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A Dark Energy Hypothesis III

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Abstract

The subject is the thermodynamics of dark energy. Thermodynamically, the ratio of dark energy to CMR temperature has the units of entropy, has a well-defined numerical value at every moment of cosmological history, and increases in time monotonically without limit. The proposal is that it is the cosmological entropy, aka dark entropy. Discussion compares it to other notions of entropy. Dark entropy is a necessary prelude to DEH IV, which is about the thermodynamics of dark matter.

Keywords

Dark Energy, Entropy, Second Law

1. Introduction

The centerpiece of a dark energy hypothesis is replacing Einstein's cosmological constant Λ with a variable cosmological parameter [1]:

$$\Lambda = \frac{1}{\eta^2 a^2} = \kappa\varepsilon$$

In this equation, η is the conformal time and "a" the scale factor, $a\mathrm{d}\eta = c\mathrm{d}t$. The quantity ε is the dark energy density and κ is the Einstein gravitational constant:

$$\kappa = \frac{8\pi G}{c^4} = 2.076\times10^{-43}\ \mathrm{m}\cdot\mathrm{J}^{-1}$$

In the subsequent text, lower case lambda, λ, plays a number of roles. It designates quantities belonging to dark energy, such as dark entropy, S_λ. It has a numerical value that depends only on time:

$$\lambda = \frac{\cosh(\eta)-1}{6\eta^2} \tag{1}$$

The hyperbolic cosine indicates that the space of a DEH is hyperbolic. And, as a function of time only, it labels a cosmological epoch. Numerical work in DEH II, which will be continued here, is based on the currently held inventory of cosmological energies:

dark energy: dark matter: baryonic matter: 70:25:5.

Hence, this cosmological epoch is $\lambda = 7/10$ from which it follows that $\eta = 5.571$.

This sets the stage for introducing the notion of dark entropy.

2. Thermodynamics of Dark Energy

A proposal.

Dark entropy is

$$S_\lambda = \frac{U_\lambda}{T} \tag{2}$$

where the numerator is dark energy, described below, and the denominator is the temperature of the cosmic microwave radiation. It has the units of entropy and behaves in the expected way, that it increases without limit as the universe expands. Cosmological epochs are isothermal. Given two epochs, $\lambda_2 > \lambda_1$, then

$$\Delta S = S_2 - S_1 = \frac{U_2}{T_2} - \frac{U_1}{T_1} > 0$$

since $U_2 > U_1$, as will be seen, and $T_2 < T_1$. As the expanding universe has no equilibrium state, ΔS will never vanish.

A comparison to non-cosmological entropies.

The point of the comparison is to ascertain how the above proposal relates to the common formulations of the second law of thermodynamics. The formulations of the second law by Kelvin and Clausius divide thermodynamic space into system and surroundings, but the universe has no surroundings. Carathéodory's derivation of the second law doesn't require the notion of surroundings, but focuses instead on the nature of the thermodynamic state space. The goal of these formulations of the second law is to find an integrating denominator that will convert heat into a property of the thermodynamic system.

Comparison to cosmological entropy.

But heat is an energy that crosses from the surroundings into the system, and since the cosmos has no surroundings, the notion of heat becomes useless on the largest scale, which leads to the idea that the universe expands adiabatically: $\mathrm{d}Q = 0$. Does cosmology even need a concept of cosmological entropy? Yes, in the context of these two papers, because it makes possible the thermodynamic exploration of the nature of dark matter. DEH IV leads to a candidate particle for dark matter.

But adiabatic behavior doesn't preclude entropy cosmologically anymore than it does in the non-cosmological setting. Consider the Clausius inequality in the latter:

$$\oint \frac{\mathrm{d}Q}{T} \le 0$$

On the indicator diagram (pV-plane) let the system expand irreversibly and adiabatically from state A to B, and then return to A reversibly. The entropy change associated with the adiabatic expansion is

$$\Delta S = \int_A^B \frac{\mathrm{d}Q_{rev}}{T}$$

In general this will be nonzero with the exception that A and B are joined by a reversible adiabat. In the cosmological setting, the problem of the integrating denominator does not arise: U_λ and T are properties of the cosmos in the sense that each has a well-defined numerical value at any epoch.

Some Dark Energy Hypothesis formulas. [2]

This is useful for the continuity of the narrative. The scale factor is

$$a = \frac{\Gamma\left(\cosh(\eta)-1\right)}{6} = \Gamma\lambda\eta^2 \tag{3}$$

Γ is the total energy of the universe expressed as a length; since total energy is conserved its numerical value is the same for all epochs: $U = \Gamma/\kappa$. In the numerical examples of DEH II, $\Gamma = 6.306 \times 10^{24}$ m.

Dark energy has various guises:

$$U_\lambda = a^3\varepsilon = \frac{a^3\Lambda}{\kappa} = \frac{a}{\kappa\eta^2} = \frac{\Gamma\lambda}{\kappa} = U\lambda \tag{4}$$

The microwave temperature obeys a conservation law:

$$Ta = T_0 a_0 \tag{5}$$

where the subscript zero means the current epoch.

DEH II argues that space is necessarily hyperbolic and that dark matter necessarily becomes dark energy. In the current epoch, dark matter is 25% of the energy inventory. Consequently a dimensionless dark matter parameter, $\chi(dm)$, with a value of 0.25 in the present epoch, is

$$M(dm)c^2 = \frac{\Gamma\chi(dm)}{\kappa} \tag{6}$$

in the manner given by Equation (4) for dark energy, leading to the conservation law

$$\lambda + \chi(dm) = 0.95 \tag{7}$$

The remainder of the energy inventory is due to baryonic matter, $\chi(b) = .05 =$ constant. Finally, the cosmic time for any epoch is

$$t = \frac{\Gamma}{6c}\left(\sinh(\eta) - \eta\right) \tag{8}$$

Behavior of dark entropy.

Given Equations (4) and (5), dark entropy is

$$S_\lambda = \frac{U_\lambda}{T} = C(\eta\lambda)^2 \tag{9a}$$

for $\lambda < 0.95$, where

$$C = \frac{\Gamma^2}{\kappa\left(a_0 T_0\right)^2}$$

For the illustration in DEH II, $a_0 = 1.37 \times 10^{26}$ m and $T_0 = 2.7255$ K. Dark entropy grows linearly until $\lambda = 0.95$ at which point dark matter disappears. After that it grows as

$$S_\lambda = C\left(0.95^2\right)\eta^2 \tag{9b}$$

Thus, dark entropy grows continuously and without limit, although the slope of its linear growth changes discontinuously at $\lambda = 0.95$, which makes it a second order phase transition in the Ehrenfest classification.

Eddington called entropy the arrow of time [3]. Dark entropy is the ultimate arrow of time, because, as Equation 9(a) & Equation 9(b) show, it depends only on the time and not on any physical processes; it encounters neither an equilibrium state nor an end to expansion that would stop its growth.

3. Comparison of Dark Entropy and the Cosmic Microwave Entropy

An interesting point of comparison is the Duhem-Margules equation for entropy density:

$$\frac{S}{V} = \frac{\mathrm{d}p}{\mathrm{d}T} - \frac{N}{V}\frac{\mathrm{d}\mu}{\mathrm{d}T}$$

For radiation, the chemical potential $\mu = 0$ whereas for dark entropy the pressure $p = 0$ as argued in DEH II. For radiation, with pressure equal to one-third of the energy density, and with the energy density given by the Stefan-Boltzmann law $\varepsilon_r = \sigma T^4$

$$\frac{S_r}{V} = \frac{4\varepsilon_r}{3T}$$

Equation (2) gives the dark entropy density as

$$\frac{S_\lambda}{V} = \frac{\varepsilon_\lambda}{T}$$

Hence the ratio of entropies is

$$\frac{S_\lambda}{S_r} = \frac{3\varepsilon_\lambda}{4\varepsilon_r} = 149$$

The numerical value is for the present epoch. A simple calculation shows that the ratio increases as time elapses.

4. The Second Law in the Dark Energy Hypothesis

1) Space is necessarily hyperbolic and dark energy necessarily grows at the expense of dark matter.

2) Dark entropy increases continuously without limit.

3) There is no equilibrium state so ΔS_λ never vanishes.

The temptation is to say that entropy increase drives the expansion. It is true that entropy increases with the expansion, just as the number of miles traveled increases during an auto trip, but to say that entropy drives the expansion is to say that miles traveled drive the auto trip. That lacks insight.

The better understanding is in the Friedmann-Lemaître equation, which takes the following form in DEH II:

$$\left(\frac{\mathrm{d}a}{\mathrm{d}\eta}\right)^2 + ka^2 = \frac{\Gamma a}{3}$$

Einstein in 1917 wrote the first relativistic paper on cosmology in which he constructed a static model of the universe of "fixed stars" [4]. Although the Friedmann-Lemaître equation did not exist at that time, it can be used to reconstruct his thinking. Choose $k = +1$ and see DEH II to find that the right-hand side of the equation consists of two terms. The result is a model of constant radius in which

$$\Lambda = \frac{\kappa\rho c^2}{2} = \frac{1}{a_E^2}$$

The third term is the curvature expressed in terms of the Einstein radius. It did not take long to realize that such a universe is metastable, meaning that if the Einstein radius is perturbed from the above condition, it never recovers it, and the radius must either increase or decrease. Many papers in early twentieth-century cosmology grapple with this instability [5]. Newton was well aware of the problem of the stability of a universe of fixed stars. In a 1692-3 letter he writes that one has to suppose that the forces acting on every star perfectly cancel, and "I reckon this as hard as to make not one Needle only, but an infinite number of them (so many as there are Particles in an infinite Space) stand accurately poised on their points." [6]

The large scale distribution of matter is intrinsically unstable: the universe's scale factor must either increase or decrease as time elapses. If it increases, then biological evolution can occur and, of course, has occurred, with $\Delta S > 0$ accompanying it. If it decreases, then $\Delta S < 0$ and there's no biological evolution.

The concept of dark entropy is useful, the next paper providing one example.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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