Journal of Electromagnetic Analysis and Applications, **2012, 4, 367-378**
http://dx.doi.org/10.4236/jemaa.2012.49052 Published Online September 2012 (http://www.SciRP.org/journal/jemaa)

The Effects of Rotation and Salt Concentration on Thermal Convection in a Linear Magneto-Fluid Layer Overlying a Porous Layer

Maram S. Al-Qurashi[1*], Abdul-Fattah K. Bukhari[2]

[1]Mathematics Department, College of Sciences, Taif University, Taif, Saudi Arabia; [2]Mathematics Department, College of Applied Sciences, Umm Al-Qura University, Mecca, Saudi Arbia.
Email: [*]mm.q777@yahoo.com

Received June 29th, 2012; revised July 30th, 2012; accepted August 15th, 2012

ABSTRACT

A linear stability analysis is applied to a system consisting of a linear magneto-fluid layer overlying a porous layer affected by rotation and salt concentration on both layers. The flow in the fluid layer is governed by Navier-Stokes's equations and while governed by Darcy-Brinkman's law in the porous medium. Numerical solutions are obtained using Legendre polynomials. These solutions are studied through two modes of instability: stationary instability and overstability when the heat and the salt concentration are effected from above and below.

Keywords: Navier-Stokes Equation; Darcy-Brinkman Law; Legendre Polynomials; Salt Concentration; Vertical Linear Magnetic Field

1. Motivations and Goals

Thermal instability theory has attracted considerable interest and has been recognized as a problem of fundamental importance in many fields of fluid dynamics. The earliest experiments to demonstrate the onset of thermal instability in fluids are those of Bernard's [1,2]. Benard worked with very thin layers of an incompressible viscous fluid standing on a levelled metallic plate maintained at a constant temperature. The upper surface which was usually free and, being in contact with the air, was at a lower temperature. In his experiments, Benard deduced that a certain critical adverse temperature gradient must be exceeded before instability can set in. The instability of a layer of fluid heated from below and subjected to Coriolis forces has been studied by Chandrasekhar [3,4] for a stationary and overstability case. He showed that the presence of these forces usually has the effect of inhibiting the onset of thermal convection. Nield [5] considered the onset of salt-finger convection in a porous layer. Taunton *et al.* [6] considered the thermohaline instability and salt-finger in a porous medium and solved the boundary value problem. Sun [7] was the first to consider such a problem, and he used a shooting method to solve the linear stability equations. Sun [7] and Nield [8] used Darcy's law in formulating the equations of porous layer. In Darcy's law of motion in porous mediums, the Darcy resistance term took the place of the Navier-stokes viscosity term, while in the modified Darcy's law (Brinkman model), suggested by Brinkman [9], the Navier- stokes viscosity term still exists. Chen & Chen [10] considered the multi-layer problem when the above layer is heated and salted from above, and the solution of the problem is obtained using a shooting method. Lindsay & Ogden [11] worked in the implementation of spectral methods resistant to the generation of spurious eigenvalues. Lamb [12] used expansion of Chebyshev polynomials to investigate an eigenvalue problem arising from a model discussing a finitely conducting inner core of the earth on magnetically driven instability. Bukhari [13] studied the effects of surface-tension in a layer of conducting fluid with an imposed magnetic field and obtained results when the free surface is deformable and non-deformable. He solved that by using Chebyshev spectral method, and thus obtained some different results from that of Chen & Chen [10]. Straughan [14] studied the thermal convection in fluid layer overlying a porous layer and considered the problem of lower layer heated from below and surface tension driven on the free top boundary of upper layer. In [15], he also dealt with the same problem considering the ratio depth of the relative layer and investigated the effect of the variation of relevant fluid and porous material properties. Allehiany [16] studied Benard convection in a horizontal porous layer

[*]Corresponding author.

permeated by a conducting fluid in the presence of magnetic field and coriolis forces. In this work, we studied the effects of rotation and salt concentration on thermal convection in a linear magneto-fluid overlying a porous layer. The numerical solution was presented in different boundary conditions solved by using Legendre polynomials.

2. The Governing Equations

We consider a fluid layer overlying a porous layer so that the top of the porous layer touches the bottom of the fluid layer. The plane interface between the two layers is $x_3 = 0,$ the upper boundary of the fluid layer is $x_3 = d_f$ and the lower boundary of the porous medium layer is $x_3 = -d_m$ where the subscripts *f* and m denote the fluid layer and porous medium layer respectively. We suppose that the upper layer is filled with an incompressible thermally and electrically conducting viscous fluid consisting of melted salt which flows in it and governed by Navier-Stokes equations. However, the lower layer is occupied by a porous medium permeated by the fluid flowing in it and governed by Darcy-Brinkman's law. Both layers subjected to a constant vertical linear magnetic field and affected by a rotation around x_3 with a constant angular velocity Ω. Gravity g acts in the negative direction of x_3 (see **Figure 1**).

Convection is driven by the temperature depending on the fluid density and salting, and damped by viscosity. The Oberbeck-Boussineq approximation is used as the density of fluid is constant everywhere except in the body force term where the density is linearly proportional to temperature and salt concentration, *i.e.*

$$\rho_f = \rho_0\left[1-\alpha\left(T-T_0\right)+\beta\left(S-S_0\right)\right]. \quad (1)$$

where *T* denotes the Kelvin temperature of the fluid, *S* is the salt concentration, ρ_0 is the density of fluid at T_0 and S_0, α (constant) is the thermal coefficient of volume expansion of the fluid and β (constant) is the salting coefficient of volume expansion of the fluid. Let V be the solenoidal velocity of the fluid.

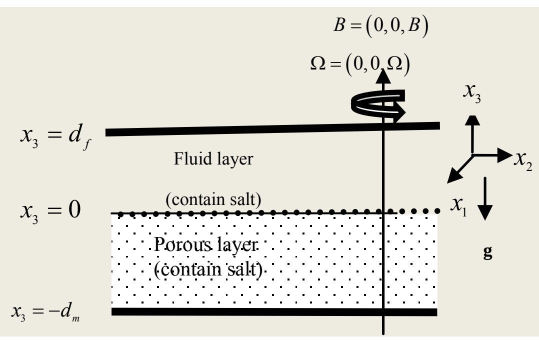

Figure 1. Schematic diagram of the problem.

Let V, H, B, J and E be respectively the solenoidal velocity of the fluid, the magnetic field, the magnetic induction, the current density and the electric field. Hence

$$div\,V = 0, div\,B = 0 \quad (2)$$

H, B, J and E connected by the relations

$$B = \mu_m H, J = \bar{\sigma}\left[E+\left(V\times B\right)\right] \quad (3)$$

where μ_m is the magnetic permeability and $\bar{\sigma}$ is the electrical conductivity. And the Maxwell equations

$$\text{curl}\,E = -\frac{\partial B}{\partial \text{t}}, J = \text{curl}H \quad (4)$$

where the displacement current has been neglected in the second of these Maxwell equations as is customary in situation when free charge is instantaneously dispersed. By substituting from (3), $(4)_2$ in to $(4)_1$ obtain by

$$\begin{aligned}\frac{\partial B}{\partial \text{t}} &= -\text{curl}\left[\frac{1}{\bar{\sigma}}J-\left(V\times B\right)\right] \\ &= -\text{curl}\left[\frac{1}{\mu_m\bar{\sigma}}\text{curl }B-\left(V\times B\right)\right] \\ &= \text{curl}\left(V\times B\right)-\eta \text{ curl curl } B\end{aligned} \quad (5)$$

where $\eta = \left(\frac{1}{\mu_m\bar{\sigma}}\right)$ is electrical resistivity. By using

$$\text{curl curl } B = \text{grad div}\left(B\right)-\nabla^2\left(B\right)$$

and

$$\text{curl}\left(V\times B\right) = \left(B\cdot\nabla\right)V-\left(V\cdot\nabla\right)B$$

then Equation (5) reduce to

$$\frac{\partial B}{\partial \text{t}} = \left(B\cdot\nabla\right)V-\left(V\cdot\nabla\right)B+\eta\,\nabla^2 B \quad (6)$$

The equation of motion is

$$\begin{aligned}&\rho_0\left(\frac{\partial V}{\partial \text{t}}+V\cdot\nabla V\right) \\ &= -\nabla\bar{P}+\mu\nabla^2 V+\rho\,g+\rho_0\left(V\times\Omega\right)+J\times B\end{aligned} \quad (7)$$

where $\bar{P}$ is hydrostatic pressure, μ is the dynamic viscosity and ∇^2 is three-dimensional Laplacian operator. And by subsituting from $(3)_1$, $(4)_2$ in to the Lorentz force $J\times B$ we obtain

$$\begin{aligned}J\times B &= \frac{1}{\mu_m}\text{curl } B\times B \\ &= \frac{1}{\mu_m}\left(B\cdot\nabla B-\frac{1}{2}\nabla B^2\right)\end{aligned} \quad (8)$$

Hence the equation of motion becomes

$$\rho_0\left(\frac{\partial V}{\partial t}+V\cdot\nabla V\right)=-\nabla\left(\bar{P}+\frac{1}{8\pi\mu_m}\nabla B^2\right)+\mu\nabla^2 V_f$$
$$+\rho g+2\rho_0\left(V\times\Omega\right)+\frac{1}{\mu_m}B\cdot\nabla B \quad (9)$$

and so the governing equations of the fluid layer are

$$\left[\frac{\partial V_f}{\partial t}+V_f\cdot\nabla V_f\right]$$
$$=-\nabla\frac{P_f}{\rho_0}+\nu\nabla^2 V_f$$
$$-g\left[1-\alpha\left(T_f-T_0\right)+\beta\left(S_f-S_0\right)\right] \quad (10)$$
$$+2\left(V_f\times\Omega_f\right)+\frac{1}{\rho_0\mu_{mf}}\left(B_f\cdot\nabla B_f\right),$$

$$\left(\rho_0 c_p\right)_f\left[\frac{\partial T_f}{\partial t}+V_f\cdot\nabla T_f\right]=k_f\nabla^2 T_f \quad (11)$$

$$\frac{\partial S_f}{\partial t}+V_f\cdot\nabla S_f=D_f\nabla^2 S_f \quad (12)$$

$$J_f=\frac{1}{\mu_{mf}}\mathrm{curl}B_f \quad (13)$$

$$\frac{\partial B_f}{\partial t}=\left(B_f\cdot\nabla\right)V_f-\left(V_f\cdot\nabla\right)B_f+\eta_f\nabla^2 B_f \quad (14)$$

and the governing equations of the porous medium layer are

$$\frac{1}{\varphi}\frac{\partial V_m}{\partial t}=-\nabla\frac{P_m}{\rho_0}-\frac{\nu}{K}V_m+\nu\nabla^2 V_m$$
$$-g\left[1-\alpha\left(T_m-T_0\right)+\beta\left(S_m-S_0\right)\right] \quad (15)$$
$$+2\left(V_m\times\Omega_m\right)+\frac{1}{\rho_0\mu_{mm}}\left(B_m\cdot\nabla B_m\right)$$

$$\left(\rho_0 c\right)_m\frac{\partial T_m}{\partial t}+\left(\rho c_p\right)_f V_m\cdot\nabla T_m=k_m\nabla^2 T_m \quad (16)$$

$$\varphi\frac{\partial S_m}{\partial t}+V_m\cdot\nabla S_m=D_m\nabla^2 S_m \quad (17)$$

$$J_m=\frac{1}{\mu_{mm}}\mathrm{curl}B_m \quad (18)$$

$$\frac{\partial B_m}{\partial t}=\left(B_m\cdot\nabla\right)V_m-\left(V_m\cdot\nabla\right)B_m+\eta_m\nabla^2 B_m \quad (19)$$

where P_f, P_m are the modified pressure of the fluid and the porous medium layers respectively and $\frac{1}{2}\left|\Omega_f\times r\right|^2$, $\frac{1}{2}\left|\Omega_m\times r\right|^2$ are the centrifugal force of the fluid and porous medium layer respectively, V_f, V_m are the solenoidal and seepage velocity respectively, $2\left(V_f\times\Omega_f\right)$, $2\left(V_m\times\Omega_m\right)$ are the coriolis acceleration of the fluid and porous medium layer respectively, T_f, T_m are the Kelvin temperature of the fluid and porous medium layer respectively, S_f, S_m are salt concentration of the fluid and porous medium layer respectively, D_f, D_m are the mass diffusivity of the fluid and porous medium layer respectively, μ_{mf}, μ_{mm} are magnetic permeability of fluid and porous layer respectively, k_f, k_m are the thermal and overall thermal conductivity of fluid and porous layers respectively, $\nu=\mu/\rho_0$ is the kinematic viscosity, K is the permeability, ϕ is its porosity and $\left(\rho_0 c_p\right)_f, \left(\rho_0 c\right)_m$ are the heat and overall heat capacity per unit volume of the fluid and porous medium layers at constant pressure. In fact

$$\left(\rho_0 c\right)_m=\varphi\left(\rho_0 c_p\right)_f+\left(1-\varphi\right)\left(\rho_0 c_p\right)_m$$

where $\left(\rho_0 c_p\right)_m$ is the heat capacity per unit volume of the porous substrate. Suppose that $x_3=d_f$ is rigid and maintained at constant temperature T_u and constant salt concentration S_u, and $x_3=-d_m$ is impenetrable and maintained at constant temperature T_l and constant salt concentration S_l, then the boundary conditions can be written as

$$w_f\left(d_f\right)=0,\frac{\partial w_f}{\partial x_3}\left(d_f\right)=0,$$
$$T_f\left(d_f\right)=T_u, S_f\left(d_f\right)=S_u \quad (20)$$
$$\zeta_{3_f}\left(d_f\right)=0, J_{3f}\left(d_f\right)=0 \quad \frac{\partial B_f}{\partial x_3}\left(d_f\right)$$

on the upper boundary, and

$$w_m\left(-d_m\right)=0,\frac{\partial w_m}{\partial x_m}\left(d_m\right)=0,$$
$$T_m\left(-d_m\right)=T_l, S_m\left(-d_m\right)=S_l \quad (21)$$
$$\zeta_{3_m}\left(d_m\right)=0, J_{3m}\left(d_m\right)=0 \quad \frac{\partial B_m}{\partial x_3}\left(d_m\right)$$

on the lower boundary, where w_f and w_m are the normal axial velocity components of the fluid in fluid layer and porous medium layer respectively, ζ_{3_f} and ζ_{3_m} are the normal axial vortecity components of the fluid in. fluid layer and porous medium layer respectively.

The boundary conditions on the interface plane $x_3 = 0$ are based on the assumption that temperature, salt concentration, heat flux, salt flux, normal and tangential fluid velocity, normal stress and tangential stress are continuous so that

$$T_f(0)=T_m(0),S_f(0)=S_m(0)$$
$$k_f\frac{\partial T_f}{\partial x_3}(0)=k_m\frac{\partial T_m}{\partial x_3}(0),D_f\frac{\partial S_f}{\partial x_3}(0)=D_m\frac{\partial S_m}{\partial x_3}(0)$$
$$w_f(0)=w_m(0),u_f(0)=u_m(0),v_f(0)=v_m(0)$$
$$-p_f(0)+2\mu\frac{\partial w_f}{\partial x_3}(0)=-p_m(0)2+\mu\frac{\partial w_m}{\partial x_3}(0) \quad (22)$$
$$\frac{\partial u_f}{\partial x_3}(0)=\frac{\partial u_m}{\partial x_3}(0),\frac{\partial v_f}{\partial x_3}(0)=\frac{\partial v_m}{\partial x_3}(0)$$
$$J_f(0)=\frac{\mu_{mf}}{\mu_{mm}}J_m(0),\frac{k_f}{\mu_{mf}}\frac{\partial J_f}{\partial x_3}(0)=\frac{k_m}{\mu_{mm}}\frac{\partial J_m}{\partial x_3}(0)$$
$$B_f(0)=B_m(0),k_f\frac{\partial B_f}{\partial x_3}(0)=k_m\frac{\partial B_m}{\partial x_3}(0)$$

Equations (10)-(19) have an equilibrium solution satisfying the boundary conditions (20)-(22) on the form

$$\begin{aligned}&V_f=0,V_m=0,\\&-\nabla P_f+\rho_f g=0,-\nabla P_m+\rho_f g=0,\\&\nabla^2 T_f=\nabla^2 T_m=0,\nabla^2 S_f=\nabla^2 S_m=0,\\&\Omega_f=(0,0,\Omega_f),\Omega_f \text{ constant},\\&\Omega_m=(0,0,\Omega_m),\ \Omega_m \text{ constant},\\&B_f=(0,0,B_f),\ B_f \text{ constant},\\&B_m=(0,0,B_m),\ B_m \text{ constant}\end{aligned} \quad (23)$$

and with the boundary conditions

$$\begin{aligned}&T_f(d_f)=T_u,T_m(-d_m)=T_l,\\&S_f(d_f)=S_u,S_m(-d_m)=S_l\end{aligned} \quad (24)$$

and the interface conditions

$$\begin{aligned}&T_f(0)=T_m(0),k_f\frac{\partial T_f}{\partial x_3}(0)=k_m\frac{\partial T_m}{\partial x_3}(0),\\&S_f(0)=S_m(0),D_f\frac{\partial S_f}{\partial x_3}(0)=D_m\frac{\partial S_m}{\partial x_3}(0),\\&P_f(0)=P_m(0)\end{aligned} \quad (25)$$

the equilibrium temperature field, hydrostatic pressure and salt concentration in the fluid layer and porous medium layer are respectively

$$\begin{aligned}&T_f=T_0-(T_0-T_u)\frac{x_3}{d_f},P_f=P_f(x_3),\\&S_f=S_0-(S_0-S_u)\frac{x_3}{d_f},0\le x_3\le d_f,\\&T_m=T_0-(T_l-T_0)\frac{x_3}{d_m},P_m=P_m(x_3),\\&S_m=S_0-(S_l-S_0)\frac{x_3}{d_m},-d_m\le x_3\le 0\end{aligned} \quad (26)$$

where $T_0=\frac{k_f d_m T_u+k_m d_f T_l}{k_f d_m+k_m d_f},S_0=\frac{D_f d_m S_u+D_m d_f S_l}{D_f d_m+D_m d_f}$.

3. Perturbed Equations

We apply the perturbation by following linear perturbation quantities

$$\begin{aligned}&V_f=0+\varepsilon v_f,P_f=P_f(x_3)+\varepsilon p_f,\\&T_f=T_0-(T_0-T_u)\frac{x_3}{d_f}+\varepsilon\theta_f,\\&S_f=S_0-(S_0-S_u)\frac{x_3}{d_f}+\varepsilon s_f,\\&J_f=0+\varepsilon j_f,B_f=B_f\ e_3+\varepsilon b_f,\\&V_m=0+\varepsilon v_m,P_m=P_m(x_3)+\varepsilon p_m,\\&T_m=T_0-(T_l-T_0)\frac{x_3}{d_m}+\varepsilon\theta_m,\\&S_m=S_0-(S_l-S_0)\frac{x_3}{d_m}+\varepsilon s_m,\\&J_m=0+\varepsilon j_m,B_m=B_m\ e_3+\varepsilon b_m,\end{aligned} \quad (27)$$

to the governing equations in the fluid layer and porous medium layer respectively and to the boundary conditions. After perturbation, the non-dimensionlisation will be apply by using

$$\begin{aligned}&t=\frac{d_f^2}{\lambda_f}t^{\bullet}{}_f,v_f=\frac{v}{d_f}v^{\bullet}{}_f,\\&x=d_f x^{\bullet}{}_f,p_f=\frac{\rho_0 v^2}{d_f^2}p^{\bullet}{}_f,\\&s_f=\frac{|S_0-S_u|v}{D_f}s^{\bullet}{}_f,\theta_f=\frac{|T_0-T_u|v}{\lambda_f}\theta^{\bullet}{}_f,\\&b_f=\frac{\rho_0\mu_{mf}v^2}{B_f d_f^2}b^{\bullet}{}_f,j_f=\frac{\rho_0 v^2}{B_f d_f^3}j^{\bullet}{}_f\end{aligned} \quad (28)$$

for the fluid layer, and by using

$$\begin{aligned}&x=d_m x^{\bullet}{}_m,v_m=\frac{v}{d_m}v^{\bullet}{}_m,\\&t=\frac{d_m^2}{\lambda_m}t^{\bullet}{}_m,p_m=\frac{\rho_0 v^2}{K}p^{\bullet}{}_m,\\&\theta_m=\frac{|T_l-T_0|v}{\lambda_m}\theta^{\bullet}{}_m,s_m=\frac{|S_l-S_0|v}{D_m}s^{\bullet}{}_m\\&j_m=\frac{\rho_0 v^2}{B_m d_m^3}j^{\bullet}{}_m,b_m=\frac{\rho_0\mu_{mm}v^2}{B_m d_m^2}b^{\bullet}{}_m\end{aligned} \quad (29)$$

for the porous medium layer, here $\lambda_m=k_m/(\rho c_p)_f$ and $\lambda_f=k_f/(\rho c_p)_f$ are the thermal diffusivity of the fluid phase and porous medium respectively ,then the Equations (10)-(14) becomes

$$\frac{1}{P_{r_f}}\frac{\partial v_f}{\partial t_f} = -\nabla p_f + \nabla^2 v_f + Rt_f\theta_f - Rs_f s_f + Ta_f\left(v_f \times e_3\right) + \frac{\partial b_f}{\partial x_3} \tag{30}$$

$$\frac{\partial \theta_f}{\partial t_f} + F_T v_f = \nabla^2\theta_f \tag{31}$$

$$\frac{1}{Le_f}\frac{\partial s_f}{\partial t_f} + F_S v_f = \nabla^2 s_f \tag{32}$$

$$J_f = \text{curl}\, b_f \tag{33}$$

$$\frac{1}{P_{m_f}}\frac{\partial b_f}{\partial t_f} = Q_f\frac{\partial v_f}{\partial x_3} + \nabla^2 b_f \tag{34}$$

where e_3 is the unit vector in the x_3-direction and $P_{r_f}, Rt_f, Rs_f, Ta_f, Q_f, Le_f$ and P_{m_f} are non-dimensional numbers denote the viscous Prandtl number, thermal Rayleigh number, salt Rayleigh number, Taylor number, Chandraskhar number, Lewis number and magnetic Prandtl number of the fluid layer and given by

$$P_{r_f} = \frac{v}{\lambda_f}, Rt_f = \frac{g\alpha\left|T_0 - T_u\right|d_f^3}{v\lambda_f}, Rs_f = \frac{g\beta\left|S_0 - S_u\right|d_f^3}{vD_f},$$
$$Ta_f = \frac{2\Omega_f d_f^2}{v}, Q_f = \frac{d_f^2 B_f^2}{\mu_{mf}\rho_0 v\eta_f},$$
$$Le_f = \frac{D_f}{\lambda_f}, P_{m_f} = \frac{\eta_f}{\lambda_f}$$

and the Equations (15)-(19) becomes

$$\frac{Da}{\varphi P_{r_m}}\frac{\partial v_m}{\partial t_m} = -\nabla p_m - v_m + Rt_m\theta_m - Rs_m s_m + Ta_m\left(v_m \times e_3\right) + Da\frac{\partial b_m}{\partial x_3} \tag{35}$$

$$G_m\frac{\partial \theta_m}{\partial t_m} + F_T v_m = \nabla^2\theta_m \tag{36}$$

$$\frac{\varphi}{Le_m}\frac{\partial s_m}{\partial t_m} + F_S v_m = \nabla^2 s_m \tag{37}$$

$$J_m = \text{curl}\, b_m \tag{38}$$

$$\frac{1}{P_{m_m}}\frac{\partial b_m}{\partial t_m} = Q_m\frac{\partial v_m}{\partial x_3} + \nabla^2 b_m \tag{39}$$

where $G_m = \left(\rho_0 c\right)_m \big/ \left(\rho_0 c_p\right)_f$ and Da, P_{rm}, Rt_m, Rs_m, Ta_m, Q_m, Le_m and P_{m_m} are non-dimensional numbers denote the Darcy number, viscous Prandtl number, thermal Rayleigh number, salt Rayleigh number, Taylor number, Chandarsekhar number, Lewis number and magnetic Prandtl number of the porous medium layer and given by

$$Da = \frac{K}{d_m^2}, P_{r_m} = \frac{v}{\lambda_m}, Rt_m = \frac{g\alpha\left|T_l - T_0\right|Kd_m}{v\lambda_m},$$
$$Rs_m = \frac{g\beta\left|S_l - S_0\right|Kd_m}{vD_m}, Ta_m = \frac{2\Omega_m K}{v},$$
$$Q_m = \frac{d_m^2 B_m^2}{\mu_{mm}\rho_0 v\eta_m}, Le_m = \frac{D_m}{\lambda_m}, P_{m_m} = \frac{\eta_m}{\lambda_m}$$

and where

$$F_T = \frac{-\left(T_0 - T_u\right)}{\left|T_0 - T_u\right|} = \frac{-\left(T_l - T_0\right)}{\left|T_l - T_0\right|} = \begin{cases} -1, & \text{when heating from below,} \\ 1, & \text{when heating from above,} \end{cases}$$

$$F_S = \frac{-\left(S_0 - S_u\right)}{\left|S_0 - S_u\right|} = \frac{-\left(S_l - S_0\right)}{\left|S_l - S_0\right|} = \begin{cases} -1, & \text{when salt concentration from below,} \\ 1, & \text{when salt concentration from above.} \end{cases}$$

The boundary conditions (20)-(22) becomes

$$\begin{aligned}
& w_f(1) = 0, \frac{\partial w_f}{\partial x_3}(1) = 0, \theta_f(1) = 0, \\
& s_f(1) = 0, \xi_{3_f}(1) = 0, \\
& \frac{\partial b_f}{\partial x_3}(1) = 0, j_{3_f}(1) = 0, \\
& \frac{\partial \theta_f}{\partial x_3}(0) = \varepsilon_T\frac{\partial \theta_m}{\partial x_3}(0), \frac{\partial s_f}{\partial x_3}(0) = \varepsilon_S\frac{\partial s_m}{\partial x_3}(0), \\
& w_f(0) = \hat{d}w_m(0), u_f(0) = \hat{d}u_m(0), v_f(0) = \hat{d}v_m(0), \\
& b_f(0) = \frac{\hat{d}^2}{\hat{m}}b_m(0), \frac{\partial b_f}{\partial x_3}(0) = \frac{\hat{d}^3}{\hat{m}\varepsilon_T}\frac{\partial b_m}{\partial x_3}(0), \\
& j_f(0) = \frac{\hat{d}^2}{\hat{m}}j_m(0), \frac{\partial j_f}{\partial x_3}(0) = \frac{\hat{d}^3}{\hat{m}\varepsilon_T}\frac{\partial j_f}{\partial x_3}(0), \\
& p_f(0) - 2\frac{\partial w_f}{\partial x_3}(0) = \frac{\hat{d}^2}{Da}p_m(0) - 2\hat{d}^2\frac{\partial w_m}{\partial x_3}(0), \\
& \frac{\partial u_f}{\partial x_3}(0) = \hat{d}^2\frac{\partial u_f}{\partial x_3}(0), \frac{\partial v_f}{\partial x_3}(0) = \hat{d}^2\frac{\partial v_f}{\partial x_3} \\
& w_m(-1) = 0, \frac{\partial w_m}{\partial x_3}(-1) = 0, \\
& \theta_m(-1) = 0, s_m(-1) = 0, \xi_{3_m}(-1) = 0, \\
& \frac{\partial b_m}{\partial x_3}(1) = 0, j_{3_m}(1) = 0
\end{aligned} \tag{40}$$

where $\hat{d}$, $\hat{n},\hat{m},\varepsilon_T,\varepsilon_S,\gamma_T$ and are given by

$$\hat{d}=\frac{d_f}{d_m},\hat{n}=\frac{\Omega_f}{\Omega_m},\hat{m}=\frac{B_m}{B_f}$$

$$\gamma_T=\frac{|T_0-T_u|}{|T_l-T_0|}=\frac{\hat{d}}{\varepsilon_T},\gamma_S=\frac{|S_0-S_u|}{|S_l-S_0|}=\frac{\hat{d}}{\varepsilon_S},$$

$$\varepsilon_T=\frac{\lambda_f}{\lambda_m},\varepsilon_S=\frac{D_f}{D_m},$$

and

$$P_{r_f}=\frac{1}{\varepsilon_T}P_{r_m},Rt_f=\frac{\hat{d}^4}{\varepsilon_T^2 Da}Rt_m,Rs_f=\frac{\hat{d}^4}{\varepsilon_S^2 Da}Rs_m,$$

$$Ta_f=\frac{\hat{n}\hat{d}^2}{Da}Ta_m,Le_f=\frac{\varepsilon_S}{\varepsilon_T}Le_m.$$

4. The Linearized Equations

Linearization will be done by neglecting all products and powers (higher than the first) of the linear perturbation quantity, and by dropping the (•) superscript, then by taking the curl of the Equations (30) and (35) we obtain

$$\frac{1}{P_{r_f}}\frac{\partial\xi_f}{\partial t_f}=\nabla^2\xi_f+Rt_f\left(\nabla\times\theta_f\right)-Rs_f\left(\nabla\times s_f\right)+Ta_f\frac{\partial v_f}{\partial x_3}+\frac{\partial J_f}{\partial x_3}\tag{41}$$

$$\frac{Da}{\varphi P_{r_m}}\frac{\partial\xi_m}{\partial t_m}=-\xi_m+Da\nabla^2\xi_m+Rt_m\left(\nabla\times\theta_m\right)-Rs_m\left(\nabla\times s_m\right)+Ta_m\frac{\partial v_m}{\partial x_3}+Da\frac{\partial J_m}{\partial x_3}\tag{42}$$

if return to the original Equations (30) and (35), but in this case we take the (curl curl). Thus

$$\frac{1}{P_{r_f}}\frac{\partial}{\partial t_f}\nabla^2 v_f=\nabla^4 v_f-Rt_f\left(\nabla\frac{\partial\theta_f}{\partial x_3}-\nabla^2\theta_f e_3\right)+Rs_f\left(\nabla\frac{\partial s_f}{\partial x_3}-\nabla^2 s_f e_3\right)-Ta_f\frac{\partial\xi_f}{\partial x_3}+\nabla^2\frac{\partial b_f}{\partial x_3}\tag{43}$$

$$\frac{Da}{\varphi P_{r_m}}\frac{\partial}{\partial t_m}\nabla^2 v_m=\nabla^2 v_m+Da\nabla^4 v_m-Rt_m\left(\nabla\frac{\partial\theta_m}{\partial x_3}-\nabla^2\theta_m e_3\right)+Rs_m\left(\nabla\frac{\partial s_m}{\partial x_3}-\nabla^2 s_m e_3\right)-Ta_m\frac{\partial\xi_m}{\partial x_3}+Da\nabla^2\frac{\partial b_m}{\partial x_3}\tag{44}$$

and if we use the curl of Equations (34) and (39) with using (33) and (38), we obtain

$$\frac{1}{P_{m_f}}\frac{\partial J_f}{\partial t_f}=Q_f\frac{\partial\xi_f}{\partial x_3}+\nabla^2 J_f\tag{45}$$

$$\frac{1}{P_{m_m}}\frac{\partial J_m}{\partial t_m}=Q_m\frac{\partial\xi_m}{\partial x_3}+\nabla^2 J_m\tag{46}$$

Now, the third components of Equations (31), (32), (34), (36), (37), (39) and (41)-(46) are

$$\frac{1}{P_{r_f}}\frac{\partial\xi_{3_f}}{\partial t_f}=\nabla^2\xi_{3_f}+Ta_f\frac{\partial w_f}{\partial x_3}+\frac{\partial j_{3_f}}{\partial x_3}\tag{47}$$

$$\frac{1}{P_{r_f}}\frac{\partial}{\partial t_f}\nabla^2 w_f=\nabla^4 w_f+Rt_f\nabla_2^2\theta_f-Rs_f\nabla_2^2 s_f-Ta_f\frac{\partial\xi_{3_f}}{\partial x_3}+\nabla^2\frac{\partial b_f}{\partial x_3}\tag{48}$$

$$\frac{\partial\theta_f}{\partial t_f}+F_T w_f=\nabla^2\theta_f\tag{49}$$

$$\frac{1}{Le_f}\frac{\partial s_f}{\partial t_f}+F_S w_f=\nabla^2 s_f\tag{50}$$

$$\frac{1}{P_{m_f}}\frac{\partial b_f}{\partial t_f}=Q_f\frac{\partial w_f}{\partial x_3}+\nabla^2 b_f\tag{51}$$

$$\frac{1}{P_{m_f}}\frac{\partial j_{3_f}}{\partial t_f}=Q_f\frac{\partial\xi_{3_f}}{\partial x_3}+\nabla^2 j_{3_f}\tag{52}$$

$$\frac{Da}{\varphi P_{r_m}}\frac{\partial\xi_{3_m}}{\partial t_m}=-\xi_{3m}+Da\nabla^2\xi_{3f}+Ta_m\frac{\partial w_m}{\partial x_3}+Da\frac{\partial j_{3_m}}{\partial x_3}\tag{53}$$

$$\frac{Da}{\varphi P_{r_m}}\frac{\partial}{\partial t_m}\nabla^2 w_m=-\nabla^2 w_m+Da\nabla^4 w_m+Rt_m\nabla_2^2\theta_m-Rs_m\nabla_2^2 s_m-Ta_m\frac{\partial\xi_{3m}}{\partial x_3}+Da\nabla^2\frac{\partial b_{fm}}{\partial x_3}\tag{54}$$

$$G_m\frac{\partial\theta_m}{\partial t_m}+F_T w_m=\nabla^2\theta_m\tag{55}$$

$$\frac{\varphi}{Le_m}\frac{\partial s_m}{\partial t_m}+F_S w_m=\nabla^2 s_m\tag{56}$$

$$\frac{1}{P_{m_{fm}}}\frac{\partial j_{3_{mf}}}{\partial t_m}=Q_f\frac{\partial\xi_{3_f}}{\partial x_3}+\nabla^2 j_{3_{mf}}\tag{57}$$

$$\frac{1}{P_{m_m}}\frac{\partial b_m}{\partial t_m} = Q_f \frac{\partial w_m}{\partial x_m} + \nabla^2 b_m \tag{58}$$

where $\nabla_2^2 = \nabla^2 - \frac{\partial^2}{\partial x_3^2}$ is tow-dimensional Laplacian operator and $\nabla^4 = \left(\nabla^2\right)^2$. We apply the normal modes solution in the form

$$\Phi(x,t) = \Phi(x_3)\exp\left[i\left(nx_1 + mx_2\right) + \sigma\, t\right]$$

with the functions $w_f, \theta_f, s_f, \xi_f, j_f, b_f, w_m, \theta_m, s_m, \xi_m$, j_m and b_m. Thus the governing equations are

$$\frac{\sigma_f}{P_{r_f}} L_f w_f = L_f^2 w_f - a_f^2 Rt_f \theta_f + a_f^2 Rs_f s_f$$
$$-Ta_f D_f \xi_{3_f} + \frac{\sigma_f}{P_{m_m}} D_f b_f - Q_f D_f^2 w_f \tag{59}$$

$$\frac{\sigma_f}{P_{r_f}} \xi_{3_f} = L_f \xi_{3_f} + Ta_f D_f w_f + D_f j_{3_f} \tag{60}$$

$$\sigma_f \theta_f + F_T w_f = L_f \theta_f \tag{61}$$

$$\frac{\sigma_f}{Le_f} s_f + F_S w_f = L_f s_f \tag{62}$$

$$\frac{\sigma_f}{P_{m_f}} j_{3_f} = Q_f D_f \xi_{3_f} + L_f j_{3_f} \tag{63}$$

$$\frac{\sigma_f}{P_{m_f}} b_f = Q_f D_f w_f + L_f b_f \tag{64}$$

$$-\frac{Da}{\varphi P_{r_m}} \sigma_m\, L_m w_f = L_m w_m + a_m^2 Rt_m \theta_m - a_m^2 Rs_m s_m$$
$$+Ta_m D\xi_{3_m} - \frac{Da}{P_{m_m}} \sigma_m D_m b_m + DaQ_m D_m^2 w_m \tag{65}$$

$$-\frac{Da}{\varphi P_{r_m}} \sigma_m \xi_{3_m} = \xi_{3_m} - DaL_m \xi_{3_m}$$
$$-Ta_m Dw_m - Da D_m j_{3_m} \tag{66}$$

$$G_m \sigma_m\, \theta_m + F_T w_m = L_m \theta_m \tag{67}$$

$$\frac{\varphi}{Le_m} \sigma_m\, s_m + F_S w_m = L_m s_m \tag{68}$$

$$\frac{\sigma_m}{P_{m_m}} j_{3_f m} = Q_m D_m \xi_{3_{fm}} + L_f j_{3_m} \tag{69}$$

$$\frac{\sigma_m}{P_{m_m}} b_m = Q_m D_m w_m + L_m b_m \tag{70}$$

where $a_f = \sqrt{n_f^2 + m_f^2}$ and $a_m = \sqrt{n_m^2 + m_m^2}$ are non-dimensional wave numbers in the fluid layer and porous medium layer respectively, σ is the growth rate and

$$a_f = \hat{d} a_m, \sigma_f = \frac{\hat{d}^2}{\varepsilon_T} \sigma_m,$$

$$D_f = \frac{\partial}{\partial x_3}, x_3 \in [0,1], D_m = \frac{\partial}{\partial x_3}, x_3 \in [-1,0],$$

$$L_f = \left(D_f^2 - a_f^2\right) \text{ and } L_m = \left(D_m^2 - a_m^2\right).$$

The boundary conditions in the final form are

$$\left.\begin{array}{l} w_f = 0, D_f w_f = 0, \theta_f = 0, \\ s_f = 0, \xi_{3_f} = 0, \\ Db_f = 0, j_{3_f} = 0 \end{array}\right\} \text{ on } x_3 = 1 \tag{71}$$

$$\left.\begin{array}{l} \gamma_T \theta_f = \varepsilon_T \theta_m, \gamma_S s_f = \varepsilon_S s_m, \\ D_f \theta_f = \varepsilon_T D_m \theta_m, D_f s_f = \varepsilon_S D_m s_m, \\ w_f = \hat{d} w_m, D_f w_f = \hat{d}^2 D_m w_m, \\ D_f^2 w_f = \hat{d}^3 D_m^2 w_m, \\ \xi_{3_f} = \hat{d}^2 \xi_{3_m}, D_f \xi_{3_f} = \hat{d}^3 D_m \xi_{3_m}, \\ b_f = \frac{\hat{d}^2}{\hat{m}} b_m, D_f b_f = \frac{\hat{d}^3}{\hat{m}\varepsilon_T} D_m b_m, \\ j_{3_f} = \frac{\hat{d}^2}{\hat{m}} j_{3_m}, D_f b_f = \frac{\hat{d}^3}{\hat{m}\varepsilon_T} D_m j_{3_m}, \\ D_f^3 w_f + 3a_f^2 D_f w_f - \frac{\sigma_f}{P_{r_f}} D_f w_f - Ta_f \xi_{3_f} - D_f^2 b_f \\ = -\frac{\hat{d}^4}{Da}\left(\frac{Da}{\varphi P_{r_m}} \sigma_m + 1 - 3a_m^2 Da - DaD_m^2 w_m \right. \\ \left. + \hat{d}^4 Da\, D_m^2 b_m\right) D_m w_m - \frac{\hat{d}^4}{Da} Ta_m \xi \end{array}\right\} \text{on } x_3 = 0 \tag{72}$$

$$\left.\begin{array}{l} w_m = 0, D_m w_m = 0, \theta_m = 0 \\ s_m = 0, \xi_{3_m} = 0, \\ Db_m = 0, j_{3_m} = 0 \end{array}\right\} \text{ on } x_3 = -1 \tag{73}$$

5. Numerical Solution

A Legendre polynomials (see Bukhari [5]) is applied to solve the Equations (59)-(70) with the relevant boundary conditions (71)-(73), and we map $x_3 \in [0,1]$ and $x_3 \in [-1,0]$ into $z \in [-1,1]$ by the transformations $z = 2x_3 - 1$ and $z = 2x_3 + 1$ respectively, and get

$$\frac{\partial}{\partial x_3} = 2\frac{\partial}{\partial z}, \text{ thus } D_f = D_m = 2\frac{\partial}{\partial z} = D,\ z \in [-1,1].$$

then, suppose that

$$y_r(z)=\sum_{k=0}^{M-1}\alpha_{kr}P_k(z), 1\le r\le 28 \quad z\in[-1,1]$$

let the variables y_r where $1\le r\le 28$ be defined by

$$y_1=w_f, y_2=D_f w_f, y_3=D_f^2 w_f, y_4=D_f^3 w_f,$$
$$y_5=\xi_{3_f}, y_6=D_f\xi_{3_f}, y_7=\theta_f, y_8=D_f\theta_f,$$
$$y_9=s_f, y_{10}=D_f s_f, y_{11}=b_f, y_{12}=Db_f$$
$$y_{13}=J_f, y_{14}=DJ_f$$
$$y_{15}=w_m, y_{16}=D_m w_m, y_{17}=D_m^2 w_m, y_{18}=D_m^3 w_m,$$
$$y_{19}=\xi_{3_m}, y_{20}=D_m\xi_{3_m}, y_{21}=\theta_m, y_{22}=D_m\theta_m,$$
$$y_{23}=s_m, y_{24}=D_m s_m, y_{25}=b_m, y_{26}=Db_m$$
$$y_{27}=J_m, y_{28}=DJ_m.$$

Then the Equations (59)-(70) can be rewritten in a system of twenty ordinary differential equations of first order, since $D_f=D_m=D$ and if we put $\sigma_m=\sigma$ then $\sigma_f=\frac{\hat{d}^2}{\varepsilon_T}\sigma$ so the eigenvalue problem can be reformulated in the form

$$\frac{\mathrm{d}Y}{\mathrm{d}z}=\boldsymbol{A}Y+\sigma\boldsymbol{B}Y, \quad z\in[-1,1]$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are real 28 × 28 matrices. The final eigenvalue problem reduces to $\boldsymbol{E}$V = σ$\boldsymbol{F}$V where matrices $\boldsymbol{E}$ and $\boldsymbol{F}$ have the block forms. The boundary conditions replace the 1 *M*th, 2 *M*th, ⋯ 28 *M*th rows of *E* and *F*.

6. Results and Remarks

Using Legendre polynomials, the eigenvalue problems (59)-(70) with the boundary conditions (71)-(73) are transformed to a system of fourteen ordinary differential equations of first order in the fluid layer and a system of fourteen ordinary differential equations of first order in the porous layer with twenty eight boundary conditions. In this work, we will discuss the numerical results through two cases—when the heat and salt concentration affected from above and below.

Case (1): the heat and the salt concentration affected from above.

Here, we put F_T = 1, F_S = 1and the value of the initial salt Rayleigh number of the porous medium Rs_m = 5000 to find the thermal Rayleigh number of the porous medium Rt_m corresponding to the wave numbers a_m for the different values of Ta_m, Q_m, Da_m, $\hat{d}$ P_{r_f}, Le_f and P_{m_f}. In this case, the eigenvalues are real, and thus the stationary instability happens, as shown in the following **Tables 1-4** and **Figures 2-5**. Therefore, we concluded that:

Table 1. The relation between a_m and Rt_m for different values of Ta_m when $\hat{d}=0.01$, Da_m = 0.0003, Q_m = 100, $P_{r_f}=Le_f=1$ and $P_{m_f}=3$.

a_m	Rt_m		
	$Ta_m=5$	$Ta_m=10$	$Ta_m=15$
1	1411.452	4404.851	15442.275
2	3791.053	1926.006	1612.915
3	4244.928	3112.021	962.857
4	4407.321	3531.430	1870.107
5	4480.673	3724.639	2290.704
6	4516.271	3826.280	2517.540
7	4532.339	3883.113	2651.554
8	4536.749	3914.926	2735.177
9	4533.343	3931.247	2788.741
10	4524.163	3937.112	2822.960
11	4510.357	3935.370	2843.926
12	4492.585	3927.696	2855.233
13	4471.240	3915.128	2859.100
14	4446.537	3898.286	2856.979
15	4418.593	3877.578	2849.797
16	4387.447	3853.229	2838.122
17	4353.093	3825.368	2822.374
18	4315.484	3794.040	2802.753
19	4274.550	3759.245	2779.416
20	4230.195	3720.946	2752.387

Table 2. The relation between a_m and Rt_m for different values of Q_m when Ta_m = 5, $\hat{d}=0.01$, Da_m = 0.0003, $P_{r_f}=Le_f=1$ and $P_{m_f}=3$.

a_m	Rt_m			
	$Q_m=100$	$Q_m=200$	$Q_m=300$	$Q_m=500$
1	1411.45	1404.08	2383.89	1973.94
2	3791.05	3792.31	4113.01	4043.10
3	4244.92	4247.22	4444.95	4428.29
4	4407.32	4409.66	4563.75	4561.69
5	4480.67	4482.88	4616.37	4619.42
6	4516.27	4518.33	4640.33	4645.42
7	4532.33	4534.26	4649.09	4655.00
8	4536.74	4538.55	4648.52	4654.70
9	4533.34	4535.04	4641.48	4647.67
10	4524.16	4525.77	4629.49	4635.56
11	4510.35	4511.89	4613.41	4619.30
12	4492.58	4494.05	4593.73	4599.37
13	4471.24	4472.63	4570.70	4576.11
14	4446.53	4447.87	4544.49	4549.64
15	4418.59	4419.87	4515.16	4520.04
16	4387.44	4388.67	4482.70	4487.33
17	4353.09	4354.26	4447.09	4451.48
18	4315.48	4316.60	4408.27	4412.42
19	4274.55	4275.62	4366.16	4370.05
20	4230.19	4231.21	4320.64	4324.32

Table 3. The relation between a_m and Rt_m for different values of Da_m when Q_m = 100, Ta_m = 5, $\hat{d} = 0.01$, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

a_m	Rt_m		
	$Da_m = 0.000003$	$Da_m = 0.00003$	$Da_m = 0.003$
1	2199.443	2261.610	3012.461
2	4045.497	4064.695	4294.012
3	4400.439	4411.751	4544.738
4	4527.884	4536.455	4636.163
5	4584.631	4591.995	4676.610
6	4610.809	4617.560	4693.868
7	4620.854	4627.285	4697.788
8	4621.115	4627.358	4692.409
9	4614.721	4620.852	4679.407
10	4603.393	4609.440	4659.348
11	4588.100	4594.085	4632.211
12	4569.475	4575.375	4597.615
13	4547.890	4553.683	4554.980
14	4523.591	4529.256	4503.524
15	4496.759	4502.256	4442.376
16	4467.546	4472.792	4370.544
17	4436.022	4440.952	4286.951
18	4402.228	4406.774	4190.427
19	4366.225	4370.297	4079.744
20	4328.078	4331.551	3953.576

Table 4. The relation between a_m and Rt_m for different values of $\hat{d}$ when Q_m = 100, Da_m = 0.0003, Ta_m = 5, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

a_m	Rt_m			
	$\hat{d} = 0.005$	$\hat{d} = 0.1$	$\hat{d} = 0.2$	$\hat{d} = 0.3$
1	2490.38	2172.34	2075.69	2037.71
2	4160.48	3774.26	3637.21	3590.73
3	4474.34	4210.74	4146.82	4136.76
4	4583.48	4411.01	4383.17	4382.40
5	4630.30	4513.35	4500.16	4500.78
6	4650.60	4567.15	4560.02	4560.55
7	4656.94	4594.47	4590.02	4590.30
8	4654.70	4605.96	4602.80	4602.90
9	4646.47	4607.13	4604.62	4604.60
10	4633.62	4600.96	4598.81	4598.71
11	4616.88	4589.15	4587.20	4587.04
12	4596.69	4572.68	4570.87	4570.64
13	4573.27	4552.16	4550.43	4550.15
14	4546.74	4527.94	4526.26	4525.93
15	4517.15	4500.22	4498.58	4498.20
16	4484.48	4469.11	4467.49	4467.07
17	4448.69	4434.63	4433.04	4432.57
18	4409.72	4396.79	4395.20	4394.70
19	4367.48	4355.50	4353.93	4353.40
20	4321.85	4310.72	4309.19	4308.56

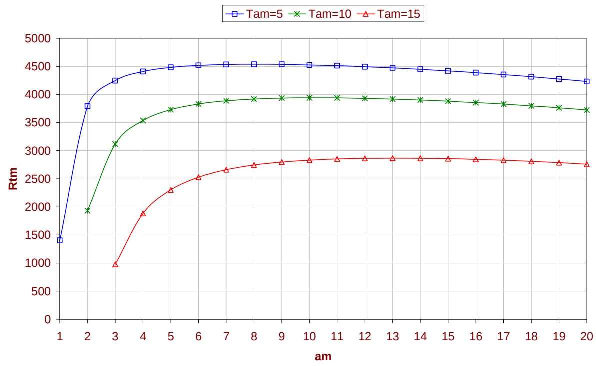

Figure 2. The relation between a_m and Rt_m for different values of Ta_m when $\hat{d} = 0.01$, Da_m = 0.0003, Q_m = 100, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

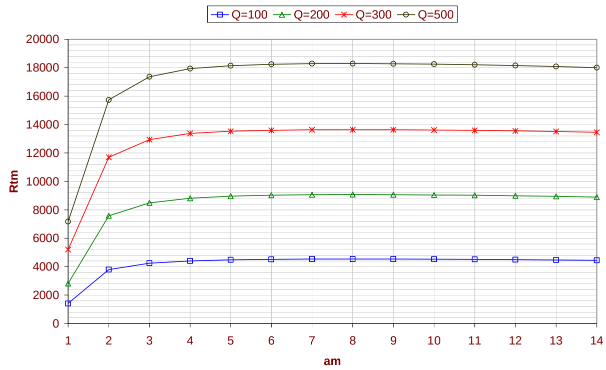

Figure 3. The relation between a_m and Rt_m for different values of Q_m when Ta_m = 5, $\hat{d} = 0.01$, Da_m = 0.0003, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

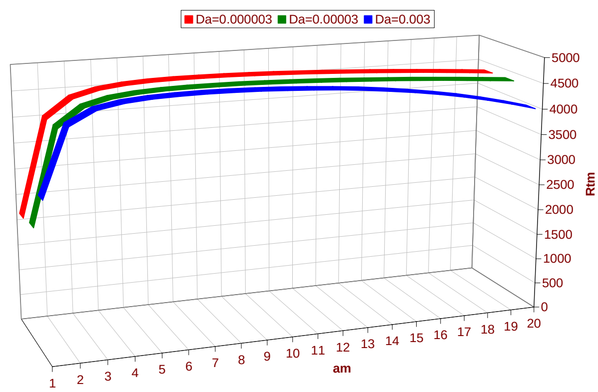

Figure 4. The relation between a_m and Rt_m for different values of Da_m when Q_m = 100, Ta_m = 5, $\hat{d} = 0.01$, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

- As Ta_m increases Rt_m decreases which means that the increase of the rotation causes the increase of the thermal convections, leading to an increase in the instability of the fluid, as shown in **Figure 2** and its numerical results in **Table 1**.

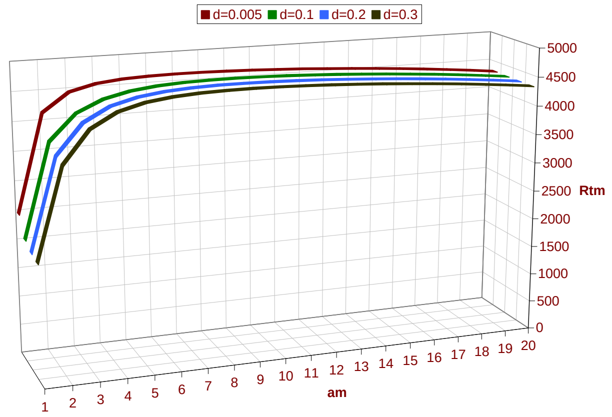

Figure 5. The relation between a_m and Rt_m for different values of $\hat{d}$ when Q_m = 100, Da_m = 0.0003, Ta_m = 5, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

- The presence of the linear magnetic field helps reduce the currents of the thermal convections, meaning that the stability will increase in the fluid, as shown in **Figure 3** and its numerical results in **Table 2**.
- The fluid becomes unstable when the permeability of the porous medium increases, as shown in **Figure 4** and its numerical results in **Table 3**.
- The increase of depth ratio between the two layers makes the fluid unstable meaning that as $\hat{d}$ increases Rt_m decreases, as shown in **Figure 5** and its numerical results in **Table 4**.

Case (2): the heat and the salt concentration affected from below.

Here, we put $F_T = -1, F_S = -1$ and the value of the initial salt Rayleigh number of a porous medium Rs_m = 10000, to find the thermal Rayleigh numbers of a porous medium, Rt_m corresponding to wave numbers, a_m for different values of Ta_m, Q_m, Da_m, $\hat{d}$ P_{r_f}, Le_f and P_{m_f}. In this case, the eigenvalues are complex, and thus the overstability happens, as shown in the following **Tables 5-8** and **Figures 6-9**. Therefore, we got the following results:

- The increase of the rotation helps reduce the currents of the thermal convections meaning that the stability will increase in the fluid, as displayed in **Figure 6** and its numerical results in **Table 5**.
- The presence of the linear magnetic field makes the fluid more stable as displayed in **Figure 7** and its numerical results in **Table 6**.
- As Da_m increases Rt_m decreases which means that the increase of the permeability causes the increase of the thermal convections, leading to an increase in the instability of the fluid, as shown in **Figure 8** and its numerical results in **Table 7**.
- As $\hat{d}$ increase Rt_m increases meaning that the in-

Table 5. The relation between a_m and Rt_m for different values of Ta_m when $\hat{d} = 0.2$, Q_m = 100, Da_m = 0.0003, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

a_m	Rt_m		
	$Ta_m = 0.5$	$Ta_m = 0.75$	$Ta_m = 1$
1	307.785	349.180	395.390
2	223.032	241.258	262.237
3	201.231	213.226	228.383
4	190.352	198.527	209.907
5	184.497	190.068	198.264
6	182.855	186.774	192.528
7	185.456	188.387	192.523
8	192.435	194.781	197.972
9	203.924	205.910	208.575
10	220.055	221.804	224.162
11	240.976	242.559	244.717
12	266.869	268.330	270.341
13	297.955	299.322	301.217
14	334.500	335.792	337.593
15	376.815	378.047	379.769

Table 6. The relation between a_m and Rt_m for different values of Q_m when Ta_m = 0.5, $\hat{d} = 0.2$, Da_m = 0.001, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

a_m	Rt_m		
	$Q_m = 10$	$Q_m = 50$	$Q_m = 100$
1	300.107	303.729	307.785
2	221.652	221.929	223.032
3	199.023	199.797	201.231
4	187.148	188.482	190.352
5	181.057	182.540	184.497
6	179.641	181.046	182.855
7	182.618	183.871	185.456
8	189.975	191.069	192.435
9	201.789	202.743	203.924
10	218.181	219.020	220.055
11	239.307	240.056	240.976
12	265.357	266.035	266.869
13	296.563	297.187	297.955
14	333.199	333.781	334.500
15	375.583	376.134	376.815

Table 7. The relation between a_m and Rt_m for different values of Da_m when Ta_m = 0.5, Q_m = 50, $\hat{d} = 0.2$, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

a_m	Rt_m		
	$Da_m = 0.00001$	$Da_m = 0.0001$	$Da_m = 0.001$
1	331.794	326.913	307.785
2	245.825	243.922	223.032
3	232.947	230.599	201.231
4	233.441	229.691	190.352
5	239.325	233.587	184.497
6	248.566	240.404	182.855
7	260.452	249.527	185.456
8	274.684	260.736	192.435
9	291.119	273.963	203.924
10	309.681	289.209	220.055
11	330.329	306.512	240.976
12	353.042	325.946	266.869
13	377.805	347.627	297.955
14	402.569	371.724	334.500
15	427.332	398.437	376.815

Table 8. The relation between a_m and Rt_m for different values of $\hat{d}$ when Ta_m = 0.5, Da_m = 0.001, Q_m = 100, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

a_m	Rt_m	
	$\hat{d} = 0.02$	$\hat{d} = 0.2$
1	325.430	307.785
2	225.413	223.032
3	203.245	201.231
4	192.852	190.352
5	187.424	184.497
6	186.039	182.855
7	188.738	185.456
8	195.706	192.435
9	207.121	203.924
10	223.150	220.055
11	243.962	240.976
12	269.748	266.869
13	300.736	297.955
14	337.192	334.500
15	379.428	376.815

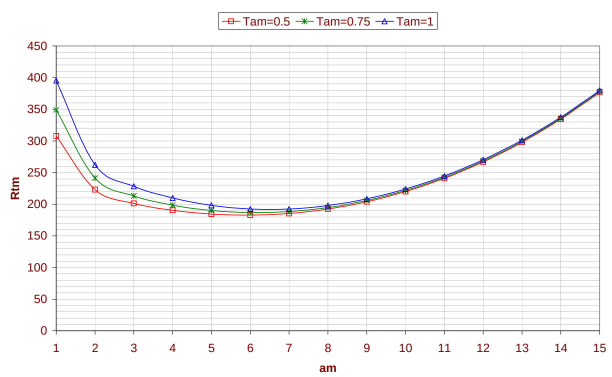

Figure 6. The relation between a_m and Rt_m for different values of Ta_m when $\hat{d} = 0.2$, Q_m = 100, Da_m = 0.0001, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

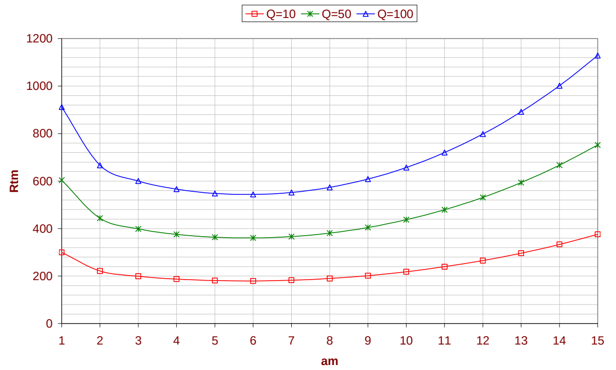

Figure 7. The relation between a_m and Rt_m for different values of Q_m when Ta_m = 0.5, $\hat{d} = 0.2$, Da_m = 0.001, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

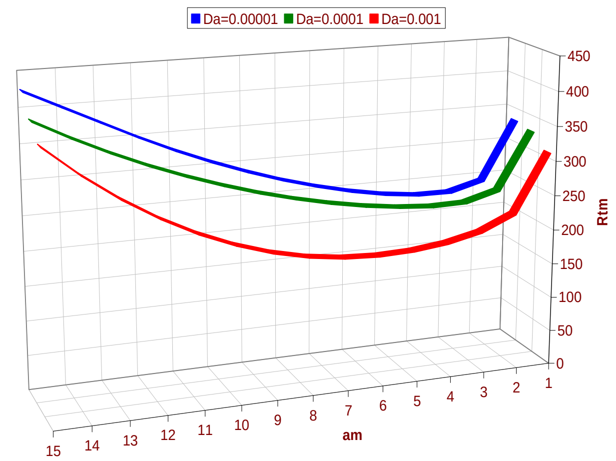

Figure 8. The relation between a_m and Rt_m for different values of Da_m when Ta_m = 0.5, Q_m = 50, $\hat{d} = 0.2$, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

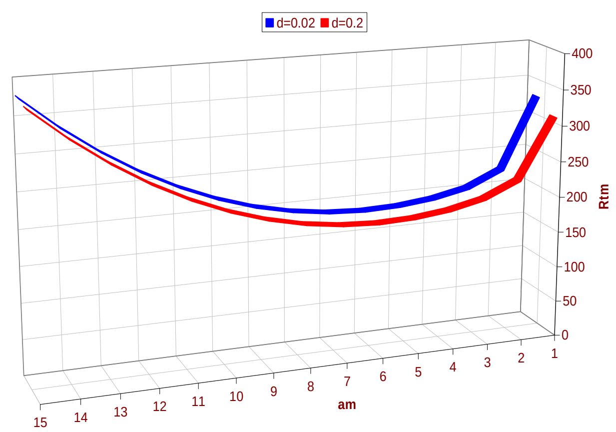

Figure 9. The relation between a_m and Rt_m for different values of $\hat{d}$ when Ta_m = 0.5, Da_m = 0.001, Q_m = 100, $P_{r_f} = Le_f = 1$ and $P_{m_f} = 3$.

crease of the depth ratio between the two layers makes the fluid unstable, as displayed in **Figure 9** and its numerical results in **Table 8**.

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