary, meaning identical to itself over time. Therefore, no matter how far into the distant past one looks, one will find a universe identical to the present one. No matter how far away one looks, one will find galaxies. The limitation is not theoretical but is linked to the capacity of the signal detectors and the type of signals used.

Furthermore, there are no different cosmological epochs (radiation, matter, and vacuum eras).

This means that **Figure 1**, which provides the scale factor as a function of normalized time, must be replaced with the figure obtained from the relation

$$a(t)=\frac{c_0}{c_0 e^{-H(t-t_0)}}=e^{H(t-t_0)}=e^{Ht_0(t_n-1)}.$$

That is, **Figure 1** provides a correct trend of $a(t)$, conceived as the inverse of $c(t)$, only in the interval in which it presents a positive curvature.

The TVSL model predicts that this trend will also extend to high values of z, contrary to the prediction of the standard model which predicts that the curvature will reverse for values of z greater than the value corresponding to the inflection point in the diagram in **Figure 1** which corresponds to the value $z=z_F=0.887$.

In fact, the relationship between redshift and scale factor is

$$z=\frac{\lambda_{obs}-\lambda_{em}}{\lambda_{em}}=\frac{\lambda_{obs}}{\lambda_{em}}-1;$$

$\frac{\lambda_{obs}}{\lambda_{em}}=z+1$. But $\frac{\lambda_{obs}}{\lambda_{em}}=\frac{\lambda(t_e)}{\lambda(t_0)}=\frac{c(t_e)}{c(t_0)}=\frac{a(t_0)}{a(t_e)}=\frac{1}{a(t)}$. So $z+1=\frac{1}{a}$. Then with $a=a_F=0.53$ we find $z_F=0.887$.

2.6. The Dark Energy Problem

The major problem for the standard cosmological model is the experimental observation, for which Perlmutter won the Nobel Prize in Physics, that the expansion of the universe accelerates over time. This conclusion was reached by carefully measuring z as a function of distance. A trend of z was found, indicated by the red dots in **Figure 2**.

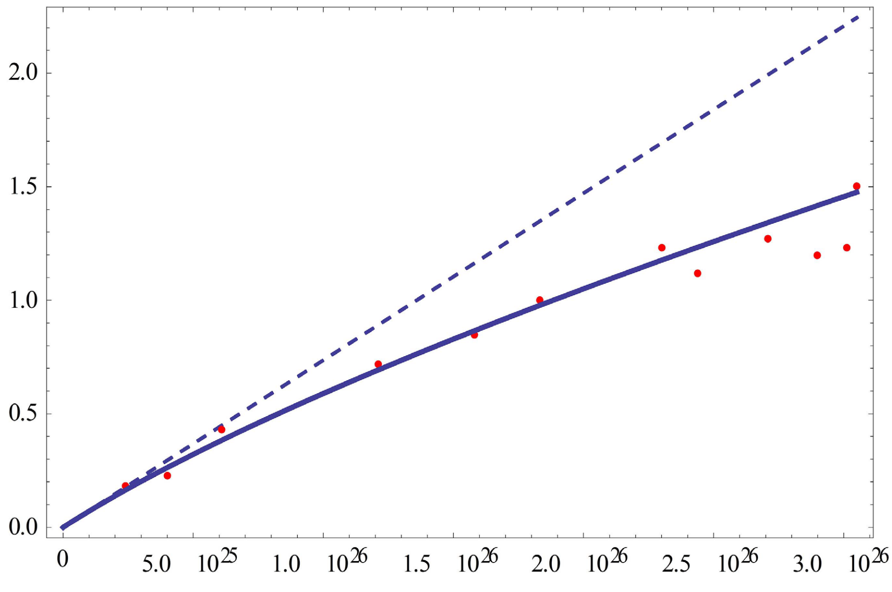

Figure 2. Redshift versus distance (in meters).

In this figure, the dashed line indicates the $z = H_0 \frac{d}{c}$ curve, which gives the value (at present time) $H_0 = \left(2.998 \times 10^8\right) / \left(1.3 \times 10^{26}\right) = 2.3 \times 10^{-18}\ \mathrm{s}^{-1}$. In this graph, the direction of time is opposite to that of distance. In fact, the more distant a galaxy is, the further back in time it has issued the observed signal. It follows that *as time increases* (*i.e. going in the left direction*), *the slope of the z = z*(*d*) *curve increases*. Since redshift measures the rate of expansion of the universe (according to the standard model), it follows that as time increases, the rate at which the universe expands increases.

This is paradoxical for two reasons. The first is due to the fact that galaxies exert an attractive gravitational force on each other. Therefore, any expansion is expected to slow down over time, not accelerate. The second is that to explain this trend, we need to think of some form of dark energy exerting a negative pressure that opposes the gravitational force, causing the universe to expand at an accelerated rate. But to explain the measured trend, we need to attribute to this form of energy a density of $\Omega_\Lambda = 0.685$. That is, approximately 70% of the universe's energy should consist of this dark energy. But the nature of this widespread dark energy is currently unknown. The hypothesis that it is due to the quantum energy of the vacuum clashes with the fact that according to quantum field theory, the energy of the vacuum should have a density many orders of magnitude (123 orders of magnitude!) greater than $\rho_\Lambda = \Omega_\Lambda \rho_{crit}$.

On the contrary, the trend indicated in **Figure 2** can be easily explained in the TVSL model. In fact, in this model, the relationship between redshift and distance is:

$$z = \mathrm{e}^{\frac{Hd}{c_e}} - 1$$

The velocity c_e can be calculated as follows. We have: $c(t) = c_e \mathrm{e}^{-H(t-t_e)}$, where c_e is the velocity at the time of emission t_e. The origin of time is arbitrary. Let's assume the current time as the origin. That is, let's assume $t_0 = 0$. It follows that $c(t_0) = c_0 = c_e \mathrm{e}^{Ht_e}$. If the origin of time is the current instant, the time of emission t_e will be a negative instant. Then $t_e = -d/c_e$.

From the relation $c_0 = c_e \mathrm{e}^{-\frac{H}{c_e}d}$ using a calculation program like Mathematica, we can calculate c_e as a function of c_0 and *d*. Substituting this into the formula that gives *z* as a function of *d*, we obtain the function *z*=*z*(*d*), whose graph is shown in **Figure 2** (thick line) and which closely approximates the experimental values found.

2.7. Resolution of the Horizon Problem in the TVSL Model

In the TVSL model, the horizon problem does not arise. In fact, redshift is not due to the expansion of the universe, but rather to the fact that the atoms that make up galaxies emit photons with decreasing wavelengths (at the same frequency) over time. Since the galaxies we observe are in their distant past, the light reaching us from galaxies was emitted millions (or billions) of years ago, when the atoms emitted photons at high velocities. The photons reaching us from these galaxies have maintained this high velocity (relative to c_0) and therefore have maintained a long wavelength. The universe, therefore, not being expanding, is in a stationary state. This presupposes that it has no age. It has therefore had an eternity of time to reach a homogeneous temperature and density in all its components.

2.8. Resolution of the Flatness Problem

The TVSL model considers the universe to be stationary, eternal, and infinite. Unlike the ΛCDM model, galaxies are not all born at the same time (within a few billion years after recombination), but are continuously born and die. On average, the number of galaxies born is equal to the number of galaxies that die. The Milky Way is one of the oldest galaxies in the universe. The speed of light depends on time. But from what time? Since the speed of light is assumed to be maximum at the instant of galaxy formation, time must be measured with reference to the instant of galaxy formation. Since z is a measure of the difference in the speed of light emitted by two galaxies, it follows that z measures the difference in the ages of the galaxies, in addition to measuring their distances. This can be deduced from the formula: $z = H(t_0 - t_e)$. If we measure time from the instant of origin of a galaxy, then t_e is the age of the galaxy at the time of emission. While t_0 is the age of the Milky Way at the time of reception. The flatness problem arises from the fact that the Big Bang model assumes that at the initial instant of the universe, it must have had infinitesimal dimensions. Therefore, around time t_{bb}, it must have been dominated by quantum physics. This presupposes that fluctuations exist that, even for a brief instant, make the curvature positive. But solving the Friedman equation with positive curvature leads to a rapid increase in curvature. Therefore, now, at time t_0, we should find ourselves with enormous curvature. This does not happen, and this gives rise to the problem. All of the previous discussion in the TVSL model is pointless. In fact, there has never been a time when the dimensions of the universe were microscopic. The universe has always been infinite and with zero curvature. And, being stationary, it will maintain this null curvature over time.

3. Part Three: Black Holes

3.1. TVSL Model of Black Holes

In the TVSL model, the speed of light undergoes very modest changes over time. These changes are important at the cosmological level, where General Relativity comes into play, but they are not at all important at the level of Special Relativity. In fact, we have $(\mathrm{d}c/\mathrm{d}t)/c = -H = -2.33 \times 10^{-18}$. The percentage change in c over time is so small that any result produced by Special Relativity can be considered exact. Therefore, to address the behaviour of black holes, we can use the latter, and in particular, we can use the result that the mass of an object varies as a function of its velocity, according to the relation $m = m_0 \big/ \sqrt{1-\beta^2}$. With m_0

being the rest mass of the object, stationary in the inertial frame in which the measurements are made, and with $\beta = V/c$ being the ratio between the velocity V of the object and the speed of light. The definition of a black hole (BH) is that of a celestial body that has an escape velocity equal to that of light. To calculate the escape velocity, the kinetic energy of the body is equalized to the potential energy due to the gravitational force. For small velocities, the kinetic energy is $E_c = 1/2\, mv^2$, while the potential energy due to the gravitational force of the mass M of the celestial body is $E_p = GM\frac{m}{R}$ (The potential energy is equal to the work required to bring m from infinity to a distance R from the center).

Then

$$\frac{1}{2}v^2 = \frac{GM}{R};\quad R = 2GM/v^2$$

If we define a Black Hole as a celestial body with an escape velocity of c, the previous formula gives the radius of the Black Hole: $R_{bh} = 2GM/c^2$. The previous formula is the same as that provided by *GR*, although it derives it differently. Let's see what happens if we take Special Relativity into account. For this, the kinetic energy is: $E_c = m_0c^2(\gamma - 1)$. While for the potential energy, keep in mind that in $E_p = GMm/R$, the mass m must be replaced with the relativistic mass $m = \gamma m_0$.

Then

$$c^2(\gamma - 1) = G\frac{M\gamma}{R};\quad \gamma = \frac{Rc^2}{c^2R - GM};$$

$$\text{Being}\quad \gamma = \frac{1}{\sqrt{1-\beta^2}}\quad \text{we find}\quad \beta^2 = \frac{2c^2RGM - G^2M^2}{R^2c^4}$$

Or:

$$v^2 = \frac{2GM}{R} - \frac{G^2M^2}{R^2c^2}$$

The previous one provides the escape velocity if special relativity is taken into account.

Note that for v to be real, it must be $\frac{2GM}{R} - \frac{G^2M^2}{R^2c^2} > 0$; Or $R > G\frac{M}{2c^2}$. Therefore, if the radius of the cosmic body is lesser than $R_{bh} = \frac{1}{2}\frac{GM}{c^2}$, then no particles, not even photons, can leave the body. Therefore, this value represents the radius of the BH in the new model. Note that for $R = R_{bh}$, the escape velocity is zero. While the escape velocity is equal to c for the value of the radius obtained from: $c^2 = \frac{2GM}{R} - \frac{G^2M^2}{R^2c^2}$. Or $R = R_{hor} = \frac{GM}{c^2} = 2R_{bh}$. We call the surface of the sphere of radius R_{hor} the horizon. *For values of R between* R_{hor} *and* R_{bh}, *the escape velocity varies from c to zero. Then any particle, even heavy ones, can leave this region* (*a circular cap with a thickness equal to the radius of the BH*). *Since from* *this region* (*spherical shell*) *particles with have sufficient energy can escape from the gravitational attraction of the BH*, *we will call it the depletion zone*.

According to this model, the luminosity observed around each black hole is not given by the radiation emitted by the matter falling into the BH, but is given by the radiation emitted, along with elementary particles, from the surface of the depletion zone surrounding the BH. Current observations of black holes indicate the presence of a bright region surrounding a central dark zone. **Figure 3** shows the BH at the centre of our galaxy, captured by the Event Horizon Telescope (EHT), an array of eight radio observatories. It should be noted that the figure is an optical reconstruction of what are actually radio signals. Therefore, the central dark region (black hole) indicates that the radiation intensity in that region is lower than in the surrounding area (depletion zone). This is evident if we consider that this central region corresponds to the BH.

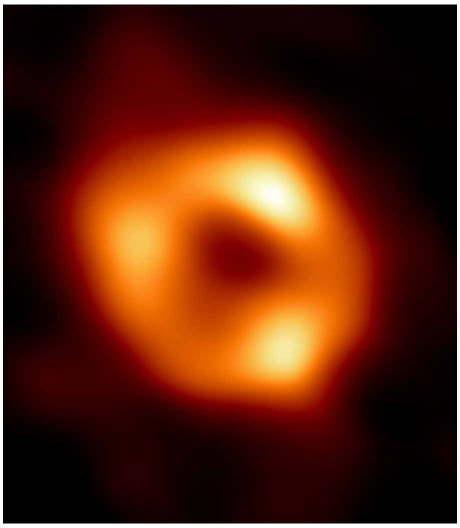

Figure 3. The first direct visual evidence of the presence of a black hole at the centre of our Galaxy, the Milky Way. It was captured by the Event Horizon Telescope (EHT), an array that connects eight existing radio observatories. The photo was published on May 12, 2022. Note how the thickness of the depletion zone equals the radius of the BH.

3.2. Redshift Arising from the Dispersive Properties of Empty Space

All the problems presented by the Big Bang model suggest that the main hypothesis from which it originates, namely that cosmological redshift is due to the expansion of space, is incorrect. It is therefore necessary to consider an alternative hypothesis.

An interesting hypothesis was recently proposed by Wenzong Zhang, who, in an interesting preprint [10], shows how Hubble's law can be deduced from the hypothesis that empty space possesses an extremely small, but non-zero, conductivity. Therefore, an electromagnetic wave propagating in empty space is subject to a loss of energy responsible for the decrease in its frequency and therefore responsible for cosmological redshift. It should be noted that this hypothesis differs from the so-called "tired light" theory [F. Zwicky, 1929]. In fact, in the latter, photons would lose energy during the journey from the source to the observer due to "collisions" with particles certainly present in interstellar space. But such collisions, due to the Compton effect, should produce deviations from the path, so the source should appear blurry, which does not happen. Instead, Zhang's theory, unlike classical electromagnetism, predicts that empty space has its own conductivity, albeit very low. This conductivity could be due to the presence of virtual particles, that is, charged particles with very short lifetimes. The existence of such particles is theoretically predicted by quantum theory and has been linked to numerous experimentally verified effects (for example, the Casimir effect). To deduce Hubble's law from this hypothesis, Zhang begins with fourth Maxwell's law $\nabla \times \boldsymbol{B} = \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$ and adds to the right side the current produced by an electric field $\boldsymbol{E}$ applied to a medium of conductivity σ (Ohm's law). Then

$$\nabla \times \boldsymbol{B} = \mu \sigma \boldsymbol{E} + \mu \varepsilon \frac{\partial \boldsymbol{E}}{\partial t}$$

In the previous equation, σ is the conductivity of the vacuum. The first three Maxwell equations remain unchanged. From Maxwell's equations, it can be deduced that if $\Psi(x,t)$ represents any component of the electric or magnetic field, it must be:

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} - \mu \varepsilon \frac{\partial^2 \Psi(x,t)}{\partial t^2} - \mu \sigma \frac{\partial \Psi(x,t)}{\partial t} = 0 \quad \textit{Zhang equation}$$

This equation admits a plane wave as a solution $\Psi(x,t) = \Psi_0 \mathrm{e}^{i(kx - \omega t)}$

From this: $\frac{\partial \Psi}{\partial x} = ik\Psi$; $\frac{\partial^2 \Psi}{\partial x^2} = -k^2 \Psi$; $\frac{\partial \Psi}{\partial t} = -k^2 \Psi$; $\frac{\partial \Psi}{\partial t} = -i\omega \Psi$; $\frac{\partial^2 \Psi}{\partial t^2} = -\omega^2 \Psi$.

Substituting into Zhang's equation we find: $k^2 = \omega^2 \mu \varepsilon + i\omega \mu \sigma$.

The wave vector k is therefore a complex number that can be written in the form $k = \alpha + i\beta$. Substituting into the expression for $\Psi(x,t)$ we find: $\Psi(x,t) = \Psi_0 \mathrm{e}^{-\beta x} \mathrm{e}^{i(\alpha x - \omega t)}$

As can be seen when introducing the conductivity of a vacuum, the electromagnetic wave undergoes a decrease in the magnitude of its electric (and magnetic) field during its propagation, since this magnitude is multiplied by the term $\mathrm{e}^{-\beta x}$.

We then derive the coefficient β. Substituting $k^2 = (\alpha + i\beta)^2$ into the expression for k^2 , and equating the real parts and the imaginary parts, we find:

$$\alpha = \omega \sqrt{\frac{\mu_0 \varepsilon_0}{2}} \left(\sqrt{1 + \frac{\sigma^2}{\omega^2 \varepsilon_0^2}} + 1 \right)^{1/2} ;$$

$$\beta = \omega \sqrt{\frac{\mu_0 \varepsilon_0}{2}} \left(\sqrt{1 + \frac{\sigma^2}{\omega^2 \varepsilon_0^2}} - 1 \right)^{1/2}$$

Since the conductivity of the vacuum σ is very small, we can set:

$$\alpha = \omega\sqrt{\frac{\mu_0\varepsilon_0}{2}}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon_0^2}}+1\right)^{1/2} \approx \omega\sqrt{\frac{\mu_0\varepsilon_0}{2}}\sqrt{2} = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c} = \frac{2\pi}{\lambda} = |k|$$

While $\beta \approx \omega\sqrt{\frac{\mu\varepsilon}{2}}\frac{\sigma}{\sqrt{2}\omega\varepsilon} = \frac{1}{2}\sigma\sqrt{\frac{\mu}{\varepsilon}}$

The electric field, as stated, is: $E(x,t) = E_0 \mathrm{e}^{-\beta x}\mathrm{e}^{i(\alpha x-\omega t)}$

And its square module is: $|E|^2 = E(x,t) * \overline{E(x,t)}$, being $\overline{E(x,t)} = E_0\mathrm{e}^{-\beta x}\mathrm{e}^{-i(\alpha x-\omega t)}$.

Then $|E|^2 = E_0^2\mathrm{e}^{-2\beta x}$. The energy of this electric field is $W(x) = \frac{1}{2}\varepsilon|E|^2 = \frac{1}{2}\varepsilon E_0^2\mathrm{e}^{-2\beta x}$ From wich:

$$W_0 = \frac{1}{2}\varepsilon|E|^2 = \frac{1}{2}\varepsilon E_0^2 \quad [W_0 = W(x=0)]$$

If the electromagnetic field is associated with a photon, the energy W of the latter is proportional to its frequency *f*. Then, indicating with f_e the frequency of emission, that is for $x = 0$ we have:

$$f(x) = \frac{1}{2}\varepsilon|E|^2 = \frac{1}{2}\varepsilon E_0^2\mathrm{e}^{-2\beta x}$$

$$f_e = \frac{1}{2}\varepsilon|E|^2 = \frac{1}{2}\varepsilon E_0^2$$

It follows:

$$f(x) = f_e\mathrm{e}^{-2\beta x}$$

The previous one provides Hubble's law. Indeed, for small distances

$$f(x) = f_e\mathrm{e}^{-2\beta x} \approx f_e(1-2\beta x)\,; \quad \frac{f}{f_e} = 1-2\beta x$$

But $z = \frac{f_e - f}{f_e} = 1 - \frac{f}{f_e} = 2\beta x$

In Hubble's law $z = \frac{Hx}{c}$. The previous one represents Hubble's law if $2\beta = \frac{H}{c}$

Being $\beta = \frac{1}{2}\sigma\sqrt{\frac{\mu}{\varepsilon}}$ we find: $H = \sigma\mu c^2 = \frac{\sigma}{\varepsilon}$.

3.3. Atomic Origin of the Cosmological Redshift

Zhang's theory is based on the concept that the speed of light remains unchanged over time. It is only the frequency of electromagnetic waves that varies during their journey from the source to the observer. But assuming that the speed of light, emitted by atoms during their energy transitions, remains constant over time is equivalent to conceiving atoms as eternal and immutable objects. However, this contradicts the fact that atoms are made up of particles like electrons that have mass. They are therefore capable of interacting with all the massive particles in the environment surrounding the atoms, such as neutrinos. That such particles exist and are very numerous is indicated by the fact that high-energy neutrinos are produced in the nuclei of stars. A neutrino is produced for every fusion reaction. By calculating the energy produced in one second by a star, we can calculate how many neutrinos are produced per second. By multiplying by the number of stars in the universe, we can calculate how many neutrinos are produced per second. Once produced, the neutrino is indestructible. It travels almost undisturbed through the universe, interacting very little with matter. Each small interaction produces a decrease in its energy, so after a long time it transforms into a low-energy neutrino (less than 5 MeV: the detection limit), constituting the cosmic neutrino background. It is precisely with the neutrinos of the neutrino background that atomic electrons interact, losing energy, thus slowing their velocity and the speed of the photons emitted during their energy transitions. In fact, the speed of light emitted by electrons during an energy transition is proportional to the speed of the electrons according to the relation: $v = \alpha c$, with $\alpha = \frac{ke^2}{\hbar c}$ (fine structure constant). Since α is dimensionless, time does not enter into its measurement. Therefore, it is independent of *t*. Therefore, a decrease in v translates into a decrease in c. Hubble's law can be derived in a similar way to Zhang's, but using not Maxwell's equation, but the Klein-Gordon equation, which provides the wave function of the electron in the hydrogen atom. By appropriately modifying this equation, one can take into account the fact that the electron loses energy due to the interaction of the neutrino background.

The Klein-Gordon equation for the hydrogen atom, modified to account for the decrease in energy, is:

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \Psi(x,t)}{\partial t^2} - \frac{m^2c^2}{\hbar^2}\frac{\partial\ \Psi(x,t)}{\partial t} = 0$$

Compared to the classical KG equation, has been added the term $-\left(\frac{m^2c^2}{\hbar^2}\right)\frac{\partial \Psi(x,t)}{\partial t}$ which refers to the energy lost through interaction with the neutrino background. In the equation the function $\Psi(x,t)$ it is the so-called wave function. It has the propriety that $|\Psi(x,t)|^2$ gives the probability of finding the electron at the point of coordinate x at time *t*. If every point in space has, at instant *t*, a non-zero probability of containing the electron, this, and therefore also its energy, can be thought as distributed in space. The quantity $|\Psi(x,t)|^2$ is then proportional to the energy of the electron at the point of coordinate x at time *t*.

The change in energy will therefore $\frac{\partial W}{\partial t} \propto \frac{\partial}{\partial t}|\Psi(x,t)|^2 \propto \frac{\partial \Psi(x,t)}{\partial t}$. (The symbol $\propto$ indicates proportionality). The constant $\left(m^2c^2\right)/\hbar^2$ was used to ensure that this term has the same physical dimensions as the other terms. Proceeding exactly as we did in the case of Zhang's equation, that is, looking for a solution of the form $\Psi = \Psi_0 e^{i(kx-\omega t)}$ we find:

$$-k^2 + \frac{\omega^2}{c^2} + \frac{i\sigma\omega}{\varepsilon c^2} = 0$$

Setting $k=\alpha+i\beta$, or $k^2=\alpha^2-\beta^2+2i\alpha\beta$ and comparing with the previous one:

$$\frac{\omega^2}{c^2}=\alpha^2-\beta^2\,;\quad \frac{\sigma\omega}{\varepsilon c^2}=2\alpha\beta$$

Similarly to what was obtained in the case of Zhang's equation we find:

$$\alpha=\omega\sqrt{\frac{\varepsilon\mu}{2}}\left[1+\sqrt{1+\frac{\sigma^2}{\varepsilon^2\omega^2}}\right]^{1/2}\,;\quad \beta\approx\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

That is, we find the same expressions found by Zhang. Therefore, we find the same expression for the frequency of the photon emitted by the atom during an energy transition:

$$f(r)=f_e\mathrm{e}^{-2\beta r}\approx f_e(1-2\beta r)$$

We thus find Hubble's law, with another interpretation. In Zhang's interpretation, the photon loses energy on its path from the source to the observer. In the second interpretation, it is the electron's wave function passing through a dispersive medium (empty space) that undergoes amplitude attenuation, which means the electron loses energy. Therefore, during a quantum jump of the electron, the emitted photon has a lower frequency.

3.4. Origin of the CMB

The TVSL model conceives the universe as stationary in time. In such a universe, radiation has all the time it needs to reach thermodynamic equilibrium with matter. Let's make a rough calculation of what the temperature of the radiation in the universe, produced by all the stars in the universe, should be. The number of stars in the universe can be calculated by imposing that the energy density of the universe corresponds to the critical energy density and that the visible matter in the universe is 5% of the total matter. We are hiring that 95% of the matter does not emit radiation in the visible field, because it corresponds to planets, cosmic dust, black holes, neutron stars, etc. This does not mean that such matter does not emit radiation, but it means that the energy of the incident radiation (in the visible range) is re-emitted with a spectrum shifted into the non-visible range (radio waves), since its temperature is much lower than that of the stars that produced the incident radiation.

This matter, which does not emit light in the visible range, can be called CM (cold matter). Is important to emphasize that this matter is not composed of mysterious exotic particles (WIMPS) but is normal matter that absorbs radiation from stars at high temperatures (3000˚ - 8000˚K) and emits it at a lower temperature, with a blackbody spectrum. It is precisely the radiation emitted by the CM that constitutes the cosmic microwave background, with a blackbody spectrum at a temperature of 2.725˚K.

To understand how the radiation of the visible universe, when it reaches thermal equilibrium with the CM, reaches this temperature, the following approximate calculation can be performed. Assuming $H = 2.33\times10^{-18}\ \text{s}^{-1}$, we find $\rho_{crit} = 9.7\times10^{-27}\ \text{kg}/\text{m}^3$. Then $\rho_{mu} = 0.05\rho_{crit} = 8.45\times10^{-28}\ \text{kg}/\text{m}^3$. The mass of the visible universe can then be found from $M_u = \rho_{mu}V_u$. The volume of the currently visible universe can be obtained from the relation $V_u = (4/3)\pi R_u^3$ where R_u is the radius of the visible universe, defined as the distance traveled by a signal with a velocity equal to c_0, in the Hubble time $T_H = 1/H$.

It follows $R_u = c_0/H = 1.28\times10^{26}\ \text{m}$; $V_u = 8.8\times10^{78}\ \text{m}^3$. Then $M_u = \rho_{mu}V_u = 4.26\times10^{51}\ \text{kg}$.

Assuming that the average mass of stars is equal to that of the Sun, the number of stars in the universe is: $N = \left(4.26\times10^{51}\right)/\left(2\times10^{30}\right) = 2.14\times10^{21}$. The Sun emits an energy of 3.8×10^{26} Joules per second. Then the energy emitted per second by *N* stars will be $E = 8.13\times10^{47}\ \text{J/s}$. This energy must pass through any closed surface surrounding all visible stars, *i.e.*, a spherical surface with radius R_u. It follows that the energy, per square meter, per second, passing through the surface of such a sphere will be:

$$B = E/S = \left(8.13\times10^{47}\right)\Big/\left[4\pi\left(1.28\times10^{26}\right)^2\right] = 3.94\times10^{-6}\ \text{W}/\text{m}^2$$

The power (per unit of area) of radiation produced by a black body, which passes through a closed surface that encloses the black body, is related to the temperature of the black body by the Stefan-Boltzmann law: $B = \sigma_B T^4$.

Since $\sigma_B = 5.67\times10^{-8}\left(\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)$, it follows $T = \sqrt[4]{B/\sigma_B} = 2.88\ \text{K}$.

Therefore, the temperature of the radiation produced by the stars, once thermalized by the cold matter, which constitutes almost all of the matter (95%), through subsequent absorption-reemission processes, reaches precisely the value measured for the CMB.

The previous calculation is only a rough calculation. However, it shows how the temperature of the radiation emitted by all visible stars, assuming that this radiation is in thermodynamic equilibrium with existing matter, has the same order as the CMB ($T_{cmb} = 2.725\ \text{K}$).

4. Conclusions

The TVSL cosmological model proposes that the cosmological redshift is due to a minimal variation in the speed of light in a vacuum $c = c(t)$. In this relationship, time is referred to the birth time of the galaxy emitting the light signals. Galaxies are not all born at the same time, but are continuously born and die. On average, the number of galaxies born is equal to the number of those dying, so as to maintain the universe in a steady state. Supermassive black holes contain plasma at extremely high temperatures and pressures, which are constantly increasing. When the pressure exceeds the gravitational force exerted between the particles of the plasma sphere, the latter "erupts" a plasma jet to restore equilibrium.

This jet passes through the depletion zone and, if the BH is rotating, is channelled, by the BH's magnetic field, along the rotation axis and projected into space.

Bright jets as long as millions of light years have been observed, emerging from the BH horizon (**Figure 4**).

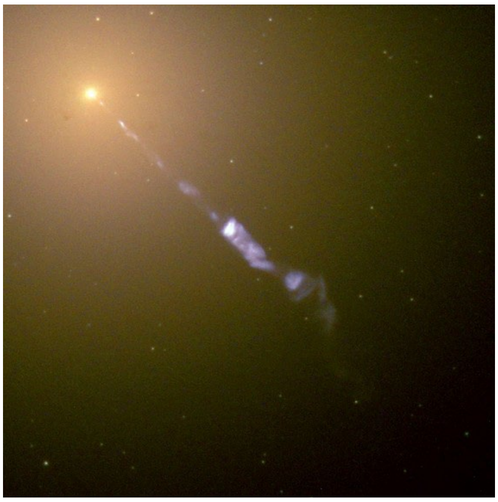

Figure 4. Hubble Telescope image of the relativistic jet ejected by the supermassive black hole in the galaxy M87. Credits: NASA and The Hubble Heritage Team (STScI/AURA).

These filaments will give rise to new stars and galaxies. The universe is shaped like a spiderweb (see **Figure 5**), because its galaxies formed along filaments erupted by supermassive BH.

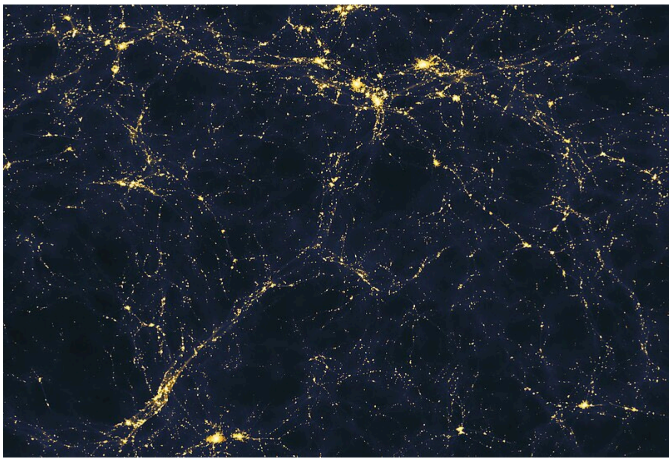

Figure 5. The image shows how galaxies are distributed along filaments which in the TVSL model are jets of plasma erupted by supermassive black holes.

An impressive detail of this cosmic spiderweb has recently been provided by the JWST space telescope [11]. This detail, shown in **Figure 6**, indicates the presence of 20 galaxies with very close redshifts ($3.43 < z < 3.45$) arranged in a long rectilinear structure called the "Cosmic vine", which extends for about 13 million light years. Only a mechanism of the type predicted can originate such a structure.

The maximum redshift difference $\Delta z = 0.02$ cannot be attributed to the difference in distance of the galaxies belonging to the Cosmic Vine. In fact, the structure is approximately 13 million light-years long, corresponding to a maximum redshift difference of 10^{-3}. Therefore, $\Delta z = 0.02$ is due to the proper motion of the galaxies. However, the fact that Δz corresponds to a velocity difference of only 6.000 km/s means that the galaxies are the same age and very young. In fact, the proper motion of a galaxy is due to its interaction with other galaxies. Young galaxies have low proper velocities (much lower than the average value of 100 km/s), having not had much time to interact with other galaxies. And the fact that they all lie on the same straight-line segment means that they were born from the same jet of relativistic plasma which, given the enormous overall mass of the galaxies, could only involve a supermassive black hole.

Figure 6. Image of the COSMIC Vine cluster. /Foto: ESA/Hubble/NASA, Fillipenko, Jansen.

The Milky Way is one of the oldest galaxies. Therefore, other galaxies are younger and emit photons at high speeds, and therefore at long wavelengths. This difference in wavelength is called redshift. Therefore, redshift is related not only to the distance but also to the age of the galaxy emitting the light. A galaxy can be near or far, young or old. Galaxies with the highest redshifts are young and distant. This is why galaxies with very high z ($z \geq 10$) appear to be very young, meaning they lack heavy elements and are small in size. However, there may be galaxies with high z that are close to us. This is the case of quasars, which are nuclei of galaxies in their initial phase of formation, immediately after the plasma filament has been emitted from the supermassive BH.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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