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A Kaluza Klein Treatment of a Graviton and Deceleration Parameter $q(z)$ as an Alternative to Standard DE and Its Possible Link to Standard DE Equation of State

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Abstract

First of all, we will reiterate a mechanism for relic early universe gravitons. This is the precursor of the main result of this paper and involves the decay of micro black holes, with relic graviton production. For the 2nd part of graviton physics, we invoke reacceleration of the universe. The case for a four-dimensional graviton mass (non-zero) influencing reacceleration of the universe in five dimensions is stated, with particular emphasis on whether five-dimensional geometries as given below give us new physical insights as to cosmological evolution. The final question is, can DM/DE be explained by a Kaluza-Klein particle construction? *i.e.*, the author presents a Kaluza Klein particle representation of a graviton mass with the first term to the right equal to a DM contribution and with the 2nd term to the right being effective DE. This is the 2nd era of DE production. Finally we in the third part invoke the methodology of a modification of 1/r potentials, and the re-acceleration of the universe as a bridge between the first and second parts of this document, in terms of the physics of DE and gravitons. This is compared with an equation of state given by reference [15], as a confirming basis for perhaps rationalizing a linkage between early universe production of gravitons, and then a subsequent Equation of state for DE, which may be able to predict reasons for quintessence.

Keywords

"Massive" Gravitons, Entropy, Kaluza Klein, Dark Matter, Dark Energy

1. PLAN of Present at ION

1st part briefly refers to production of DE due to as an example, graviton release from 100 or more black holes, due to relic conditions: (source for this paper is provided). It is already accepted by JHEPGC and is summed up in [1].

2nd part refers to DE as created by Gravitons, creating reacceleration of the Universe one billion years ago (jerk calculation). This is also an accepted JHEPGC paper, which will be published later [2].

3rd part is a modification of gravity, which may produce DE. This involves the modification of 1/r potentials for the reason brought up in this document. This is also an accepted JHEPGC paper, which will be published later [3].

The conclusion of this document compares both part 1 and part 2, while asking if there exists a bridge between the initial production of gravitons, possibly by the mechanism (Black hole in a micro scale, leading to their rapid decay and delivery of subsequent gravitons) brought up in the 1st part, with the subsequent asking of pertinent links to the reinflation era, which is $z = 0.43$, *i.e.* one billion years ago. This is also compared to a result given by [4], as to reinflation, which may be the bridge between the first and second parts of graviton production. *i.e.*, it is the author's consideration that reconciling the [3] bridge between the 1st and the 2nd part mechanisms may entail answering and fine-tuning the details given in part 3, which will be attempted later.

2. Mini Black Holes Decaying, and Graviton Production

i.e., look at what is given in [1], in its conclusion, which we will cite here. *i.e.*, the idea is based upon the formation of a finite number of black holes, which decay. Quoting [1], we have that we will be looking at the following.

Quote,

In Corda's recent work [5], we have a so-called Horizon volume, where n is the so-called quantum number n put in where Planck mass is normalized to 1, so then, if there are 10^2 black holes of mass 10^2 times Planck mass (will set Planck mass, as 1).

$$\begin{aligned} & M \approx 10^2 m_{\text{Planck}} \\ & \Delta V_{n-1\to n}\big|_{\text{Total}} \approx 10^2 \times \Delta V_{n-1\to n} \\ & \qquad \approx 10^2 \times 16\pi \left| \sqrt{\sqrt{\left(10^2 m_{\text{Planck}}\right)^2 - \frac{n}{2}} - \sqrt{\left(10^2 m_{\text{Planck}}\right)^2 - \frac{n-1}{2}}} \right| \end{aligned} \tag{1}$$

Here, we make the following simplification of Equation (47) to read as

$$\begin{aligned} \Delta V_{n-1\to n}\big|_{\text{Total}} & \approx 10^2 \times \Delta V_{n-1\to n} \\ & \approx 10^3 \times 16\pi \cdot \frac{n}{4\times 10^4} \cdot \left| \sqrt{1-\frac{n-1}{n}} \right| \end{aligned} \tag{2}$$

Our supposition is that there are 10^2 mini black holes, and a mass of 10^2 times Planck mass, per each black hole, so that we perform the following normalization, *i.e.*, find n for quantum number, so that to first approximation

$$16\pi \cdot \frac{n}{4\times 10^4} \cdot \left| \sqrt{1-\frac{n-1}{n}} \right| \approx 10^6 \tag{3}$$

i.e., that say 1000 times Planck length, we have the beginning of say creation of 100 mini black holes, each of mass about 100 times Planck mass which would put a huge restriction upon the admissible value, *n*, whereas giving a quantum value, *n*, for the enhanced quantum perturbation, used for penetration of the initial quantum state so assumed in this document as we go from Pre Planckian to Planckian physics, by emergent field construction. Equation (3) could be used to ascertain a quantum value, *n*, which would be for quantization level used to penetrate beyond the shell used to create the cosmological constant.

End of quote.

What we have is in Equation (3) the initial production of say 10^6 relic gravitons, and we are specifying here that relic graviton production is due, maybe even before the electroweak regime of spacetime, and that this may serve as a template for to initial relic graviton production. If we have graviton production this way, we do have a way to ascertain if we have a new candidate to form DE, as we will state next.

3. Gravitons with A Non-Zero Rest Mass. The KK Treatment

Consider if there is then also a small graviton mass, *i.e.*, as stated by Beckwith [6] [7]:

$$m_n(\text{Graviton}) = \sqrt{\frac{n^2}{L^2} + \left(m_{\text{graviton rest mass}} = 10^{-65}\text{ grams}\right)^2} = \frac{n}{L} + 10^{-65}\text{ grams} \quad (4)$$

Note that Rubakov (2002) works with KK gravitons, without the tiny mass term for a 4-dimensional rest mass included in Equation (7). To obtain the KK graviton/DM candidate representation along RS dS brane world, Rubakov obtains his values for graviton mass and graviton physical states in space-time after using the following normalization: $\int \frac{\mathrm{d}z}{a(z)} \cdot \left[h_m(z) \cdot h_{\tilde{m}}(z)\right] \equiv \delta(m - \tilde{m})$. Rubakov [8] (2002) uses J_1, J_2, N_1, N_2 which are different forms of Bessel functions. His representation of a graviton state is given by Equation (4), which is almost completely acceptable for our problem, since the rest mass of a graviton in four dimensions is so small. If so, then the wave function for a graviton with a tiny 4-dimensional space-time rest mass can be written as [8].

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2\left([m/k] \cdot \exp(k \cdot z)\right) - N_1(m/k) \cdot J_2\left([m/k] \cdot \exp(k \cdot z)\right)}{\sqrt{\left[J_1(m/k)\right]^2 + \left[N_1(m/k)\right]^2}} \quad (5)$$

Equation (5) is for KK gravitons having a TeV magnitude mass $M_Z \sim k$ (*i.e.*, for mass values at 0.5 TeV to above 1 TeV) on a negative tension RS brane. It would be useful to relate this KK graviton, which is moving with a speed proportional to H^{-1} with regards to the negative tension brane with $h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ as an initial starting value for the KK graviton mass. If Equation (5) is for a "massive" graviton with a small 4-dimensional graviton rest mass and if

$h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ represents an initial state, then one may relate the mass of the KK graviton moving at high speed with the initial rest mass of the graviton. This rest mass of a graviton is $m_{\text{graviton}}(\text{4-Dim GR}) \sim 10^{-48}\ \text{eV}$, opposed to $M_X \sim M_{\text{KK Graviton}} \sim 0.5 \times 10^9\ \text{eV}$. Whatever the range of the graviton mass, it may be a way to make sense of what was presented by Dubovsky *et al.* [9], who argue for a graviton mass, using CMBR measurements, of $M_{\text{KK Graviton}} \sim 10^{-20}\ \text{eV}$. Also, Equation (6) will be the starting point used for a KK tower version of Equation (6). So from Maartens [10],

$$\dot{a}^2 = \left[\left(\frac{\tilde{\kappa}^2}{3}\left[\rho + \frac{\rho^2}{2\lambda}\right]\right)a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K\right] \tag{6}$$

Maartens [10] also gives a 2nd Friedman equation:.

$$\dot{H}^2 = \left[-\left(\frac{\tilde{\kappa}^2}{2} \cdot [p + \rho] \cdot \left[1 + \frac{\rho^2}{\lambda}\right]\right) + \frac{\Lambda \cdot a^2}{3} - 2\frac{m}{a^4} + \frac{K}{a^2}\right] \tag{7}$$

Also, an observer is in the low redshift regime for cosmology, for which $\rho \cong -P$, for red-shift values z from zero to 1.0 - 1.5. One obtains exact equality, $\rho = -P$, for z between zero to 0.5. The net effect will be to obtain, based on Equation (7), assuming $\Lambda = 0 = K$ and using $a \equiv [a_0 = 1]/(1+z)$ to get a deceleration parameter q as given in Equation (8).

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{2}{1 + \tilde{\kappa}^2[\rho/m] \cdot (1+z)^4 \cdot (1 + \rho/2\lambda)} \approx -1 + \frac{2}{2 + \delta(z)} \tag{8}$$

These set values allow a graviton-based substitute for DE. $\Lambda = 0 = K$ plus a small rest mass for a graviton in four dimensions allows for "massive gravitons" that behave the same as DE. Setting $\Lambda = 0 = K$, while having a modified behavior for the density expression, for a Friedman equation with a small 4-dimensional graviton mass, means that dark energy is being replaced by a small 4-dimensional rest mass for a graviton.

CONSEQUENCES OF SMALL GRAVITON MASS FOR REACCELERATION OF THE UNIVERSE

In a revision of Alves *et al.* [11], Beckwith [6] [7] used a higher-dimensional model of the brane world combined with KK graviton towers per Maartens [10]. The energy density ρ of the brane world in the Friedman equation is used in a form similar to Alves *et al.* [11] by Beckwith [6] [7] for a non-zero graviton:

$$\rho \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g \cdot (c=1)^6}{8\pi G(\hbar = 1)^2}\right] \cdot \left(\frac{1}{14 \cdot (1+z)^3} + \frac{2}{5 \cdot (1+z)^2} - \frac{1}{2}\right) \tag{9}$$

Beckwith [6] [7] suggests that at $z \sim 0.4$, a billion years ago, acceleration of the universe increased, as shown in **Figure 1**. **Figure 1** is, if confirmed, a good verification of the Ng [12] hypothesis, and would be a starting point to investigate the role of gravitons in cosmology. The author notes that Buonnano [13] assumes a much lower range of initial frequencies for relic GW than the author does. Beckwith [6] [7] obtained a re-acceleration of the universe result as given in **Figure 2**. The contribution of a low rest mass for 4-dimensional gravitons, as given in equation 7, leads to a speed-up of acceleration of the expansion of the universe a billion years ago, *i.e.*, for a red shift slightly smaller than 0.5. **Figure 2** below is predicated upon a small 4-dimensional rest mass (stated in Equation (7) for a graviton behaving the same as dark energy)... We will state in our discussions section what is needed to give experimental confirmation as to what is a current for a "massive" graviton, which is appropriate for explaining, in part, **Figure 2** below.

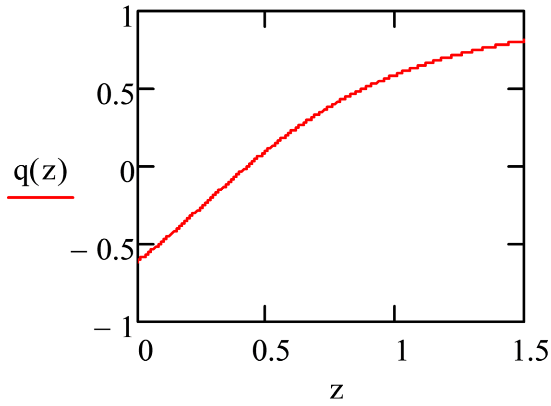

Figure 1. Re-acceleration of the universe based on Beckwith [6]; (note that $q(z) < 0$ if $z < 0.423$).

Reconciling both part 1 and part 2. *i.e.*, what we are considering is a bridge between part 1 and part 2, *i.e.*, how can we reconcile both of these regimes?

From [3], consider the following, namely:

Chaotic inflation would be using the approximation that

$$V(\varphi) \approx \frac{k^2}{a^2} \cdot \varphi^2 \tag{10}$$

Use the approximation that the time derivative is $\mathrm{d}/\mathrm{d}\tau$, and $\varphi \equiv \varphi_k$, and if so, then

$$\begin{aligned} &\frac{\dot{\varphi}_k^2}{2} = \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 - \rho \right) \\ &\& \\ &\frac{\dot{\varphi}_k^2}{2} = \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 - \frac{16}{3} \cdot c_1 \cdot B^4 \right) \end{aligned} \tag{11}$$

The last line of Equation (11) states that, if we apply it to the Pre Planckian to Planckian regime, that there will be a change in the energy, which we will call

$$\Delta E \approx \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 - \frac{16}{3} \cdot c_1 \cdot B^4 \right) \tag{12}$$

We then will call this shift in energy, as equivalent to a change in KINETIC energy, and then reference the Virial theorem, which in a general form, will be interpreted as

$$\langle \psi \mid \text{Kinetic Energy} \mid \psi \rangle = \langle \psi \mid r \cdot \nabla V(\text{Potential energy}) \mid \psi \rangle \tag{13}$$

Leading to

$$\begin{aligned} &\left\langle \psi \,\middle|\, \left[\text{Kinetic Energy} \approx \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 - \frac{16}{3} \cdot c_1 \cdot B^4 \right)\right] \middle|\, \psi \right\rangle \\ &\approx \left\langle \psi \,\middle|\, \left(r \cdot \nabla \left[V(\text{Potential energy}) \approx c_2 / r^{\alpha} \right] \right) \middle|\, \psi \right\rangle \end{aligned} \tag{14}$$

In the Pre Planckian to Planckian space time, we will approximate, in the instant before time is initialized, formally, the mean value theorem as to the computed values of both the Left and right hand sides of Equations (13) and (14), with the results that we obtain

$$\begin{aligned} &\left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 - \frac{16}{3} \cdot c_1 \cdot B^4 \right) \approx -\alpha / r^{\alpha} \equiv -\alpha / (\text{Planck length})^{\alpha} \\ &\Leftrightarrow \left[\alpha / (\text{Planck length})^{\alpha} \right] \approx \frac{16}{3} \cdot c_1 \cdot B^4 - \left(\frac{3}{8\pi G} \cdot \frac{\kappa}{a^2} + \frac{k^2}{a^2} \cdot \varphi_k^2 \right) \end{aligned} \tag{15}$$

THIS VARIANT OF DARK ENERGY should be compared with the main scheme, which has reacceleration of the Universe as its main constituent point. *i.e.*, what is equivalent to DE is given below.

This shows in part that α *is no longer strictly real valued but is strongly influenced by the input from* φ_k , *i.e.*, *which has real and imaginary components*

4. Conclusions

To put it mildly, we are concerned about making a bridge between the initial and second physics realms as to graviton physics and cosmology. In a nutshell, we are proposing a modification of the 1/r gravitational potential for gravitational waves, which in themselves will have gravitons, which is a way to not only get at the DE puzzle, but to answer specifics as to the interaction of gravitons with gravitational waves. *i.e.*, if the 2nd part of our presentation is accurate, we are setting the grounds for examining quantization of gravity. The gravitational field gμν also has field equations, like those of electromagnetism (the gravitational equations are much more complicated). When we quantize the gravitational field as an effective field theory, we find that it too comes in "quanta," called gravitons. In short, we argue that to come up with a graviton-based model of DE with reacceleration of the universe, and to have it commensurate with the modification of the 1/r potential, we are really coming up with a program of finding out if gravity can be quantized. In addition, what we have done is also to specify a relic black hole genesis for early universe gravitons, which complements the picture of turbulence in the electroweak era, which in turn is relatable to the question of whether micro black holes could contribute to cosmology. This, in turn, means comparison with [4] and keeping in mind the following, *i.e.*, the diagram given by Abbott *et al.* [14] (2009), which shows the relation between GW frequency and GW energy density for different cosmological models.

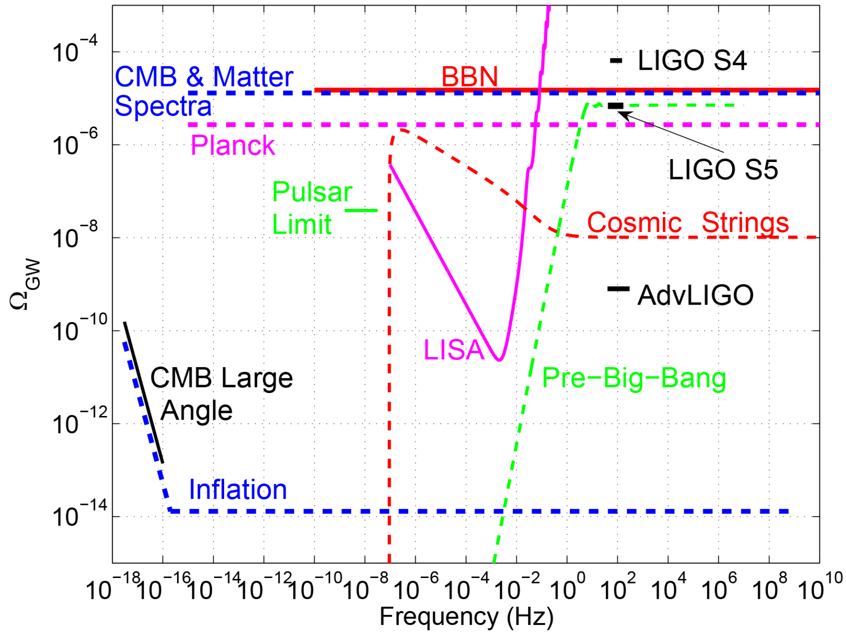

Figure 2. Abbott *et al.* [14] (2009) show the relation between GW frequency and GW energy density for different cosmological models.

At first glance, this looks hopeless. *i.e.*, the models are incommensurate with each other and we do have a huge problem. A way of having reconciliation may be to consider what is brought up on pages 114 to 115 of Li, Li, Wang, and Wang, [15], as we have, then an examination of the equation of state for DE, which is commensurate with reacceleration of the universe, as reading,

$$\omega_{\text{equation of state DE}} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \approx \frac{\dot{\phi}^2 - 2V_0 \exp\left(-\sqrt{\frac{2}{\tilde{m}}}\frac{\phi}{m_{\text{Plank}}}\right)}{\dot{\phi}^2 + 2V_0 \exp\left(-\sqrt{\frac{2}{\tilde{m}}}\frac{\phi}{m_{\text{Plank}}}\right)} \quad (16)$$

for acceleration of universe $\tilde{m} > 1$

Again, this looks like a very hard problem, but in a nutshell, what we need to accomplish is having the following identification made, which may allow for us to make a concrete bridge between formalisms which otherwise look like they have no linkage to each other, *i.e.* do the following, namely is there a way to link ϕ of Equation (16) with φ_k of Equation (11) and Equation (12) so as to come up with an acceptable form of ϕ which would satisfy the following equation, given in Equation (18) below, while keeping in mind that [15] gives the following form for a "scalar field" based potential for an accelerating universe, which is called in [15], Equation (17) immediately below this paragraph

Here, we need for acceleration of universe $\tilde{m} > 1$

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{2}{\tilde{m}}}\frac{\phi}{m_{\text{Plank}}}\right) \quad (17)$$

Then, we should try to reconcile the following, if is there a way to link ϕ of

Equation (16) with φ_k of Equation (11) and Equation (12)

$$\omega_{\text{equation of state DE}} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \approx \frac{\dot{\phi}^2 - 2V_0 \exp\left(-\sqrt{\frac{2}{\tilde{m}}}\frac{\phi}{m_{\text{Plank}}}\right)}{\dot{\phi}^2 + 2V_0 \exp\left(-\sqrt{\frac{2}{\tilde{m}}}\frac{\phi}{m_{\text{Plank}}}\right)}$$

$$\text{for acceleration of universe } \tilde{m} > 1$$

$$\text{reconcile above with} \tag{18}$$

$$\left(\frac{3}{8\pi G}\cdot\frac{\kappa}{a^2} + \frac{k^2}{a^2}\cdot\varphi_k^2 - \frac{16}{3}\cdot c_1\cdot B^4\right) \approx -\alpha/r^\alpha \equiv -\alpha/(\text{Planck length})^\alpha$$

$$\Leftrightarrow \left[\alpha/(\text{Planck length})^\alpha\right] \approx \frac{16}{3}\cdot c_1\cdot B^4 - \left(\frac{3}{8\pi G}\cdot\frac{\kappa}{a^2} + \frac{k^2}{a^2}\cdot\varphi_k^2\right)$$

In doing so, we would, after a great deal of work, perhaps be able to make linkage to the scale factor a which shows up in Equation (15) above. *i.e.*, having results in terms of the scale factor, and ϕ while making full identification with the Equation (18) above, may be a way to reformulate the linkage between early DE, which we want. In addition, such restraints upon scale factor a may also begin to explain some of the scale factor behavior which may be implied by Equation (7) and Equation (8) above. Also, if we do this, the final equation of state constraint we have to keep in mind is also given in [15] as

$$-2\cdot(m_{\text{Plank}})^2\cdot\dot{H} = \dot{\phi}^2 \tag{19}$$

This should be compared against Equation (7) and justified, if possible, as part of the bridge between early universe expressions of DE and the reacceleration of the universe.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Beckwith, A. (2018) Using "Enhanced Quantization" to Bound the Cosmological Constant, and Computing Quantum Number N for Production of 100 Relic Mini Black Holes in a Spherical Region of Emergent Space-Time. http://vixra.org/abs/1805.0378

[2] Beckwith, A. (2011) Identifying a Kaluza Klein Treatment of a Graviton Permitting a Deceleration Parameter Q(Z) as an Alternative to Standard DE. https://www.researchgate.net/publication/236335939_Identifying_a_Kaluza_Klein_Treat-ment_of_a_Graviton_Permitting_a_Deceleration_Parameter_QZ_As_An_Alternative_to_Standard_DE

[3] Beckwith, A. (2018) Structure Formation in the Early Universe as a Result of Non-

Linear Electrodynamics Influencing Scale Factor Size with Attendant Changes in Gravitational Potential and Its Relationship to the 3 Body Problem. http://vixra.org/abs/1805.0357

[4] Inomata, K., Kawasaki, M., Mukaida, K. and Yanagida, T.T. (2018) Double Inflation as a Single Origin of Primordial Black Holes for All Dark Matter and LIGO Observations. *Physical Review D*, **97**, Article ID: 043514. https://doi.org/10.1103/physrevd.97.043514

[5] Corda, C. (2018) Spacetime Can Be neither Discrete nor Continuous. *Modern Physics Letters A*, **33**, Article ID: 1850069. https://doi.org/10.1142/s0217732318500694

[6] Beckwith, A.W. (2010) Applications of Euclidian Snyder Geometry to the Foundations of Space Time Physics. *EJTP*, **7**, 241-266. http://vixra.org/abs/0912.0012

[7] Beckwith, A.W. (2011) Energy Content of Gravitation as a Way to Quantify both Entropy and Information Generation in the Early Universe. *Journal of Modern Physics*, **2**, 58-61. https://doi.org/10.4236/jmp.2011.22010

[8] Rubakov, V.A. (2002) Classical Theory of Gauge Fields. Princeton University Press.

[9] Dubovsky, S., *et al.* (2009) Signatures of a Graviton Mass in the Cosmic Microwave Background. *Physical Review D*, **81**, Article ID: 023523. http://arxiv.org/abs/0907.1658

[10] Maartens, R. (2004) Brane-World Gravity. *Living Reviews in Relativity*, **7**, Article No. 7. https://doi.org/10.12942/lrr-2004-7

[11] Alves, E., Miranda, O. and de Araujo, J. (2010) Can Massive Gravitons Be an Alternative to Dark Energy? http://arxiv.org/pdf/0907.5190

[12] Ng, Y.J. (2008) Spacetime Foam: From Entropy and Holography to Infinite Statistics and Nonlocality. *Entropy*, **10**, 441-461. https://doi.org/10.3390/e10040441

[13] Buonanno, A. (2007) Gravitational Waves. In: *Particle Physics and Cosmology*, *the Fabric of Space-Time*, Elsevier, 3-5, 7-52. https://doi.org/10.1016/s0924-8099(07)80027-8

[14] Abbott, B.P., *et al.* (2009) An Upper Limit on the Stochastic Gravitational-Wave Background of Cosmological Origin. *Nature*, **460**, 990-994. https://doi.org/10.1038/nature08278

[15] Li, M., Li, X.-D., Wang, S. and Wang, Y. (2015) Dark Energy. Peking University World Scientific Advance Physics Series, Volume 1, World Scientific.