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Effect of Exponentially Temperature-Dependent Viscosity on the Onset of Penetrative Ferro-Thermal-Convection in a Saturated Porous Layer via Internal Heating

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Abstract

The effect of viscosity depending exponentially on temperature on the onset of penetrative ferro-thermal-convection (FTC) in a saturated horizontal porous layer in the presence of vertical magnetic field is investigated. The bounding surface of the ferrofluid layer is considered to be rigid-rigid and insulated to temperature perturbations. The resulting eigenvalue problem is solved numerically using the Galerkin technique and also analytically by a regular perturbation technique with wave number as a perturbation parameter. The analytical and numerical results are found to be concurrence. The characteristics of stability of the system are strongly dependent on the viscosity parameter *B*. The effect of *B* on the onset of ferroconvection in a porous layer is dual in nature depending on the choices of physical parameters and a sublayer starts to form at higher values of *B*. Whereas, increase in magnetic number M_1 and the Darcy number *Da* is to advance the onset of ferroconvection in a porous layer. The nonlinearity of fluid magnetization M_3 is found to have no influence on the onset of ferroconvection.

Keywords

Ferroconvection, Internal Heating, Variable Viscosity, Porous Media, Galerkin Technique, Insulated Boundary

1. Introduction

Ferrofluids are stable colloidal suspensions of magnetic nanoparticles in a carrier fluid such as water, hydrocarbon (mineral oil or kerosene), or fluorocarbon. The weirdness of these fluids is the combination of normal liquid behaviour with a magnetic control of their flow and properties. Presently, these fluids are in wide use in seals, bearings, magnetostatic support, jet printers, separation of non-magnetic particles, flow control and drag reduction, dampers, actuators, sensors, transducers, and medical applications. An authoritative introduction to this fascinating subject along with their applications is provided in [1] [2] [3].

The magnetization of ferrofluids depends on the magnetic field, temperature, and density. Hence, any variations of these quantities induce change of body force distribution in the fluid and eventually give rise to convection in ferrofluids in the presence of a gradient of magnetic field. There have been numerous studies on thermal convection in a ferrofluid layer called ferroconvection analogous to Rayleigh-Bénard convection in ordinary viscous fluids. The theory of thermal convective instability in a ferrofluid layer began with Finlayson [4] and extensively continued over the years [5] [6] [7] [8]. Nanjundappa and Shivakumara [9] have studied a variety of velocity and temperature boundary conditions on the onset of ferroconvection in an initially quiescent ferrofluid layer. Singh and Bajaj [10] have investigated a time-periodic modulation in temperatures of two horizontal rigid planes containing an initially quiescent ferrofluid layer induces time-periodic oscillations in the fluid layer at the onset of instability.

Thermal convection of ferrofluids saturating a porous medium has also attracted considerable attention in the literature owing to its importance in controlled emplacement of liquids or treatment of chemicals, and emplacement of geophysically imageable liquids into particular zones for subsequent imaging etc. Rosensweig *et al.* [11] have studied the stabilization of fluid penetration through a porous medium using magnetizable fluids. The stability of the magnetic fluid penetration through a porous medium in high uniform magnetic field oblique to the interface is studied. Zahn and Rosensweig [12]. The thermal convection of a ferrofluid saturating a porous medium in the presence of a vertical magnetic field is studied by Vaidyanathan *et al.* [13]. The laboratory-scale experimental results of the behavior of ferrofluids in porous media consisting of sands and sediments are presented by Borglin *et al.* [14]. Sunil *et al.* [15] have dealt with the theoretical investigation of the double-diffusive convection in a micropolar ferromagnetic fluid layer heated and soluted from below saturating a porous medium. Nanjundappa *et al.* [16] have explored a model for penetrative ferroconvection in saturated porous layer via internal heat generation.

Majority of ferrofluids are either water-based or oil-based. The viscosity of water is far more sensitive to temperature variations and oils are known to have viscosity decreasing exponentially with temperature rather than linearly. Realizing the importance, several investigators have considered exponential variation in viscosity with temperature in analyzing thermal convective instability in a horizontal fluid layer but the studies are limited to ordinary viscous fluids [17] [18] [19] [20] as well as in a layer of saturated porous medium [21] [22] [23]. To our knowledge, due attention has not been given to investigate convective instability problems involving ferrofluids despite its relevance and importance in many heat transfer applications. Shivakumara *et al.* [24] have investigated the onset of thermogravitational convection in a horizontal ferrofluid layer with viscosity depending exponentially on temperature.

The intent of the present study is to analyze the influence of viscosity varying exponentially with temperature on the onset of penetrative FTC in a ferrofluid saturated porous layer via internal heating in the presence of a uniform vertical magnetic field. In investigating the problem, the boundaries of the ferrofluid layer is considered to be rigid-ferromagnetic with insulated to temperature perturbations. The resulting eigenvalue problem is solved numerically by the Galerkin technique and analytically by a regular perturbation technique.

2. Mathematical Formulation

The physical configuration considered is horizontal layer of an incompressible ferrofluid of characteristic thickness d in the presence of an imposed spatially uniform magnetic field H_0 in the vertical direction (see Figure 1).

The lower and upper boundaries are maintained at constant but different temperatures T_l and $T_u\left(<T_l\right)$, respectively. A Cartesian co-ordinate system (*x*, *y*, *z*) is used with the origin at the bottom and *z*-axis is directed vertically upward. Gravity acts in the negative *z*-direction, $\boldsymbol{g}=-g\hat{k}$, where $\hat{k}$ is the unit vector in the *z*-direction. The variation of viscosity η of the ferrofluid with temperature is assumed to be exponential, given by

$$\eta=\eta_0\exp\left[-\gamma\left(T-T_r\right)\right] \tag{1}$$

where, T is the temperature, η_0 is the reference value at the reference temperature T_r and γ is a positive constant.

The governing equations under the Oberbeck-Boussinesq approximation are given by the following:

Mass balance:

$$\nabla\cdot\boldsymbol{q}=0. \tag{2}$$

Linear momentum balance:

$$\begin{aligned}&\rho_0\left[\frac{1}{\varepsilon}\frac{\partial\boldsymbol{q}}{\partial t}+\frac{1}{\varepsilon^2}\left(\boldsymbol{q}\cdot\nabla\right)\boldsymbol{q}\right]\\&=-\nabla p+\rho\boldsymbol{g}+\nabla\cdot\left[\frac{\eta}{\varepsilon}\left(\nabla\boldsymbol{q}+\nabla\boldsymbol{q}^{\mathrm{T}}\right)\right]+\mu_0\left(\boldsymbol{M}\cdot\nabla\right)\boldsymbol{H}-\frac{\eta}{k}\boldsymbol{q}\end{aligned} \tag{3}$$

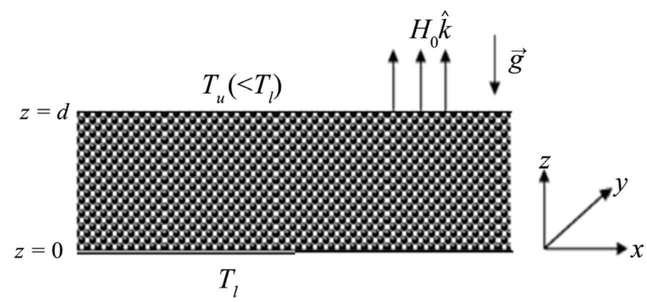

Figure 1. Physical configuration.

Energy balance:

$$\varepsilon\left[\rho_0 C_{V,H}-\mu_0 \boldsymbol{H}\cdot\left(\frac{\partial \boldsymbol{M}}{\partial T}\right)_{V,H}\right]\frac{DT}{Dt}+(1-\varepsilon)(\rho_0 C)_S\frac{\partial T}{\partial t} + \mu_0 T\left(\frac{\partial \boldsymbol{M}}{\partial T}\right)_{V,H}\cdot\frac{D\boldsymbol{H}}{Dt}=k_1\nabla^2 T \quad (4)$$

Equation of state:

$$\rho=\rho_0\left[1-\alpha_t\left(T-T_a\right)\right] \quad (5)$$

Maxwell's equations in the magnetostatic limit:

$$\nabla\cdot\boldsymbol{B}=0,\quad \nabla\times\boldsymbol{H}=0, \quad (6a,b)$$

$$\boldsymbol{B}=\mu_0\left(\boldsymbol{M}+\boldsymbol{H}\right). \quad (6c)$$

Here, $\boldsymbol{q}=(u,v,w)$ the seepage velocity vector, p the pressure, ρ the fluid density, $\boldsymbol{B}$ the magnetic induction, $\boldsymbol{M}$ the magnetization, $\boldsymbol{H}$ the magnetic field intensity, μ_0 the magnetic permeability of vacuum, k the permeability of the porous medium, $T_a=\left(T_l+T_u\right)/2$ the average temperature, ε the porosity of the porous medium, k_t the thermal conductivity, $C_{V,H}$ the specific heat at constant volume and magnetic field, ρ_0 the reference density, α_t the thermal expansion coefficient and the subscript s represents the solid. In view of Equation (6b), $\boldsymbol{H}$ can be expressed as

$$\boldsymbol{H}=\nabla\varphi \quad (7)$$

where, φ is the magnetic potential.

Since the magnetization depends on the magnitude of magnetic field and temperature, we have

$$\boldsymbol{M}=\frac{\boldsymbol{H}}{H}M\left(H,T\right) \quad (8)$$

The linearized equation of magnetic state about H_0 and T_a is

$$M=M_0+\chi\left(H-H_0\right)-K\left(T-T_a\right) \quad (9)$$

where, $M_0=M\left(H_0,T_a\right)$ is the saturation magnetization, $\chi=\left(\partial M/\partial H\right)_{H_0,T_a}$ the magnetic susceptibility, $K=-\left(\partial M/\partial T\right)_{H_0,T_a}$ the pyromagnetic co-efficient, $H=\left|\boldsymbol{H}\right|$ and $M=\left|\boldsymbol{M}\right|$.

It is clear that there exists the following solution for the quiescent basic state:

$$\boldsymbol{q}_b=0$$

$$p_b(z)=p_0-\rho_0 gz-\frac{1}{2}\rho_0\alpha_t g\beta z(z-d)-\frac{\mu_0 M_0 K\beta}{1+\chi}z-\frac{\mu_0 K^2\beta^2}{2(1+\chi)^2}z(z-d)$$

$$T_b(z)=T_a-\beta\left(z-\frac{d}{2}\right)$$

$$\boldsymbol{H}_b(z)=\left[H_0-\frac{K\beta}{1+\chi}\left(z-\frac{d}{2}\right)\right]\hat{k}$$

$$\boldsymbol{M}_b(z)=\left[M_0+\frac{K\beta}{1+\chi}\left(z-\frac{d}{2}\right)\right]\hat{k} \tag{10}$$

where, $\beta=(T_l-T_u)/d$ is the temperature gradient and the subscript b denotes the basic state. To investigate the conditions under which the quiescent solution is stable against small disturbances, we consider a perturbed state such that

$$\begin{aligned}&\boldsymbol{q}=\boldsymbol{q}',\ p=p_b(z)+p',\ \eta=\eta_b(z)+\eta',\\&T=T_b(z)+T',\ \boldsymbol{H}=\boldsymbol{H}_b(z)+\boldsymbol{H}',\ \boldsymbol{M}=\boldsymbol{M}_b(z)+\boldsymbol{M}'\end{aligned} \tag{11}$$

where, $\boldsymbol{q}'$, p', η', T', $\boldsymbol{H}'$ and $\boldsymbol{M}'$ are perturbed variables and are assumed to be small. Then, we note that

$$\eta=\eta_0\exp\left[\gamma\beta(z-d/2)+\gamma(T_r-T_a)-\gamma T'\right] \tag{12}$$

Substituting Equation (9) into Equation (8) and using Equation (9) and assuming the $K\beta z\ll(1+\chi)H_0$, we get (after dropping the primes)

$$H_i+M_i=\left(1+\frac{M_0}{H_0}\right)H_i,\ i=1,2 \tag{13}$$

$$M_3=\chi H_3-KT \tag{14a}$$

Hence on using Equation (14), we get

$$M_3+H_3=\chi H_3-KT+H_3=(1+\chi)H_3-KT$$

Again substituting Equation (11) into momentum Equation (3), linearizing, eliminating the pressure term by operating curl twice and using Equation (13) the z-component of the resulting equation can be obtained as (after dropping the primes)

$$\begin{aligned}\rho_0\frac{\partial}{\partial t}\left(\nabla^2 w\right)=&\ \eta(z)\nabla^4 w+\rho_0\alpha_t g\nabla_h^2 T+2\frac{\partial\eta(z)}{\partial z}\nabla^2\left(\frac{\partial w}{\partial z}\right)\\&-\frac{\eta(z)}{k}\nabla^2 w+\frac{\partial^2\eta(z)}{\partial z^2}\left(\nabla^2 w-2\nabla_h^2 w\right)\\&-\mu_0 K\beta\frac{\partial}{\partial z}\left(\nabla_h^2\varphi\right)-\frac{1}{k}\frac{\partial w}{\partial z}\frac{\partial\eta(z)}{\partial z}+\frac{\mu_0K^2\beta}{1+\chi}\nabla_h^2 T\end{aligned} \tag{14b}$$

where, $\eta(z)=\eta_0\exp\left[\gamma\beta\left(z-\frac{d}{2}\right)+\gamma(T_r-T_a)\right]$ and $\nabla_h^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the horizontal Laplacian operator.

The energy balance Equation (4) after using Equation (11) and linearizing, takes the form (after dropping the primes)

$$(\rho_0 C)_1\frac{\partial T}{\partial t}-\mu_0 T_0 K\frac{\partial}{\partial t}\left(\frac{\partial\varphi}{\partial z}\right)=k_1\nabla^2 T+\left[(\rho_0 C)_2-\frac{\mu_0 T_0 K^2}{1+\chi}\right]w\beta \tag{15}$$

where $(\rho_0 C)_1=\varepsilon\rho_0 C_{V,H}+\varepsilon\mu_0 H_0 K+(1-\varepsilon)(\rho_0 C)_s$, $(\rho_0 C)_2=\varepsilon\rho_0 C_{V,H}+\varepsilon\mu_0 H_0 K$ and we have assumed $\beta d\ll T_0$.

Equations 6(a, b), after substituting Equation (11) and using Equation (13), may be written as (after dropping the primes)

$$\left(1+\frac{M_0}{H_0}\right)\nabla_h^2\varphi+(1+\chi)\frac{\partial^2\varphi}{\partial z^2}-K\frac{\partial T}{\partial z}=0. \tag{16}$$

Since the principle of exchange of stability is valid [5], the normal mode expansion of the dependent variables is assumed in the form

$$\begin{Bmatrix} w \\ T \\ \varphi \end{Bmatrix} = \begin{Bmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{Bmatrix} \exp\left[i\left(\ell x + my\right)\right] \tag{17}$$

where, ℓ and m are wave numbers in the x and y directions, respectively.

On substituting Equation (17) into Equations (14)-(16) and non-dimesionalizing the variables by setting

$$\begin{aligned} &\left(x^*, y^*, z^*\right) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), \\ &W^* = \frac{d}{\nu A} W, \\ &\Theta^* = \frac{\kappa}{\beta \nu d}\Theta, \\ &\Phi^* = \frac{(1+\chi)\kappa}{K\beta\nu d^2}\Phi, \\ &f^*(z) = \frac{\eta(z)}{\eta_0} \end{aligned} \tag{18}$$

where, kinematic viscosity, $\nu = \eta_0/\rho_0$, effective thermal diffusivity $\kappa = k_1/\left(\rho_0 C\right)_2$ and $A = \left(\rho_0 C\right)_1/\left(\rho_0 C\right)_2$.

Following the classical lines of linear stability theory as presented by Chandrasekhar [25], neglecting the asterisk, the linearized and dimensionless governing equation can be written as:

$$\begin{aligned} &f\left(D^2 - a^2\right)^2 W + 2Df\left(D^2 - a^2\right)DW + D^2 f\left(D^2 + a^2\right)W \\ &- f\,Da^{-1}\left(D^2 - a^2\right)W - Df\,Da^{-1}DW \\ &= R_t a^2\Theta - R_m a^2\left(D\Phi - \Theta\right) \end{aligned} \tag{19}$$

$$\left(D^2 - a^2\right)\Theta = -\left(1 - M_2 A\right)W \tag{20}$$

$$\left(D^2 - a^2 M_3\right)\Phi - D\Theta = 0\,. \tag{21}$$

Here, $D = \mathrm{d}/\mathrm{dz}$ is the differential operator, $a = \sqrt{\ell^2 + m^2}$ is the overall horizontal wavenumber, W is the amplitude of vertical component of velocity, Θ is the amplitude of temperature, Φ is the amplitude of magnetic potential, $R_t = \alpha_t g\beta d^4/\nu\kappa A$ is the thermal Raleigh number, $R_m = R_t M_1 = \mu_0 K^2\beta^2 d^4/(1+\chi)\mu\kappa A$ is the magnetic Rayleigh number, $M_1 = \mu_0 K^2\beta/(1+\chi)\alpha_t\rho_0 g$ is the magnetic number, $M_2 = \mu_0 T_a K^2/(1+\chi)\rho_0 C$ is the magnetic parameter, $M_3 = \left(1 + M_0/H_0\right)/(1+\chi)$ is the measure of nonlinearity of fluid magnetization parameter. The typical value of M_2 for magnetic fluids with different carrier liquids turns out to be of the order of 10^{-6} and hence its effect is neglected as compared to unity and $f(z)$ is given by

$$f(z) = \exp\left[\Lambda\left(z - \frac{1}{2}\right) + \frac{\left(T_r - T_a\right)}{\beta d}\right] \tag{22}$$

where, $B=\gamma\beta d$ is the dimensionless viscosity parameter. If the reference temperature $T_r = T_a$, then

$$f(z) = \exp\left[B(z-1/2)\right]. \tag{23}$$

The boundaries are considered to be rigid-ferromagnetic and they are insulated to temperature perturbations:

$$W = DW = \Phi = D\Theta = 0 \quad \text{at} \quad z = 0,1 \tag{24}$$

3. Method of Solution

Equations (19)-(21) together with the corresponding boundary condition (24) constitute an eigenvalue problem with R_t or R_m as an eigenvalue. The eigenvalue problem is solved numerically using the Galerkin technique as well as analytically using a regular perturbation technique and the results so obtained are compared to know the accuracy of the methods employed.

3.1. Solution by the Galerkin Technique

The Galerkin method is used to solve this problem as explained in the book by Finlayson [26]. In this method, the test (weighted) functions are the same as the base (trial) functions. Accordingly, W, Θ and Φ are written as

$$\begin{aligned} W &= \sum_{i=1}^{n} A_i W_i(z) \\ \Theta &= \sum_{i=1}^{n} C_i \Theta_i(z) \\ \Phi &= \sum_{i=1}^{n} D_i \Phi_i(z) \end{aligned} \tag{25}$$

where, A_i, C_i and D_i are unknown constants to be determined. The base functions $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ are generally chosen such that they satisfy the boundary condition (24) but not the differential equations. For this boundary, they are chosen as

$$\begin{aligned} W_i &= \left(z^4 - 2z^3 + z\right) T_{i-1}^*, \\ \Theta_i &= z(1 - z/2) T_{i-1}^*, \\ \Phi_i &= z^2(1 - 2z/3) T_{i-1}^* \end{aligned} \tag{26}$$

where, T_i^*'s are the modified Chebyshev polynomials. The above trial functions satisfy the boundary condition. Multiplying Equation (19) by $W_i(z)$, Equation (20) by $\Theta_i(z)$ and Equation (21) by $\Phi_i(z)$; performing the integration by parts with respect to z between $z=0$ and $z=1$ and using the boundary conditions, we obtain a system of linear homogeneous algebraic equations in A_i, C_i and D_i. Nontrivial solution exists if and only if the characteristic determinant is equal to zero. This leads to a relation involving the characteristic equation in the form

$$F\left(R_t, R_m, Da^{-1}, M_1, M_3, B, a\right) = 0.$$

The critical values of R_{tc} or R_{mc} are found as a function of wave number a for various values of physical parameters. It is observed that the convergence is achieved with six terms in the series expansion.

3.2. Solution by Regular Perturbation Technique

It is known that for insulated boundary conditions the onset of convection corresponds to a vanishingly small wave number (*i.e.* unicellular convection). The numerical calculations carried out in the previous section also corroborate this fact. Therefore, an attempt is being made to exploit this fact to obtain an analytical formula for the onset of convection using a regular perturbation technique with wave number a as a perturbation parameter. Accordingly, the variables W, Θ and Φ are expanded in powers of a^2 as

$$\left(W,\Theta,\Phi\right)=\left(W_0,\Theta_0,\Phi_0\right)+a^2\left(W_1,\Theta_1,\Phi_1\right)+\cdots \tag{27}$$

Substituting Equation (27) into Equations (19)-(21) and also in the boundary conditions, and collecting the terms of zero-th order, we obtain

$$D^4W_0+2BD^3W_0+\left(B^2-Da^{-1}\right)D^2W_0-BDa^{-1}DW_0=0, \tag{28a}$$

$$D^2\Theta_0+W_0=0, \tag{28b}$$

$$D^2\Phi_0+D\Theta_0=0 \tag{28c}$$

with the boundary condition $W_0=DW_0=0=D\Theta_0=\Phi_0$. The solution to the zero-th order equations is: $W_0=0,\Theta_0=1$, and $\Phi_0=0$.

The first order equations are then

$$D^4W_1+2BD^3W_1+\left(B^2-Da^{-1}\right)D^2W_1-BDa^{-1}DW_1=R_t\left(1+M_1\right)\mathrm{e}^{-B(z-1/2)}, \tag{29a}$$

$$D^2\Theta_1+W_1=1, \tag{29b}$$

$$D^2\Phi_1-D\Theta_1=0 \tag{29c}$$

with the boundary conditions $W_1=DW_1=\Phi_1=D\Theta_1=0$.

The general solution of Equation (29a) is given by

$$W_1=c_1+c_2\mathrm{e}^{-Bz}+c_3\mathrm{e}^{\delta_1 z}+c_4\mathrm{e}^{\delta_2 z}+\frac{R_t\left(1+M_1\right)}{BDa^{-1}}z\mathrm{e}^{-B(z-1/2)} \tag{30}$$

where,

$$\delta_1=\frac{-B+\sqrt{B^2+4Da^{-1}}}{2},\quad \delta_2=\frac{-B-\sqrt{B^2+4Da^{-1}}}{2},\quad c_1=-c_2-c_3-c_4,$$

$$c_2=\frac{-BDa^{-1}\left[\left(1-\mathrm{e}^{\delta_1}\right)c_3+\left(1-\mathrm{e}^{\delta_2}\right)c_4\right]+\tau}{B\left(1-\mathrm{e}^{-B}\right)Da^{-1}},\quad c_3=\frac{\tau\left[\left(B+1-\mathrm{e}^{B}\right)\Delta_4-B\mathrm{e}^{B}\Delta_2\right]}{BDa^{-1}\left(\Delta_1\Delta_4-\Delta_2\Delta_3\right)},$$

$$c_4=\frac{\tau\left[-\left(B+1-\mathrm{e}^{B}\right)\Delta_3+B\mathrm{e}^{-B}\Delta_1\right]}{BDa^{-1}\left(\Delta_1\Delta_4-\Delta_2\Delta_3\right)},\quad \tau=R_t\left(1+M_1\right)\mathrm{e}^{-B/2}$$

with

$$\Delta_1=B\left(1-\mathrm{e}^{\delta_1}\right)+\delta_1\left(1-\mathrm{e}^{-B}\right),$$

$$\Delta_2 = B\left(1-e^{\delta_2}\right)+\delta_2\left(1-e^{-B}\right),$$

$$\Delta_3 = Be^{-B}\left(1-e^{\delta_1}\right)+\delta_1 e^{\delta_1}\left(1-e^{-B}\right),$$

$$\Delta_4 = Be^{-B}\left(1-e^{\delta_2}\right)+\delta_2 e^{\delta_2}\left(1-e^{-B}\right).$$

From Equation (29b), it follows that

$$1=\int_0^1 W_1 \mathrm{d}z\,. \tag{31}$$

Substituting for W_1 from Equation (30) into Equation (31) and carrying out the integration leads to an expression for the critical Rayleigh number R_{tc} in the form

$$R_{tc}=\frac{2B^2 Da^{-1}\left[\lambda_1\eta_1-\lambda_2\eta_2\right]}{\left(1+M_1\right)\left[4B\lambda_1 P_1+\eta_3\sinh\delta+\eta_4\cosh\delta-2\lambda_6 P_2\right]}$$

where

$$P_1=\cosh\left(B/2\right)$$
$$P_2=\sinh\left(B/2\right)$$
$$\eta_1=-1+P_1\cosh\delta$$
$$\eta_2=P_2\sinh\delta$$
$$\eta_3=\lambda_3-\lambda_4\cosh B$$
$$\eta_4=\lambda_5+\lambda_6\sinh B$$

with

$$\lambda_1=-4\delta B\left(-4\delta^2+B^2\right),$$

$$\lambda_2=-16\delta^4+B^4,$$

$$\lambda_3=-8\delta^2B^2\left(-6+B^2\right)+\left(16\delta^4+B^4\right)\left(2+B^2\right),$$

$$\lambda_4=2\left(16\delta^4+24\delta^2B^2+B^4\right),$$

$$\lambda_5=16\delta B\left(-4\delta^2B^2+B^3\right),$$

$$\lambda_6=32\delta B\left(4\delta^2+B^2\right),$$

$$\delta=\sqrt{B^2+4Da^{-1}}\,.$$

As $B\to 0$ and $Da^{-1}\to 0$, Equation (32), reduce to

$$R_{tc}=\frac{720}{1+M_1} \tag{33}$$

These are the results for constant viscosity ferrofluids and coincide with Nanjundappa and Shivakumara [16]. When $M_1=0$ (*i.e.* ordinary viscous fluid), Equation (33) reduce to the critical Rayleigh number of $R_{tc}=720$, which is he known exact value documented in the literature. Equation (33) further reveal that the nonlinearity of fluid magnetization (*i.e.* M_3) has no effect on the onset of ferroconvection; a result which is observed by numerical computations carried out in the previous section. This result is similar to the one noticed in the case of constant viscosity ferrofluids [16]. Since at the onset of convection $a_c = 0$ (very large wave length), one would expect that M_3 has no effect on the stability of the system.

4. Results and Discussion

The linear stability analysis is carried out with viscosity depending exponentially on temperature on the onset of FTC in a ferrofluid saturated porous layer. The bounding surface of the ferrofluid layer is rigid-ferromagnetic and insulated to temperature perturbations. The critical eigenvalue R_{tc} or R_{mc} and the corresponding wave number a_c are computed numerically by the Galerkin method as well as analytically by employing a regular perturbation technique for different R_m, M_1, Da^{-1} and *B*. It is noted that the critical wave number is vanishingly small and this fact is exploited to obtain an analytic expression for R_{tc} using a regular perturbation technique with wave number *a* as a perturbation parameter. Such a study also helps in knowing the accuracy of the numerical method employed in solving the problem. The stability characteristics of the system are found to be independent of the nonlinearity of fluid magnetization parameter M_3. The salient features of the physical parameters on the onset of FTC are exhibited in **Figures 2-7**.

Figure 2 shows the variation of R_{tc} as a function of viscosity parameter B for different magnetic parameter M_1 when $Da^{-1} = 50$. The figure clearly illustrates the strong influence of viscosity parameter B on FTC.

As a result of viscosity variation, two distinguish regions are shown, R_{tc} increases negligibly small for B up to 5.17019, at which maximum value of R_{tc} are reached; rapidly decreasing trends are found for $M_1 = 0$ (*i.e.* absence of magnetic force) and decreasing slowly for $M_1 > 0$ (*i.e.* presence of magnetic force). At maximum value of R_{tc} a sublayer starts to form. It is seen that maximum R_{tc} exists at $B = 0$ and R_{tc} decreases monotonically with increasing B indicating

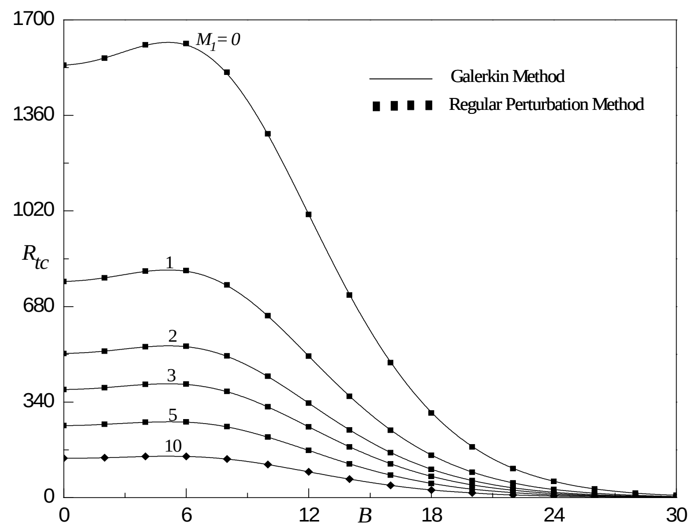

Figure 2. Variation of R_{tc} versus B for different M_1 when $Da^{-1} = 50$.

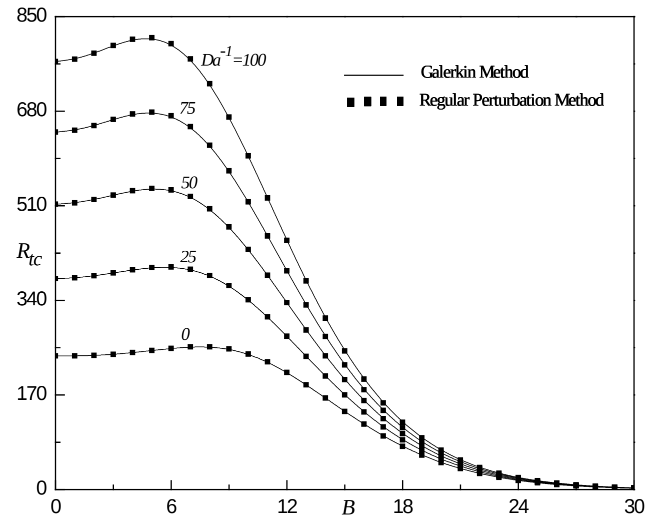

Figure 3. Variation of R_{tc} versus B for different Da^{-1} when $M_1 = 2$.

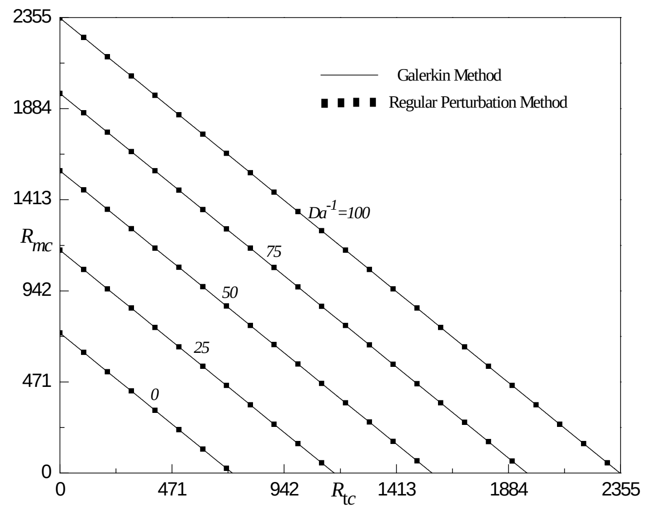

Figure 4. Locus of R_{tc} versus Ma_c for different Da^{-1} for $B = 2$.

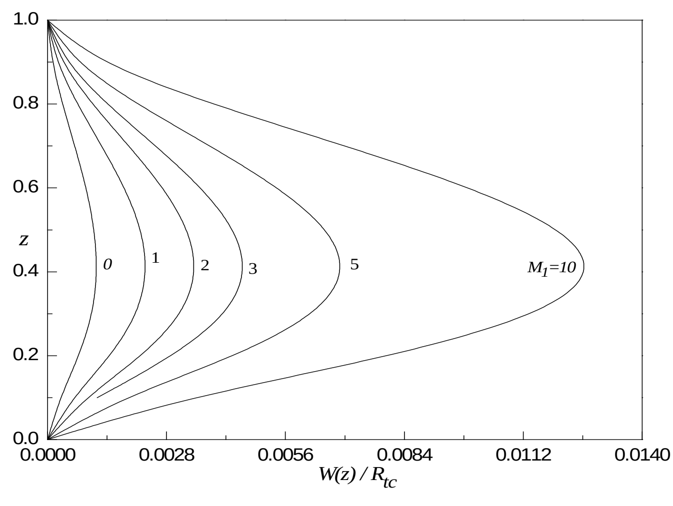

Figure 5. Vertical velocity eigenfunction for $B = 2$ and $Da^{-1} = 50$.

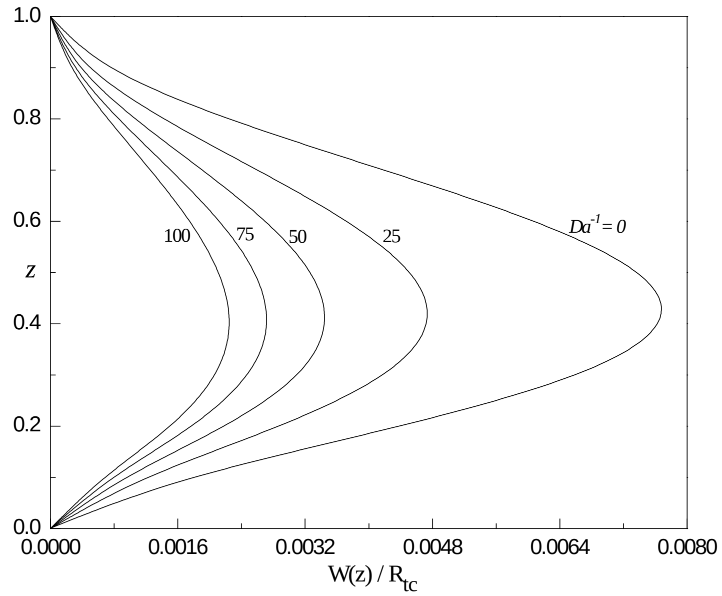

Figure 6. Vertical velocity eigenfunction for $B = 2$ and $M_1 = 2$.

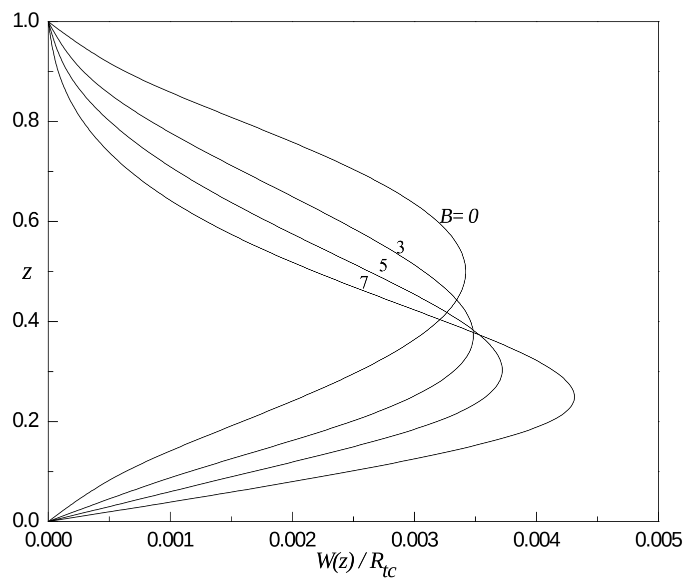

Figure 7. Vertical velocity eigenfunction for $Da^{-1} = 50$ and $M_1 = 2$.

its effect is to destabilize the system. This is due to the decrease in viscosity of the ferrofluid with temperature. Moreover, R_{tc} decreases quite rapidly at first then slowly and finally the curves of different M_1 merge with increasing *B*. It is more so with an increase in the value of M_1 and this is due to additive reinforcement of destabilizing magnetic force. The results for $M_1 = 0$ correspond to ordinary viscous fluid and it is observed that higher heating is required to have instability in this case. Thus the combined effect of temperature dependent viscosity and magnetic forces is to reinforce together and to hasten the onset of FTC compared to their effect in isolation. The value of $B = 5.17019$ at which R_{tc} attains its maximum value $(R_{tc})_{\max}$ are tabulated in **Table 1** for different values of M_1 when $Da^{-1} = 50$. From Table 1 it is seen that $(R_{tc})_{\max}$ decreases with increasing M_1.

Figure 3 shows variation of R_{tc} with variation parameter B for various values of Da^{-1} when $M_1 = 2$. It is seen that R_{tc} increases with increasing Da^{-1} and hence its effect is to delay the onset of FTC.

For a fixed thickness of the porous layer, increase in the value of Da^{-1} leads to decrease in the permeability of the porous medium which in turn retards the flow of ferrofluid. Therefore, higher heating and hence higher value of R_t is required for the onset of R_{tc} in a ferrofluid saturated porous medium. For different Da^{-1}, Figure 3 demonstrates two distinct characteristics and which is same situation in the presence of magnetic forces with increase in B.

The value of B at which R_{tc} attains its maximum value $(R_{tc})_{\max}$ are tabulated in Table 2 for different Da^{-1} when $M_1 = 2$. From Table 2 it is seen that $(R_{tc})_{\max}$ and Da^{-1} decreases with increasing B.

The locus of R_{tc} and R_{mc} is shown in Figure 4 for different Da^{-1} with $B = 2$ to know the simultaneous presence of buoyancy and magnetic forces on the stability of the system.

From the figures it is obvious that there is a strong coupling between R_{tc} and R_{mc} and the curves are slightly convex. That is, when the buoyancy force is predominant the magnetic force becomes negligible and vice-versa. From Figure 4, it is seen that an increase in Da^{-1} is to increase in R_{tc} as well as R_{mc} and thus its effect is to delay the onset of ferroconvection.

The perturbed vertical velocity eigenfunctions are presented in Figures 5-7 for different values of M_1, B and Da^{-1}, respectively.

Table 1. Values of $(R_{tc})_{\max}$ occurring at B for different values of M_1 when $Da^{-1} = 50$.

M_1	$(R_{tc})_{\max}$	B
0	1623.48	5.17019
1	811.74	5.17019
2	541.16	5.17019
3	405.87	5.17019
5	270.58	5.17019
10	147.589	5.17019

Table 2. Values of $(R_{tc})_{\max}$ occurring at B for different values of Da^{-1} when $M_1 = 2$.

Da^{-1}	$(R_{tc})_{\max}$	B
0	256.737	7.56313
25	400.153	5.72418
50	541.16	5.17019
75	678.309	4.91311
100	812.433	4.76013

As can be seen, the shape of the eigenfunction is parabolic in nature. Increasing M_1, decreasing B and Da^{-1} is to increase the vigor of the ferrofluid flow and hence their effect is to hasten the onset of ferroconvection.

5. Conclusion

The onset of ferroconvection in a ferofluid saturated porous layer with viscosity varying exponentially with temperature is studied. The viscosity parameter B exhibits a dual effect on the stability characteristics of the system. It shows stabilizing effect on the system viscosity. The viscosity parameter B exhibits a dual effect on the stability characteristics of the system. It shows stabilizing effect on the system initially but displays a reverse trend after exceeding certain value of B. The critical Rayleigh number R_{tc} attains maximum value at some intermediate values of B. The effect of the increase in R_m and the Darcy number Da is to hasten the onset of FTC. The buoyancy and magnetic forces reinforce each other in hastening the onset of FTC. The nonlinearity of fluid magnetization parameter M_3 has no effect on the onset of FTC. The critical eigenvalues were obtained by a regular perturbation technique and computed numerically using the Galerkin method complement with each other indicating the analytical solutions obtained are exact.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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