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Chaos Synchronization of Lorenz System Using an Observer Backstepping Finite-Time Controller Design

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Abstract

This paper presents an observer backstepping control strategy to achieve finite-time Chaos Synchronization for uncertain Lorenz system. To compensate for unknown behavior dynamics, radial basis function neural networks (RBFNN) is employed to approximate the uncertain terms system depending on the slave behavior of the system independent of the master system and adaptive laws are derived to update the network weight online within the Lyapunov framework. The authors use Lyapunov stability theory analysis to design an observer controller that ensures the synchronization error converges to zero in a finite-time even with unknown system parameters constants and external disturbances. The effectiveness of the proposed method is demonstrated through numerical simulations.

Keywords

Chaotic Lorenz System, Finite-Time Synchronization, Neural Network, Observer Backstepping Controller, Uncertainty

1. Introduction

The first classical chaotic system was found by Edward Lorenz when he studied the atmospheric convection in 1963, in [1]. It is a nonlinear system of three differential equations. With the most commonly used values of three parameters, there are two unstable critical points. The solutions remain bounded, but orbit chaotically around these two points. For a more in-depth study in 1990, the US Naval Research Laboratory researchers L. M. Pecora and T. L. Caroll first proposed in the international response to synchronous master principles and methods, and the circuits experiment chaotic synchronization as in [1]-[28].

The synchronization of chaotic systems has been widely studied due to its importance in secure communications, nonlinear science, and complex networked systems. Among chaotic models, the Lorenz system serves as a benchmark due to its strong nonlinearity and sensitivity to initial conditions. Classical synchronization strategies typically guarantee asymptotic convergence, which may be too slow for practical applications. Finite-time control techniques improve convergence speed but still depend on initial conditions. In this context, observer-based adaptive backstepping control provides a systematic framework to handle nonlinearities, uncertainties, and partial state measurements.

Among chaotic systems, the Lorenz system is one of the most widely studied due to its strong nonlinear behavior, sensitivity to initial conditions, and complex chaotic attractors.

The objective is to design a suitable controller such that the slave system tracks the states of the master system despite uncertainties, disturbances, or unknown parameters. The most common suggestion was found to use the observer backstepping finite-time controller design method.

This approach is an advanced and convenient for design, yet its performance was susceptible to disturbance or noise, by using an observer looks at synchronization in [1]. Since the measurement disturbance was also unavoidably amplified by the proportional gain as in [2].

This paper focuses on finite-time synchronization of a Lorenz master-slave system using an observer-based adaptive backstepping controller.

To improve the noise performance of Lorenz, the observer backstepping finite-time controller approach has been recommended in Master-Response system, then being extended to a class of nonlinear systems.

In this paper, a new scheme of an observer backstepping finite-time controller design of chaos synchronization is proposed for a class of chaotic system such as in nonlinear control and chaos synchronization and based on an advanced control scheme, some new sufficient conditions are derived for chaos synchronization, proved by the Lyapunov stability theory.

The time evolution of the nonlinear dynamical system responses is described in phase portraits, states phase, the occurrence and nature of chaotic attractors are verified by Lyapunov Exponents and Lyapunov dimensions. The presence of chaotic behavior is generic for suitable nonlinearity, ranges of parameters and external forces, where one wishes to avoid or control so as to improve the dynamical system. Sometimes chaos is useful, as in mixing process transfer system.

However, chaos is always undesirable. Clearly that, the ability to chaos control, that's to convert chaotic oscillations into desired regular ones with periodic time dependence, would be important in working with a particular system.

It is thus of great practical application to develop suitable control theory which implies an observer backstepping control strategy to achieve the finite-time Chaos-Synchronization of uncertain Lorenz system.

This motivation breaking research such chaos used in electronics, information and communications and other engineering fields as chaos synchronization, chaotic secure communications and state estimation for uncertain systems schemes have been proposed. Studies have shown that not only is chaotic dynamic can be achieved based control and synchronization, but can also serve as information transmission and processing of system as in [1]-[29].

Master-response synchronization is characterized by the existence of two nonlinear dynamics systems driven relationship with the response as in [19]-[21]. Depending on the slave behavior of the system independent of the master system, the drive system behavior and slave system behavior.

The synchronization method only way to transmit encrypted signal through the channel, and is self-synchronous mode, when for some reason to re-step self-synchronization, viability with existing communication transmission, in sensing and control communication, in the circuit, DSP or ARM, MSP430 technology the practical application. Let a n-dimensional autonomous power system, as follows as in [1]-[6].

Rossler carried out a most important work which brought the interest inaccurate nonlinear of dynamic system in 1976, as in [2]. Rossler himself proposed an advanced system in 1979, as in [3]. Otto Grebogi *et al.*, controlling chaos as in [4]. Sprott embarked upon an extensive search as in [5] for autonomous three states chaotic systems. Chen and Dong made another chaotic system as in [6], which nevertheless is not structurally equivalent to the Lorenz' system as in [1]-[8]. A chaotic system can be chaotic dynamic whenever its evolution sensitively depends on the initial conditions as in [9]. Chaos control refers to manipulating the dynamical behavior of chaotic system, in which the goal is to suppress chaos when it is harmful or create chaos when it is beneficial as in [1]-[9].

When the wheel or disc spins, it exhibits properties of angular momentum, which helps it resist changes in orientation. Chaotic systems are used in various applications as follows as a particular form of nonlinear system, including navigation systems, aircraft and spacecraft control, stabilization systems for cameras and sensors, and even in some consumer devices like smartphones for motion sensing. They play a crucial role in maintaining stability and accuracy in these systems by providing a reference for orientation and angular velocity, which have been widely to evaluate control schemes of chaotic system as in [8]-[10]. A variety of approaches have been proposed for solving the gyros chaos control problem. These methods include active control in [11], based on dynamical behaviors and chaos control as in [12], based on variable structure control as in [13], based on fuzzy sliding mode control as in [14], based on via backstepping control in [15]. Based on an improved backstepping method as in [16]. Designing to stabilize gyro chaotic system. Chaos control and modified projective synchronization of unknown heavy symmetric chaotic system as in [17]. Based on adaptive control for the stabilization and synchronization of nonlinear gyroscopes as in [18]. Based on robust nonlinear dynamic inversion with finite-time as in [19]. Based on adaptive robust finite-time as in [20]. Xiaomin Tian *et al.* designed on finite-time adaptive synchronization of two different fractional-order Gyroscope Systems with dead-zone nonlinear inputs as in [21]. Guo Yong *et al.* designed adaptive finite-time backstepping control for attitude tracking of spacecraft based on rotation matrix as in [22]. Loembe-Souamy *et al.* designed on backstepping control design as in [23]. Loembe-Souamy *et al.* based on adaptive backstepping control design as in [24], Uğur Erkin Kocamaz *et al.* designed on secure communication with Chaos and electronic circuit design using passivity-based Synchronization as in [25]. Serdar Çiçek *et al.* designed on secure communication with a Chaotic system owning logic element as in [26]. Alinaghi Hosseinabadi P. *et al.* designed adaptive finite-time sliding mode backstepping controller for double-integrator systems with mismatched uncertainties and external disturbances as in [27], Abdullah Gokyildirim *et al.* designed a novel five-term 3D chaotic system with cubic nonlinearity and its microcontroller-based secure communication implementation as in [28]. Mahougnon Jeannette Aguessivognon *et al.* designed an effect of biharmonic excitation on complex dynamics of a two degree-of freedom heavy symmetric gyroscope as in [29]. Loembe Souamy *et al.* based on an Enhanced of Synchronization of Chaos to the Applications of the Drive Principle of a Synchronized Responsive based a Reference Observer of DCSK by Comparing of the Performance Between GCS-DCSKI and GCSDCSKII over AWGN chaotic system as in [30], is for a conference proceedings, Loembe Souamy *et al.* based on adaptive backstepping finite-time controller design as in [31], and others as in [25]-[31], etc.

This paper investigates the problem of synchronization for a class of uncertain chaotic systems with unknown parameters. A novel observer backstepping control scheme is proposed to achieve synchronization between master and slave systems. The designed observer controller and parameter update laws ensure that all signals in the closed-loop system remain bounded while the synchronization error converges to zero asymptotically, based on backstepping control design system, which is different from the existing methods as in [9]-[28].

The proposed method shows a novel of an observer backstepping controller can reduce the complexity of Lorenz chaos control and increase the effectiveness and feasibility of an observer backstepping controller design technique, which will be supported by theoretical analysis and simulations results.

The rest parts of this paper are organized as follows. In Section 2, a brief description of the Lorenz system with some uncertainties are introduced. In Section 3, we discuss, the problem formulation of design. In Section 4, we discuss also the design of the observer based on adaptive backstepping finite-time controller design and verify the stability of the error system by using the Lyapunov stability theory. In Section 5, and Section 6, the design of the observer backstepping finite-time controller design based on adaptive and robust, verify the estimation of the error system by using the Lyapunov stability theory. In Section 7, numerical simulations are given for illustration of the effectiveness of the backstepping control technique. Some conclusions are presented in Section 8.

2. Mathematical of Modeling of Lorenz Chaotic System

2.1. Description of Lorenz System

This is a common issue in control theory, paper is described by: Presenting a very general framework and then applying it to a specific system, leaving the researcher to mentally bridge the gap. Here is how to explain and resolve this lack of clarify. This section establishes the connection between the theoretical development and the application can be expressed as:

$$\dot{x} = v(x) + \omega(x)\delta + h(x)u \tag{1}$$

A general model takes the form of Lorenz chaotic system; we design the specific Lorenz system can be expressed as:

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = \rho x - y - xz \\ \dot{z} = xy - \beta z \end{cases} \tag{2}$$

To facilitate controller design, the nonlinear system is reformulated into a backstepping compatible structure (v, ω, δ, h) to these specific systems (x, y, z) equations with (σ, ρ, β). A common challenge in nonlinear control design is the presence of system uncertainties and unmeasured states.

We explain that a bridge between the general theory and the specific Lorenz system, the system dynamics are given by: According to the general framework Section 2, the Lorenz system Equations (2) can be expressed as the general form Equation (1). For the slave system with control input is given by:

$$u = [u_1, u_2, u_3]^{\mathrm{T}} \tag{3}$$

Added to each (state equation), the mapping takes the form the mappings. Let state vector is described by:

$$x = [x_1, x_2, x_3]^{\mathrm{T}} = [x, y, z]^{\mathrm{T}} \tag{4}$$

And the unknown parameter vector takes the form:

$$\delta = [\sigma, \rho, \beta]^{\mathrm{T}} \tag{5}$$

The known nonlinear function $v(x)$ can be expressed and the parameter regressor matrix $\omega(x)$ are constructed into the specific Lorenz system. We investigate the problem of the finite-time on chaos synchronization of three uncertain chaotic non-linear Lorenz systems that are discussed below.

2.2. Lorenz Chaotic System

This section establishes the connection between the theoretical development and the application of the proposed method. The effects of behavior of model uncertainties nonlinear systems, we define that an advanced based on observer controller verifies the stability of the error system by using proper Lyapunov functions, then we design a controller term to synchronize the master-slave system asymptotically stables at origin. For example, this explicit mapping allows the general (v, ω, δ, h) to these specific (x, y, z) equations.

Let an n-dimensional autonomous power system, is defined by:

$$\dot{u} = f(u), t \in R, u(t) \in R^n, f : R^n \to R^n \tag{6}$$

We will be decomposed into two subsystems as in [15]-[18], it follows as:

$$\begin{cases} \dot{v}^{(1)} = \delta\left(v^{(1)}, \omega^{(1)}\right) \\ \dot{\omega}^{(1)} = h\left(v^{(1)}, \omega^{(1)}\right) \end{cases} \tag{7}$$

Among them, we define that a controller measures proportionately more than the systems are defined by:

$$\begin{cases} u = [u_1, u_2, u_3, \cdots, u_m]^{\mathrm{T}} \\ v^{(1)} = [v_1, v_2, v_3, \cdots, v_m]^{\mathrm{T}} \\ \omega^{(1)} = [u_{m+1}, u_{m+2}, u_{m+3}, \cdots, u_n]^{\mathrm{T}} \end{cases} \tag{8}$$

$$\begin{cases} f = [f_1, f_2, f_3, \cdots, f_n]^{\mathrm{T}} \\ \delta = [f_1, f_2, f_3, \cdots, f_m]^{\mathrm{T}} \\ h = [f_{m+1}, f_{m+2}, f_{m+3}, \cdots, f_n]^{\mathrm{T}} \end{cases} \tag{9}$$

Added into Equation (7) where in the master is called active drive system chaotic signal $v^{(1)}$ to drive response system, is defined by:

$$\begin{cases} \dot{v}^{(2)} = \delta\left(v^{(2)}, \omega^{(2)}\right) \\ \dot{\omega}^{(2)} = h\left(v^{(1)}, \omega^{(2)}\right) \end{cases} \tag{10}$$

Note that the above the formula and master system in response to the system has the same form, but in using the second Equation, Equation (10) type of drive signal $v^{(1)}$ replaces the original signal $v^{(2)}$. Similarly, we can obtain the drive system in the chaotic signal $\omega^{(1)}$ to drive a response subsystem is designed:

$$\begin{cases} \dot{v}^{(2)} = \delta\left(v^{(2)}, \omega^{(1)}\right) \\ \dot{\omega}^{(2)} = h\left(v^{(2)}, \omega^{(2)}\right) \end{cases} \tag{11}$$

Similarly, in response to the above formula and having a drive system in exactly the same form, with only the first Equation (6) type of drive signal $\omega^{(1)}$ replaces the original signal $\omega^{(2)}$. Pecora and Caroll principle of stability theory and synchronization subsystem response were analyzed and the stability of chaos synchronization theory, the so called conditional Lyapunov exponential stability criterion and proved and given only when the response subsystem, type of Lyapunov exponents are negative, the response system to achieve synchronization with the drive system, namely:

$$\begin{cases} \Delta v(t) = \lim\limits_{t\to\infty} \left\| v^{(2)}(t) - v^{(1)}(t) \right\| \\ \Delta\omega(t) = \lim\limits_{t\to\infty} \left\| \omega^{(2)}(t) - \omega^{(1)}(t) \right\| \end{cases} \tag{12}$$

Similarly, according to Equation (12) formula, and give the corresponding Δv and $\Delta\omega$ linearized equation is defined by:

$$\begin{cases} \Delta\dot{v} = \left.\dfrac{\partial g\left(v^{(2)},\omega^{(2)}\right)}{\partial v}\right|_{v^{(2)}=v^{(1)},\omega^{(2)}=\omega^{(1)}} \cdot \Delta v, \Delta v = \Delta v^{(2)} - \Delta v^{(1)} \\ \Delta\dot{\omega} = \left.\dfrac{\partial h\left(v^{(1)},\omega^{(1)}\right)}{\partial \omega}\right|_{\omega^{(2)}=\omega^{(1)}} \cdot \Delta\omega, \Delta\omega = \Delta\omega^{(2)} - \Delta\omega^{(1)} \end{cases} \tag{13}$$

where

$$\begin{cases} \Delta\dot{v} = \left.\dfrac{\partial g\left(v^{(2)},\omega^{(2)}\right)}{\partial v}\right|_{v^{(2)}=v^{(1)}} \cdot \Delta v, \Delta v = \Delta v^{(2)} - \Delta v^{(1)} \\ \Delta\dot{\omega} = \left.\dfrac{\partial h\left(v^{(1)},\omega^{(1)}\right)}{\partial \omega}\right|_{v^{(2)}=v^{(1)},\omega^{(2)}=\omega^{(1)}} \cdot \Delta\omega, \Delta\omega = \Delta\omega^{(2)} - \Delta\omega^{(1)} \end{cases} \tag{14}$$

The so called synchronous stability criterion are designed: all index above two linear equations are negative that is Equation (13) and Equation (14) where all the conditions under driving conditions Lipschitz index are negative, the synchronization asymptotically stable. The general forms of the Jacobian matrix are designed by:

$$\begin{cases} D_v g = \left.\dfrac{\partial g}{\partial v}\right|_{v^{(2)}=v^{(1)},\omega^{(2)}=\omega^{(1)}} = \begin{pmatrix} \partial f_1/\partial u_1 & \partial f_1/\partial u_2 & \cdots & \partial f_1/\partial u_m \\ \partial f_2/\partial u_1 & \partial f_2/\partial u_2 & \cdots & \partial f_2/\partial u_m \\ \vdots & \vdots & & \vdots \\ \partial f_m/\partial u_1 & \partial f_m/\partial u_2 & \cdots & \partial f_m/\partial u_m \end{pmatrix}_{v^{(2)}=v^{(1)},\omega^{(2)}=\omega^{(1)}} \\ D_v h = \left.\dfrac{\partial h}{\partial v}\right|_{v^{(2)}=v^{(1)},\omega^{(2)}=\omega^{(1)}} = \begin{pmatrix} \partial f_1/\partial u_1 & \partial f_1/\partial u_2 & \cdots & \partial f_1/\partial u_m \\ \partial f_2/\partial u_1 & \partial f_2/\partial u_2 & \cdots & \partial f_2/\partial u_m \\ \vdots & \vdots & & \vdots \\ \partial f_m/\partial u_1 & \partial f_m/\partial u_2 & \cdots & \partial f_m/\partial u_m \end{pmatrix}_{v^{(2)}=v^{(1)},\omega^{(2)}=\omega^{(1)}} \end{cases} \tag{15}$$

It should be stressed that not all chaotic systems can achieve a response synchronous drive. Specially, only when all the conditions Lyapunov exponent response system Equations (15) have the formula is negative, in order to achieve synchronization. In addition to rigorous theoretical proof, but in practical application is mainly based on the simulation results to determine the two chaotic systems can really achieve the synchronous mode. The driving principle of a synchronized responsive to the third-order chaotic system, for example are constructed by $x^{(1)}$ variables, $y^{(1)}$ variables, and $z^{(1)}$ variables as the driving variable three synchronized manner, as shown in **Figure 1**, also shown in **Figure 2** and in **Figure 3**.

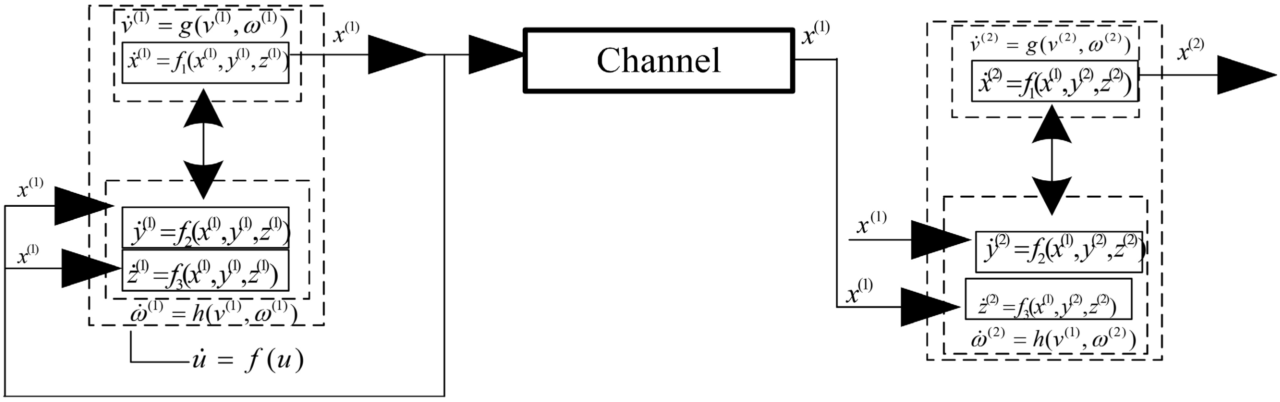


Figure 1. $x^{(1)}$ variables driving synchronize system.

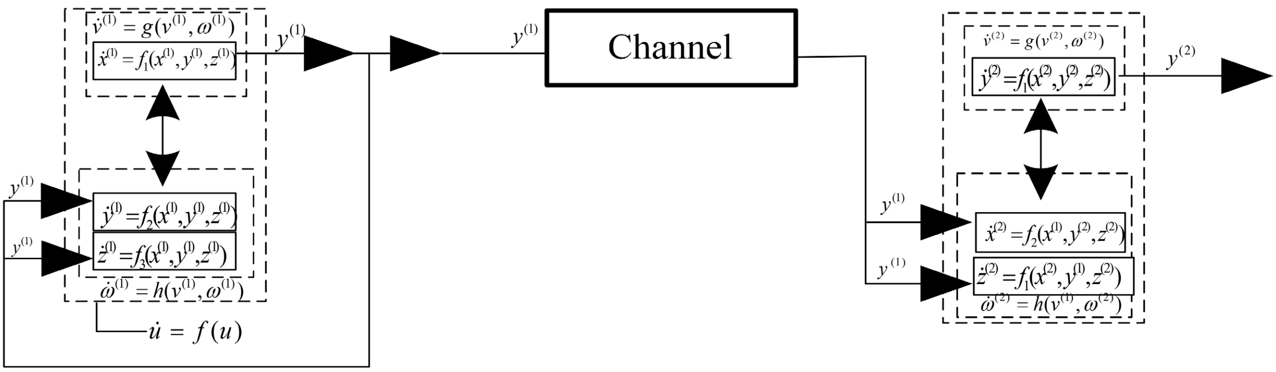


Figure 2. $y^{(1)}$ variables driving synchronize system.

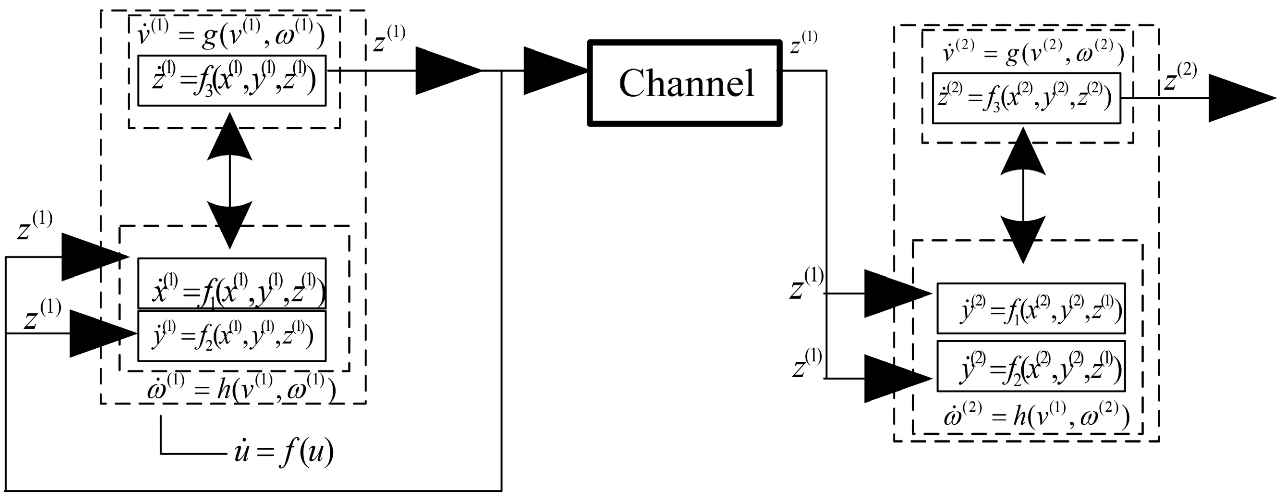


Figure 3. $z^{(1)}$ variables driving synchronize system.

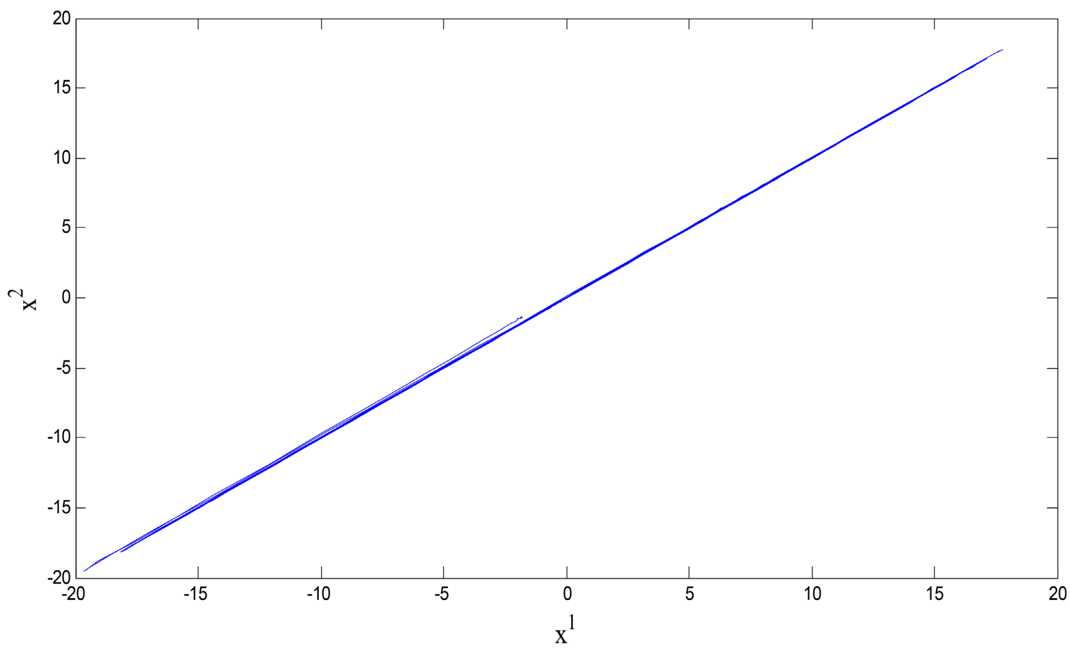
Figures 1-3 double arrows represent between two subsystems are not independent, but rather an interaction between variables. It should be noted that the two selected subsystems are varied, where only one of the three options given and number of combinations for each subsystem equation selects the equation and is also flexible. Lorenz system for Driving-Response Synchronization. In the above analysis, the type of synchronization and parameters of the driving system and response system assumes exactly the same, belonging to the same structure synchronization of chaotic systems as in [7]-[30]. As shown in **Figure 1** with the variable $x^{(1)}$ as the master synchronization system must drive system equation of state is defined by:

$$\begin{cases} \mathrm{d}x^{(1)}/\mathrm{d}t = -a\left(x^{(1)} - y^{(1)}\right) \\ \mathrm{d}y^{(1)}/\mathrm{d}t = bx^{(1)} - x^{(1)}z^{(1)} - y^{(1)} \\ \mathrm{d}z^{(1)}/\mathrm{d}t = -cz^{(1)} + x^{(1)}y^{(1)} \end{cases} \tag{16}$$

where $x^{(1)}, y^{(1)}, z^{(1)}$ in the master system parameters as $a = 10, b = 30, c = 8/3$, the drive system of the three state variables. When using variable $x^{(1)}$ as a signal to give a response to state equation is defined by:

$$\begin{cases} \mathrm{d}x^{(2)}/\mathrm{d}t = -a\left(x^{(2)} - y^{(2)}\right) \\ \mathrm{d}y^{(2)}/\mathrm{d}t = bx^{(1)} - x^{(1)}z^{(2)} - y^{(2)} \\ \mathrm{d}z^{(2)}/\mathrm{d}t = -cz^{(2)} + x^{(1)}y^{(2)} \end{cases} \tag{17}$$

where $x^{(2)}, y^{(2)}, z^{(2)}$ are the drive systems in response to the same system parameters in response to the system three are state variables. The proposed method's simulation results are shown as in **Figure 4**. Since the synchronous phase diagrams are strictly diagonal, synchronization errors, it can be achieved synchronization. As shown with the variable $y^{(1)}$ as drive synchronization system to give the state of the drive system shown in **Figure 3** then Equation (18) is defined by:



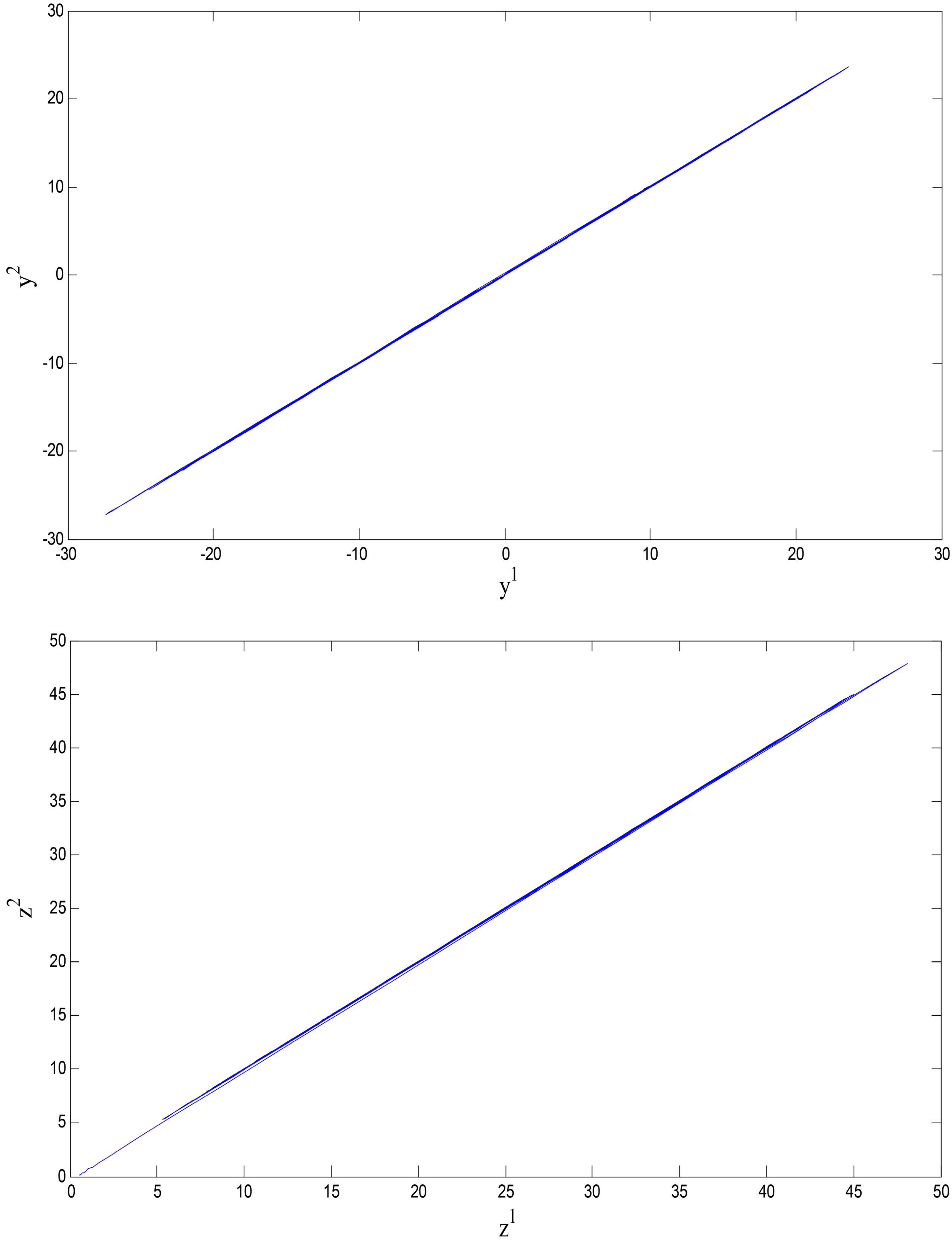


Figure 4. $x^{(1)}$ variables driving synchronized system.

$$\begin{cases} \mathrm{d}x^{(1)}/\mathrm{dt} = -a\left(x^{(1)} - y^{(1)}\right) \\ \mathrm{d}y^{(1)}/\mathrm{dt} = bx^{(1)} - x^{(1)}z^{(1)} - y^{(1)} \\ \mathrm{d}z^{(1)}/\mathrm{dt} = -cz^{(1)} + x^{(1)}y^{(1)} \end{cases} \tag{18}$$

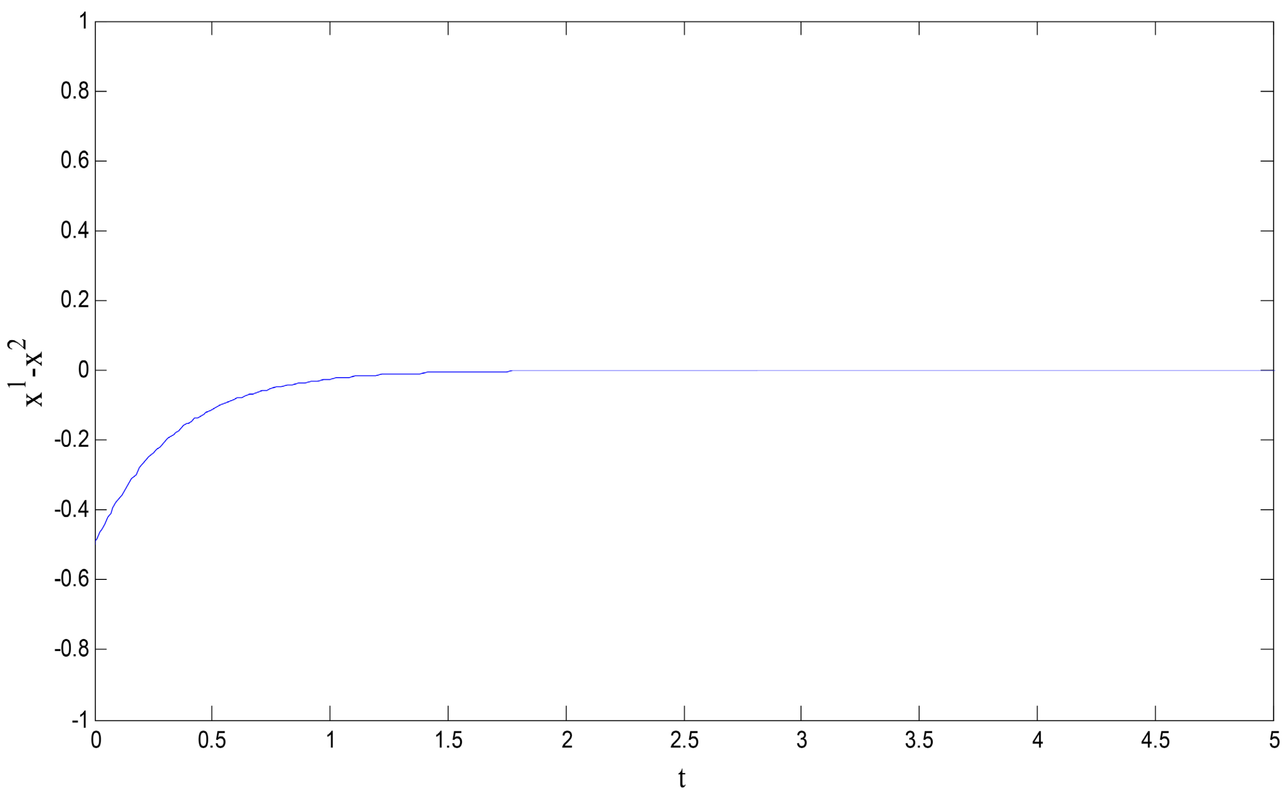
When using the variable $y^{(1)}$ as a signal to give a response to the state equation, it is defined by:

$$\begin{cases} \mathrm{d}x^{(2)}/\mathrm{dt} = -a\left(x^{(2)} - y^{(1)}\right) \\ \mathrm{d}y^{(2)}/\mathrm{dt} = bx^{(2)} - x^{(2)}z^{(2)} - y^{(2)} \\ \mathrm{d}z^{(2)}/\mathrm{dt} = -cz^{(2)} + x^{(2)}y^{(1)} \end{cases} \tag{19}$$

Applications of the drive principle of synchronization responsive on Matlab programming simulation results as shown in **Figure 5**, synchronization errors, it can be achieved synchronization. As shown with the variable $y^{(1)}$ as drive synchronization system to give the status of the drive system shown in **Figure 5** then Equation (20) is defined by:

$$\begin{cases} \mathrm{d}x^{(1)}/\mathrm{dt} = -a\left(x^{(1)} - y^{(1)}\right) \\ \mathrm{d}y^{(1)}/\mathrm{dt} = bx^{(1)} - x^{(1)}z^{(1)} - y^{(1)} \\ \mathrm{d}z^{(1)}/\mathrm{dt} = -cz^{(1)} + x^{(1)}y^{(1)} \end{cases} \tag{20}$$

When using the variable $z^{(1)}$ as a signal to give a response to the sate Equation (21) is defined by:



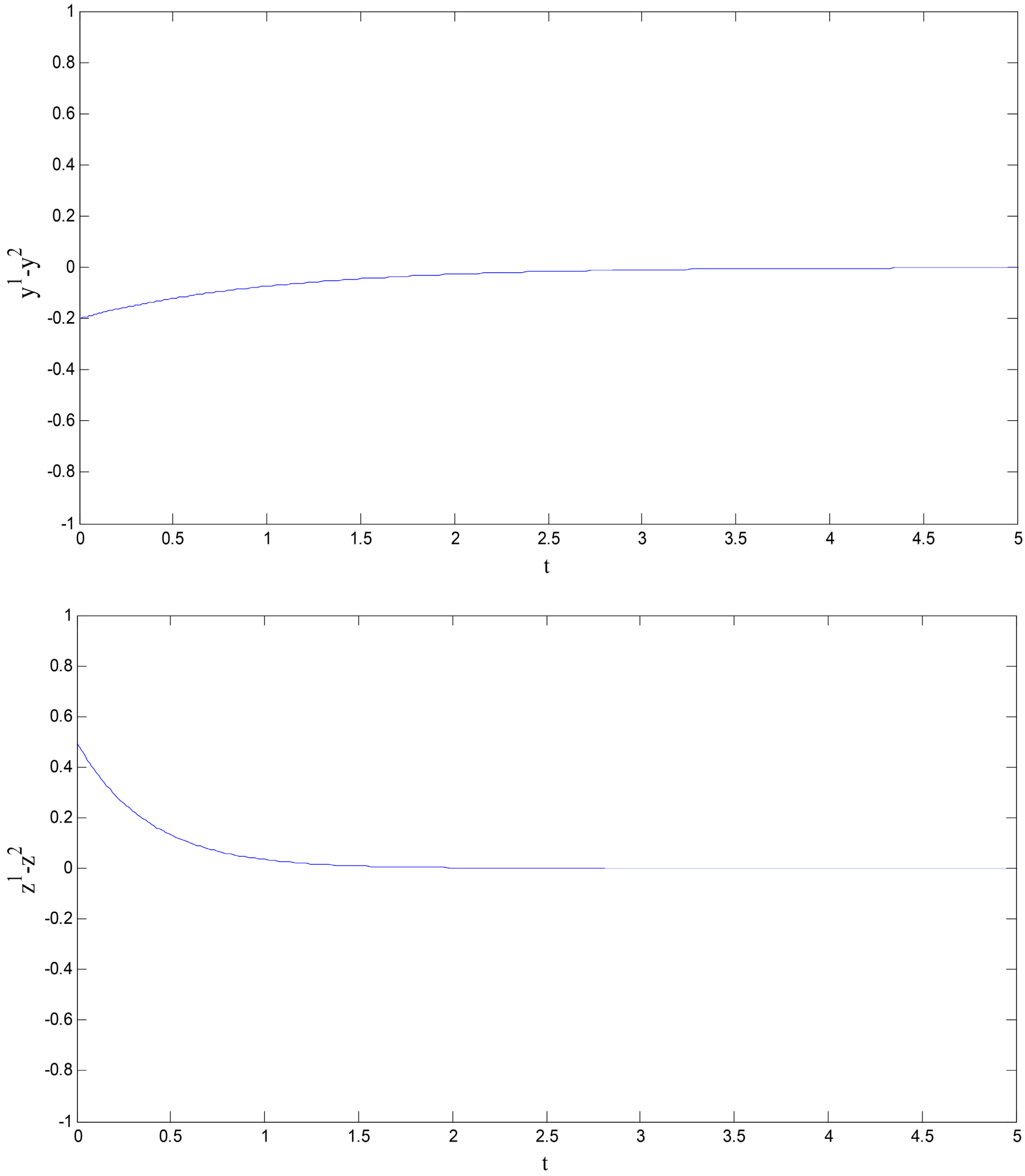


Figure 5. $y^{(1)}$ variables driving synchronized system.

$$\begin{cases} \mathrm{d}x^{(2)}/\mathrm{d}t = -a\left(x^{(2)} - y^{(2)}\right) \\ \mathrm{d}y^{(2)}/\mathrm{d}t = bx^{(2)} - x^{(2)}z^{(1)} - y^{(2)} \\ \mathrm{d}z^{(2)}/\mathrm{d}t = -cz^{(2)} + x^{(2)}y^{(2)} \end{cases} \tag{21}$$

Above the system so mapping allows the general (v,ω,δ,h) to these specific (x,y,z) equations synchronization errors realize, it can be achieved synchronization system as in [1]-[30]. This general form facilitates the Lyapunov based design in the next section.

2.3. Principle of the Chaos and Synchronization of the Lorenz System

Applications of the drive principle of synchronization responsive on Matlab simulation results as shown in **Figure 6**, synchronization errors don't realize, it can't be achieved synchronization, as follows as in **Figure 6**.

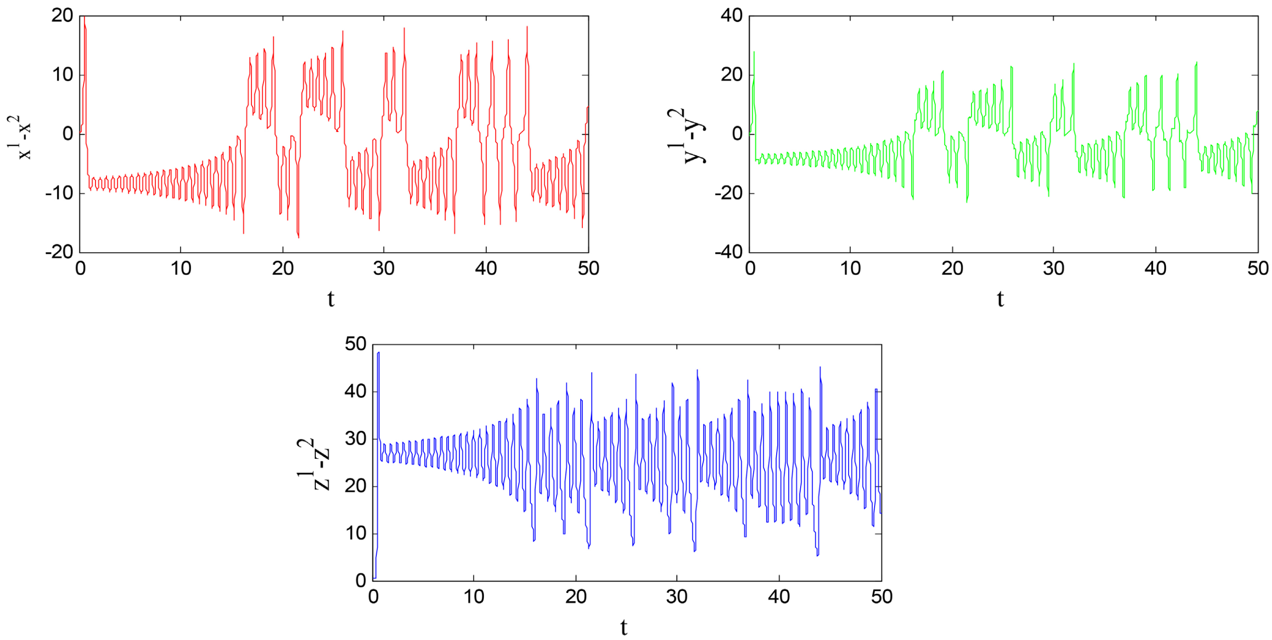


Figure 6. $z^{(1)}$ variables driving error are not synchronized system.

Lorenz system drives response synchronized, Lorenz stability theory analysis. Lorenz system variable x drive signal and other drive system and response system divided into subsystem Equation (20), then Equation (21) then we design the drive system response system become the errors signals are defined by:

$$\begin{cases} e_x = x^{(1)} - x^{(2)} \\ e_y = y^{(1)} - y^{(2)} \\ e_z = z^{(1)} - z^{(2)} \end{cases} \tag{22}$$

Adding from Equation (20) using subtracting Equation (21) according to Equation (22), then we obtain the dynamics errors states are defined by:

$$\begin{cases} \dot{e}_x = -a\left(e_x - e_y\right) \\ \dot{e}_y = -x^{(1)}e_z - e_y \\ \dot{e}_z = -ce_z + x^{(1)}e_y \end{cases} \tag{23}$$

We choose a candidate of Lyapunov function, as follows as:

$$V(e)=\frac{A}{2}e_x^2+\frac{B}{2}e_y^2+\frac{D}{2}e_z^2 \tag{24}$$

where A,B,D are parameters positive, adding Equation (23) and Equation (24), are defined by:

$$\begin{aligned}\dot{V}(e)&=Ae_x\dot{e}_x+Be_y\dot{e}_y+De_z\dot{e}_z\\&=Ae_x\left[-a\left(e_x-e_y\right)\right]+Be_y\left[-x^{(1)}e_z-e_y\right]+De_z\left[ce_z+x^{(1)}e_y\right]\\&=-aAe_x^2+aAe_xe_y-x^{(1)}Be_ye_z-Be_y^2-cDe_z^2+x^{(1)}De_ye_z\end{aligned} \tag{25}$$

Let $D=B$, above the analysis then proceed, the derivative of system, is defined by:

$$\dot{V}(e)=-aAe_x^2+aAe_xe_y-Be_y^2-cDe_z^2 \tag{26}$$

Let, first eliminate the cross terms aAe_xe_y, and then proceed with the recipe, the dynamics of systems are defined by:

$$\dot{V}(e)=-\left[\sqrt{aAe_x} \quad -\frac{\sqrt{aA}}{2}e_y\right]^2-\left[B \quad \frac{aA}{4}\right]e_y^2-cDe_z^2 \tag{27}$$

If the dynamic of Equation (27) is defined by:

$$A>0, B-\frac{aA}{4}>0, B=D>0. \tag{28}$$

It follows when $\dot{V}(e)$ is negative, then error states Equation (18) is asymptotically stable so $e_x(t)\to 0, e_y(t)\to 0, e_z(t)\to 0$. While the drive system and response system are synchronized. Take $A=1/4, B=D=1$, satisfied a condition of analysis system Equation (28) then we obtain the Lyapunov function are defined by:

$$V(e)=\frac{1}{8}e_x^2+\frac{1}{2}e_y^2+\frac{1}{2}e_z^2 \tag{29}$$

Above the system $\dot{V}(e)$ is negative so $e_x\to 0, e_y\to 0, e_z\to 0$. Synchronization errors realize, it can be achieved synchronization system as in [1]-[31].

As an example shows that, consider the observer based adaptive backstepping structure is a hierarchical nonlinear control framework that combines three key components. Consider the uncertain nonlinear system is given by:

$$\dot{x}=f(x,\theta)+Bu(t) \tag{30}$$

where:

$x\in\mathbb{R}^n$: State vector

θ: Unknown parameters

$u(t)$: Control input

$f(\cdot)$: Nonlinear dynamics

The slave system is defined by:

$$\dot{y}=f(y,\theta)+Bu(t)+d(t) \tag{31}$$

Define synchronization error system, is defined as:

$$e(t) = y(t) - x(t) \tag{32}$$

Observer design (state estimation), since not all states are not measurable, construct an observer is defined as:

$$\dot{\hat{x}} = Lf\left(\hat{x}, \hat{\theta}\right) + Bu(t) + L(x - \hat{x}) \tag{33}$$

where:

L : is an observer gain matrix, ensures that $\hat{x} \to x$. Definition estimation error is given by:

$$\tilde{x} = x - \hat{x} \tag{34}$$

Adaptive Law (Parameter estimation) to handle unknown parameters is defined by:

$$\dot{\hat{\theta}} = \Gamma \Phi(x) e^{\mathrm{T}} \tag{35}$$

where:

$x \in \mathbb{R}^n$: State vector

$\Gamma > 0$: Adaptation gain

$\Phi(x)$: regression vector or RBFNN radial basis function neuron network

e^{T} : Synchronization error

The slave system is defined by: Backstepping control design is defined as:

Step 1: Virtual control, define first error variable system $z_1 = e^{\mathrm{T}}$. Choose virtual stabilizing function is defined by:

$$\alpha_1 = -k_1 z_1$$

Step 2: Error transformation $z_2 = \dot{z}_1 - \alpha_1$

Step 3: Actual control law, final control input is given by:

$$u(t) = \alpha\left(x, \hat{x}, \hat{\theta}\right) - k_2 z_2 \tag{36}$$

where:

- Nonlinear compensation uses estimated states
- Adaptive parameter cancels uncertainties
- Finite-time convergence claim

2.4. Finite-Time Synchronization

Consider the master-slave Lorenz system with an observer based adaptive controller. Let the synchronization error is defined as: $e(t) = y(t) - x(t)$

Assume

- System nonlinearities are smooth and bounded.
- RBFNN approximation error is bounded.
- Adaptive laws are properly designed, then under the proposed control law, the synchronization error $e(t)$ converges to zero in finite time, *i.e.*, there exists a settling time $T(x_o)$ such that: $e(t) = 0, \forall t \geq T(x_o)$.

We choose a Lyapunov Function is defined by:

Define:

$$V=\frac{1}{2}e^{\mathrm{T}}e+\frac{1}{2}\tilde{W}^{\mathrm{T}}\Gamma^{-1}\tilde{W} \tag{37}$$

Key finite-time condition shows that the closed-loop system satisfies:

$$\dot{V}(t)\le -\mathrm{C}V^{\alpha}(t) \tag{38}$$

with $0<\alpha<1$.

From the above inequality, integrating both sides give.

$$T(x_o)\le\frac{V(0)^{1-\alpha}}{C(1-\alpha)} \tag{39}$$

Convergence occurs in finite-time. Settling time depends on initial condition $V(0)$.

2.5. Observer Design System (Unmeasured States System)

This section bridges the theoretical development of the observer based on backstepping controller with its numerical validation on the Lorenz system. Assume only x_1 is measured. construct a nonlinear observer is defined by: Master (Drive) Lorenz System. Chaotic systems have attracted significant attention in nonlinear science and control engineering due to their complex dynamical behaviors, high sensitivity to initial conditions, and broad engineering applications. Since the pioneering work of Lorenz in 1963, chaotic models have been extensively investigated in fields such as secure communications, robotics, aerospace engineering, neural networks, biological systems, and intelligent control. Among these models, the Lorenz system remains one of the most representative chaotic systems because of its rich nonlinear dynamics and butterfly-shaped attractor. The classical Lorenz chaotic system is defined by:

$$\begin{cases}\dot{x}_1=\sigma(x_2-x_1)\\ \dot{x}_2=x_1(\rho-x_3)-x_2\\ \dot{x}_3=x_1\cdot x_2-\beta\cdot x_3\end{cases} \tag{40}$$

where $\sigma=10$, $\rho=28$, $\beta=\frac{8}{3}$ are chaotic parameters unknown constant values.

Let $x=\begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}^{\mathrm{T}}$ are slave states vector.

The controlled slave is defined by:

$$\begin{cases}\dot{y}_1=\sigma(y_2-y_1)+u_1(t)\\ \dot{y}_2=y_1(\rho-y_3)-y_2+u_2(t)\\ \dot{y}_3=y_1y_2-\beta y_3+u_3(t)\end{cases} \tag{41}$$

Goal:

We define that $\lim\limits_{t\to T_f}\left(y_i(t)-x_i(t)\right)=0$. The error dynamical system (42) can be obtained directly by subtracting Equation (41) from Equation (40). Observer the error dynamical system (42). Some global asymptotic synchronization conditions are obtained. Define synchronization errors is defined by:

$$e_i = y_i - \hat{x}_i$$

Then

$$\begin{cases} \dot{e}_1 = \sigma\left(e_2 - e_1\right) + u_1\left(t\right) \\ \dot{e}_2 = e_1\left(\rho - y_3\right) - y_1 e_3 - e_2 + u_2\left(t\right) \\ \dot{e}_3 = y_1 e_2 + x_2 e_1 - \beta e_3 + u_3\left(t\right) \end{cases} \tag{42}$$

An Observer Design System (Unmeasured States System) is defined by:

$$\begin{cases} \dot{\hat{x}}_1 = \sigma\left(\hat{x}_2 - \hat{x}_1\right) + L_1\left(x_1 - \hat{x}_1\right) \\ \dot{\hat{x}}_2 = \hat{x}_1\left(\rho - \hat{x}_3\right) - \hat{x}_2 + L_2\left(x_1 - \hat{x}_1\right) \\ \dot{\hat{x}}_3 = \hat{x}_1 \cdot \hat{x}_2 - \beta \cdot \hat{x}_3 + L_3\left(x_1 - \hat{x}_1\right) \end{cases} \tag{43}$$

Observer error converges exponentially for suitable L_1 L_2, L_3. based on backstepping controller design. Where: $L_1 = 15$; $L_2 = 20$, $L_3 = 25$. Finite-time Backstepping Controller. Define finite-time sliding variables by:

$$s_1 = e_1$$

$$s_2 = e_2 + c_1\left|e_1\right|^{\alpha}\operatorname{sign}\left(e_1\right)$$

$$s_3 = e_3 + c_2\left|s_2\right|^{\alpha}\operatorname{sign}\left(s_2\right) \tag{44}$$

with $0 < \alpha < 1$.

Control Law is defined by:

$$u_1\left(t\right) = -\sigma\left(e_2 - e_1\right) - k_1\left|s_1\right|^{\alpha}\operatorname{sign}\left(s_1\right)$$

$$u_2\left(t\right) = -e_1\left(\rho - y_3\right) + y_1 e_3 + e_2 - k_2\left|s_2\right|^{\alpha}\operatorname{sign}\left(s_2\right)$$

$$u_3\left(t\right) = -y_1 e_2 - x_2 e_1 + \beta e_3 - k_3\left|s_3\right|^{\alpha}\operatorname{sign}\left(s_3\right) \tag{45}$$

We define a finite-time stability Proof Lyapunov function as:

We choose Lyapunov function, is defined as:

$$V\left(t\right) = \frac{1}{2}\left(S_1^2 + S_2^2 + S_3^2\right) \tag{46}$$

Then

$$\dot{V}\left(t\right) \le -kV^{\frac{1+\alpha}{2}} \tag{47}$$

Which implies finite-time convergence:

$$T \le \frac{2V\left(0\right)^{\frac{1-\alpha}{2}}}{k\left(1-\alpha\right)} \tag{48}$$

Thus, the observer is globally asymptotically stable.

3. Problem Formulation of Design

This section presents the mathematical formulation of the chaotic Lorenz synchronization framework, including the master system, controlled slave system, parametric uncertainties, and external disturbances. To ensure consistency with the claim online parameter identification, the adaptive laws for estimating the unknown Lorenz parameters are explicitly defined by:

3.1. Master System (Drive System)

The master (reference) Lorenz system is defined by:

$$\begin{cases} \dot{x}_1 = \sigma(x_2 - x_1) \\ \dot{x}_2 = x_1(\rho - x_3) - x_2 \\ \dot{x}_3 = x_1 \cdot x_2 - \beta \cdot x_3 \end{cases} \tag{49}$$

where:

$x = [x_1 \quad x_2 \quad x_3]^{\mathrm{T}} \in \mathbb{R}^3$ are master states vector.

σ, ρ, β are unknown constant parameters.

3.2. Slave System (Response System with Control and Disturbance)

This section presents the mathematical formulation of the chaotic Lorenz synchronization framework, including, the controlled slave system, parametric uncertainties, and external disturbances. A common challenge in nonlinear control design is the presence of system uncertainties and unmeasured states then controlled slave system is defined by:

$$\begin{cases} \dot{y}_1 = \sigma(y_2 - y_1) + u_1(t) + d_1(t) \\ \dot{y}_2 = y_1(\rho - y_3) - y_2 + u_2(t) + d_2(t) \\ \dot{y}_3 = y_1 y_2 - \beta y_3 + u_3(t) + d_3(t) \end{cases} \tag{50}$$

where:

$y = [y_1 \quad y_2 \quad y_3]^{\mathrm{T}} \in \mathbb{R}^3$ are slave state vector.

$u(t) = [u_1(t) \quad u_2(t) \quad u_3(t)]^{\mathrm{T}} \in \mathbb{R}^3$ are control input.

$d(t) = [d_1(t) \quad d_2(t) \quad d_3(t)]^{\mathrm{T}} \in \mathbb{R}^3$ are external bounded disturbance.

Assumption 3.1

$$d(t) \le d_{\max}$$

3.3. Measured Output

Only partial state measurement is assumed by:

$$y_m = Cy \tag{51}$$

where typically:

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Only y_1, y_2 are measurable, y_3 must be reconstructed by observer.

3.4. Parameter Uncertainty

The system parameters are unknown but constant is defined by:

$$\theta = \begin{bmatrix} \sigma \\ \rho \\ \beta \end{bmatrix} \tag{52}$$

With estimates:

$$\hat{\theta} = \begin{bmatrix} \hat{\sigma} \\ \hat{\rho} \\ \hat{\beta} \end{bmatrix} \tag{53}$$

And estimation error is given by:

$$\tilde{\theta} = \hat{\theta} - \theta \tag{54}$$

3.5. Regressor Formulation

The Lorenz dynamics are rewritten in linear in parameters form:

$$f(x,\theta) = Y(x)\theta \tag{55}$$

where the regressor matrix is defined by:

$$Y(x) = \begin{bmatrix} x_2 - x_1 & 0 & 0 \\ 0 & x_1 & 0 \\ 0 & 0 & -x_3 \end{bmatrix} \tag{56}$$

3.6. Adaptive Update Law

The parameter adaptation law is defined by:

$$\dot{\hat{\theta}} = -\Gamma Y^{\mathrm{T}}(x) e \tag{57}$$

where:

$\Gamma = \Gamma^{\mathrm{T}} > 0$ is an adaptation gain matrix.

$e = y - x$ is a synchronization error.

3.7. Component Wise Explicit Laws

To avoid ambiguity, each parameter update is written explicitly:

Master (unknown parameters), is defined by: $\dot{x} = f(x,\theta)$ (58)

Slave $\dot{y} = f\left(y,\hat{\theta}\right) + u(t)$ (59)

Observer is defined by: $\dot{\hat{x}} = f\left(\hat{x},\hat{\theta}\right) + L\left(x_1 - \hat{x}_1\right)$ (60)

Define error is defined by: $e = y - \hat{x}$ (61)

Fixed time backstepping structure, is defined by:

$$\begin{aligned} z_1 &= e_1 \\ z_2 &= e_2 + c_1 |e_1|^{\alpha} \operatorname{sign}(e_1) \\ z_3 &= e_3 + c_2 |z_2|^{\alpha} \operatorname{sign}(z_2) \end{aligned} \tag{62}$$

Fixed time control law. To guarantee fixed time, we combine two powers are defined by:

$$u_i(t) = -k_{i1} |z_i|^{\alpha} \operatorname{sign}(z_i) - k_{i2} |z_i|^{\beta} \operatorname{sign}(z_i)$$

where $0 < \alpha < 1, \beta > 1$.

$$\dot{\hat{\theta}} = -\Gamma Y^{\mathrm{T}} xe \quad (63)$$

Sigma update is defined by:

$$\dot{\hat{\sigma}} = -\gamma_1 \left(\hat{x}_2 - \hat{x}_1 \right) e_1 \quad (64)$$

Rho update is defined by:

$$\dot{\hat{\rho}} = -\gamma_2 \hat{x}_1 e_2 \quad (65)$$

Beta update is defined by:

$$\dot{\hat{\beta}} = -\gamma_3 \hat{x}_3 e_3 \quad (66)$$

3.8. Lyapunov Analysis Conditions

We choose Lyapunov function is defined by:

$$V = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2 \right) + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta} \quad (67)$$

Derivation of Lyapunov function after substitution is defined by:

$$\dot{V} \le -aV^{\frac{1+\alpha}{2}} - bV^{\frac{1+\beta}{2}} \quad (68)$$

Fixed-time convergence theorem is defined by:

$$T \le \frac{1}{a(1-\alpha)} + \frac{1}{b(\beta - 1)} \quad (69)$$

Independent of initial conditions, stronger than finite-time.

3.9. Lyapunov Consistency Conditions

With the proposed of method Lyapunov function is given by:

$$V = \frac{1}{2} e^{\mathrm{T}} e + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta} \quad (70)$$

The adaptive law ensures cancellation of parameter dependent terms as:

$$\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\hat{\theta}} + e^{\mathrm{T}} Y(x)\tilde{\theta} = 0 \quad (71)$$

So parameter terms vanish in $\dot{V}$. Disturbance bounding is defined by: Using Cauchy-Schwarz is defined as:

$$e^{\mathrm{T}} d(t) \le \|e\| \|d(t)\| \le d_{\max} \|e\|$$

Main inequality construction after substitution and cancellations are defined by:

$$\dot{V} \le -k_1 \sum |e_i|^{1+\alpha} - k_2 \sum |e_i|^{1+\beta} + d_{\max} \|e\| \quad (72)$$

Under the above adaptation law is defined as: $\dot{\hat{\sigma}}$, $\dot{\hat{\rho}}$, $\dot{\hat{\beta}}$.

The parameter estimates, $\tilde{\sigma}$, $\tilde{\rho}$, $\tilde{\beta}$ are guaranteed to remain bounded, and convergence is achieved under persistent excitation conditions. The adaptive laws ensure boundedness of the parameter estimates and cancellation of parameter-dependent terms in the Lyapunov derivative.

However, exact parameter convergence is guaranteed only under the persistence of excitation condition, which is not required for stability of the closed-loop system. Parameter estimates are bounded.

Estimation errors satisfy as:

$$\tilde{\theta} \in L_\infty \tag{73}$$

Convergence is achieved in the sense of Lyapunov stability but the parameter estimates are guaranteed to remain bounded and convergence is achieved.

3.10. Control Objective

Design a control law system $u(t)$ and an observer such that synchronization objective is defined by:

$$\lim_{t \to T} \|y(t) - x(t)\| = 0 \tag{74}$$

Finite-time or fixed-time convergence is given by:

$$T < \infty \tag{75}$$

State estimation is defined by:

$$\hat{y}(t) \to y(t) \tag{76}$$

Parameter estimation boundedness is defined by:

$$\hat{\theta}(t) \to \theta \tag{77}$$

3.11. Synchronization Error System

Synchronization error system is defined by:

$$e = y - x \tag{78}$$

Then

$$\dot{e} = f(y,\theta) - f(x,\theta) + u(t) + d(t) \tag{79}$$

This is the core system used for controller design.

4. Finite-Time Synchronization for the Observer Adaptive Backstepping Controller Design

This section bridges the theoretical development of the observer based on adaptive backstepping controller with its numerical validation on the Lorenz system. The parameter estimates are guaranteed to remain bounded, and convergence is achieved under persistent excitation conditions. We start strictly form the measured slave system with uncertainties and disturbance are defined by:

$$\dot{y} = f(y,\theta) + u(t) + d(t) \tag{80}$$

with only partial measurements is defined by:

$$y_m = Cy \tag{81}$$

4.1. Structural Decomposition

We rewrite the nonlinear dynamics are defined by:

$$\dot{y} = Ay + \Phi(y)\theta + u(t) + d(t) \tag{82}$$

where:

Ay : linear part of Lorenz structure

$\Phi(y)\theta$: parameterized nonlinearities

$d(t)$: bounded disturbance

4.2. Nonlinear Observer Structure

We design a Luenberger type nonlinear adaptive observer is defined by:

$$\dot{\hat{y}} = A\hat{y} + \Phi(\hat{y})\hat{\theta} + u(t) + L(y_m - \hat{y}_m) \tag{83}$$

where:

$\hat{y}$: estimated state

$\hat{\theta}$: estimated parameters

L : observer gain matrix

$\hat{y}_m = C\hat{y}$: estimated measured output

4.3. Observer Error Dynamics

We define estimation error is given by:

$$\tilde{y} = y - \hat{y} \tag{84}$$

Then

$$\dot{\tilde{y}} = (A - LC)\tilde{y} + \Delta_\Phi + d(t) \tag{85}$$

where:

$$\Delta_\Phi = \Phi(y)\theta - \Phi(\hat{y})\hat{\theta}$$

4.4. Main Design Conditions

We define L such that:

$$A - LC \quad \text{is Hurwitz} \tag{86}$$

Ensures exponential decay of linear part.

4.5. Adaptive Parameter Observer Law

We define some unknown parameters that are defined by:

$$\dot{\hat{\theta}} = -\Gamma\Phi^{\mathrm{T}}\hat{y}C^{\mathrm{T}}\tilde{y} \tag{87}$$

where:

$$\dot{\hat{\theta}} = -\Gamma\Phi^{\mathrm{T}}(\hat{y})C^{\mathrm{T}}(y_m - \hat{y}_m) \tag{88}$$

4.6. Lyapunov Based on Observer Stability

We choose a Lyapunov function defined by:

$$V = \tilde{y}^{\mathrm{T}}P\tilde{y} + \tilde{\theta}\Gamma^{-1}\tilde{\theta} \tag{89}$$

Time is defined by:

$$\dot{V} = -\tilde{y}^{\mathrm{T}}\left(A - LC\right)^{\mathrm{T}} P + P\left(A - LC\right)\tilde{y} + \Phi\left(y\right)\theta - \Phi\left(\hat{y}\right)\hat{\theta} + d\left(t\right) \tag{90}$$

Main results:

If

- $\left(A - LC\right)$ is Hurwitz
- Adaptive law cancels parameter coupling then:

$$\dot{V}\left(t\right) \le -\lambda_{\min}\left(Q\right)\left\|\tilde{y}\right\|^2 \tag{91}$$

4.7. Final Observer Property

State convergence is defined by:

$$\tilde{y} \to 0 \tag{92}$$

Parameter boundedness is defined by:

$$\hat{\theta} \in L_{\infty} \tag{93}$$

$$\text{Robustness} \quad d\left(t\right) \quad \text{bounded} \quad \Rightarrow \tilde{y} \quad \text{bounded} \tag{94}$$

The nonlinear observer guarantees that all unmeasured states are accurately reconstructed, estimation errors converge to zero, the observer remains stable under nonlinear coupling, and the estimated states can be safely used in the adaptive finite time backstepping controller are defined by:

$$\begin{gathered} z_1 = e_1 \\ z_2 = e_2 + c_1\left|e_1\right|^{\alpha} \operatorname{sign}\left(e_1\right) \\ z_3 = e_3 + c_2\left|z_2\right|^{\alpha} \operatorname{sign}\left(z_2\right) \end{gathered} \tag{95}$$

where $0 < \alpha < 1$.

Adaptive finite-time control law is defined by:

$$\begin{gathered} u_1\left(t\right) = -\sigma\left(e_2 - e_1\right) - k_1\left|z_1\right|^{\alpha} \operatorname{sign}\left(z_1\right) \\ u_2\left(t\right) = -e_1\left(\rho - y_3\right) + y_1 e_3 + e_2 - k_2\left|z_2\right|^{\alpha} \operatorname{sign}\left(z_2\right) \\ u_3\left(t\right) = -y_1 e_2 - \hat{x}_2 e_1 + \beta e_3 - k_3\left|z_3\right|^{\alpha} \operatorname{sign}\left(z_3\right) \end{gathered} \tag{96}$$

Adaptive Law

Define parameters errors systems that are defined by:

$$\begin{gathered} \tilde{\sigma} = \hat{\sigma} - \sigma, \\ \tilde{\rho} = \hat{\rho} - \rho, \\ \tilde{\beta} = \hat{\beta} - \beta, \end{gathered} \tag{97}$$

Adaptive update laws are given by:

$$\dot{\hat{\sigma}} = -\gamma_1 z_1\left(\hat{x}_2 - \hat{x}_1\right) \tag{98}$$

$$\dot{\hat{\rho}} = -\gamma_2 z_2 \hat{x}_1 \tag{99}$$

$$\dot{\hat{\beta}} = -\gamma_3 z_3 \hat{x}_3 \tag{100}$$

Cancels parameter uncertainty terms guarantees bounded estimation compatible with finite-time convergence.

4.8. Lyapunov Stability

Define Lyapunov functions, is defined by:

$$V = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2\right) + \frac{1}{2\gamma_1}\tilde{\sigma}^2 + \frac{1}{2\gamma_2}\tilde{\rho}^2 + \frac{1}{2\gamma_3}\tilde{\beta}^2$$

Derivative of Lyapunov function, is given by:

$$\dot{V}(t) = -\left(z_1\dot{z}_1 + z_2\dot{z}_2 + z_3\dot{z}_3\right) + \frac{1}{\gamma_1}\tilde{\sigma}\left(-\dot{\hat{\sigma}}\right) + \frac{1}{\gamma_2}\tilde{\rho}\left(-\dot{\hat{\rho}}\right) + \frac{1}{\gamma_3}\tilde{\beta}\left(-\dot{\hat{\beta}}\right)$$

Substituting the control law yields, we get:

$$\dot{V}(t) \le -CV^{\frac{1+\alpha}{2}}$$

where $0 < \alpha < 1$.

This guarantees

$$T \le \frac{V(0)^{\frac{1-\alpha}{2}}}{C(1-\alpha)}$$

This makes the controller observer based on realistic system with finite-time backstepping controller.

Finite-time synchronization remains bounded parameter estimates.

The above error dynamical that are system (74, 78) remains asymptotically stable at the origin when $t \to 0$. Also, the all's unknown parameters σ, ρ, β are success fully estimated by $\hat{\sigma}, \hat{\rho}, \hat{\beta}$. In general, the construction of observers for nonlinear systems depends on the solvability of a linear matrix inequality involving system matrices, and it is based on the system's nonlinearity. Therefore, the type of nonlinearity presents in the system heavily affects the observer design process. There are significant developments in the literature for observer design for descriptor systems with various types of nonlinearity.

5. Finite-Time Synchronization for the Observer Based on Backstepping Controller Design

This section bridges the theoretical development of the observer based on backstepping controller design with its numerical validation on the Lorenz system. Assume only x_1 is measured. construct a nonlinear observer is defined by: The proposed observer based on backstepping controller guarantees finite-time synchronization of the Lorenz system under parameter uncertainties and partial state measurements. The stability of the closed loop system is rigorously established using Lyapunov theory.

Theorem 5.1

Under the proposed observer-based adaptive finite-time backstepping controller, the proposed of methods based on Lorenz system is: globally bounded, ultra – faster adaptive to uncertainties and disturbances, finite-time stable, and achieves exact synchronization.

Theorem 5.2

Under the proposed observer-based robust adaptive finite-time backstepping controller, the proposed of methods based on Lorenz system is globally bounded, robust to uncertainties and disturbances, stronger finite-time stable, and achieves exact synchronization.

Theorem 5.3

Under the proposed observer-based finite-time backstepping controller, the proposed methods based on Lorenz system are: globally bounded, faster and finite-time, asymptotically stable, and achieves exact synchronization.

Assumptions 5.1

The structure of the Lorenz system is known, including the nonlinear functional form of the dynamics. However, the system parameters are σ, ρ, β unknown constant values.

Assumptions 5.2

Only partial state measurements are available. Specifically, the output is given by $y_m = Cy$ where typically $C = [1 \quad 0 \quad 0; 0 \quad 1 \quad 0]$, meaning that only y_1 and y_2 are measurable, while y_3 is unmeasured and must be estimated via an observer.

Assumptions 5.3

The slave system is affected by external disturbances $d(t)$ which are assumed to be unknown but bounded $d(t) \le d_{\max}$.

Assumptions 5.4

The nonlinear vector field $f(x)$ of the Lorenz system is assumed to be locally Lipschitz continuous, ensuring existence and uniqueness of solutions.

Assumptions 5.5

The unknown parameters σ, ρ, β are assumed to belong to known compact set *i.e.*

$$\sigma \in \Omega_\sigma, \quad \rho \in \Omega_\rho, \quad \beta \in \Omega_\beta \quad \text{o}$$

Assumptions 5.6

All system states and estimation errors are assumed to be bounded at $t = 0$.

As an example shows that, in this section, a nonlinear state observer is constructed to estimate the unmeasured states of the Lorenz system. The observer is designed to ensure accurate reconstruction of system states and provide reliable signals for the adaptive finite-time backstepping controller.

The proposed model designs a simulate Chaos Synchronization of the Lorenz System using an observer based on finite-time backstepping controller. We rewrite the Equations (40), (41), (42), (43), (44) are given Master (Drive) Lorenz Chaotic system is defined by

$$\begin{cases} \dot{x}_1 = \sigma(x_2 - x_1) \\ \dot{x}_2 = x_1(\rho - x_3) - x_2 \\ \dot{x}_3 = x_1 \cdot x_2 - \beta \cdot x_3 \end{cases} \tag{101}$$

Parameters (chaotic regime) are defined by: $\sigma = 10$, $\rho = 28$, $\beta = \frac{8}{3}$.

Slave (response) system with control input

$$\begin{cases} \dot{y}_1 = \sigma\left(y_2 - y_1\right) + u_1\left(t\right) \\ \dot{y}_2 = y_1\left(\rho - y_3\right) - y_2 + u_2\left(t\right) \\ \dot{y}_3 = y_1 y_2 - \beta y_3 + u_3\left(t\right) \end{cases} \tag{102}$$

5.1. Synchronization Errors Dynamics

We define that $\lim_{t \to T_f}\left(y_i\left(t\right) - x_i\left(t\right)\right) = 0$. Observer design only x_1 is measured.

Assume only x_1 is available. Synchronization error dynamics is defined by:

$$e_i\left(t\right) = y_i\left(t\right) - \hat{x}_i\left(t\right) \tag{103}$$

This makes the controller observer based on realistic system with finite-time backstepping controller.

5.2. Finite-Time Controller Design

We design a finite-time synchronization for the Lorenz system using observer based on backstepping control.

Step 1: Virtual error is given:

$$z_1 = e_1$$

Step 2:

$$z_2 = e_2 + c_1\left|e_1\right|^{\alpha}\operatorname{sign}\left(e_1\right)$$

Step 3:

$$z_3 = e_3 + c_2\left|z_2\right|^{\alpha}\operatorname{sign}\left(z_2\right) \tag{104}$$

with $0 < \alpha < 1$.

5.3. Control and Law System

The proposed controller guarantees: finite-time synchronization, robustness against uncertainties, compensation of nonlinear Lorenz dynamics, observer parameter estimation, Lyapunov stability, fast convergence of tracking errors. Control law is defined as finite-time backstepping controller, we design a finite-time controller is defined by:

$$u_1\left(t\right) = -\sigma\left(e_2 - e_1\right) - k_1\left|z_1\right|^{\alpha}\operatorname{sign}\left(z_1\right)$$

$$u_2\left(t\right) = -e_1\left(\rho - y_3\right) + y_1 e_3 + e_2 - k_2\left|z_2\right|^{\alpha}\operatorname{sign}\left(z_2\right)$$

$$u_3\left(t\right) = -y_1 e_2 - \hat{x}_2 e_1 + \beta e_3 - k_3\left|z_3\right|^{\alpha}\operatorname{sign}\left(z_3\right) \tag{105}$$

5.4. Finite-Time Stability

We choose a Lyapunov function defined by:

$$V\left(t\right) = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2\right) \tag{106}$$

Derivative of Lyapunov function, is given by:

$$\dot{V}(t) = -\left(z_1\dot{z}_1 - z_2\dot{z}_2 - z_3\dot{z}_3\right)$$

Substituting the control law yields, we get:

$$\dot{V}(t) \le -\lambda V^{\frac{1+\alpha}{2}} \tag{107}$$

where $0 < \alpha < 1$.

Which implies Finite-time convergence and synchronization guaranteed chaos synchronization of the Lorenz system using observer based in finite-time backstepping controller. Which implies finite-time convergence, we rewrite this Equation (48), is defined by:

$$T \le \frac{2V(0)^{\frac{1-\alpha}{2}}}{\lambda(1-\alpha)} \tag{108}$$

Which proves a finite-time stability and observer. There, the error of synchronization is verified to be asymptotically stable at origin (0, 0). Finite-time is convergence of nearby trajectory. It's observed based on Chaos and Synchronization.

The nonlinear observer guarantees that: all unmeasured states are accurately reconstructed estimation errors converge to zero, the observer remains stable under nonlinear coupling and the estimated states can be safely used in the adaptive finite-time backstepping controller.

As an example we illustrate this structure, is exactly what is expected in nonlinear control and chaos synchronization We reconsider master and slave dynamics are defined as in [7]-[19] [20] [21] [23].

Remark

The finite-time enhancement provides: ultra-fast synchronization of Lorenz chaotic systems, Stronger, robustness to uncertainties as in [19], improved disturbance attenuation, finite settling-time guarantees, better performance for secure communication and intelligent nonlinear systems.

6. Expanded Stability Proof (Finite-Time Derivation) for the Observer Design

Step 1: Closed-Loop Error Dynamics

Define synchronization error is defined by:

$$e(t) = y(t) - x(t) \tag{109}$$

From the master and slave are defined by:

$$\dot{e}(t) = f(y,\theta) - f(x,\theta) + u(t) + d(t) \tag{110}$$

Step 2: Lyapunov Candidate is defined as:

$$V = \frac{1}{2}e^{\mathrm{T}}e + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta} \tag{111}$$

Step 3: Time Derivative is defined as:

$$\dot{V}(t) = e^{\mathrm{T}}\dot{e} + \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\tilde{\theta}} \tag{112}$$

Substituting dynamics are defined by:

$$\dot{V}(t)=e^{\mathrm{T}}\left(f(y)-f(x)\right)+e^{\mathrm{T}}u(t)+e^{\mathrm{T}}d(t)+\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\tilde{\theta}} \tag{113}$$

Step 4: Backstepping Control Substitution

Using the controller law system is defined by:

$$u(t)=-k_1\left|e_i\right|^{\alpha}\operatorname{sign}(e)-k_1\left|e_i\right|^{\beta}\operatorname{sign}(e)+\hat{f}(x)-\hat{f}(y) \tag{114}$$

Step 5: Errors

We obtain:

$$e^{\mathrm{T}}\left(f(y)-f(x)\right)+e^{\mathrm{T}}\left(\hat{f}(x)-\hat{f}(y)\right)=e^{\mathrm{T}}\hat{f} \tag{115}$$

where:

$\hat{f}$ is the approximation error.

The adaptive law ensures cancellation of parameter dependent terms:

$$\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\tilde{\theta}}+e^{\mathrm{T}}Y(x)\tilde{\theta}=0 \tag{116}$$

So parameter terms vanish in $\dot{V}$.

Step 6: Disturbance bounding is defined by: Using Cauchy-Schwarz is defined by:

$$e^{\mathrm{T}}d(t)\le\|e\|\|d(t)\|\le d_{\max}\|e\|$$

Step 7: Main inequality construction after substitution and cancellations are defined by:

$$\dot{V}\le-k_1\sum\left|e_i\right|^{1+\alpha}-k_2\sum\left|e_i\right|^{1+\beta}+d_{\max}e \tag{117}$$

Step 8: Norm-to-Lyapunov Transformation Using norm equivalence:

Using norm equivalence is defined as:

$$e^2=2V\Rightarrow e=\sqrt{2V} \tag{118}$$

and inequalities are defined by:

$$\sum\left|e_i\right|^{1+\alpha}\ge c_1V^{\frac{1+\alpha}{2}}$$

$$\sum\left|e_i\right|^{1+\beta}\ge c_2V^{\frac{1+\alpha}{2}}$$

Step 9: Final Differential Inequality is defined by:

$$\dot{V}(t)\le-aV^{\gamma}-bV^{\delta}+c\sqrt{V} \tag{119}$$

where:

$$\gamma=\frac{1+\alpha}{2}\in(0,1)$$

$$\delta=\frac{1+\beta}{2}>1$$

Step 10: Finite-Time Dominance Condition

For sufficiently large gains is defined by:

$$aV^{\gamma}+bV^{\delta}\ge c\sqrt{V}$$

so disturbance term is dominated, yielding:

$$\dot{V}(t) \le -\lambda_1 V^{\gamma} - \lambda_2 V^{\delta}$$

Step 11: Standard Finite-Time Reduction

For Lyapunov analysis near convergence, dominant term is defined by:

$$\dot{V}(t) = -\lambda V^{\gamma}, \quad 0 < \gamma < 1$$

$$\dot{V}(t) \le -\lambda_1 V^{\gamma} - \lambda_2 V^{\delta} \tag{120}$$

7. Finite-Time Settling Time

Integrating is defined by:

$$\frac{\mathrm{d}V}{V^{\gamma}} \le -\lambda \mathrm{d}t$$

Yields also and we get:

$$\int_{V(0)}^{0} V^{-\gamma} \mathrm{d}V = -\lambda \int_{0}^{T} V \mathrm{d}t$$

$$T \le \frac{V(0)^{1-\gamma}}{\lambda(1-\gamma)} \tag{121}$$

The proposed controller guarantees: finite-time synchronization, robustness against uncertainties, compensation of nonlinear Lorenz dynamics, adaptive parameter estimation Lyapunov stability, fast convergence of tracking errors.

8. Simulation and Analysis Stability

In this section, the chaotic behavior and synchronization performance of Lorenz system under the proposed of chaos and synchronization of Lorenz system using an observer backstepping finite-time control design, is evaluating by the proposed of method to observer with those analytics results in Sections 3, 4, 5 various system including the variables of synchronization errors of Lorenz system in neural network, phase portrait of master system and slave system, and parameters estimates. Initially, the master and slave states are widely different due to different initial conditions and chaotic nature of the system. Upon activating the proposed observer based on backstepping finite-time controller at $t = 5$ s, the slave system's states rapidly begin to track the master's states.

This improvement is considered a strong contribution in modern nonlinear control and is widely used in neural-network synchronization, and intelligent observer systems. To verify the effectiveness of the proposed observer based on adaptive finite-time backstepping controller.

Numerical simulations are conducted on the Lorenz chaotic system with unknown parameters. Only the first state of the master system is assumed to be measurable systems, stronger robustness to uncertainties, improved disturbance attenuation, finite settling-time guarantees, better performance for secure communication and intelligent nonlinear systems.

Figure 7 illustrates the synchronization errors. It can be observed that all errors converge to zero within approximately 3 s, which confirms the finite-time synchronization property of the proposed observer-based on backstepping controller.

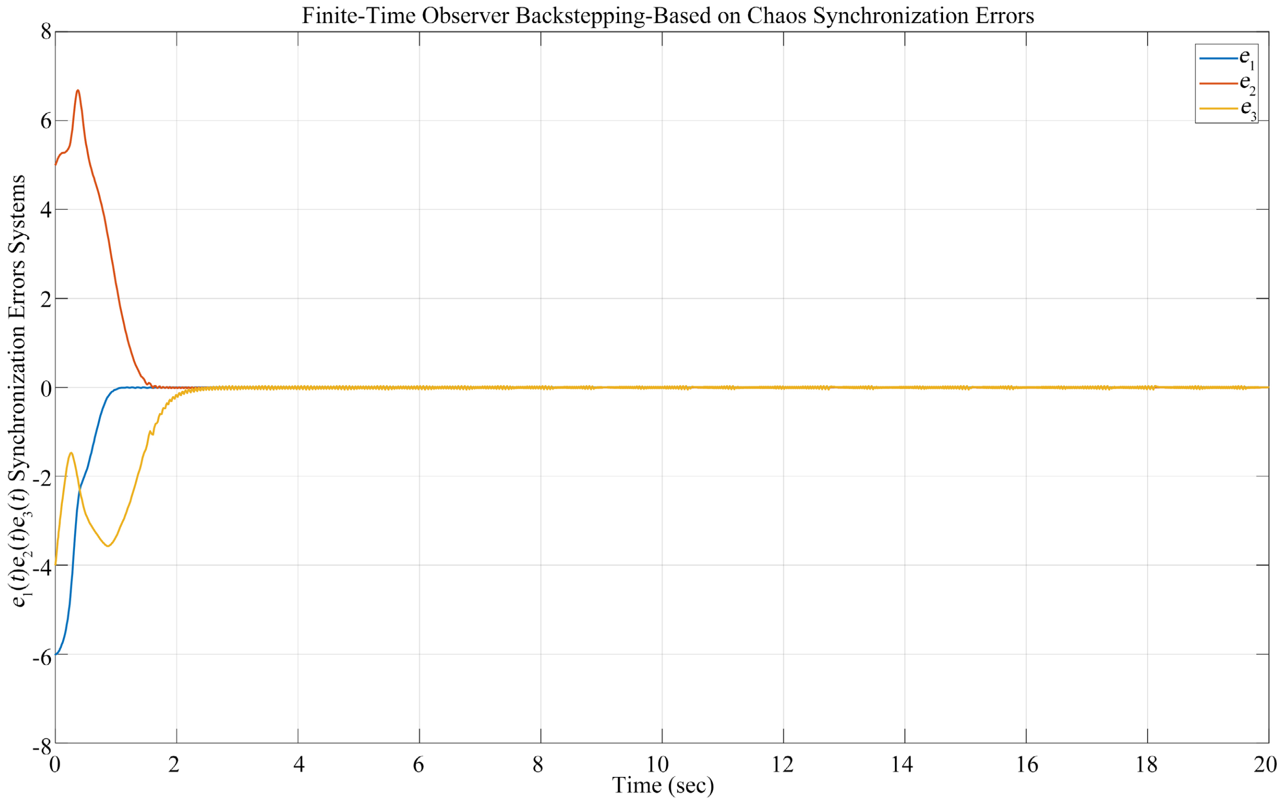


Figure 7. $e(t)$ errors are synchronized system based on observer system analysis and stability.

Figure 8 illustrates the synchronization errors. It can be observed that all errors converge to zero within approximately 5 s, which confirms the finite-time synchronization property of the proposed observer-based adaptive backstepping controller.

Figure 7 illustrates the synchronization errors, this analysis results in observability in the presence of perturbation; is exactly what is expected in nonlinear control and Chaos Synchronization, where it is observed that all errors converge to zero with approximately 2 seconds, confirming finite-time observer backstepping control on chaos synchronization error system.

A comparative study was conducted against a conventional adaptive backstepping controller. As shown in Figure 8, the proposed finite time controller achieves perfect synchronization in a significantly shorter compared to the asymptotic convergence of the conventional controller.

This demonstrates the key advantage of the proposed method under the adaptive law which is superior convergence speed and guaranteed performance within a predefined time. A rigorous analysis has shown that by introducing an observer finite-time backstepping control design techniques. Using adaptive backstepping

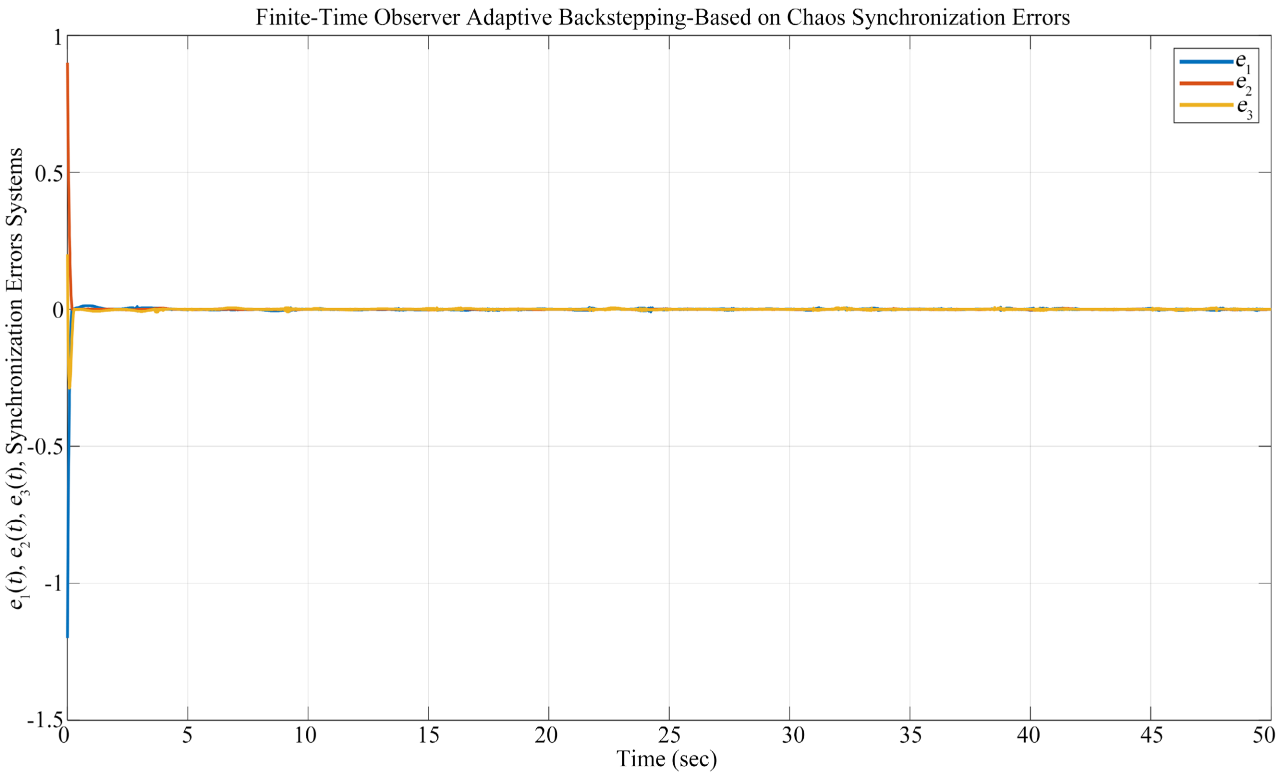


Figure 8. $e(t)$ errors are synchronized system based on adaptive observer system analysis and stability.

control law, the control of the parameters estimates of chaotic system can be achieved. A numerical simulation shows the effectiveness and feasibility of the proposed adaptive controller based on backstepping control design.

Figure 9 shows the adaptive backstepping, which remain bounded and converge close to their true values despite fast initial synchronization errors system and estimation of perturbation is defined. As shown in Figure 8, these results demonstrate that the proposed method achieves fast synchronization, adaptive state estimation and effective online parameter identification.

A comparative study was conducted against a conventional adaptive backstepping controller. As shown in Figure 9, the proposed finite time controller achieves perfect synchronization in a significantly shorter compared to the asymptotic convergence of the conventional controller.

Figure 8 illustrates the synchronization errors, this analysis results stability, is exactly what is expected in nonlinear control and behavior of chaos synchronization on Lorenz chaotic system, where it is observed that all errors converge to zero within approximately 5 seconds, confirming on adaptive observer backstepping finite-time control designs, on chaos synchronization.

The finite-time enhancement provides: Ultra-fast synchronization of Lorenz chaotic systems, stronger robustness to uncertainties, improved disturbance attenuation, finite settling-time guarantees, better performance for secure communication and intelligent nonlinear systems with radial basis function neural network as shown in Figure 8.

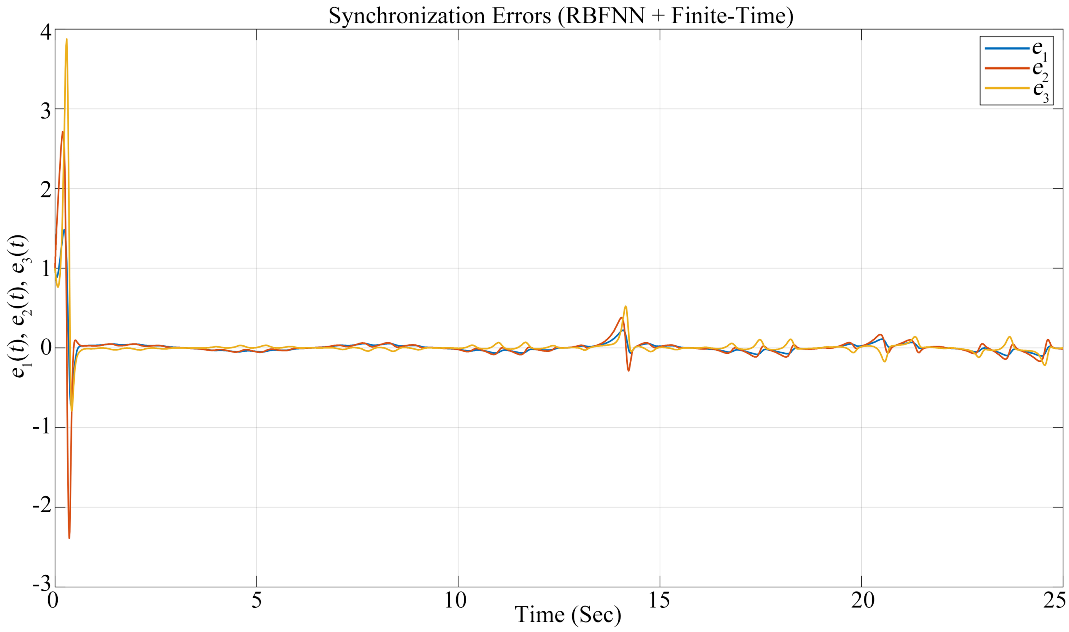


Figure 9. $e(t)$ errors are synchronized system based on robust adaptive observer system analysis and parameter estimation of perturbation is defined.

These results demonstrate that the proposed method achieves stronger synchronization, robust state estimation and effective online parameter identification. In this study, we address quasi-synchronization in nonlinear coupled neural networks characterized by multi-weight connections, structural heterogeneity, parameter uncertainties, and mixed time delays as shown in **Figure 9**.

This demonstrates the key advantage of the proposed method under the robust adaptive control law which is superior convergence speed and guaranteed performance within a predefined time. The proposed observer-based adaptive fixed-time backstepping controller guarantees synchronization within a predefined time, independent of initial conditions, while maintaining robustness against external disturbances and parameter uncertainties. A rigorous analysis has shown that by introducing an observer finite-time backstepping control design techniques.

Using robust adaptive backstepping control law, the control of the parameters estimates of chaotic system can be achieved. A numerical simulation shows the effectiveness and feasibility of the proposed robust controller based on backstepping control design finite-time.

Figure 9 shows the robust adaptive backstepping, which remain bounded and converge close to their true values despite stronger initial synchronization errors system and estimation of perturbation is defined. As shown in **Figure 9**, these results demonstrate that the proposed method achieves fast synchronization, robust state estimation and effective online parameter identification. Despite the complex, chaotic behavior of the master, the slave converges and perfectly synchronizes after a very short transient period. Finite-time controller error converges to zero at sometime $T_{\max} = 25$ s.

The finite-time enhancement provides: Ultra-fast synchronization of Lorenz chaotic systems, stronger robustness to uncertainties, improved disturbance attenuation, finite settling-time guarantees, better performance for secure communication and intelligent nonlinear systems with radial basis function neural network as shown in **Figure 9**.

These results demonstrate that the proposed method achieves stronger synchronization, robust state estimation and effective online parameter identification.

In this study, we address quasi-synchronization in nonlinear coupled neural networks characterized by multi-weight connections, structural heterogeneity, parameter uncertainties, and mixed time delays (**Figure 10**).

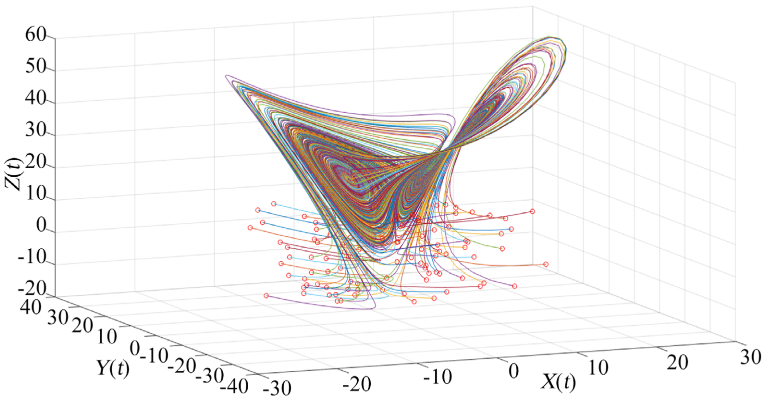


Figure 10. $X(t)$, $Y(t)$, $Z(t)$ variables driving error are not synchronized system with neural network system.

For the parameters $\sigma = 10$, $\rho = 28$, and $\beta = 8 = 3$, all trajectories collapse to an attractor. These trajectories, generated from a diverse set of initial data, are used to train a neural network to learn the nonlinear mapping from x_k to x_{k+1}.

We introduce the network architecture used to train NN on the trajectory data system is defined by:

Over 1000 epochs of training, accuracies on the order of 10^{-5} are produced. The NN is also cross-validated in the process.

Comparison of the time evolution of the Lorenz system (solid line) with the NN prediction (dotted line) for two randomly chosen initial conditions (red dots) are defined by:

Therefore, the type of nonlinearity presents in the system heavily affects the observer design process based on neural network, we found a strong correlation in our experiment. The results were analyzed as shown in **Figure 11**.

There are significant developments in the literature for observer design for descriptor systems with various types of nonlinearity. A performance shows that a neural network is being developed in this research. A new strategy is being developed, the results were analyzed, as shown in **Figure 12**.

The experiment shows a strong correlation in The NN prediction stays close to the dynamical trajectory of the Lorenz model as shown in **Figure 13**.

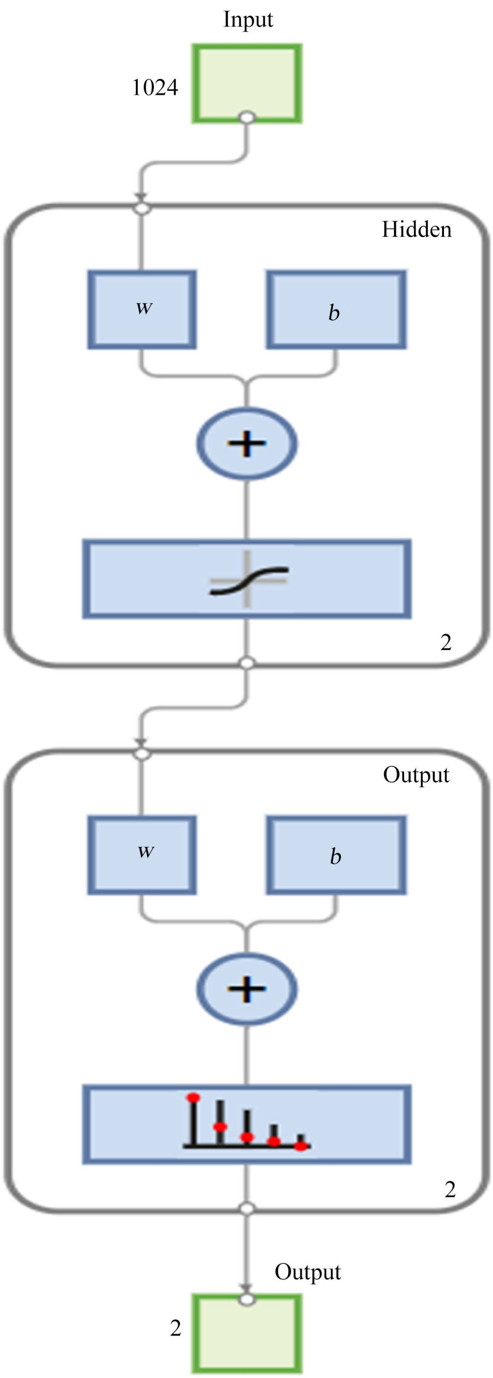


Figure 11. Network architecture used to train NN on the trajectory data system.

The Maximum Lyapunov Exponent (MLE) is a key concept in the study of dynamical systems, particularly in understanding chaotic behavior. It quantifies the rate of divergence or convergence of nearby trajectories and is defined as the largest Lyapunov exponent. A positive MLE indicates that nearby trajectories diverge exponentially, which is a hallmark of chaotic behavior. The MLE is crucial in determining the stability of a system and can be used to predict the behavior of dynamical systems under various conditions as shown in **Figure 14**.

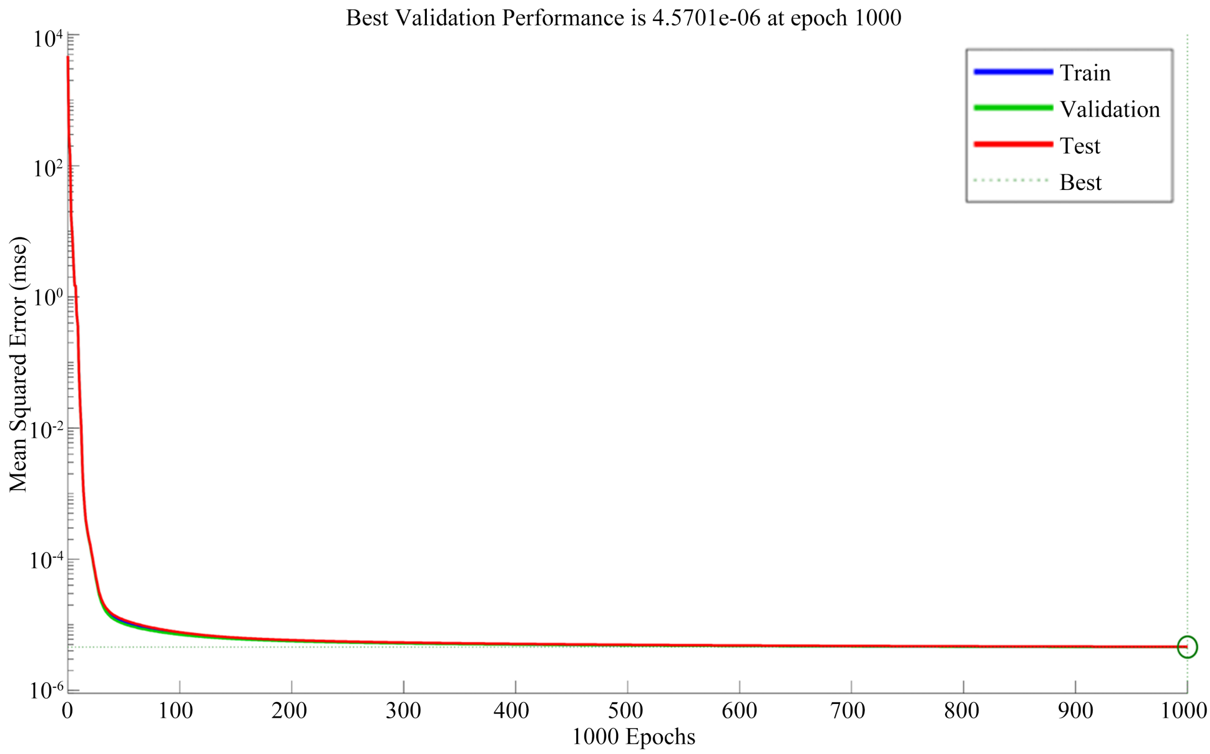


Figure 12. Performance summary of the NN optimization algorithm.

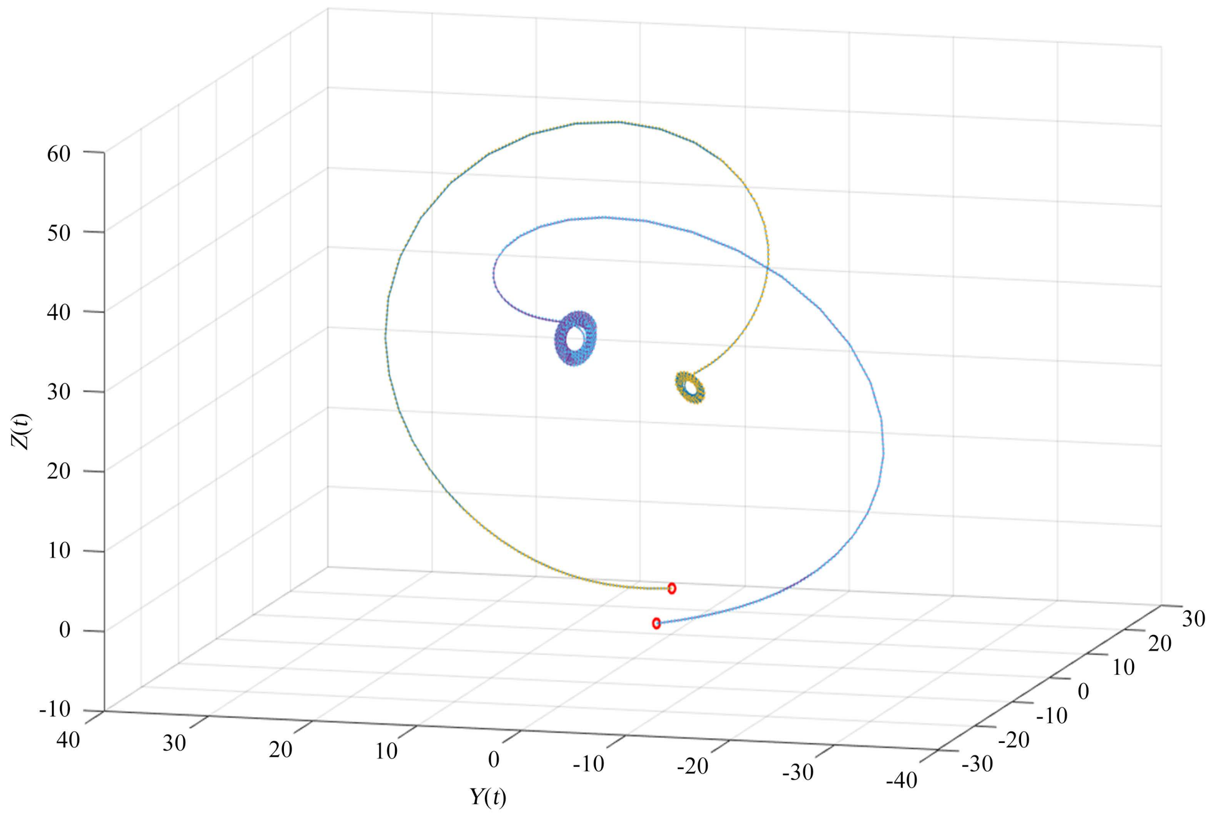


Figure 13. Comparison of the time evolution of the Lorenz system (solid line) with the NN prediction (dotted line) for two randomly chosen initial conditions (red dots). The NN prediction stays close to the dynamical trajectory of the Lorenz model.

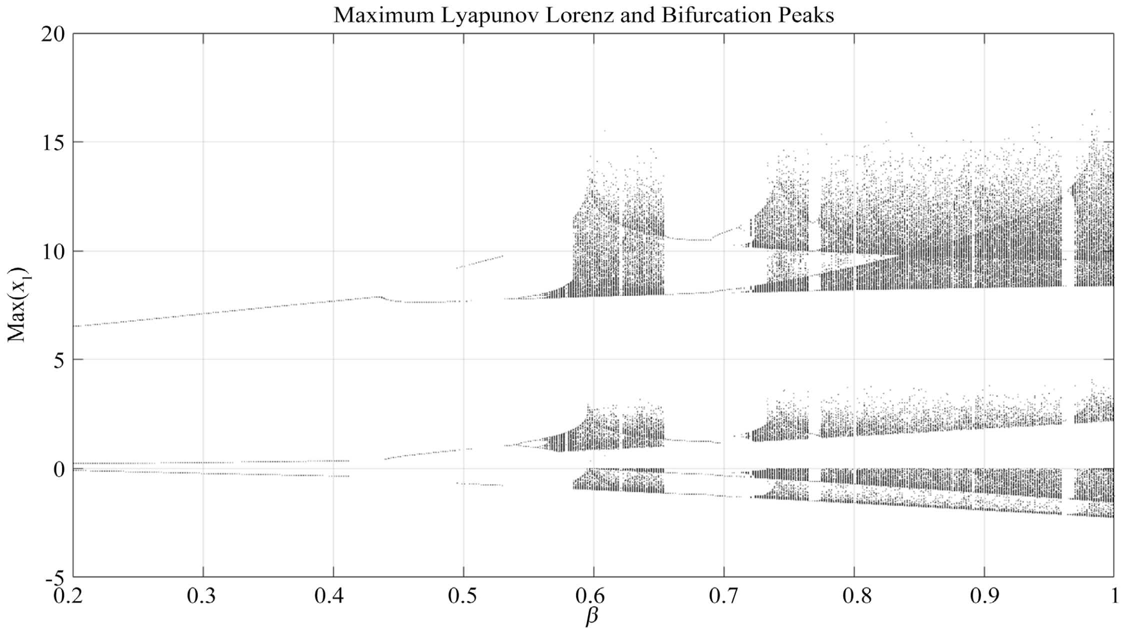


Figure 14. x maximum Lyapunov Lorenz and Bifurcation.

Bifurcations are significant transitions in dynamical systems, and the MLE plays a vital role in understanding these transitions. For instance, in the context of pitchfork bifurcations and Hopf bifurcations, the MLE can indicate the stability of the system and the nature of the bifurcation. Stochastic perturbations can also influence the bifurcation behavior of dynamical systems, and the MLE is used to assess the stochastic stability of such systems. In summary, the MLE and bifurcations are interconnected in the study of dynamical systems. The MLE helps in assessing the stability and chaotic nature of systems, while bifurcations describe the transitions that occur within these systems. Understanding these concepts is essential for predicting and analyzing the behavior of complex dynamical systems.

The driving principle of a synchronized responsive to the third-order chaotic system, for example are reconstructed by $x^{(1)}$ variables, $y^{(1)}$ variables, and $z^{(1)}$ variables as the driving variable three synchronized manner, their system is observed as shown in **Figure 9**. To satisfy the demands of different applications, many chaos synchronization, chaotic secure communications and state estimation for uncertain system schemes have been proposed as shown in **Figure 15**.

An observer chaotic sequence is the time-series produced by an observer designed to estimate or reconstruct a behavior of chaotic system's dynamics. It is observer's version of chaotic signal used for synchronization, estimation, or decoding. An example is the concerned attention to study, the emergence of collective synchronized dynamics in complex networks from the point of point.

It can be observed that system converges to the true parameters value ($\alpha > 0$), demonstrating the learning capability of the controller. This observed settling time aligns with the theoretical upper bound $T_{\max} = 50$ s, calculated from our Lyapuov analysis. The control effort is large initially to capture the diverging slave system and force it onto the master's trajectory.

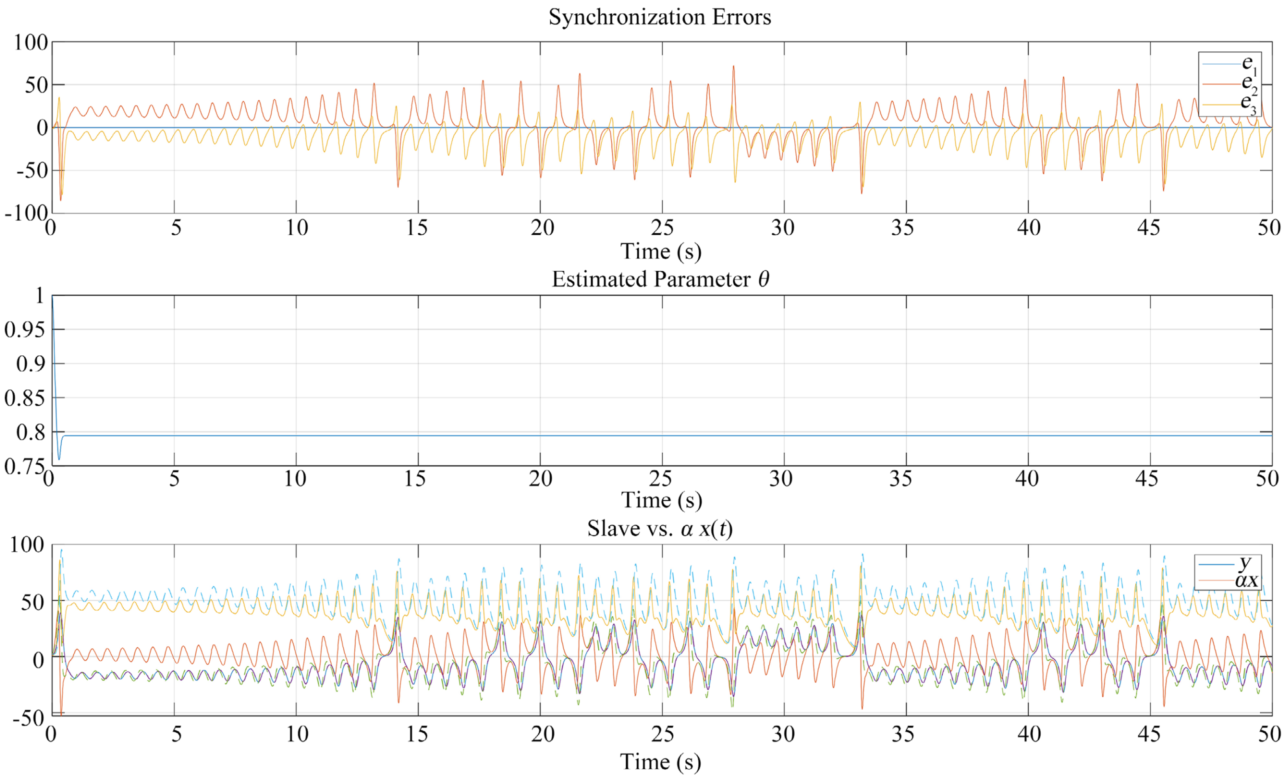


Figure 15. Driving principle of error are synchronized of Lorenz system based on observer analysis.

9. Conclusions

This work addressed the challenge of synchronizing chaotic Lorenz systems with unknown parameters based on observer backstepping finite-time controller design. A novel control strategy combining observer control, backstepping, and finite-time theory was designed and analyzed. The slave system perfectly synchronizes with the master. Synchronization is achieved in a finite, and shorter time, as proven on observer including between adaptive and robust using a comparison.

Theoretically and validated numerically while observer tracks states estimation $e(t) \to 0$. The unknown system parameter is accurately identified. The controller outperforms conventional asymptotic methods in convergence speed. The simulation results conclusively validate the theoretical design, proving it to be an effective and efficient analysis for the finite-time Chaos Synchronization problem based on observer backstepping controller design. Based on Lyapunov stability theory, an observer finite-time backstepping controller which employed the backstepping control design. The backstepping technique, we have applied, allows for the flexibility in the controller design and global stability based on the appropriate choice of Lyapunov functions. Some useful results are achieved on the Lorenz chaotic systems and synchronization including control in this paper. However, a Lorenz system is important in dynamically system because of their behaviors in nonlinear system have not research clearly that's why this makes a newly of the controller observer based on realistic when this system has been increasingly focused on observer backstepping finite-time controller design.

We will contribute continuously to the Lorenz system in future and develop our research work from the following of the proposed method. In order to eliminate the negative effect of behavior of Lorenz system, a novel control scheme is needed. In the future, we will consider more dynamical systems, such as the adaptive observer of control fault diagnose and tolerance of diagnostic system, adaptive fuzzy backstepping control system and others, etc. The simulation results show that the Lorenz chaos and synchronization system schemes of the backstepping approach are effective and have lower complexity. Compared with the existing Lorenz chaos control scheme, our design avoids the complexity of behavior on the chaos and synchronization based on Lorenz system, adaptive of controller and therefore, has a stronger implementation.

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Available of Data and Materials

All data generated or analyzed during this study are included in this published article.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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