

# A Correspondence between the Tensions $H_0$ and $S_8$ , and the Fundamental Constants $G$ and $c$

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## Abstract

Since the discovery of the Hubble tension and the Sigma 8 tension, the scientific world is still puzzling over the possible causes. We have noticed an inhomogeneous nature of the measurements between the two ends of the scale, and extrapolated a similar amplitude for both, through the fractional change formula. We have matched the resulting value over the age of universe, with the ratio between the constant of universal gravitation  $G$  and the constant of the speed of light in vacuum  $c$ . The magnitude factor over the age of universe was shown to match with the ratio between  $G$  and  $c$ , hinting to a divergence of cosmological nature. A further correspondence found with the Weinberg angle from the ratio of  $G$  over the radial acceleration scale value of Mond theories, the same as the Radial Acceleration Relation, supports the hypothesis of a discrepancy caused by a phase transition, like that of the electroweak theory, responsible for the emergence of the mass in  $Z$  boson, and of the photon.

## Keywords

Gravitation, Cosmological Parameters, Electroweak Theory, Cosmological Tensions, Dark Matter, Dark Energy

## 1. Introduction

Recent astrophysical measurements achieved a fine-tuning, unthinkable only a few decades ago. Theoretical physics has developed several cosmological models to match them. The Lambda Cold Dark Matter Model ( $\Lambda$ CDM) has resisted as the most credible until recently. Recent observations have been challenging it with inexplicable mismatches, the so-called Hubble ( $H_0$ ) tension and the amplitude of density fluctuations, the so-called Sigma 8 ( $S_8$ ) tension [1], among others. Those are discrepancies in the measurements from the most varied sources, often totally independent from each other, dealing respectively on the acceleration of universe

expansion and on the matter density fluctuation, aka the lumpiness of universe. However, to date, any theory made up to alleviate, or to solve the tensions, has not yet been endorsed by the general scientific consensus as a satisfactory solution.

After evaluating the nature of the measurements, we have calculated the average amplitude of the tensions. We have then put in relation the fundamental constants  $G$  and  $c$ , inspired by Planck's natural units, and provided with an additive justification for the results emerged from the calculations. We have shown how the tensions may stem from a well-known cosmological phase transition. In the conclusion, we have dared an approach to the dark energy and dark matter issues, summoning how the results fit into an equation of state, consistent with other historical precedents in cosmology, while suggesting a future perspective and the predictability that our theory offers.

## 2. Discussion

### 2.1. The Nature of the Measurements

While the  $H_0$  tension lower values are strongly constrained around  $67.4 \pm 0.5$  Km/s/Mpc [2], its higher values are rather scattered along the scale, grossly ranging between 73 and 76 km/s/Mpc [3]. Also,  $S_8$  tension showed a constrained value at one end, while scattered at the other, though in reverse order with respect to  $H_0$ . The measurements by PLANCK, TT, TE, EE have reported a higher tip of  $0.831 \pm 0.013$ , while the lower end ranges from about  $0.721 \pm 0.043$  [4] to about  $0.800^{+0.029}_{-0.027}$  [5]. Both tensions grossly show a discrepancy of 10% [6].

The classification of the measurements in function of the universe age, as early and late sources, showed no apparent proportion with time.

A further classification has been based on the methods applied, as indirect observations, inferred from the Microwave Cosmic Background (MCB), and direct observations, from EM signals, which on the contrary exhibit the tensions as a common thread. To explore a possible reason, we have focused on the nature of the measurements, which characterize the lower, the middle and the higher values:

- 1) The measurements extrapolated from the micro-differences in temperature of the CMB, and from the Baryonic Acoustic Oscillation (BAO) were derived from an estimation of baryon density and of power spectral density parameters. It is important to keep in mind that both expansion parameter and the fluctuation of matter density parameter might have emerged much earlier than the formation of the CMB, which dates 300,000 years from the Big Bang (BB). They might have arisen already in the first second of universe life, at least at the time of the formation of the Cosmic Neutrinos Background (CvB) [7]. Those concern respectively the  $H_0$  tension lower values, and the  $S_8$  tension higher values (BOSS, Planck).
- 2) The values in the midway have been gained by taking an average value through a joint method from multiple observations [8], as also from standard sirens [9], aka multi-messenger, based on the signals received by gravitational waves.
- 3) The values at the opposite end of the scale were obtained by the so-called cosmic ladders, mainly based on electromagnetic (EM) signals, like luminosity,

brightness, cosmological redshift, and lensing. They correspond respectively to the higher values of the  $H_0$  tension (MIRAS, HOLiCOW, SH0ES, etc.), and to the lower values of the  $S_8$  tension (KiDs, DES, VIKING, etc.).

4) There is a unique case, in which a single method has provided with measurements from the bottom to the end of the scale, as in the tip of the red giant branch (TRGB). Indeed, under the same umbrella of the TRGB, different measurements apply distinct approaches, choosing different types of sources with the relevant calibrations, with the result of a higher uncertainty in the measurements [10] [11]. If the different calibrations do not directly lead to a bias, they are not certainly free from it, due to the differently parametrized matter density and luminosity of the sources.

This analysis enhances that density parameters dominate at one end of the scale, while EM signals at the other, leaving the middle values to a mixing of the two. An aspect, which apparently had not been raised in recent proceedings [12].

## 2.2. The Magnitude of the Tensions

Concerning the  $H_0$  tension, we have averaged between the two extremes of the higher tip of the scale  $73.2 \pm 1.3$  km/s/Mpc [13] and  $76 \pm 2.3$  [14], obtaining 74.6 km/s/Mpc (fine-tuned to 0.0359, profitably aligned with the  $S_8$  tension down below). Then we have applied the formula of the fractional change:

$$\frac{\Delta x}{x} \quad (1)$$

and assigned to  $x$  the higher value for each tension. Thus:

$$\frac{74.6359 - 67.4}{74.6359} = 0.096949 \quad (2)$$

Concerning  $S_8$ , we have fine-tuned the higher value to 0.836(963); and chose for the lower a fair representative  $0.755^{+0.019}_{-0.021}$  from KV450+DES-Y1 [15], which also works well as an average between the two tips of  $S_8$  tension lower values. Again, fine-tuning to 0.00082, always within the margins of error, and applying the fractional change formula in the same way, we have obtained:

$$\frac{0.836963 - 0.75582}{0.836963} = 0.096949 \quad (3)$$

Even applying further slight adjustments, the result would have kept on the same magnitude. Two identical results from distinct phenomena appeared to us as the two facets of a single medal, supporting the hypothesis that the origin of the two tensions must be common.

## 2.3. The Fundamental Constants $G$ and $c$

We have decided then to focus on the relation running between the different nature of data, *i.e.* mass density and EM signals, though more radically than in the stellar mass-to-light ratio. Since  $G$ , and  $c$ , are respectively directly related to mass and EM signals, we have put the two fundamental constants in relation. There already exists a precedent in Planck's natural units [16]. Planck's equations showed

intriguing relations between  $G$  and  $c$ , *i.e.* the difference of  $c^{-2}$  between length  $l_p$  and time  $t_p$ , or the interchange of  $c^5$  and  $G$  between time  $t_p$  and energy  $E_p$ , denoting an inverse proportionality, a central aspect in the concepts of evolution and entropy [17]. Since cosmological tensions obviously lie on a cosmological scale, we decided to let aside  $\hbar$ , typical of the quantum mechanical scale, and to focus only on the correlation of  $G$  with  $c$ . We noted that both constants stand on the opposite sides of a fraction bar. Despite their distinct nature and the huge scale difference, it would not come naturally to relate them. Instead, we have followed the examples of Planck's length, and Planck's time (mimicking together the essentials of spacetime), though without exponentiation:

$$\frac{G}{c} = ? \quad (4)$$

which through dimensional analysis becomes:

$$\frac{\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}}{\text{m} \cdot \text{s}^{-1}} = \text{m}^2 \cdot \text{kg}^{-1} \cdot \text{s}^{-1} \quad (5)$$

We have obtained an unknown quantity, unless it is decomposed into the ratio of specific surface area ( $A$ ) over time ( $t$ ):

$$\text{m}^2 \cdot \text{kg}^{-1} \cdot \text{s}^{-1} = \frac{\text{m}^2 \cdot \text{kg}^{-1}}{\text{s}} \quad (6)$$

which leads us to conclude:

$$\frac{G}{c} = \frac{A}{t} \quad (7)$$

We have obtained that the ratio between  $G$  and  $c$  equals, at least dimensionally, the ratio between  $A$  (surface over its volume or mass, or also the reciprocal of surface density  $\rho_a$ ) and  $t$ . Assumed  $A$  corresponded to the amplitude of the tensions, then we could infer  $t$ , and discover what it could be:

$$\frac{cA}{G} = t \quad (8a)$$

$$\frac{299792458 \times 0.096949}{6.67430 \times 10^{-11}} = 4.3547 \times 10^{+17} \quad (8b)$$

in light-years the factor time ends up being 13,799,213,880, within the margins of error of the age of universe currently adopted in  $13,787,000,000 \pm 0.020$ .

Extrapolating the fractional change from the tensions, and identifying it with an  $A$ , a massive quantity, we have ended up to the age of universe confirmed by distinct sources (WMAP, Planck). A good result from just the empirical measures of  $G$ ,  $c$ ,  $H_0$ , and  $S_8$ , and independent from any theoretical model.

## 2.4. Deriving the $G$ Constant

Since uncertainties on the measurements of the tensions have been ruled out with several sigma, we argued that the discrepancy is physically real. Given the common factor, our hypothesis of a common origin has grown stronger. The two fundamental constants  $G$  and  $c$  might have diverted from a common stem, shaping

also the discrepancy of the tensions.

In the search for a contextualization of the factor  $A$ , we have looked at Newton's universal law of gravitation. Its mass over the distance squared reproduces fairly well the dimensional units of a surface density ( $\rho_a$ ), which is the reciprocal of  $A$ . Therefore, instead of deriving  $G$  from Newton's formula, we made it to derive from an acceleration ( $\mathbf{a}$ ) over  $\rho_a$ :

$$G = \frac{\mathbf{a}}{\rho_a} \tag{9}$$

or also, better fitting our case, from the product between  $\mathbf{a}$  and  $A$ :

$$G = \mathbf{a}A \tag{10}$$

Assuming  $A$  as the factor inferred from the tensions, then  $\mathbf{a}$  should amount to:

$$\frac{G}{A} = \mathbf{a} \tag{11a}$$

$$\frac{6.67430 \times 10^{-11}}{0.096949} = 6.88434 \times 10^{-10} \tag{11b}$$

We will see further on, whether this value may be of any cosmological relevance.

### 2.5. The Electroweak Cosmological Phase Transition

Speaking of gravitation, anomalies have been observed even at galactic scale, particularly in the spiral galaxies' rotational velocity. Vera Rubin's discovery [18] of the galaxies flat rotation curves signed the discrepancy between the expected radial acceleration, according to the Newtonian gravity, and the actual observations. The anomalous radial velocity in spiral galaxies led to the hypothesis of a mass discrepancy, today ascribed to the dark matter (DM) invisible mass. While DM is at the base of the current  $\Lambda$ CDM model, the alternative theories of Modified Newtonian Dynamics (MOND) modify  $G$  in various ways. It is important to note that the radial acceleration scale value  $g_{\ddagger} 1.20 \times 10^{-10} \pm 0.026 \text{ m}\cdot\text{s}^{-2}$  [19]-[21] of MOND theories equals that of the radial acceleration relation (RAR) [22]. The RAR concerns another important tension between baryon density and EM luminosity. We have decided to explore whether the relation between  $G$  and  $g_{\ddagger}$  could hopefully provide with a hint to the eventual cosmological PT at the origin of the tensions. With  $g_{\ddagger}$  fine-tuned to +0.040 (well inside its margins of error) we have found:

$$\frac{G}{g_{\ddagger}} = \tan(\varphi_W) \tag{12a}$$

$$\frac{6.67430 \times 10^{-11}}{1.20(40) \times 10^{-10}} = 0.5543439 \tag{12b}$$

a value, which coincides with the tangent of an angle of  $29.00153^\circ$ . Speaking of PTs, the weak mixing angle, or the Weinberg angle ( $\varphi_W$ ) of the electroweak theory, ranges between  $28.7^\circ$  and  $29.3^\circ$ . We could therefore obtain the scale acceleration value simply from:

$$g_{\ddagger} = \frac{G}{\tan(\varphi_W)} \tag{13}$$

Instead of a mere empirical value, for the first time, the scale acceleration value has found a physical reason to be, originating directly from the ratio between  $G$  and  $\tan(\varphi_W)$ . Both gravity and the essential element  $\varphi_W$  of the EW symmetry break theory, seem to share an intimate relation. After all, mass represents a common actor for both the EW theory and the gravitational interaction. To be noted that a comparable value, that we would identify with the tangent of  $\varphi_W$ , has also been observed in the constant stellar surface density [23] and, at planetary level, in the low-mass-to-radius relation [24].

## 2.6. Dark Energy and Dark Matter from the Fundamental Constants $G$ and $c$ ?

Notwithstanding the character of acceleration of  $G$ , we have demonstrated that  $A$  can be intimately linked to the gravitational constant. However, whether  $\mathbf{a}$  might concern dark energy (DE) remains an open question. In such a case, DE dimensional units, alias energy density over a given volume, must be equaled with  $\mathbf{a}$  over  $A$ :

$$\frac{E}{V} = U = \frac{\mathbf{a}}{A} \quad (14a)$$

$$\frac{\text{m}^2 \cdot \text{kg} \cdot \text{s}^{-2}}{\text{m}^3} = \text{m}^{-1} \cdot \text{kg} \cdot \text{s}^{-2} = \frac{\text{m} \cdot \text{s}^{-2}}{\text{m}^2 \cdot \text{kg}^{-1}} \quad (14b)$$

$$\frac{6.88434 \times 10^{-10}}{0.096949} = 7.10099 \times 10^{-9} \quad (14c)$$

Nevertheless, the DE density estimations from PLANCK and WMAP range within the same magnitude of  $6 \times 10^{-10}$  as in  $\mathbf{a}$ .

Finally, we could even notice that  $\sin \varphi_W$  over  $A$ :

$$\frac{\sin(\varphi_W)}{A} \quad (15a)$$

$$\frac{0.4848}{0.096949} \cong 5 \quad (15b)$$

returns the number of times, by which DM exceeds baryonic matter. As if  $A$  becomes the mass, hidden or refracted or multiplied by the electroweak PT (in the time domain).

## 3. Conclusions

1) Both tensions share the same discrepancy, which also emerged over the age of universe, from the ratio between the constant of universal gravitation and the speed of light in vacuum.

2) The values that we have assigned to the  $x$  of fractional change formula derive respectively from EM signals in the case  $H_0$ , and from matter density parameters in the case of  $S_8$ . In fact, to always obtain a positive value, we have assigned to  $x$  always the major value. We interpreted it more as a dichotomy, than as a real inhomogeneity in the nature of the data. The same dichotomy, which opposes  $A$

to  $\rho_a$  or the term of EM signals to the term of matter density as in:

$$\frac{c}{t} - \frac{G}{A} = 0 \quad (16a)$$

$$\frac{299792458}{4.3547 \times 10^{+17}} - \frac{6.67430 \times 10^{-11}}{0.096949} = 0 \quad (16b)$$

$$\frac{\text{m} \cdot \text{s}^{-1}}{\text{s}} - \frac{\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}}{\text{m}^2 \cdot \text{kg}^{-1}} = \text{m} \cdot \text{s}^{-2} - \text{m} \cdot \text{s}^{-2} \quad (16c)$$

from which we understand that:

- The ratio  $c/t$  returns an acceleration concerning radiation.
- $G$  over  $A$  returns the same acceleration, though with opposite orientation, concerning mass.
- Both balance each other.

While it certainly does not sound new that the first term may stand for a cosmological constant balanced by gravity, as in Fridman's and Einstein's equations, the novelty is that here acceleration emerges spontaneously from the observation on the EM signals travelling along the time dimension ( $c/t$ ).

3) The observations would have not normally misaligned the measurements unless a mismatching factor was already hiding behind them (Equation (7)). Dimensional analysis has enhanced that this mismatching element has the dimension of  $A$ , while its reciprocal  $\rho_a$  is a fundamental quantity in observational astrophysics. Both can be identified as factors within  $G$  itself (Equation (10)).

4) We have averaged the measurements to the most representative because we were aiming for an eventual underlying overall structure, without entering the issue about the accuracy of the measurements. The  $A$  value should not necessarily be taken as an exact quantity, but rather as a trace of a factor, which was unknown until today.

5) The massive quantity  $A$  describes an interphase [25], thus relevant to PTs. We believe that this factor had spurted from a cosmological PT, occurred in the very early universe, diverting  $G$  from  $c$ . With the help of the RAR then, we have pointed out a possible link between  $G$  and the Weinberg angle of the EW theory (Equation (13)), the mechanism explaining the emergence of the photon, the mediating particle for EM, on one side, and of the mass of the  $Z$  boson on the other. This correspondence supports the hypothesis that the EW symmetry break is the most suitable cosmological PT, triggering the divergence between the two fundamental constants, and consequently the tensions.

Our conclusion favors the hypothesis, which looks at DM and DE as consequences of a single phenomenon, similarly to other theories [26]-[31]. In this regard, it is interesting a recent study, according to which circular velocities show no deviation from flatness, even at hundreds of kiloparsec from the nuclei of galaxies [32].

In our opinion, the importance of our study does not lie much in providing with a solution of the tensions, but rather in the establishment of a relation between EM and gravity. Such connection, *i.e.* between the quantum mechanics of EM and that

of general relativity of  $G$ , should be further probed exploring the underlying geometry of the electroweak theory, which might finally shed light on the dark sector.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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