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The Law of Conservation of Momentum and the Contribution of No Potential Forces to the Equations for Continuum Mechanics and Kinetics

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Abstract

The most common systems are open non-stationary systems. From the previously formulated equations and some experiments, the connection between the gradients of physical quantities and the moment of momentum (force) is traced. The article investigates this trace. The use of the Hamiltonian formalism and the dependence of the force only on the distance between particles limit the study. In the collision integral, for example, for a rarefied gas, the Lennard-Jones potential is often used, which is not of the type considered. The foregoing forces us to turn to the study of the influence of forces of a more general form on the equations of mechanics. Hamilton's formalism traces the behavior of closed systems. The general form of boundary conditions and forces changes the theory proposed in the works by N.N. Bogolyubov. The results of the reformulation are discussed. Even in classical theory, after taking moments, we arrive at Boltzmann's theory at no symmetric stress tensor. The symmetric tensor is obtained after the assumption of a small effect of no symmetry and from the condition of the balance of forces. The requirement of simultaneous fulfillment of the laws of conservation of forces and moments of forces leads to the existence of two solutions. To take into account the moment, in addition to the conditions for the equilibrium of forces, the law of equilibrium of the moments of forces is required in the calculations. From it, the degree of no symmetry of the stress tensor is determined. The work illustrates the contribution of the distributed moment of force to the problems of continuum mechanics and the kinetic theory. Examples of the solution to the problem of fluid mechanics, the theory of elasticity and kinetic theory are given. A correspondence is established between the terms of the Liouville equation with more general and traditional forces. Previously, the influence of moments in boundary layer problems, jet problems, and the simplest problems of elasticity theory was considered. The work proves the important role of distributed moments in the formation of new structures.

Keywords

Angular Momentum, Non-Symmetrical Stress Tensor, Hamiltonian Formalism

1. Introduction

The most common systems are open non-stationary systems. The accumulated experimental facts led to the hypothesis of the importance of spatial gradients. From the record of the modified equations and some experiments, the connection between the gradients of physical quantities and the moment under consideration is traced. The original (classical) formulation of the conservation laws was based on the closed elementary volumes. Being an open system, the elementary volume exchanges components of physical quantities not only along the normal, but also along the tangential direction. It is hardly possible to use a stream tube for a turbulent flow regime when deriving the continuity equation. Mathematical modeling of such objects causes difficulties since their description is represented by a system of non-linear non-stationary equations. An important component of research is the choice of model. Previous works have shown that classical models do not include one of the most important laws—the law of conservation of angular momentum, if the moment does not act as a given force effect [1]-[6]. When the action of the distributed moment is taken into account, the system of equations is supplemented by the moment conservation law. In classical theory, since the symmetry of the stress tensor is postulated, the law is identically satisfied due to the vanishing of the sum of stresses under equilibrium conditions. The paper shows that no symmetric tensor corresponds to two types of solutions. As a rule, the analysis of the mathematical properties of such systems is performed without analyzing the role of the initial and boundary conditions. So, for example, it is difficult to choose a problem that satisfies the group properties of the equations of gas dynamics, elasticity theory, kinetic equations, etc. The self-similar solutions obtained from the stretch group are, in fact, a consequence of singular initial conditions. In practice, the required conditions can rarely be met. Thus, the number of groups used in practice is limited not only by the properties of the equations but also by the initial and boundary conditions. The same is true for closed and open systems. The performed mathematical analysis of the equations of continuum mechanics with the no symmetric stress tensor showed that in the plane case for four unknowns in the classical formulation, we have three equations: two equations from the stress equilibrium condition and one equation—the moment equilibrium condition. Thus, we need to close the problem with an additional condition. In the classical version, such a condition is the condition of symmetry of the stress tensor. From the stretch of pressure, both from the classical Boltzmann equation and the modified one, it does not follow that the hydrostatic pressure is one-third of the sum of the pressures on the coordinate areas. Using Pascal's law for equilibrium, we choose a pressure equal to one-third of the pressures on the coordinate sites. However, the theory remains the same when determining different pressures at each of the sites, *i.e.* p_x, p_y, p_z. The use of one pressure is possible under equilibrium conditions (Pascal's law), but for non-equilibrium conditions, the fact is not obvious. Neglecting the integral term when taking integrals by parts (the Ostrogradsky-Gauss theorem) is possible only for slow laminar flows. The pressure difference is indicated by the analysis of the determination of pressures when comparing the results of its determination through potential velocities by substitution into the Euler equations.

Writing separately the laws of equilibrium for forces and separately for moments of forces without taking into account the mutual influence, although the moment creates an additional force, we come to the conclusion about the symmetry of the stress tensor. If we consider equal pressures in different directions, we lose the moment of force but the pressure gradient is a force. Analyzing the results of solving the Euler equations [3] [4] and calculating potential flows, we obtain a vortex sheet, which indicates the existence of a moment. The numerical results were processed by the authors without averaging the values over the sides of the elementary volume. An analysis of the correspondence between the solutions for the potential flow and for the Euler equations indicates their no coincidence. In stochastic processes of open systems, the motion of fast molecules is accompanied by a change in the position of the center of inertia, which is accompanied by the appearance of a moment. The appearance of torque leads to a change in the direction of velocities and the formation of local structures. It can be assumed that the moment will make a significant contribution to the equation of the state of liquids and gases at high pressures (virial equation) since even for equilibrium distribution functions there are fast and slow molecules. It is possible to obtain a solution to the problem with no symmetric tensor by applying an iterative procedure.

To do this, you must first solve the problem with a symmetric tensor, leave one solution obtained, and then use the second equation and solve it as a differential one. This would be one of the possible solutions. In three-dimensional problems, three stress equilibrium equations and three equations for moments are solved. In the classical version, the symmetry of the stress tensor condition again closes the problem. The algorithm for solving a three-dimensional problem is the same as for a two-dimensional one. The class of systems considered in statistical mechanics textbooks includes systems described by Hamiltonian dynamics. In the modern version, this class includes all systems described by the laws of classical or quantum mechanics. Since in our theory an important role is assigned to the law of conservation of the moment of force (momentum), it is necessary to study how the properties and structure of the system change when using the law of conservation of moment, since it was not included in statistical theories. We have already noted that a feature of classical theories is the identification of the properties of systems without studying the influence of boundaries and initial conditions on their state. The basic equation in kinetic theory is the Liouville equation for a closed system. As already noted, the mathematical apparatus for deriving the equation is Hamilton's formalism. Questions arise about changing the character of equations for open systems. The definition of the distribution function of an open system for a non-stationary problem will depend on the initial and boundary conditions. For example, in the theory of N. N. Bogolyubov, boundary conditions are set with the requirement that the derivatives of the distribution function vanish at infinity. A special type of force action $\Phi\left(r_i - r_j\right)$ is used [7] [8]. On this basis, a number of terms are assumed to be equal to zero. When considering flying aircraft and missiles, the boundaries of the perturbation region are limited and the behavior of the generalized coordinates and impulses is different. Perturbations can also exist at infinity. An important component is the circulation component of the velocity, which ceases to be potential and additional dissipation occurs. The resulting stresses create a global circulation motion around the apparatus and lead to beats during movement, especially in the case of no symmetric design. To illustrate the action of the moment, model problems of fluid mechanics and the theory of elasticity are considered. For the problem of the first type, a self-similar problem was chosen, which was solved by A.G. Petrov, in the classical formulation [9]: Exact solution of the Navier-Stokes equations in a liquid layer between plates moving in parallel. As a second example, we consider the problem of a concentrated force F on a half-space at an arbitrary point with coordinates x,z. Correspondence between individual force terms of a general form and terms in the Liouville and Boltzmann equations and the equations of aeromechanics is clarified. Thus, the article demonstrates the contribution of the moment to the processes of aeromechanics and the theory of elasticity. The local interaction of molecules leads to the collective behavior of particles. From the point of view of statistics, the large size of the system turns out to be useful, since then it is possible to consider the average values of physical quantities. The trajectory of molecules with a small number of particles turns out to be confusing. However, the statistical description does not include the action of some factors, for example, the action of flows at the border. Correspondence between individual force terms of a general form and terms in the Liouville and Boltzmann equations and the equations of aeromechanics are clarified. Thus, the article demonstrates the contribution of the moment to the processes of aeromechanics, the theory of elasticity and kinetics. In fact, questions of the influence of no conservative forces and their contributions to macro equations are discussed. The results of the study the work consist in illustrating the influence of the moment on the examples of solving the problem for fluid and solid mechanics, developing a general methodology for determining a no symmetric operator if a solution with a symmetric operator is found, constructing the more general kinetic theory based on the Liouville equation with a complicated type of acting forces.

2. Examples of Solution Equations with Influence Angular Momentum

2.1. Fluid between Parallel Moving Plates

In previous works, the general equations of continuum mechanics were obtained taking into account the influence of the angular momentum (force). There are few analytical solutions of viscous fluid problems, even two-dimensional ones. For one-dimensional problems, the moment does not work. Therefore, we restrict ourselves to the simplest two-dimensional ones. Consider, as an example, the unsteady motion of a fluid between two moving plates. A.G. Petrov showed the self-similarity of the problem in the classical formulation and obtained an analytical solution [9]. Let us try to determine the influence of the no symmetric of the stress tensor. We leave one solution with a symmetric stress tensor.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right),$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\nu\frac{\partial}{\partial y}\left(y\frac{\partial^2 v}{\partial y^2}\right), \quad (1)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0,$$

$$x\left(\frac{\partial \sigma_{xz}}{\partial x}+\frac{\partial \sigma_{zz}}{\partial z}\right)-z\left(\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{zx}}{\partial z}\right)+\sigma_{zx}-\sigma_{xz}=0.$$

With boundary conditions

$$u(t,x,0)=u(t,x,h)=0,\ v(t,x,0)=0,\ v(t,x,h)=\dot{h}.$$

Here, ν—kinematic viscosity, the density is considered equal to one. σ_{ij}—projections of the stress tensor onto the corresponding axes; u, v—speeds in x, y coordinates. p—pressure. The equations differ from those of A. G. Petrov by the last term, which describes the action of the moment. We are looking for a new solution, leaving a functional depend about the kinetic description.

$$v(t,x,h)=\dot{h}V(\eta),\ u=\frac{\dot{h}}{h}xU(\eta),\ p=\dot{h}^2\left[b\frac{x^2}{2h^2}+P(t,\eta)\right]+p_0,\ \eta=\frac{y}{h}. \quad (2)$$

After substitution, we get

$$U+V'=0,\ -aU-\eta U'+U^2+VU'+b-\frac{1}{Re}U''+\frac{1}{Re}(\eta U'')'=0. \quad (3)$$

$$U(0)=V(0)=0,\ U(1)=0,\ V(1)=1,$$

$$a(t)=1-\frac{\ddot{h}h}{\dot{h}^2},\ Re(t)=\frac{h\dot{h}}{\nu}. \quad (4)$$

In the case of plate motion according to the law $h=k\sqrt{|t-t_0|}$ we get

$$P=\eta V-\frac{V^2}{2}+\frac{1}{Re}V'(\eta). \tag{5}$$

$$u=\frac{\dot{h}}{h}x\left[\cos\left(2n\pi\frac{y}{h}\right)-1\right], \tag{6}$$

$$v=\dot{h}\left[\eta-\frac{1}{2\pi n}\sin(2\pi n\eta)\right] \tag{7}$$

Let's leave one old solution

$$x\left(\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}\right)-y\left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{yx}}{\partial y}\right)+\sigma_{yx}-\sigma_{xy}=0,$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\nu\left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{yx}+\Delta\sigma_{yx}}{\partial x}\right), \tag{8}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\nu\left(\frac{\partial\sigma_{yx}+\Delta\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial x}\right),$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0$$

Then, leaving the old solution along the x axis, let's look at the value of the addition to the no symmetric tensor.

$$x\left(\frac{\partial\left(\sigma_{yx}+\Delta_{xy}\right)}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}\right)-\Delta_{xy}=0$$

$$x\frac{\partial\sigma_{yx}}{\partial x}+x\frac{\partial\Delta_{xy}}{\partial x}-\Delta_{xy}=0$$

$$\frac{\partial\Delta_{xy}}{\partial x}-\frac{1}{x}\Delta_{xy}+\frac{\partial\sigma_{yx}}{\partial x}=0 \tag{9}$$

$$\frac{\partial\sigma_{yx}}{\partial x}=-\frac{\dot{h}}{h}\sin\left(2n\pi\frac{y}{h}\right)$$

$$\Delta_{xy}=\mathrm{e}^{-\int\frac{1}{x}\mathrm{d}x}\left[-\int\frac{\partial\sigma_{yx}}{\partial x}\mathrm{e}^{\int\frac{1}{x}\mathrm{d}x}\mathrm{d}x+C\right],\quad \Delta_{xy}=\frac{1}{x}\int\frac{\dot{h}}{h}\sin\left(2n\pi\frac{y}{h}\right)\cdot x\mathrm{d}x$$

Here Δ_{xy} is the difference between $\sigma_{zx}-\sigma_{xz}$.

The solution of the new equation differs slightly in the coefficients at η. New values are determined using iterations. Most likely, the moment along the x axis $\left(\nu\frac{\partial}{\partial y}\left(y\frac{\partial^2 u}{\partial x^2}\right)\right)$ plays the greatest role. Calculations are done in the same way, but more iterations are required.

2.2. Examples of the Influence of the Moment in the Theory of Elasticity

The problems are taken from the book [10]. Consider the problem when a concentrated force is given on a half-plane. Let us find the stress distribution with and without taking into account the influence of the moment (**Figure 1**).

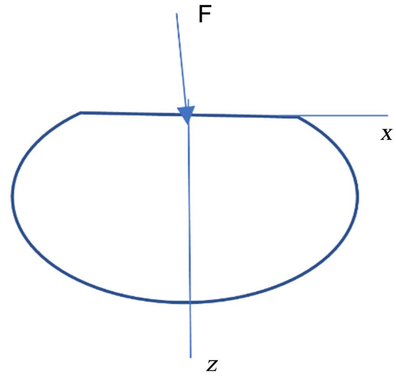

Figure 1. Formulation of the problem.

General view of equations

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\sigma_{zx}}{\partial z}=0,\quad \frac{\partial\sigma_z}{\partial z}+\frac{\partial\sigma_{xz}}{\partial x}=0,$$

$$x\left(\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{zz}}{\partial z}\right)-z\left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{zx}}{\partial z}\right)+\sigma_{zx}-\sigma_{xz}=0. \tag{10}$$

Suggested that $\sigma_{zx}=-A\frac{x}{x^2+z^2}$, option for case $\sigma_{xz}=\sigma_{zx}$ and $\sigma_z=-A\frac{z^3}{\left(x^2+z^2\right)^2}$ was considered in [10]. First, we solve the problem when the symmetry condition of the stress tensor is satisfied $\sigma_{xz}=\sigma_{zx}$.

$$\frac{\partial\sigma_{zx}}{\partial z}=-A\frac{2xz}{\left(x^2+z^2\right)^2},$$

$$\sigma_x=-2Az\int_0^x\frac{x\mathrm{d}x}{\left(x^2+z^2\right)^2}-Az\int_0^y\frac{\mathrm{d}y}{\left(y+z^2\right)^2}=Az\frac{1}{\left(z^2+y\right)}+f(z),$$

$$\sigma_x=Az\frac{1}{z^2+x^2}+f(z). \tag{11}$$

$$\frac{\partial\sigma_{xz}}{\partial x}=-A\left(\frac{1}{x^2+z^2}-\frac{2x^2}{\left(x^2+z^2\right)^2}\right)=-A\left(-\frac{z^2-x^2}{\left(x^2+z^2\right)^2}\right).$$

$$\begin{aligned}\sigma_z&=-A\int_0^{z^2}\left(\frac{1}{x^2+z^2}-\frac{2x^2}{\left(x^2+z^2\right)^2}\right)\mathrm{d}z\\&=-A\frac{1}{x}arctgz-A\left(\frac{1}{x}\frac{z}{1+z^2}+\frac{1}{x}arctgz\right)\\&=-A\frac{1}{x}\frac{z}{1+z^2}+f(x)\end{aligned}$$

Let σ_x and σ_{zx} remain the same. Then from the equation for the moments it is possible to determine the difference a new value σ_{xz}

$$\frac{\partial\sigma_{xz}}{\partial x}-\frac{1}{x}\sigma_{xz}+\frac{\partial\sigma_{zz}}{\partial z}=0. \tag{12}$$

$$x\left(\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{zz}}{\partial z}\right)+\sigma_{zx}-\sigma_{xz}=0\,. \tag{13}$$

Then a new value σ_z ($\sigma_z=\sigma_{zz}$) determined. An example is given in order to demonstrate the many possible solutions in the two-dimensional case. Consequently, the solution of the two-dimensional problem under the condition of symmetry of the stress tensor is only one of the options for solving the problem. Algorithm for finding stresses with no symmetric tensor easily programmed using previously obtained results.

2.3. Kinetic Theory

The macroequations of continuum mechanics, the special cases of which we have considered, can theoretically be obtained from the Boltzmann kinetic equation, which, in turn, can be derived from the Liouville equation. In practice, continuum equations are obtained experimentally. Since in the experiment we work with material objects, and not with points, the corresponding experimental data can be represented in integral form. After taking moments and averaging, we arrive at the equations of continuum mechanics. However, in the integral formulation, the individual components of the forces and their contribution to the individual terms of the equations are not single out. Mathematical translation equations from integral form to different is means the transition from a piecewise continuous space to a continuous one. In addition, the use of the Hamiltonian formalism and the dependence of the force only on the distance between particles limits the study; in the collision integral, for example, for a rarefied gas, the Lennard-Jones potential is often used, which does not belong to the type under consideration. The foregoing makes us to the study of the influence of forces of another type on equations of mechanics. Let us consider new members for force: $\boldsymbol{M}=\sum_i m_i\left[\boldsymbol{x}_i,\dot{\boldsymbol{x}}_i\right]$. In the most frequently studied case $U=U\left(x_i-x_j\right)$

$$\frac{\mathrm{d}\boldsymbol{M}}{\mathrm{d}t}=-\sum_{i,j}\frac{\mathrm{d}U_{ij}}{\mathrm{d}r_{ij}}\frac{\left[x_i,x_j\right]}{r_{ij}},\ L=T-U,\ \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{q}_k}-\frac{\partial L}{\partial q_k}=0 \tag{14}$$

T—kinetic energy, *U*—potential.

Usually the Hamiltonian contains only potential forces. Actually generalized forces represent the sum of forces [11]

$$Q=-B-Cq,\ \dot{q}_i=0,\ B=D+\Gamma,\ D\Gamma=D,\ \Gamma^{\mathrm{T}}=-\Gamma \tag{15}$$

Forces $-Dq$ dissipative, Gq-gyroscopic, C positional.

$$-\Gamma\dot{q}=\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial V}{\partial\dot{q}}-\frac{\partial V}{\partial q},\quad V\left(q,\dot{q}\right)=-\frac{1}{2}q^{\mathrm{T}}\Gamma q. \tag{16}$$

The matrix of positional forces C also decomposes into the sum of symmetric and skew-symmetric parts.

$$C=K+N,\ K^{\mathrm{T}}=K,\ N^{\mathrm{T}}=-N \tag{17}$$

Conservative forces have potential

$$-Kq = \frac{\partial U}{\partial q},\ U = -\frac{1}{2} q^{\mathrm{T}} K q. \tag{18}$$

Consequently, $L = T - Q_r$, where the latter is the work of the generalized forces.

Formally, one can define, as in classical mechanics

$$\dot{q}_i = \frac{\partial H(q,\dot{q})}{\partial \dot{q}_i},\ \ddot{q}_i = -\frac{\partial H(q,\dot{q})}{\partial q_i}$$

However, they often work with generalized velocities

$$(q_1,\cdots,q_n,\dot{q}_1,\cdots,\dot{q}_n) = (q_1,\cdots,q_n,\dot{q}_1,\cdots,\dot{q}_l,p_{l+1},\cdots,p_n),\ p_k = \frac{\partial T}{\partial \dot{q}_k} \tag{19}$$

$$H = \sum\nolimits_{1\le i\le N}\left(\frac{p_i^2}{2m} + U(r_i,t)\right) + \frac{1}{2}\sum\nolimits_{1\le i,j\le N,i\ne j}\Phi\left(\left|r_i - r_j\right|\right). \tag{20}$$

Thus, in statistical mechanics, a particular case of forces with a Coulomb potential is considered. An interesting feature of modern kinetic theories is the use of other potentials in the integral, for example, Lennard-Jones potential. In this case, the acting force is the derivative of a complex function. The presence of other molecules j is partially taken into account. Another assumption is the potential nature of the action of circulation forces. This discards the dissipation of the finely vortex motion in the theory of turbulence. In addition, the action of the moment of forces arising from the movement of molecules is not taken into account. Thus, modern representations require a revision of the equations and the inclusion in the theory of initial and boundary conditions.

3. The Effect of Dispersion in the Boltzmann Equation

Consider the equations for the s-particle distribution function F_s [8],

$$\begin{aligned}
\frac{1}{V^s}\frac{\partial F_s}{\partial t} &= -\frac{1}{V^s}\sum_{i=1}^{s}\frac{\boldsymbol{p}_i}{m}\left(\frac{\partial F_s}{\partial \boldsymbol{r}_i} + \frac{\partial}{\partial \boldsymbol{r}_i}\boldsymbol{r}_j\frac{\partial F_s}{\partial \boldsymbol{r}_j}\right) + \frac{1}{2V^s}\sum_{\substack{i,j=1\\(j\ne i)}}^{s}\frac{\partial Q}{\partial \boldsymbol{r}_i}\frac{\partial F_s}{\partial \boldsymbol{\xi}_i}\\
&+\frac{1}{V^{s+1}}\sum_{i=1}^{s}\int\sum_{j=s+1}^{N}\frac{\partial Q}{\partial \boldsymbol{r}_i}\frac{\partial F_{s+1}(t,\boldsymbol{r}_1,\cdots,\boldsymbol{r}_s,\boldsymbol{r}_j,\boldsymbol{\xi}_1,\cdots,\boldsymbol{\xi}_s,\boldsymbol{\xi}_j)}{\partial \boldsymbol{p}_i}\mathrm{d}\boldsymbol{r}_j\mathrm{d}\boldsymbol{\xi}_j\\
&+\frac{1}{V^{s+1}}\sum_{s+1}^{N}\int\frac{\boldsymbol{\xi}_j}{m}\frac{\partial F_{s+1}(t,\boldsymbol{r}_1,\cdots,\boldsymbol{r}_s,\boldsymbol{r}_j,\boldsymbol{\xi},\cdots,\boldsymbol{\xi}_s,\boldsymbol{\xi}_j)}{\partial \boldsymbol{r}_j}\mathrm{d}\boldsymbol{r}_j\mathrm{d}\boldsymbol{\xi}_j\\
&+\frac{1}{V^{s+1}}\sum_{i=1}^{s}\sum_{j=s+1}^{N}\int\frac{\partial Q}{\partial \boldsymbol{r}_j}\frac{\partial F_{s+1}(t,\boldsymbol{r}_1,\cdots,\boldsymbol{r}_s,\boldsymbol{r}_j,\boldsymbol{\xi},\cdots,\boldsymbol{\xi}_s,\boldsymbol{\xi}_j)}{\partial \boldsymbol{p}_j}\mathrm{d}\boldsymbol{r}_j\mathrm{d}\boldsymbol{\xi}_j\\
&+\frac{1}{2V^{s+2}}\sum_{\substack{i,j=s+1\\(j\ne i)}}^{s}\int\frac{\partial Q}{\partial \boldsymbol{r}_i}\frac{\partial F_{s+2}(t,\boldsymbol{r}_1,\cdots,\boldsymbol{r}_s,\boldsymbol{r}_j,\boldsymbol{\xi}_1,\cdots,\boldsymbol{\xi}_s,\boldsymbol{\xi}_j)}{\partial \boldsymbol{p}_j}\mathrm{d}\boldsymbol{r}_i\mathrm{d}\boldsymbol{r}_j\mathrm{d}\boldsymbol{\xi}_i\mathrm{d}\boldsymbol{\xi}_j
\end{aligned} \tag{21}$$

It is usually assumed that when $\left|r_j\right| \to \infty$ $\frac{\partial}{\partial r_j}F_n \to 0$, $\left|\xi_j\right| \to \infty$ $\frac{\partial}{\partial \xi_j}F_n \to 0$.

This is possible only for the equilibrium state at infinity. Otherwise, perturbations of the distribution function will create flows and form a circulation motion around the apparatus, giving an additional force. Such a situation is possible when the apparatus enters the environment created by another flying body. If we trace the correspondence of the terms of the Liouville equation to the terms Boltzmann equations, and then macroequations, it turns out that the Euler equation, taking into account the moment for pressure, corresponds to all the potential forces included in Q.

a. Let define exact solution for kinetic theory and receive the barometric Boltzmann formula. Gas is at stationary condition in field of force which has potential φ (this is analogy tasks from [12]):

$$\xi_i \frac{\partial f}{\partial x_i} + \xi_i \frac{\partial}{\partial x_i} x_i \frac{\partial f}{\partial x_i} - \frac{1}{m} \frac{\partial \varphi}{\partial x_i} \frac{\partial f}{\partial \xi_i} = J(f, f). \tag{22}$$

As before solution we shall be look for as $f = A(x) \mathrm{e}^{-B(x)\xi^2}$. We receive the old result $B =$ Const. For $A(x)$ we have equation

$$\frac{\mathrm{d}A}{\mathrm{d}x} + \frac{\mathrm{d}}{\mathrm{d}x_i} x_i \frac{\mathrm{d}A}{\mathrm{d}x_i} + 2\frac{A \cdot B}{m} \frac{\partial \varphi}{\partial x_i} = 0.$$

We have old result that is common for one-dimension tasks:

$$f = n_0 \left(\frac{m}{2\pi kT} \right)^{3/2} \mathrm{e}^{-\frac{\varphi}{kT}} \mathrm{e}^{-\frac{m}{2kT}\xi^2}. \tag{23}$$

General local-Maxwell distribution is $f = n \left(\frac{m}{2\pi kT} \right)^{3/2} \exp\left\{ -\frac{m}{2kT} c^2 \right\}$, $c = \xi - u$.

Modification the Boltzmann equation is

$$\xi_i \frac{\partial f}{\partial x_i} + \xi_i \frac{\partial}{\partial x_i} x_i \frac{\partial f}{\partial x_i} - g_i \frac{\partial f}{\partial \xi_i} = J(f, f), \tag{24}$$

$g = \frac{X}{m}$ -acceleration of molecules.

Let local-Maxwell solution of the equation will be considered as in old algorithm

$$\ln f = \gamma_0 + \gamma_i \xi_i + \gamma_4 \xi^2. \tag{25}$$

Then we receive old equation and the changing

$$\frac{\partial \gamma_0}{\partial t} + g_i \gamma_i = 0,$$

$$\frac{\partial \gamma_i}{\partial t} + 2g_i\gamma_4 + \frac{\partial \gamma_0}{\partial x_i} + \frac{\partial \gamma_0}{\partial x_i} + \frac{1}{2} x_i \frac{\partial \gamma_0^2}{\partial x_i^2} + \frac{\partial}{\partial x_i} x_i \frac{\partial \gamma_0}{\partial x_i} = 0, \tag{26}$$

$$\frac{\partial \gamma_4}{\partial t}\delta_{ij} + \frac{1}{2}\left(\frac{\partial \gamma_i}{\partial x_j} + \frac{\partial \gamma_j}{\partial x_i} \right) + \frac{1}{2}\left(\frac{\partial \gamma_i}{\partial x_j} + \frac{\partial \gamma_j}{\partial x_i} \right) + + \frac{1}{2} * \frac{1}{2}\left(x_i + x_j \right)\left(\frac{\partial \gamma_i}{\partial x_j} + \frac{\partial \gamma_j}{\partial x_i} \right)\frac{\partial \gamma_0}{\partial x_i}$$
$$+ \frac{1}{2}\left(x_i + x_j \right)\frac{1}{2}\left(\frac{\partial}{\partial x_j}\left(\frac{\partial \gamma_i}{\partial x_j} + \frac{\partial \gamma_j}{\partial x_i} \right) + \frac{\partial}{\partial x_i}\left(\frac{\partial \gamma_i}{\partial x_j} + \frac{\partial \gamma_j}{\partial x_i} \right) \right) = 0$$

As before

$$\frac{\partial \gamma_4}{\partial x_i} = 0, \ \ T = const.$$

As the result we shall receive modified gas-dynamic equations but without viscosity and thermal conductivity.

4. Conclusion

The most common systems are open non-stationary systems. From the previously formulated equations and some experiments, the connection between the gradients of physical quantities of quantities and the angular momentum (force) under consideration is traced. The work illustrates the contribution of the distributed moment of force to the problems of continuum mechanics and the kinetic theory. Examples of the solution to the problem of fluid mechanics, the theory of elasticity and kinetic theory are given. A correspondence is established between the terms of the Liouville equation with more general and traditional forces. Previously, the influence of moments in boundary layer problems, jet problems, and the simplest problems of elasticity theory was considered. The work proves the important role of distributed moments in the formation of the new structure. Further progress of the research consists of checking the results on more complex examples, solved both analytically and numerically.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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