Journal of High Energy Physics, Gravitation and Cosmology, 2025, 11(2), 600-606
https://www.scirp.org/journal/jhepgc
ISSN Online: 2380-4335
ISSN Print: 2380-4327

# A Dark Energy Hypothesis VI

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**How to cite this paper:** Togeas, J. (2025) A Dark Energy Hypothesis VI. *Journal of High Energy Physics, Gravitation and Cosmology*, **11**, 600-606.
https://doi.org/10.4236/jhepgc.2025.112042

**Received:** March 6, 2025
**Accepted:** April 26, 2025
**Published:** April 29, 2025



## Abstract

The broad subject is the interaction of dark matter and baryonic matter. The specific problem analyzed is the effect of dark matter on hydrogen recombination in the early universe. The Saha equation governs recombination, and a modified Saha equation describes recombination in the presence of dark matter. Derivation of the latter is in the Appendix.

## Keywords

Dark Matter, Recombination, Saha Equation

## 1. Introduction

All numerical work is based on DEH formalism and the cosmological energy inventory: dark energy:dark matter:baryonic matter::70:25:5. The formalism is in DEH II - IV [1]-[3] but the intention here is to give a short summary of the essentials of that formalism that is sufficiently complete so that those who wish to closely follow the arguments in this paper can do so without referring to that earlier work.

In the DEH formalism, the cosmological baryonic energy is a constant, whereas dark matter changes into dark energy from one cosmological epoch to the following, the two standing in relation as free energy to entropy, resp. At any epoch, the scale factor "*a*" and the cosmic time *t* are found by

$$a = \frac{\Gamma}{6}\left(\cosh(\eta) - 1\right) \quad \& \quad ct = \frac{\Gamma}{6}\left(\sinh(\eta) - \eta\right)$$

The occurrence of hyperbolic functions means that the cosmological space in the DEH formalism is negatively curved, *i.e.*, hyperbolic. The symbol $\eta$ is the conformal time given by $\eta\mathrm{d}a = c\mathrm{d}t$. An epoch in a DEH is given by a dimensionless quantity $\lambda$ for the dark energy:

$$\lambda = \frac{\cosh(\eta) - 1}{6\eta^2}$$

From the energy inventory, $\lambda = 7/10$ for this epoch, giving its conformal time as $\eta = 5.571$. The total energy is conserved in a DEH, which in the preceding equations is given by a length, $\Gamma = 6.306 \times 10^{24}$ m, and hence applies to all epochs. To convert to an energy, divide by the Einstein gravitational constant:

$$k = \frac{8\pi G}{c^4} = 2.076 \times 10^{-43}\ \mathrm{m} \cdot \mathrm{J}^{-1}$$

$G$ of course is the Newtonian gravitational constant. A dimensionless parameter for the dark matter mass at any epoch is

$$\chi(dm) + \lambda = 0.95$$

The baryonic parameter then is $\chi(b) = 0.05$ = constant. The conversion of dark matter into dark energy means that $\mathrm{d}\lambda = -\mathrm{d}\chi(dm) > 0$. The dark matter mass in a given epoch is

$$M(dm) = \frac{\chi(dm)\Gamma}{\kappa c^2}$$

The early universe is the setting for the interaction of dark matter and baryonic matter; it is defined by $\eta < 1$, or more precisely by the conformal time test that $\cosh(\eta) - 1 \approx \eta^2/2$. Recognizing invariants is crucial to the analysis.

1) Time and temperature:

$$tT^2 = \zeta = \text{constant} \tag{1}$$

Since it is the early universe, time and temperature can safely be related through the thermal radiation law:

$$e(r) = \frac{3c^2}{32\pi G t^2} = \sigma T^4$$

where the first term is the radiation density and $\sigma$ is the Stefan-Boltzmann constant. This gives $\zeta = 2.305 \times 10^{20}$ s K$^2$.

2) Scale factor and temperature

For small values of the conformal time,

$$a = \frac{\Gamma}{12}\eta^2 \quad \& \quad ct = \frac{\Gamma}{36}\eta^3$$

Now eliminate the time t between 1) and 2) to give

$$a^3 T^4 = \frac{3}{4}(c\zeta)^2 \Gamma = 2.258 \times 10^{82}\ \mathrm{m}^3 \cdot \mathrm{K}^4 \tag{2}$$

Thus, in the early universe, the scale factor and temperature are an invariant pair, which is not true in the later universe. This invariance greatly simplifies the subsequent analysis.

Finally, the proposed equation of state for dark matter in Ref. [3] is

$$\frac{p}{nk_B T} = \frac{2\phi}{\sqrt{\pi}} \tag{3}$$

where $n$ is the number density and $\phi$ is the fugacity. Fugacity for either dark matter or baryonic matter is

$$\phi = n\Lambda_B^3 = \exp(\beta\mu) \tag{4}$$

where

$$\Lambda_B = \frac{h}{\sqrt{2\pi m k_B T}} \tag{5}$$

is the de Broglie thermal wavelength, $\mu$ is the chemical potential, and $\beta = 1/k_B T$ as usual. The equation of state is for an ideal, non-classical gas: ideal because by hypothesis dark matter particles experience no interparticle forces, being spinless bosons that obey Bose-Einstein statistics; and non-classical because for an ideal classical gas, the right-hand side would be unity. The deviation from unity is a quantum effect.

## 2. The Goal

It is to study the effect of dark matter on the temperature of hydrogen recombination:

$$\mathrm{H} + \gamma = \mathrm{p} + \mathrm{e}$$

The procedure is to first review recombination in the absence of dark matter and then to introduce dark matter into the recombination problem through its equation of state. The first step brings in the well-known Saha equation and the second a modified Saha equation.

## 3. The Recombination Problem

The degree of ionization, $f$. This is given in terms of the stoichiometry of recombination. Suppose that initially hydrogen is not dissociated with a number density $n^*$. At equilibrium, let the number densities of the proton and electron be $n$, leading to a hydrogen number density of $n^* - n$. Then the degree of ionization, $f$, that is, the fraction of hydrogen that has ionized is just $f = n/n^*$.

The Saha equation.

$$\frac{n_p n_e}{n_H} = \frac{n^2}{n^* - n} = \frac{f^2 n^*}{1-f} = \frac{1}{\Lambda_e^3}\exp(-B/k_B T) \equiv S \tag{6}$$

where the denominator of $S$ is the cube of the electron's de Broglie thermal wavelength and $B$ is the binding energy (aka ionization energy) of the hydrogen atom: $B = 2.179 \times 10^{-18}$ J. The binding energy may also be written as a temperature: $\Theta = B/k_B = 1.578 \times 10^5$ K. Solving for the degree of ionization gives

$$f = \frac{2}{\xi}\left[(1+\xi)^{1/2} - 1\right] \tag{7}$$

where $\xi = 4n^*/S$. By definition, the recombination temperature corresponds to $f = 1/2$, that is, $\xi = 8$ or $n^* = 2S$.

The number density of preionized hydrogen atoms is $n^* = N(\text{H-atoms})/a^3$. For convenience, suppose that all of the baryons are hydrogen. The baryon mass is $M(b) = 0.05\Gamma/\kappa c^2 = 1.690 \times 10^{49}$ kg. Divide by the mass of a hydrogen atom, $1.674 \times 10^{-27}$ kg to get $N(\text{H-atoms}) = 1.010 \times 10^{76}$.

Recombination in the absence of dark matter. The equation $n^* = 2S$ depends on the scale factor "*a*" and the temperature *T*, but these are not independent variables as shown by Equation (2). Let Equation (6) be a function of *T* alone by eliminating the scale factor. Then, collecting all of the constants into the symbol D and rearranging gives

$$1 = \frac{D}{T^{5/2}} \exp\left[-\frac{\Theta}{T}\right] \tag{8}$$

where $D = 1.080 \times 10^{28}\ \mathrm{K}^{5/2}$. The root of this equation is the recombination temperature: $T = 3579$ K. The time and scale factor at recombination are $t = 0.570$ Myr and $a = 5.163 \times 10^{22}$ m; from the latter, the redshift is

$$z = \frac{a_0}{a} - 1 = \frac{1.37 \times 10^{26}\ \mathrm{m}}{a} = 2653$$

with the current scale factor given in Ref. [1]. By comparison, in the ΛCDM theory, $T = 3760$ K and $z = 1380$ [4]; the temperatures are comparable, while the difference in redshifts results from the Euclidean space of ΛCDM theory and the hyperbolic space of a DEH.

Recombination in the presence of dark matter. As shown in the Appendix,

$$1 = \frac{D}{T^{5/2}} \exp\left[\frac{4\phi}{\sqrt{\pi}} - \frac{\Theta}{T}\right] \tag{9}$$

where $\phi$ is the fugacity of dark matter. That it enters the exponential with a positive sign means that dark matter will oppose recombination, that is, favor the ionized state. Once again, the expression depends on the scale factor and the temperature, which are not independent quantities; by Equations (2) and (4)

$$1 = \frac{D}{T^{5/2}} \exp\left[\gamma \left(\frac{T}{m}\right)^{5/2} - \frac{\Theta}{T}\right] \tag{10}$$

where $\gamma = 1.054 \times 10^{-98}\ \mathrm{kg}^{5/2}\ \mathrm{K}^{-5/2}$. This result argues that the smaller the mass of a dark matter particle, the more strongly dark matter opposes recombination, that is, the more greatly it favors the ionized state (see **Table 1**).

**Table 1.** Recombination temperature as a function dark matter mass.

| *m* (kg) | *T* (K) | *z* |
|---|---|---|
| $10^{-35}$ - $10^{-30}$ | 3579 | 2653 |
| $10^{-36}$ | 3140 | 2228 |
| $5 \times 10^{-37}$ | 2488 | 1633 |
| $10^{-37}$ | 1012 | 491 |
| $7 \times 10^{-38}$ | 803 | 362 |
| $5.1 \times 10^{-38}$ | 692 | 296 |

The largest mass is essentially the electron mass. Evidently, the largest masses have no effect on recombination. A mass of $5.0 \times 10^{-38}$ kg returns no solution. The smallest redshift still meets the conformal time test, $\cosh(\eta) - 1 = 0.47 \approx \eta^2/2 = 0.44$ and hence belongs to the early universe. The onset of the dark matter effect occurs suddenly over a short range of masses as the table shows and the following graph further illustrates (see **Figure 1**).

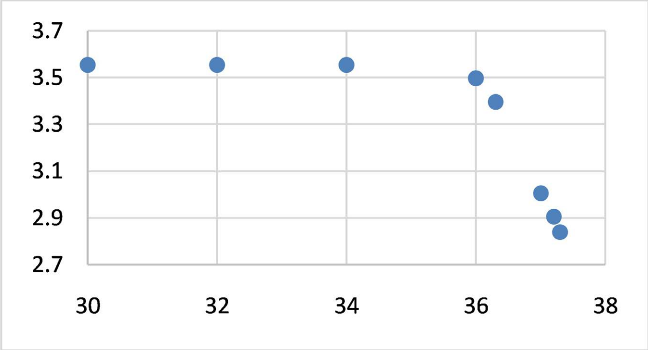


**Figure 1.** $\log_{10}(T)$ vs. $-\log_{10}(m)$.

Evidently, increasing mass size runs right to left.

The case of $m = 1 \times 10^{-40}$ kg illustrates the failure to find a solution for the smaller masses. This mass is of interest because in Ref. [3], it is given as a possible dark matter particle mass by the method of fluctuations, but it evidently is not suitable here. In Equation (10)

$$\gamma\left(\frac{T}{m}\right)^{5/2} - \frac{\Theta}{T} = 0$$

for $T = 8.08$ K, so a recombination temperature would have to be at $T \ll 8\,\mathrm{K}$. Hence, this mass can have nothing to do with the current state of the universe, which then is likely the case of the masses that refuse solutions.

Why should the ionized state be favored by smaller dark energy masses, that is, why do smaller masses oppose recombination? In the absence of dark matter, recombination is opposed by Thomson scattering, that is, electron-photon scattering [5]. The following is conjecture. Perhaps in the presence of dark matter, Thomson scattering is supplemented by gravitational scattering, that is, proton-dark matter scattering. In a given epoch, $\lambda$, the total dark matter mass is fixed. The number of dark matter particles is $M(dm)/m$: the smaller the value of m, the greater the number of dark matter particles. Hence, smaller mass corresponds to a greater number of centers to scatter protons.

If hydrogen is in its ionized state, the universe is opaque. If it were the case that $m = 10^{-37}$ kg, then according to the DEH the universe would be opaque for $z > 500$. The onset of opacity means that information about the earlier states of the universe is unavailable, and must be inferred from the information at hand and theory. A consideration of the onset of opacity is beyond the scope of this article.

## 4. Note on Dark Matter in a Dark Energy Hypothesis

Primordial black holes are candidates for dark matter in particle physics research.

Black holes evaporate by the Hawking mechanism. According to Ref. [1], dark matter disappears into dark energy over cosmic time. However, the dark matter masses listed in **Table 1** cannot be primordial black holes because black hole masses of that size have lifetimes shorter than the age of the universe [5].

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

## References


[1] Togeas, J. (2024) A Dark Energy Hypothesis II. *Journal of High Energy Physics, Gravitation and Cosmology*, **10**, 1142-1151. https://doi.org/10.4236/jhepgc.2024.103069

[2] Togeas, J. (2025) A Dark Energy Hypothesis III. *Journal of High Energy Physics, Gravitation and Cosmology*, **11**, 39-44. https://doi.org/10.4236/jhepgc.2025.111005

[3] Togeas, J. (2025) A Dark Energy Hypothesis IV. *Journal of High Energy Physics, Gravitation and Cosmology*, **11**, 45-55. https://doi.org/10.4236/jhepgc.2025.111006

[4] Ryden, B. (2017) Introduction to Cosmology. 2nd Edition. Cambridge University Press, 157.

[5] Marsh, D., Ellis, D. and Mehta, V. (2024) Dark Matter: Evidence, Theory, and Constraints. Princeton University Press, Ch. 16.

## Appendix. The Modified Saha Equation

The condition for chemical equilibrium of ionized hydrogen is

$$\mu_p + \mu_e - \mu_H = 0$$

Each chemical potential by Equation (4) can be written

$$\mu = \mu^0 + k_B T \ln\left(n\Lambda^3\right)$$

where the first term on the right side is a standard reference chemical potential: if the argument of the logarithmic term is unity, $\mu = \mu^0$. In order to get meaningful numerical results all energies must refer to a common zero of energy. To that end, let the standard hydrogen chemical potential vanish, $\mu_H^0 = 0$, which means that $\mu_p^0 + \mu_e^0 = B$, the atom's binding energy. This leads to the Saha equation, Equation (6):

$$\frac{n_p n_e}{n_H} = \frac{1}{\Lambda_e^3}\exp\left(-B/k_B T\right) \equiv S$$

The de Broglie wavelengths of the proton and hydrogen atom have been cancelled because their masses are essentially the same.

In the presence of dark matter, the equilibrium condition changes:

$$\mu_p + \mu_e - \mu_H = \mu\left(dm\right)$$

If the physical state changes and equilibrium is to be maintained, the differentials of the preceding equation must be equal:

$$\mathrm{d}\left(\mu_p + \mu_e - \mu_H\right) = \mathrm{d}\mu\left(dm\right)$$

The differential on the right-hand side can be obtained from the Duhem-Margules relation for the Gibbs free energy, $\Phi$ ($G$ in the chemist's notation):

$$\Phi = \mu N \quad \text{and} \quad \mathrm{d}\Phi = V\mathrm{d}p - S\mathrm{d}T + \mu\mathrm{d}N$$

In what follows, only isothermal changes need to be considered because the state of every cosmological epoch, $\lambda$, is isothermal. Then, $\mathrm{d}\mu = V\mathrm{d}p/N$ and

$$\mathrm{d}\left(\mu_p + \mu_e - \mu_H\right) = \frac{\mathrm{d}p}{n} = \frac{4k_B T \Lambda_B^3 \mathrm{d}n}{\sqrt{\pi}}$$

using dark matter's equation of state, Equations (3)-(4). Integrating with $T$ = constant gives

$$k_B T \ln\left[\frac{n_p n_e \Lambda_e^3}{n_H}\right] = \frac{4k_B T n \Lambda_B^3}{\sqrt{\pi}} + C$$

If $n = 0$, meaning there's no dark matter, $C = -B$. This is the modified Saha equation, Equation (9).