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# The Method of Thermomechanical Dynamics (TMD) and the Application to the Concept of Maxwell's Demon

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## Abstract

The method of thermomechanical dynamics (TMD) has successfully explained mechanisms of heat engines and heat-energy flows, time-dependent physical quantities and the phase transition from thermodynamic equilibrium to nonequilibrium irreversible states (NISs). In the current paper, the TMD is applied to study the physical meaning and mathematical expression of Maxwell's demon. The analysis results in the extension of the traditional, disordered, static thermodynamic equilibrium to accommodate stable, structured and closed nonequilibrium thermodynamic systems. The new image of thermodynamic equilibrium is termed as the *homeostatic equilibrium*, which shall be an appropriate foundation for nonequilibrium irreversible dynamics of the thermal, macroscopic world. The interrelated fundamental problems, such as the *concept of time* and *causality*, the thermodynamic time and the mechanical clock time are discussed in detail by the method of TMD.

## Keywords

Thermomechanical Dynamics (TMD), Maxwell's Demon, Measure for Nonequilibrium Irreversible States, Homeostatic Equilibrium, Thermodynamic Time and Mechanical Clock Time

## 1. Introduction

Thermodynamics has evolved from equilibrium thermodynamics to the theory of nonequilibrium irreversible thermodynamics [1]-[4]. The content of modern thermodynamics has radically helped progress fundamental physical concepts, such as *the law of entropy*, *phase transition* and *emergent phenomena* [5] [6] in almost all fields of research, which is applied from hadron, chemical, molecular interactions to biological systems, electromagnetic radiations, heat engines and astrophysical problems [7]-[9]. The exchanges of heat and energy among dynamical systems of dissipative structure [10] [11] in classical and quantum systems have revealed interesting physical, mathematical and technological problems. It demands fundamental knowledge and careful investigations of physical meanings of thermodynamic laws to study nonequilibrium irreversible thermodynamics [12]-[14].

The thermomechanical dynamics (TMD) is a theory for nonequilibrium irreversible thermodynamics, which has been applied to mechanisms of heat engines, technologies of mechanoelectric power conversions, resulting in successful descriptions of thermomechanical motions of heat engines [15]-[17]. The TMD analysis of time-dependent processes in nonequilibrium states helped us study a variety of technological possibilities for energy activations and conversions of a low temperature waste heat and energy. It is also applicable to microscopic thermal phenomena at small length and short time scales [18], energy transport and radiation. The produced electricity would be collected and used for electrolysis, for example, to produce basic useful chemicals ($H_2$, $O_2$, C, COOH, $CH_3COOH$, *etc.*), to support sustainable development goals [19]-[21].

The TMD method is applied to study the zeroth law (the physical interpretation of temperature) and the third law of thermodynamics (the unattainability of the absolute temperature $T=0$) [22] [23]. Traditionally, the zeroth law is introduced to explain the existence of temperature, but the method of TMD explained the zeroth law as a proposition deduced from the law of entropy [15]. The measure $\tau(t)$ of the third proposition in the method of TMD explained the physical reason of the unattainability of absolute temperature $T=0$, resulting in the conclusion that the thermodynamic state $T=0$ does not exist in reality as a thermodynamic state. In other words, the attaining absolute temperature $T=0$ state is mathematically identical to the process of self-reference [15]; thus, the absolute $T=0$ state cannot be obtained nor exist. The zeroth law and the third law of thermodynamics are essentially interrelated with the concept of entropy and entropy-variation, $\Delta S(T,V,N)$. The first law and the second law of thermodynamics only suffice as the fundamental laws of thermodynamics.

The physical and technical analyses of the phase transition from thermal equilibrium to nonequilibrium irreversible states (NISs) of heat engines are specifically discussed by the method of TMD. The time-dependent nonlinear dissipative equation of motion for the system of heat engines precisely produced the time-dependent changes of entropy, $\mathcal{S}(t)$, internal energy $\mathcal{E}(t)$, thermodynamic work $W(t)$ and temperature $\tilde{T}(t)=T_0\tau(t)$ with the measure $\tau(t)$ of NISs. The time-dependent quantities are consistent with the time flows given by Gibbs' relation to thermodynamic equilibrium. In addition, the analysis of time-dependent quantities contributed to study fundamental relations among the concept of *time* and *time's arrow*, *causality* and *irreversibility* [14].

The idea of *Maxwell's demon* in general is considered to be a working definition or a *thought experiment* to discover energy-efficient thermal processes and experimental devices, such as heat engines, transistors and quantum engines. The assumption of Maxwell's demon helps find possible physical processes and applications, by assuming as if certain thermal processes were not obeying the second law of thermodynamics. The assumption is applied to examine the physics of phase transitions from a thermodynamic equilibrium state to a nonequilibrium state in order to study the relation between the Maxwell's demon and the law of entropy. Based on the TMD method, a possible mathematical expression and physical meanings of Maxwell's demon in thermodynamics are specifically studied and investigated in the paper.

The concept of Maxwell's demon and the law of maximum entropy are explained in sec. 2. The thermomechanical dynamics (TMD), the nonlinear dissipative equation of motion with time-dependent coefficients (NDT-TC), heat-energy flow and time-dependent temperature in NISs are reviewed in sec. 3. The work of Maxwell's demon is specifically explained by the dissipative equation of motion for heat engines, such as a drinking bird and a low temperature Stirling engine. Then, the mathematical expression of Maxwell's demon in terms of the measure $\tau(t)$ is discussed in sec. 4. The TMD analyses produced a new picture of thermodynamic equilibrium which is termed as a *homeostatic equilibrium*, different from a conventional, chaotic and disordered equilibrium state, and consequently, a hierarchical metastable structure at thermodynamic equilibrium for solid, liquid, gas phases is discussed in sec. 5. The concept of *time and causality* and the fundamentally different physical concepts between the mechanical clock time and the thermodynamic time are discussed in sec. 6. Conclusions and perspectives are in sec. 7.

## 2. The Concept of Maxwell's Demon and the Law of Entropy

The macroscopic world changes according to thermodynamic laws, the first law (conservation of total heat-energy) and the second law of thermodynamics (evolution to the maximum entropy). A thermal state naturally progresses from an ordered state (State 1) to a disordered state (State 2) as shown in **Figure 1** and never goes backward. Maxwell's demon works reversely as shown in **Figure 2**, from thermodynamic equilibrium to a lower entropy, nonequilibrium state, which is an artificial fiction or imagination on thermomechanical motions; however, it is an enlightening physical and thermodynamic thought-experiment.

The time-dependent progresses from thermodynamic equilibrium to NISs and vice versa are often explained by the work of Maxwell's demon which is a fictitious microscopic being to induce dynamic changes of a thermal state. This is a metaphor for the thought-experiment to study heat-energy transporting processes and changes of thermal states. If such a mechanical being exists, dynamical changes of macroscopic world seem to be naturally created. Though this is not realistic, the idea of Maxwell's demon is very useful for a working definition to find classical or quantum energy-efficient information processes and energy-conversion devices, by eliminating errors or causes of failure.

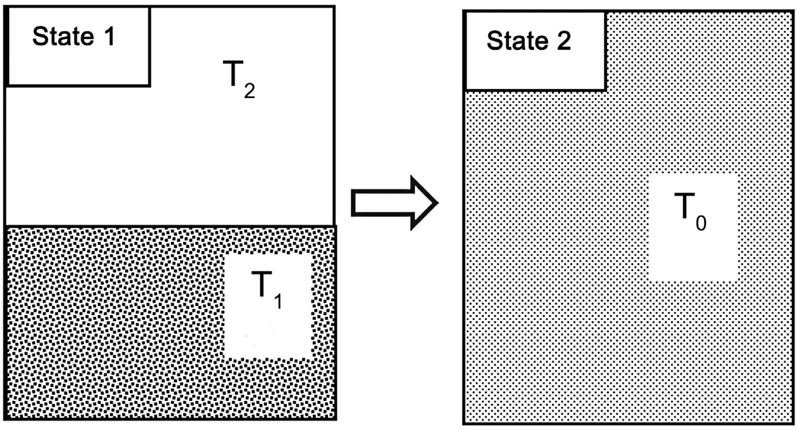


**Figure 1.** The direction of a thermal state into the state of maximum entropy, $T_1 < T_0 < T_2$ .

The well-known technological device related to Maxwell's demon would be a refrigerator, which is an application of the second law of thermodynamics to an open thermal system. A refrigerator in an open system never breaks the second law of thermodynamics, and it is tempting to suppose a heat engine without the restriction of the *law of entropy*, or without requiring any energy and work. The fictitious demon's work could enable one to operate a steam engine continuously without any fuel, and it could be useful to search for an efficient heat engine. The concept of Maxwell's demon is applied to information theory and the intricate energy requirements of computer devices [20] [21] and investigations for quantum heat engines, power generators [24] [25] and time's arrow [26] [27].

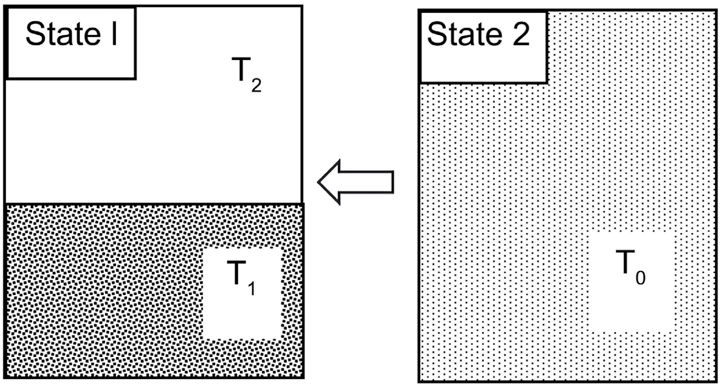


**Figure 2.** The direction of thermal state by the work of Maxwell's demon, $T_1 < T_0 < T_2$ .

Maxwell's demon can distinguish and sort particles of black or white particles by using information on kinetic energies or temperature of particles and create a low entropy thermal state with a constant and homogeneous temperature $T_0$ at thermodynamic equilibrium, but it should be understood that $T_0$ is only determined after the maximum entropy is achieved. This is the fundamental property of thermodynamics, and the thermodynamic requirement proves that the maximum law of entropy is more fundamental than the action of Maxwell's demon. In other words, Maxwell's demon allows freedom to a thought experiment, but it must be confined within thermal states and systems allowed by the maximum law of entropy. The thermal states of heat engines progress to NISs and produce time-dependent thermodynamic quantities with the heat-energy conservation law.

The time-dependent changes of state are simultaneously created by changes of entropy $\mathrm{d}S(t)$. Temperature is determined at the end of thermal changes, which is not directly related to order and disorder of matter and particles, which is explained precisely by the following example. When thermal systems of A (gas) and B (liquid) are in the same temperature, one cannot distinguish A (gas) and B (liquid) in terms of temperature, but one can distinguish A (gas) and B (liquid) in terms of entropy. This is one of the reasons why the second law of entropy is more fundamental than temperature in our macroscopic thermal world [15]. Maxwell's demon is supposed to distinguish hot or cold, fast or slow particles to achieve a low entropy state. The entropy can specify changes of thermal states, order of events and streamlines of heat energy flows. Therefore, the work of Maxwell's demon should be understood by the law of entropy, which is specifically explained using heat engines in section 4.

## 3. The Fundamental Propositions in Thermomechanical Dynamics (TMD)

The method of thermomechanical dynamics (TMD) has consistently solved two independent problems of heat engines: a drinking bird and a low temperature Stirling engine. The thermomechanical motion of heat engines, phase transitions from thermodynamic equilibrium to NISs and time-dependent changes of thermal quantities are self-consistently solved [15]-[17]. The TMD analysis has yielded a new lightweight, low speed, low temperature device for thermoelectric energy conversion by incorporating a disk-magnet electromagnetic induction (DM-EMI) [28] [29]. Moreover, the method of TMD revealed that the zeroth law and the third law of thermodynamics are auxiliary problems to understand and examine profound physical meanings of the first law and the second law of thermodynamics [15]; in other words, the zeroth law and the third law of thermodynamics are integrated in the first law and the second law of thermodynamics. The method of TMD model is thermodynamically consistent and hence, applied to clarify the concept of Maxwell's demon.

There are three propositions in the TMD method: 1) the construction of the dissipative equation of motion, 2) the conservation of the total heat-energy flows, and 3) the definition of measure or the time-dependent temperature $\tilde{T}(t)$.

1) The construction of the *dissipative equation of motion* for thermomechanical motion.

The dissipative equation of motion for thermomechanical motion must be investigated by considering mechanical motion and thermal effects such as frictional variations, time-dependent changes of physical parameters and mechanical components, thermal conductivity and efficiency of working fluid.

The dissipative equation of motion explicitly breaks time-symmetry and reversibility of thermal states. Hence, there is no Euler-Lagrange equation to construct the equation of motion by employing Lagrangian or Hamiltonian method, but the mechanical methods can be employed to assist finding a possible dissipative equation of motion.

A thermal state is observed as either a *metastable state* or a nonequilibrium state. A metastable state is defined as $\mathrm{d}S(t)/\mathrm{d}t \sim 0$, considered sufficient to be thermodynamic equilibrium by observers, and the true thermodynamic equilibrium is defined by $\mathrm{d}S(t)/\mathrm{d}t = 0$. Thermodynamic work is prepared or produced by variations of entropy from thermodynamic equilibrium to a metastable state:

$$\mathrm{d}S(t)/\mathrm{d}t = 0 \to \mathrm{d}S(t)/\mathrm{d}t \sim 0, \tag{1}$$

and the metastable state must perform the reverse process to produce thermodynamic work. The process from a metastable state to thermodynamic equilibrium is given by:

$$\mathrm{d}S(t)/\mathrm{d}t \sim 0 \to \mathrm{d}S(t)/\mathrm{d}t = 0, \tag{2}$$

Therefore, the work of Maxwell's demon is specified by the mathematical expression (1), and the (2) expresses the time-dependent direction of thermal states (the law of maximum entropy). The thermomechanical processes from (1) to (2) are recurrently used for heat engines to extract thermodynamic work efficiently.

For practical applications, the cyclicity of motion is the fundamental key for macroscopic, thermomechanical phenomena. The thought-experiment of Maxwell's demon is expressed by equations from (1) to (2), which corresponds to **Figure 1** and **Figure 2**. Therefore, the concepts of Maxwell's demon, arrow of time and time-dependent physical quantities are all intertwined with a fundamental principle of the law of entropy [12]-[14] [18] [25], and propositions of the TMD method help clarify interrelations among the fundamental physical concepts.

The classical and quantum mechanical equations of motion are nonlinear differential equations with constant coefficients (NDE-CC) derived from Lagrangian or Hamiltonian, and time-symmetry is strictly maintained. However, the dissipative equation of motion in thermomechanical dynamics is the nonlinear differential equation with *time-dependent coefficients* (NDE-TC), not subject to time-symmetry. The dissipative equation of motion with NDE-TC conveys phase transitions from thermodynamic equilibrium to NISs and produces independent solutions which do not exist in NDE-CC; the independent solutions of NDE-TC are termed as the *bifurcation-integration* solutions [30]. The transition from the time-symmetric NDE-CC to the time-asymmetric NDE-TC is profoundly related to the work of Maxwell's demon, thermodynamic time and phase transition to NISs.

The conservation law of heat-energy flows regulates the dissipative equation of motion with NDE-TC, which is established in the second proposition as the conservation law of total heat-energy flow:

2) The conservation law of total heat-energy flow at time $t$.

The time-dependent progress of thermodynamic work $\mathrm{d}W_{th}(t)$, internal energy $\mathrm{d}\mathcal{E}(t)$ and total entropy $T(t)\mathrm{d}S(t)$ must maintain the conservation law of total heat-energy flow at time $t$ as:

$$\mathrm{d}\mathcal{E}(t)/\mathrm{d}t = T(t)\mathrm{d}S(t)/\mathrm{d}t + \mathrm{d}W_{th}(t)/\mathrm{d}t. \quad (3)$$

Since the thermodynamic quantities and functions are often explained separately as mechanical energy (*Joule*) and heat energy (*Calorie*), the conventional explanations of thermodynamic quantities obscure fundamental distinctions among mechanical energies and dissipative energies. Therefore, the heat picture, $Q(t)$-picture, is introduced for prudent and convenient analyses [16] [17].

In nonequilibrium processes, dissipations of heat and energy occur simultaneously with internal energy and thermodynamic work (friction, viscosity among working fluid, wear and warming-up of internal systems, *etc.*). Therefore, in order to make discussions clear, we rewrite internal energy as: $\mathcal{E}(t) \equiv Q_\varepsilon(t)$, entropy flow or the total heat dissipation as: $T(t)\mathrm{d}S(t)/\mathrm{d}t \equiv \mathrm{d}Q_d(t)/\mathrm{d}t$ and thermodynamic work as: $W_{th}(t) \equiv Q_w(t)$. The differential form of time-dependent total heat-energy flow (3) is rewritten as:

$$\mathrm{d}Q_\varepsilon(t)/\mathrm{d}t = \mathrm{d}Q_d(t)/\mathrm{d}t + \mathrm{d}Q_w(t)/\mathrm{d}t. \quad (4)$$

This is the $Q(t)$-picture, which is referred to as the total heat-energy conservation law at time $t$. Boundary conditions of the initial and final states are assumed in thermodynamic equilibrium $\mathrm{d}Q_d(t)/\mathrm{d}t = 0$. The $Q(t)$-picture (*calorie* unit) is useful for theoretical investigations in NISs and consistently used in the analyses of disk-magnet electromagnetic induction (DM-EMI) and thermodynamic consistency of solutions [28]-[31].

It is fundamental to realize that the heat flows of all thermodynamic quantities must be separated into two constituent parts: the *thermally conserved* and *dissipating* parts. Therefore, it is taken for granted that all thermodynamic quantities are separated into two parts as;

$$Q(t) = \text{thermally conserved energy} + \text{thermally dissipating energy}. \quad (5)$$

The decomposition is useful for studying a phase transition, thermal flows and interactions, which can be readily tractable in the $Q(t)$-picture.

The time-dependent internal energy $Q_\varepsilon(t)$ is separated as:

$$Q_\varepsilon(t) = Q_{\varepsilon i}(t) + Q_{\varepsilon d}(t). \quad (6)$$

The heat $Q_{\varepsilon i}(t)$ is the conserved, thermal internal energy, and $Q_{\varepsilon d}(t)$ is the *associated* heat dissipation. Similarly, thermodynamic work $Q_w(t)$ is separated as;

$$Q_w(t) = Q_{wk}(t) + Q_{wd}(t). \quad (7)$$

The heat $Q_{wk}(t)$ is the conserved, thermal kinetic energy, and $Q_{wd}(t)$ is the *associated* heat dissipation.

Denoting the total in-coming heat as $Q_{in}(t)$, the heat-energy conservation law is written by:

$$\begin{aligned} Q_{in}(t) &= Q_{\varepsilon}(t) + Q_{w}(t) \\ &= Q_{\varepsilon i}(t) + Q_{\varepsilon d}(t) + Q_{wk}(t) + Q_{wd}(t) \\ &= Q_{\varepsilon i}(t) + Q_{wk}(t) + Q_{d}(t). \end{aligned} \tag{8}$$

The total heat dissipation $Q_d(t)$ is defined as

$$Q_d(t) = Q_{\varepsilon d}(t) + Q_{wd}(t), \tag{9}$$

with $Q_d(0) = 0$. In $Q(t)$-picture, the direct expressions of non-equilibrium thermodynamic variables, such as pressure, volume, friction, stress, chemical potentials are suppressed. However, time-dependent internal energy $\mathcal{E}(t)$, thermodynamic work $W_{th}(t)$, the total entropy (heat dissipation) $S(t)$ can be explicitly determined [28]-[31], which can be extended by including exchanges of particles and electromagnetic interactions.

The third fundamental proposition in TMD establishes the *measure* for NISs, and the measure is fundamentally related to the time-dependent temperature $\tilde{T}(t)$.

3) The definition of measure or the time-dependent temperature $\tilde{T}(t)$ in NISs.

The measure of a nonequilibrium irreversible state is defined by the ratio of entropy-flow against internal energy-flow:

$$\tau(t) = \frac{\mathrm{d}Q_d(t)/\mathrm{d}t}{\mathrm{d}Q_{\varepsilon}(t)/\mathrm{d}t}. \tag{10}$$

The value of $\tau(t)$ is a dimensionless, positive-definite function, $\tau(t) > 0$.

Thermodynamic equilibrium is defined when the measure $\tau(t) = 1$ holds identically for all time [3], resulting in;

$$\frac{\mathrm{d}Q_d(t)}{\mathrm{d}t} = \frac{\mathrm{d}Q_{\varepsilon}(t)}{\mathrm{d}t}, \tag{11}$$

The Equation (11) is equivalent to claim the thermodynamics equilibrium state: $\mathrm{d}S(t)/\mathrm{d}t = 0$. The definition of thermodynamic equilibrium (11) and the conservation law of heat-energy flow (4) declare the condition of thermodynamic equilibrium:

Neither thermodynamic work nor heat dissipation into an external system should exist in thermodynamic equilibrium.

The *time-dependent temperature* $\tilde{T}(t)$, is defined by the measure $\tau(t)$ as,

$$\tilde{T}(t) \equiv T_0 \tau(t), \tag{12}$$

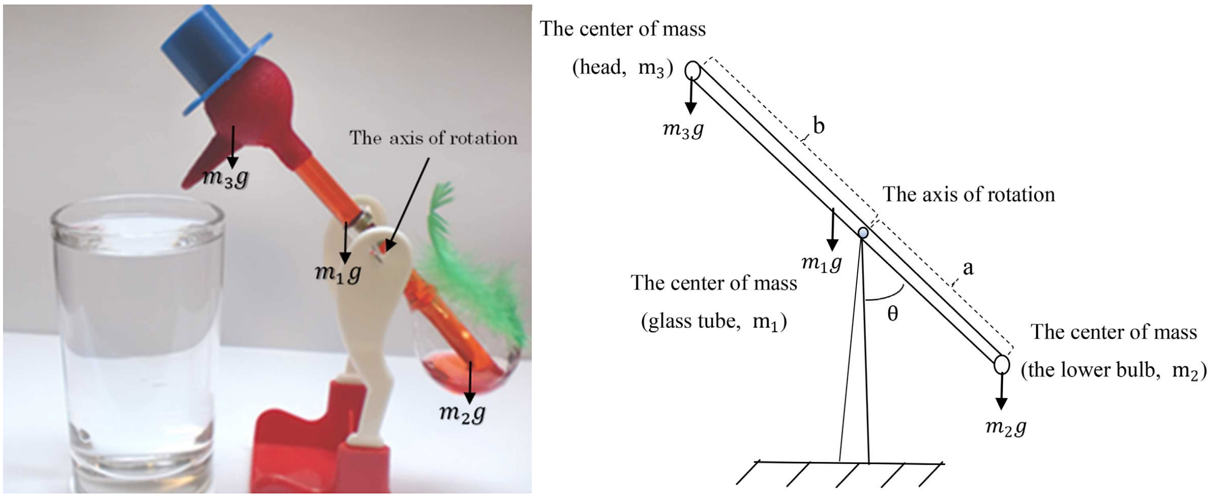
where the initial condition is assumed at $t = 0$, $\tilde{T}(0) = T_0$ and $\tau(0) = 1$. The time-dependent progress of entropy flow is rigorously stated as $\tilde{T}(t) \to T_0$ and $\tau(t) \to 1$, after a sufficiently long time.

The conditions of near equilibrium, local equilibrium defined by linearity of fluxes [1] [2] and physical quantities are studied in TMD method, by the *near equilibrium condition* as,

$$\tau(t) = \left(\mathrm{d}Q_d(t)/\mathrm{d}t\right)/\left(\mathrm{d}Q_\varepsilon(t)/\mathrm{d}t\right) \sim 1. \tag{13}$$

The measure or nonequilibrium temperature $\tilde{T}(t) = T_0\tau(t)$ reproduces empirically well-known thermodynamic quantities ($\mathcal{E}(t)$, $W_{th}(t)$, $S(t)$, $\tilde{T}(t)$) in heat engines. The empirical properties of heat engines, such as ignition and detonation, fuel-injection and combustion timings, are consistently calculated [28]-[31]. The time-dependent temperature $\tilde{T}(t)$ can calibrate and express empirical mechanical-motion of heat engines.

The dissipative equation of motion, arrow of time, time-irreversibility are intrinsically intertwined with phase transition phenomena, Maxwell's demon and the law of entropy. Thermomechanical motion destroys information of initial conditions, and the arrow of time or the direction of time appears naturally in thermal systems. The work of Maxwell's demon is precisely demonstrated by heat engines, such as a drinking bird and a low temperature Stirling engine, which is explained by way of the TMD method in the following section.



(a) (b)

**Figure 3.** A mechanical modeling and deformation of a drinking bird. (a) drinking bird (DB) divided into the head $m_3$, glass tube $m_1$, and bottom $m_2$ with volatile water. (b) A topological deformation for the drinking bird of (a) by employing the concept of centers of mass.

## 4. Maxwell's Demon Working in Motions of a Drinking Bird and a Low Temperature Stirling Engine

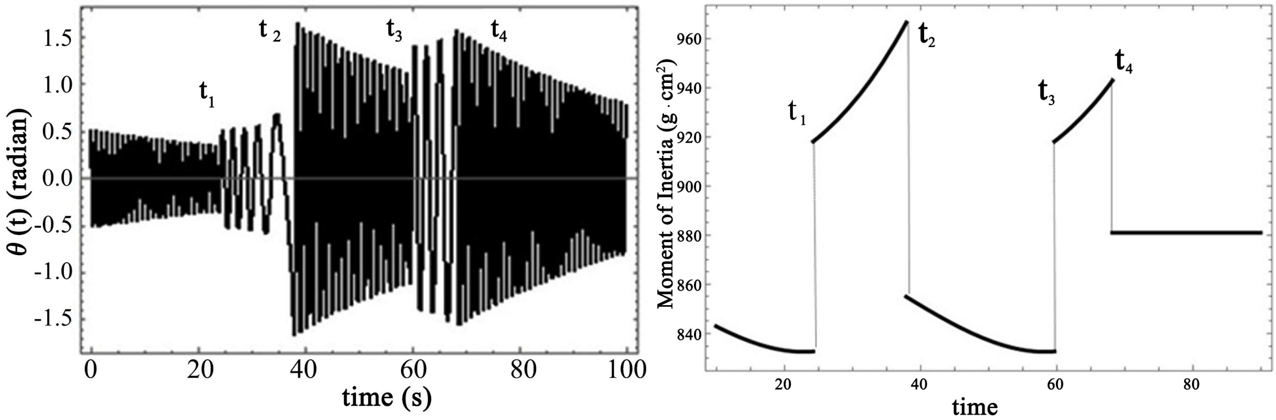
The work of Maxwell's demon is demonstrated by examining nonequilibrium motions of a drinking bird (DB) and a low temperature Stirling engine (LTSE). The changes of thermal states of DB and LTSE are precisely obtained by the dissipative equations of motion, time-evolution of the internal energy, $\mathcal{E}(t)$, entropy $S(t)$, thermodynamic work $W(t)$ and measure $\tau(t)$.

### 4.1. Maxwell's Demon Working in the Drinking Bird Motion

A mechanical modeling and deformation of a drinking bird are shown in **Figure 3(a)** and **Figure 3(b)**. The dissipative equation of motion for the drinking bird (DB) is given by,

$$\ddot{\theta} + c\dot{\theta} + \frac{glm^*(t)}{I(t)}\sin\theta = 0. \quad (14)$$

As emphasized in (1) of sec. 3, the dissipative equation of motion (14) is the nonlinear differential equation with time-dependent coefficients (NDE-TC). The nonlinear differential equation with constant coefficients (NDE-CC) is formally written by changing $I(t) \to I$ and $m^*(t) \to m$. Although the NDE-CC can produce a simple swinging motion, it cannot produce solutions of the drinking bird motion. The NDE-CC and NDE-TC are similar in the form of equation, but the time-dependent coefficient is only different. However, the difference produces the fundamentally distinct thermal states. The transition from the time-symmetric NDE-CC to the time-asymmetric NDE-TC (14) is essentially related to the emergence of thermodynamic time, phase transition to NISs and the work of Maxwell's demon.



(a) (b)

**Figure 4.** The solutions to the dissipative equation of motion (14) and the time-dependent moment of inertia $I(t)$. (a) The solution to the dissipative equation of motion (14); (b) The time-dependent moment of inertia $I(t)$.

The numerical calculations of the dissipative equation of motion (14) and the time-dependent moment of inertia $I(t)$ are shown in **Figure 4(a)** and **Figure 4(b)** until the drinking bird's second drinking motion is completed. The sudden discrete changes of figures at times, $t_1, t_2, t_3$ and $t_4$, are respectively phase transitions from a simple back-and-forth swinging motion to a drinking motion and *vice versa*. It should be noticed that the discreet lines at times, $t_1, t_2, t_3$ and $t_4$, are produced by employing piecewise continuous step-functions for $m^*(t)$ and $I(t)$ to establish the numerical calculations[16] [17]. The changes of lines would become smooth and continuous when Fourier's heat conduction method is integrated in the TMD method.

Maxwell's demon is considered working at times, $t_1, t_2, t_3$ and $t_4$ in **Figure 4(a)** and **Figure 4(b)**, producing thermomechanical work and phase transitions from a metastable state to NISs. Therefore, Maxwell's demon's work is mathematically expressed by the proposition (3) of TMD as:

$$\left.\frac{T\partial S(\mathcal{E},V)}{\partial \mathcal{E}}\right|_{V,N,\cdots}=1\Bigg|_{t\le t_c}\xrightarrow{t_c<t}\left.\frac{T(t)\partial S\big(t,\mathcal{E}(t),W(t)\big)}{\partial \mathcal{E}\big(t,W(t)\big)}\right|_{t\le t_d}\sim 1, \tag{15}$$

where the time $t_c$ is a "critical time" (onset time) for the transition, and $t_d$ is the corresponding drinking (dipping) time. The work of Maxwell's demon is precisely produced by the equation (15). The transition from a nonequilibrium metastable state after dipping to thermodynamic equilibrium is similarly given by:

$$\left.\frac{T(t)\partial S\big(t,\mathcal{E}(t),W(t)\big)}{\partial \mathcal{E}\big(t,W(t)\big)}\right|_{t\le t_d}\sim 1\xrightarrow{t_d<t}\left.\frac{T\partial S(\mathcal{E},V)}{\partial \mathcal{E}}\right|_{V,N,\cdots}=1. \tag{16}$$

The Equation (16) corresponds to the mathematical expression of the flow of work generated by Maxwell's demon.

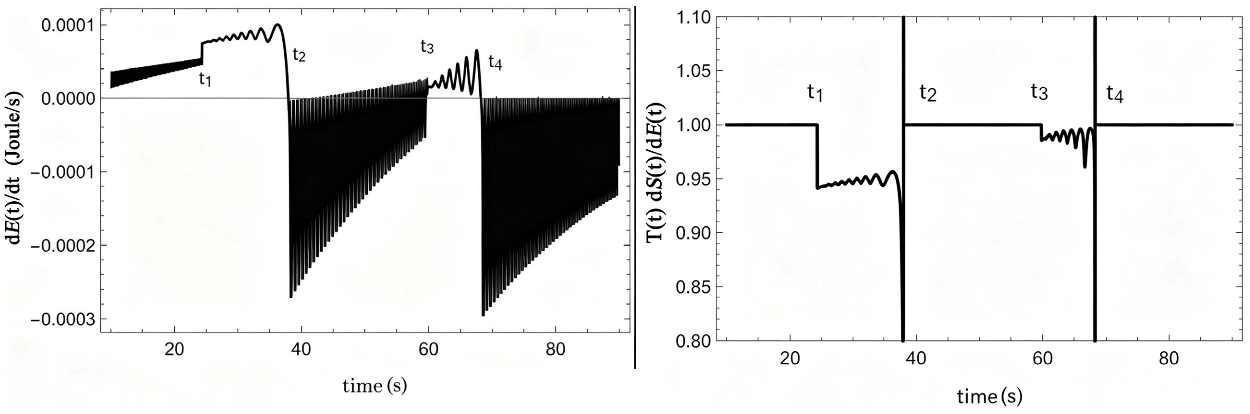
The time-dependent effective mass $m^*(t)$ and moment of inertia $I(t)$, time-evolution of thermal quantities such as $E(t)$, thermodynamic work $W(t)$ and entropy $S(t)$ are calculated with the conservation law of heat-energy flow [17]. The measure $\tau(t)$ is simultaneously obtained by $\mathrm{d}\mathcal{E}(t)/\mathrm{d}t$ and $T(t)\mathrm{d}S(t)/\mathrm{d}t$ and (10), resulting in the self-consistent description of the drinking bird. The numerical calculations of time-dependent physical quantities, $\mathrm{d}\mathcal{E}(t)/\mathrm{d}t$ and $\tau(t)=\big(T(t)\mathrm{d}S(t)/\mathrm{d}t\big)/\big(\mathrm{d}\mathcal{E}(t)/\mathrm{d}t\big)$ are respectively shown in **Figure 5(a)** and **Figure 5(b)**. The smooth thermal changes of states are neglected, which is considered to disregard time-retardation conduction effects for the analyses as a whole. The time-dependent progresses of thermal states are thermodynamically consistent, which is an important consequence in the TMD method.

### 4.2. Maxwell's Demon Working in a Low Temperature Stirling Engine

The time-evolution of measure $\tau(t)$ and temperature $\tilde{T}(t)$ for a low temperature Stirling engine (LTSE) are discussed in detail [31]. The appropriate *dissipative equation of motion* for LTSE is written by [16]:

$$I_0\theta''(t)+c\theta'(t)-\lambda_w Q_w(t)\left|\sin\theta(t)\right|=0. \tag{17}$$

The schematic structure of LTSE and rotation angle $\theta(t)$ are shown in **Figure 6** and **Figure 7**. The LTSE dissipative equation of motion (17) has constant $c\theta'(t)$ for the overall friction and the dimensionless coupling constant $\lambda_w$ for heat and mechanical work. The piecewise continuous function $\left|\sin\theta(t)\right|$ is induced by the viscous pumping and friction of gas inside the displacer room, which is assumed to couple with the heat of work $Q_w(t)$. The initial conditions of (17) is chosen as $\theta(0)=0$ and $\theta'(0)=0.1\pi$ (rad/s). Thermal quantities $\mathcal{E}(t)$, thermodynamic work $W(t)$ and entropy $S(t)$ are calculated with the conservation law of heat-energy flow and the definition of measure $\tau(t)$, resulting in the self-consistent description of motion of LTSE.



(a) (b)

**Figure 5.** The internal energy flow $\mathrm{d}\mathcal{E}(t)/\mathrm{d}t$ and measure $\tau(t)$ until the second drinking process.(a) The internal energy flow $\mathrm{d}\mathcal{E}(t)/\mathrm{d}t$ until the second drinking process. The changes of oscillation mode observed in time ranges $t_1<t<t_2$ and $t_3<t<t_4$ signify the motion to a drinking process. (b) $\tau(t)=\left(T(t)\mathrm{d}S(t)/\mathrm{d}t\right)/\left(\mathrm{d}\mathcal{E}(t)/\mathrm{d}t\right)$ until the second drinking process. The straight horizontal line $\tau(t)=1$, shows mechanical and thermal equilibriums.

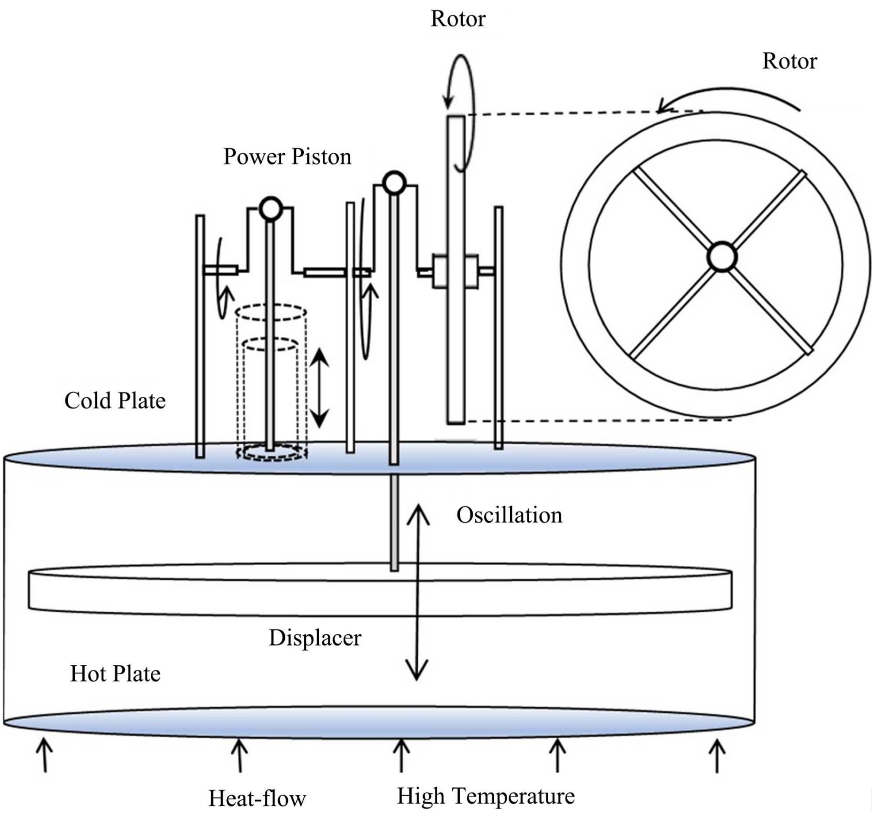


**Figure 6.** A schematic structure of a low temperature Stirling engine (LTSE).

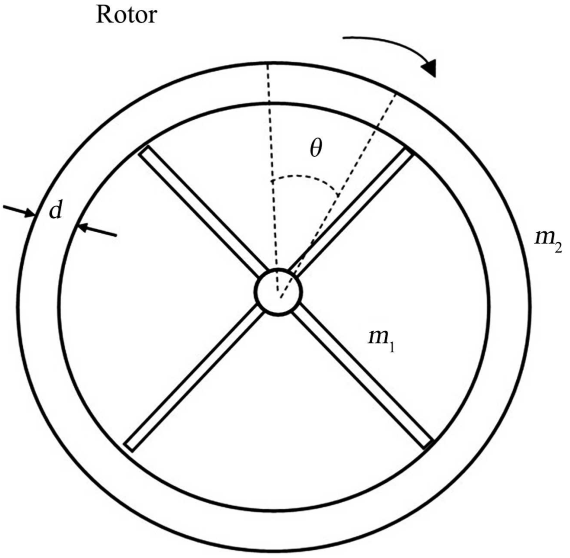


**Figure 7.** The rotational angle $\theta$ from the vertical axis of the rotor in **Figure 6**.

The kinetic energy of flywheel rotation, $I_0'\theta'(t)^2/2$ (joule) reaches a stable motion in $t\sim500$ (s) and persists for a long time, shown in **Figure 8**. The corresponding dissipation of heat is shown in **Figure 9**, and the dissipation of heat increases for the first period of time, which would be interpreted as the initial time of engine-idling. After the engine-idling, the dissipation of heat comes to a stable minimum. It is remarkable that the dissipation of heat becomes a minimum value when the flywheel rotation reaches a stable maximum rotation, compatible with well-known empirical engine motion. The internal energy $Q_\varepsilon(t)$ and the total heat dissipation $Q_d(t)$ are shown in **Figure 10** and **Figure 11**. Thermodynamic quantities, $Q_\varepsilon(t)$, $Q_d(t)$ and $Q_w(t)$ reach a stable thermal state respectively as the flywheel rotation reaches a stable maximum speed shown in **Figure 8**. The results are useful to understand *efficiency* and *stability* of motion of heat engines.

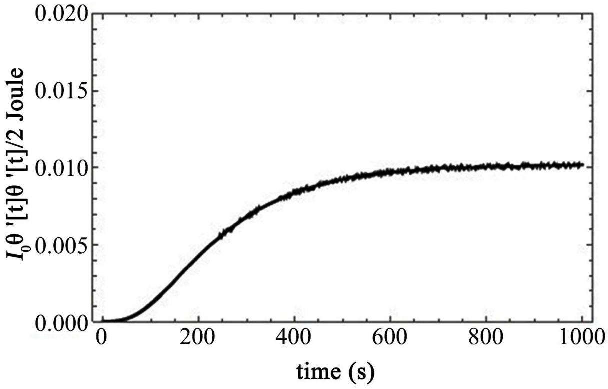


**Figure 8.** Thermomechanical work, $I_0'\theta'(t)^2/2$ (joule) reaches a stable maximum value ($0<t<1000$ (s)) [16].

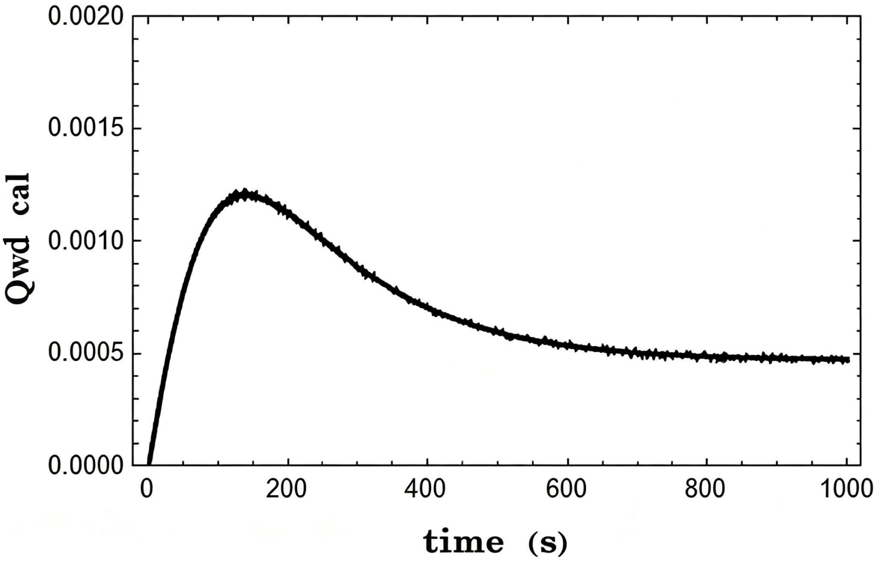


**Figure 9.** The dissipation of heat, $Q_{wd}(t)$, from thermodynamic work reaches a minimum stable value after certain rotations [16].

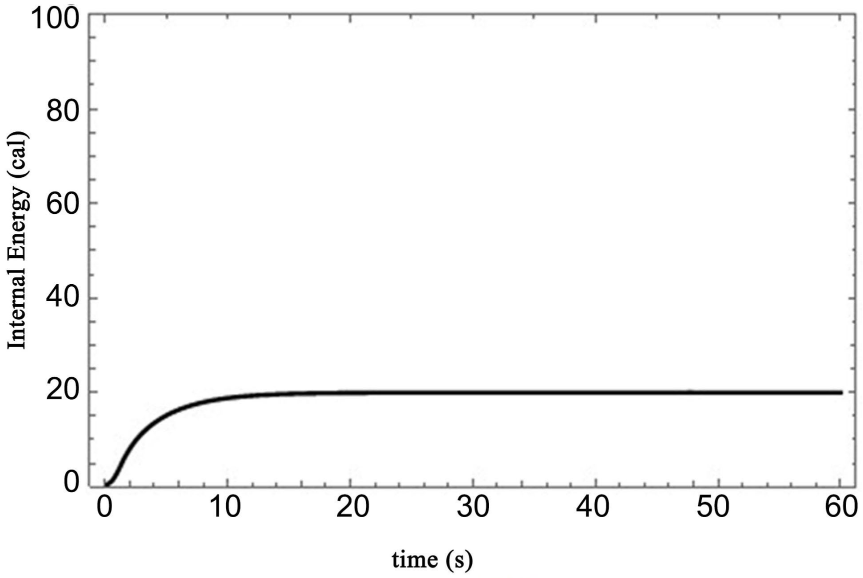


**Figure 10.** The internal energy, $Q_{\varepsilon}(t) = Q_{\varepsilon i}(t) + Q_{\varepsilon d}(t)$, for a rapidly-decreasing, short-time uniform heat flow.

The short and unstable rotation and the emergence of instability before a halt of engine rotations is shown by the measure $\tau(t)$ in **Figure 12**. The figure shows that the thermal state slowly becomes unstable from $t \sim 30$ and diverges above $t > 40$, meaning the Starling engine has no solutions above $t > 40$. The divergence signifies a halt of engine-rotation, because the dynamic relation between $S(t)$ and $\mathcal{E}(t)$ breaks down when a heat engine terminates. The time-dependent temperature $\tilde{T}(t) = T_0\tau(t)$ is sensitive to thermal states and mechanical changes of heat engines.

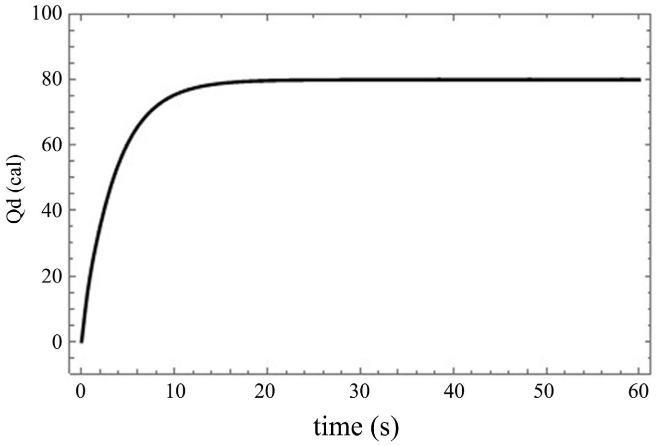


**Figure 11.** The total heat dissipation, $Q_d(t)$, for a rapidly-decreasing, short-time uniform heat flow.

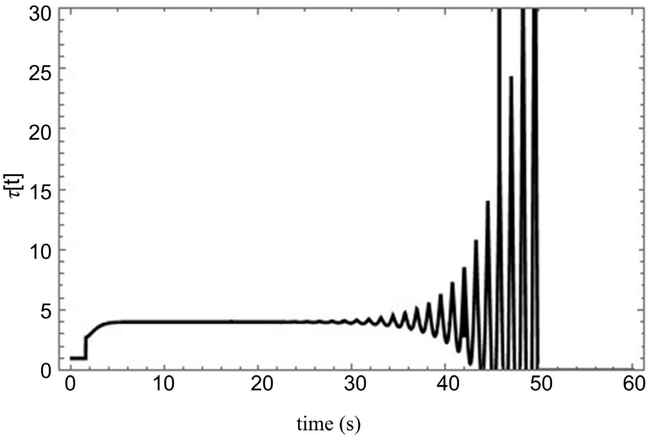


**Figure 12.** The emergence of instability before a halt of rotations, shown by $\tau(t)$. The LTSE is adjusted to become unstable and diverges for $t > 40$ (a halt of rotation). Note that the measure $\tau(t)$ must be positive definite: $\tau(t) > 0$ [17].

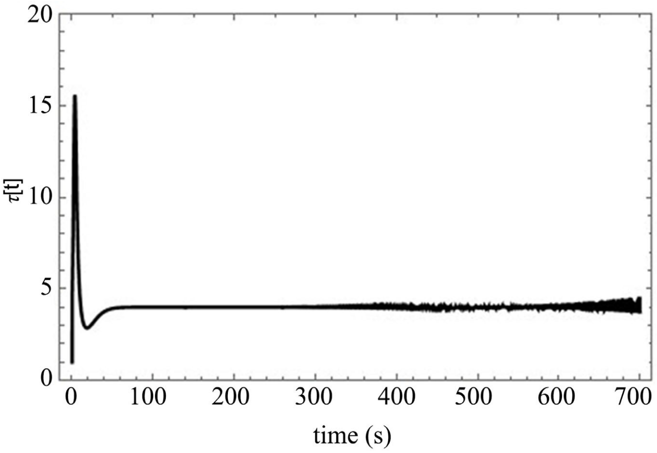


**Figure 13.** The variation of $\tau(t)$ or temperature $\tilde{T}(t) = T\tau(t)$ in a long-time rotation. Note the variation and fluctuations at the beginning and intermediate time.

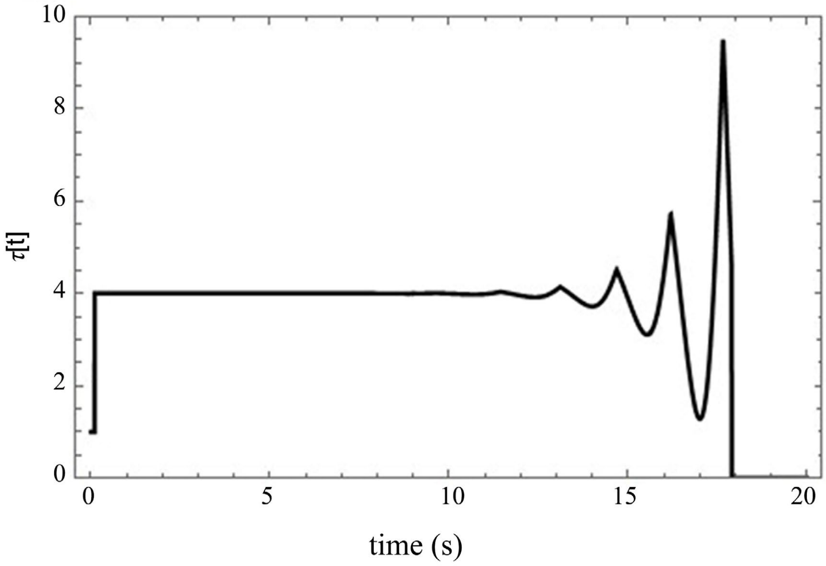


**Figure 14.** The variation of $\tau(t)$ or temperature in a short-time rotation; rotations terminate at $t \sim 18$. The function, $\tau(t)$, diverges when a heat engine terminates [16].

The variation of $\tau(t)$ or temperature $\tilde{T}(t) = T\tau(t)$ shown in **Figure 13** changes suddenly at the beginning and reaches a stable state for a long time; then, thermal fluctuations and unstable states appear before the engine halts. The heat engine becomes unstable and stops abruptly when heat flow is weak to make rotations, shown in $\tau(t)$ in **Figure 14** around $10 < t < 18$. The characteristic properties of $\tilde{T}(t)$ at the beginning for ignition, intermediate stable thermal state, unstable thermal fluctuations before final termination of heat engines are compatible with empirical motion of heat engines, which is applied to study the analysis of ignition, the optimal fuel-injection and combustion timings [16] [31].

The LTSE uses the energy flow from the high temperature plate to the cold plate. It should be noticed that the flywheel and displacer motions are repeated continuously to extract work, and a cyclic quality or *cyclicity of work* must be maintained in a mechanical system, which is not usually explained as the work of Maxwell's demon. The cyclic quality, unstable thermal fluctuations, ignition, stability and efficiency of heat engines are consistently explained in the method of TMD.

### 4.3. The work of Maxwell's Demon and the Measure $\tau(t)$ in TMD

The dynamics of a thermal state inside heat engines is consistently examined by time-dependent physical quantities, and it is noticeable that temperature $\tilde{T}(t) = T_0\tau(t)$ is used as a consistent function to study thermodynamic work, changes of thermal states and stability [16]. In other words, the time-dependent temperature $\tilde{T}(t)$ explains the work of Maxwell's demon: time changes of thermodynamic work and thermal states toward a metastable state.

Thermodynamic equilibrium is written by the fundamental relation in thermodynamics [3] [4]:

$$\left.\frac{\partial S(\mathcal{E},V)}{\partial \mathcal{E}}\right|_{V,N,\cdots}=\frac{1}{T}, \tag{18}$$

with the entropy $S$, internal energy $\mathcal{E}$, constant temperature $T$, at fixed volume $V$ and particle number $N,\cdots$, at thermodynamic equilibrium (Boltzmann constant is set as $k_B \equiv 1$). The fundamental relation (18) is generalized into the *measure* $\tau(t)$, in order to include NISs and time evolutions to thermomechanical states.

The work of Maxwell's demon can be understood as the work for generating low entropy states against the maximum entropy state $\Delta S(t)=0$, and so, Maxwell's demon is supposed to know a low entropy thermal state, in advance. However, in reality, there should be many possible low entropy states and structures observed in matter, molecules, cells and biological systems. The time-evolution from a metastable complex structure to the maximum entropy state is inevitable in the macroscopic world, and a metastable low entropy thermal state would not be uniquely determined as time-reversible work of Maxwell's demon, which means that the concept of entropy is more fundamental than Maxwell's demon in thermodynamics [15].

The states of fast and low velocities of particles respectively correspond to a high temperature and a low temperature state, meaning that a high temperature and a low temperature state are an observer-dependent physical concept. So, it is necessary for the demon to determine a physical way of distinguishing particles and thermal states, which suggests that a measure for NISs is needed. The measure $\tau(t)$ is defined by the ratio of the total heat-flow $\mathrm{d}Q_d(t)/\mathrm{d}t$ against the internal energy-flow $\mathrm{d}Q_\varepsilon(t)/\mathrm{d}t$, which incorporates the knowledge of *direction of heat-flow*, as well as kinetic energy of particles. It determines both the heat-flow and the energy-flow in order to progress to a metastable equilibrium, and this is the fundamental property of the measure $\tau(t)$.

Thermodynamic equilibrium is consistently defined by $\tau(t)\equiv 1$, and changes of temperature in nonequilibrium irreversible states are expressed in two cases [17]:

(1) $0<\tau(t)<1\to 1$: the thermal flow from a low temperature state to a high temperature metastable state.

(2) $1<\tau(t)\to 1$: the thermal flow from a high temperature state to a low temperature metastable state.

The property of $\tau(t)$ is very helpful to understand time-evolution of thermal states and heat-energy flows of matter and particles.

In the case of (1) for $\tilde{T}(t)$ lower than $T_0$, the temperature $\tilde{T}(t)$ will gradually increase and progress to the maximum temperature at thermodynamic equilibrium $T_0$ as $0<\tilde{T}(t)/T_0<\tau(t)\to 1$. The typical example is that ice melts in liquid and converges to $T_0$ at a metastable equilibrium. Similarly, in the case of (2) for $\tilde{T}(t)$ higher than $T_0$, the temperature $\tilde{T}(t)$ will gradually decrease and progress to $T_0$ as $\tilde{T}(t)/T_0>\tau(t)\to 1$. The typical example is that the Sun ultimately disintegrates into a homogeneous infinite matter with a low temperature $T_0$ in the future. Thus, the measure $\tau(t)$ is consistent with the *law of maximum entropy* and empirical changes of thermal states and matter.

The measure $\tau(t)$ defined by the entropy and the internal energy signifies properties of thermal states in NISs, and the time-dependent temperature $\tilde{T}(t)$ is very useful in physics, as well as applications in engineering and sciences in general. Temperature progresses to the maximum state of entropy, and the conditions of near equilibrium, far equilibrium, stability of a thermal state of heat engines are studied in detail. In addition, the method of TMD proved that zeroth law and the third law of thermodynamics are auxiliary lemmas to understand the fundamental concept of entropy [15]. The measure $\tau(t)$ or equivalently time-dependent temperature $\tilde{T}(t)$ defined by the entropy and internal energy is fundamental quantity in thermodynamics.

## 5. The Hierarchical Metastable Structures as Mixtures of Solid, Liquid, Gas Phases (Homeostatic Equilibrium)

It is remarkable that the measure $\tau(t)$ explains much more on thermodynamic states and systems than Maxwell's demon speaks, and therefore, physical consequences of the measure $\tau(t)$ should be summarized in the section.

[Remark 1] The measure $\tau(t)$ helps the analysis of stability and instability, ignition and detonation mechanisms.

The measure $\tau(t)$ explains properties of thermodynamic state of heat engines, energy-entropy flows, fuel-injection and combustion timings, thermal stability, ignitions and jet-stream like detonations [31].

[Remark 2] The constant temperature $T_0$ at thermodynamic equilibrium ($\tilde{T}(t) \to T_0$) signifies two cases of transitions to fundamental equilibrium structures.

The property of constant temperature is explained in the section 4.3, in the two cases as $\tilde{T}(t) \searrow T_0$ and $\tilde{T}(t) \nearrow T_0$. The characteristic metastable states for the two cases are respectively abundant in reality, such as gas-liquid-solid mixed metastable matter in the macroscopic universe and the microscopic structures of cells, microorganisms and living creatures. The results create different views to thermodynamic equilibrium state at $T_0$, which is often explained as a chaotic, random state. Thermodynamic equilibrium state can be more complex than a single chaotic state, and the thermodynamic equilibrium state of the universe can have hierarchical structures regardless that the universe is open or closed, which must exist as metastable thermal states of mixed solid, liquid and gas phases.

The analysis demands a new metastable equilibrium state to expand the realm of thermodynamics to nonequilibrium irreversible thermodynamics. Hence, in addition to thermodynamic equilibrium, the concept of *homeostatic* equilibrium is explained.

[Remark 3] The definition of thermodynamic equilibrium indicated by the method of TMD.

Thermodynamic equilibrium is expressed in TMD as $\tau(t)=\left(\mathrm{d}Q_d(t)/\mathrm{d}t\right)/\left(\mathrm{d}Q_\varepsilon(t)/\mathrm{d}t\right)=1$. The total heat-energy flow (4) and $\tau(t)=1$ result in the conclusion: $\mathrm{d}Q_w(t)/\mathrm{d}t=0$ at thermodynamic equilibrium, which is directly declared in the Equation (11) as:

Neither thermodynamic work nor heat dissipation into external systems should exist in thermodynamic equilibrium.

The mathematical definition of thermodynamic equilibrium, $\tau(t)=1$ or $\mathrm{d}Q_d(t)/\mathrm{d}t=\mathrm{d}Q_\varepsilon(t)/\mathrm{d}t$, extends thermodynamic equilibrium to a structured thermal metastable system. The metastable equilibrium is introduced as *homeostatic equilibrium* in the following remark.

[Remark 4] The metastable equilibrium of structured stable thermal systems can be possible, if thermal fluctuations in internal systems are counterbalanced with heat-energy fluctuations as $\mathrm{d}Q_d(t)/\mathrm{d}t=\mathrm{d}Q_\varepsilon(t)/\mathrm{d}t$ so that the total entropy is maintained as $\mathrm{d}S(t)/\mathrm{d}t=0$. The metastable equilibrium state is termed as *homeostatic equilibrium*.

In homeostatic equilibrium states, the energy fluctuations and heat-energy flows can be counterbalanced by maintaining $\mathrm{d}Q_d(t)/\mathrm{d}t=\mathrm{d}Q_\varepsilon(t)/\mathrm{d}t$ and $\mathrm{d}S(t)/\mathrm{d}t=0$ as a whole. The result is fundamental for equilibrium states of complex systems of matter, cells and microbiology. The mixed solid, liquid and gas phases of matter, the earth and planets, systems of living beings are typical examples. The stable and structural views of macroscopic systems given by $\tau(t)$ are more general and consistent with dynamical views in nature and the universe.

Thermodynamic equilibrium is expressed by the maximum entropy, $\mathrm{d}S(t)/\mathrm{d}t=0$. However, it is expressed as a metastable system of $n$ internal systems as:

$$\mathrm{d}S(t)/\mathrm{d}t=\sum_{i=1}^{n}\mathrm{d}S_i(t)/\mathrm{d}t=0, \tag{19}$$

where $n$ is a finite number for representing structured metastable internal states. Though the sum of entropy should progress to thermodynamic equilibrium $\mathrm{d}S(t)/\mathrm{d}t=0$ as a whole, the entropy variations of internal systems do not have to be $\mathrm{d}S_i(t)/\mathrm{d}t=0$ respectively, but they are required to be a metastable state, $\mathrm{d}S_i/\mathrm{d}t\sim 0$. The internal systems mutually can have possible freedom to realize $\mathrm{d}S_i/\mathrm{d}t\sim 0$, provided that the internal energy $\mathrm{d}Q_{i\varepsilon}(t)/\mathrm{d}t$ and $\mathrm{d}S_i/\mathrm{d}t$ are counterbalanced as $\mathrm{d}S_i/\mathrm{d}t\sim \mathrm{d}Q_{i\varepsilon}(t)/\mathrm{d}t$. Therefore, the measure $\tau(t)\sim 1$ indicates the existence of possible systems of structured metastable equilibrium by maintaining the sum of the total entropy as $\mathrm{d}S(t)/\mathrm{d}t=0$.

The measure $\tau(t)$ suggests that a homeostatic equilibrium for dynamic states could be possible if thermal and energy fluctuations are given by $\mathrm{d}Q_d(t)/\mathrm{d}t=\mathrm{d}Q_\varepsilon(t)/\mathrm{d}t$. Such a stable thermal structure could be shown schematically in **Figure 15**, which is possible to exist in the environment $E$ with the overall constant temperature $T$ and $\mathrm{d}S(t)/\mathrm{d}t=0$. The new image of semi-active view of a *homeostatic equilibrium* is stable and compatible with the dynamic state of the universe, galaxies and planets, systems of living beings. The concept of a homeostatic equilibrium given by $\mathrm{d}S_i/\mathrm{d}t \sim 0$ and the total entropy $\mathrm{d}S(t)/\mathrm{d}t = 0$ as a whole is consistent with the observed nature in reality. The picture is different from a disordered, static thermodynamic equilibrium often explained in thermodynamic literatures.

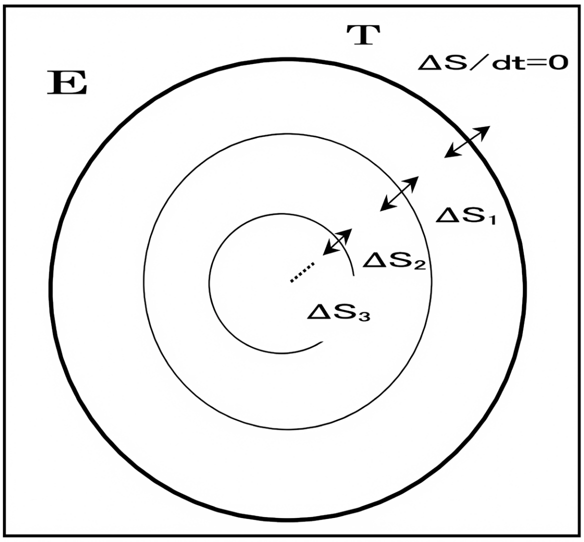


**Figure 15.** The structured thermal metastable system in a closed environment $E$. It is satisfactory to be $\mathrm{d}S_i/\mathrm{d}t \sim 0$ in each internal system, as long as the condition: $\sum_{i=1}^{n}\mathrm{d}S_i(t)/\mathrm{d}t = \mathrm{d}S(t)/\mathrm{d}t = 0$ in the total system $E$ is maintained. Maxwell's demons are supposed to be working as the counterbalance between entropy and energy flow: $\mathrm{d}Q_{id}(t)/\mathrm{d}t = \mathrm{d}Q_{i\varepsilon}(t)/\mathrm{d}t$ in each internal system, resulting in $\mathrm{d}S(t)/\mathrm{d}t = 0$ as a whole.

Lake Hillier shown in **Figure 16** is a saline lake on the edge of Middle Island, off the south coast of Western Australia. The lake is known as the coexistence of conspicuous surface between a colorless and a pink colored strata. The vibrant pink color is stable in the natural environment of NISs and does not alter for a very long time, which is believed due to the presence of microorganisms. The natural phenomenon is compatible with the conclusion of Remark 4, which explains that the metastable equilibrium in a stratified system is possible in nature, with the counterbalancing condition, $\mathrm{d}Q_d(t)/\mathrm{d}t = \mathrm{d}Q_\varepsilon(t)/\mathrm{d}t$ and the over all condition, $\sum_{i=1}^{n}\mathrm{d}S_i(t)/\mathrm{d}t = \mathrm{d}S(t)/\mathrm{d}t = 0$.

The conventional thermodynamics equilibrium $\mathrm{d}S(t)/\mathrm{d}t = 0$ and the *homeostatic equilibrium* $\mathrm{d}S_i(t)/\mathrm{d}t \sim 0$ with overall condition $\sum_{i=1}^{n}\mathrm{d}S_i(t)/\mathrm{d}t = \mathrm{d}S(t)/\mathrm{d}t = 0$ are fundamental in thermodynamics. The concept of a structured metastable state in **Figure 15** would be a physical foundation of NISs and could be the basis for stable biological systems and coordinated regulations. These systems have internal stable structure, steady physical and chemical flows of heat and energies. The stratified stable thermal systems could support the foundation of *homeostasis* for all actions of living beings. The measure $\tau(t)$ is useful and can speak physics more than Maxwell's demon invokes for thermodynamic properties and phenomena.

**Figure 16.** The Lake Hillier: a stratified colored lake, the south coast of Western Australia. The vibrant color is considered to be permanent and does not alter for a very long time. It could be an example of the structured thermal metastable system, but it is in open environment and so, it would be eventually dissipated. No dissipation of heat nor temperature difference between the internal borderline surface would be observed, as signified as $\mathrm{d}S_i/\mathrm{d}t \sim 0$.

## 6. The Mechanical Clock Time vs the Thermodynamic Time; Time and Causality

The discussions of Maxwell's demon in terms of TMD are useful and enlightening to understand theoretical possibilities and technical applications of thermodynamics in NISs. The macroscopic world of thermodynamics is characterized by *entropy productions*, *time-symmetry breaking* and *causality*. Hence, it is inevitable to confront with the fundamental problem of the *origin of time*, induced by the entropy production, time-symmetry breaking and time-dependent progresses of thermal quantities. In order to avoid confusion and difficulty on the concept of time, the difference between the *mechanical clock time* and the *thermodynamic time* should be explained in detail.

The mechanical clock time is a precise, objective time-calibration for ordering of events in a macroscopic thermal system by independent observers external to an observing system. The independent observers have precise timekeeping instruments, such as the cesium Cs-clock time calibration. On the contrary, the thermodynamic time is subjective time recognized by *internal observers* or thermodynamic system itself, such as living beings. The internal observers could be extended to creatures including human beings, because different creatures have different lifetime, corresponding to the process of motions, actions and homeostasis of structured thermal bodies. It is important to distinguish the two types of time for correct scientific understandings and discussions. Interested readers are also advised to read discussions in the paper [15].

[Remark 5] The definition of mechanical clock time

The mechanical clock time is the time commonly used to specify events in everyday life. The mechanical time must be exactly reproduced for most practical purposes and scientific measurements, and the invention of the cesium Cs atomic clock in 1955 has facilitated a precise time-calibration in modern society. Hence, the mechanical time is necessary to record variations and order of events for data analyses. The recording of events or data by the mechanical clock time is just one to one correspondence between events and scales of time.

The mechanical time is just a parameter to denote ordering of events, and so, the recording is only one to one correspondence of events with numbers of the time-axis, which has nothing to do with causality and arrow of time. Although the mechanical time is extended to the space-time coordinate in the theory of relativity, it has yet an intrinsic problem that the time-coordinate is restricted to one direction from the past to the future. The concept of time, causality and arrow of time become important when physical meanings and fundamental dynamics are studied and investigated. It should be emphasized that the internal time is measured and calibrated by an observer essentially outside a metastable system; however, the external observer exists apparently in a time-varying nonequilibrium state outside a metastable internal system. It is not possible for external observers to signify the ordering of events inside a metastable system, and an external observer can only record series of events by employing the mechanical clock time.

The temperature progresses to a corresponding metastable state as $\tilde{T}(t) \to T_0$, and the constant homogeneous temperature $T_0$ is determined after a metastable state or the maximum entropy state is accomplished as, $\mathrm{d}S(t)/\mathrm{d}t = \sum_{i=1}^{n} \mathrm{d}S_i(t)/\mathrm{d}t = 0$. The time variable $t$ will disappear when $\tilde{T}(t)$ reaches $T_0$. The changes of state and temperature determine the *arrow of time*, or *causality* which makes an ordering of events possible from the past to the future [14]. It is not possible to record changes of states inside a stable thermal system in **Figure 15**, because the change of state is counterbalanced with $\mathrm{d}Q_d(t)/\mathrm{d}t = \mathrm{d}Q_\varepsilon(t)/\mathrm{d}t$. There exists no concept of time to distinguish the past and the future inside the system E in **Figure 15**, because the time-symmetry is maintained in all metastable states inside E with $\mathrm{d}S(t)/\mathrm{d}t = 0$ as a whole. The external observer can record time by the standard mechanical clock time; however, it is irrelevant for the internal system E, whether the internal time is passing by according to the external observer's mechanical time.

The mechanical clock time is assumed and used in fundamental classical and quantum equations of motion, and the order of events is recorded by using the mechanical clock time and scales of mathematical coordinate. It is meaningful to record the order of events, but it is meaningless to think the existence of the future and the past in the mathematical coordinate axis of time. The mathematical coordinate time axis is essentially different from space coordinates, because reverse direction is strictly restricted in the macroscopic real world due to the law of entropy. The concept of the thermodynamic time is fundamentally different from that of classical and quantum mechanics. The time in classical and quantum mechanics is used as the auxiliary variable or the parameter to order events of dynamical motions, events and changes of states. The dynamics of nature and nonequilibrium irreversible thermodynamics demand the arrow of time, ordering of time and causality, time-asymmetry of thermal states. The thermodynamic time is fundamental when the physical meanings and dynamics are investigated.

[Remark 6] The genesis of thermodynamic time

The thermodynamic time is clearly recognized by measuring the events of time variations of thermal quantities and changes in nonequilibrium thermal states. Temperature variations are readily recorded according to a sequence of time from a beginning to an end; however, the concept of the beginning and end of time is the characteristic property only in NISs. The temperature variations and time, $\tilde{T}(t) \to T_0$, define simultaneously the characteristic thermodynamic time and the direction of time. The thermodynamic time has its own characteristic length of time, depending on changes of heat-energy flows and homeostatic systems. Therefore, the thermodynamic time is determined and characterized with thermal structures and homeostasis, motions and actions induced by nonequilibrium irreversible thermodynamics.

The time evolutions in TMD are produced by the time-dependent dissipative equation of motion, which is the nonlinear differential equation with time-dependent coefficients (NDE-TC). The solutions of NDE-TC is fundamentally different from the corresponding nonlinear differential equations of motion with constant coefficients (NDE-CC) produced by Lagrangian or Hamiltonian method. The equations of motion are usually produced as NDE-CC in classical mechanics, electromagnetism and Schrödinger equations in quantum mechanics. It should be emphasized that the nonlinear equations with time-dependent coefficients (NDE-TC) have independent solutions to those of corresponding NDE-CC. Therefore, it is fundamental to distinguish the class of NDE-TC from NDE-CC mathematically for dynamical discussions [30]. The thermomechanical time emerges from entropy variations and phase transition, resulting in NDE-TC.

The time-dependence produced by the dissipative equation of motion is succeeded to the measure $\tau(t)$ with the conservation of heat-energy flows. The phase transitions from thermodynamic equilibrium to NISs and time variations of dynamical quantities, such as internal energy $E(t)$, thermal work $W(t)$, entropy $S(t)$ and nonequilibrium temperature $\tilde{T}(t)$ are all thermodynamically consistent. The phase transition phenomena and the concept of time originate simultaneously in NISs, and evolutions of time-dependent events and physical quantities are induced simultaneously.

The thermodynamic time signifies a characteristic time span intrinsic in a homeostatic system, and the thermal changes of a homeostatic system are usually very stable and long even in natural environment, as explained in the colored lake in **Figure 16**. The nonequilibrium thermodynamic time defined by $\tilde{T}(t) \to T_0$, could support long lasting, stable systems observed in nature, such as cells, biological systems and living beings. The concept of the thermodynamic time for the beginning and end is physically fundamental, related to causality and the direction of the maximum entropy. Hence, the thermodynamic time embodies important scientific, physical meanings of dynamics and existence.

The mechanical clock time is a mathematical construct given by a calibration of time axis, and no conceptual problem exists on recording ordered events by the mechanical clock as data of space coordinates. However, if the mechanical clock time is considered produced by dynamics of causality or equivalent with the thermodynamic time, it would cause confusions on the concept of time. The concept of the beginning and end, characteristic length of time and causality are inherent in the thermodynamic time, which is fundamentally related to the law of entropy, dynamics of macroscopic systems and states.

It is remarkable that the measure $\tau(t)$ in TMD explains the work of Maxwell's demon mathematically and extends the picture of thermodynamic equilibrium into structured metastable states (homeostatic equilibrium), which helps understand the phase transition from thermodynamic equilibrium to NISs. It demands the distinction of the mechanical clock time from the thermodynamic time perceived subjectively by human beings. The distinction is essential to understand the physical meanings of Maxwell's demon, the law of entropy, causality and time. The analysis of NISs in terms of the method of TMD is useful to understand the concept of time, thermodynamic equilibrium and the law of entropy.

## 7. Conclusions and Perspectives

The mathematical expression and physical concept of Maxwell's demon are discussed in detail in terms of the method of TMD, resulting in precise and enlightening extensions of thermodynamics to nonequilibrium irreversible thermodynamics. The measure $\tau(t)$ can explain the role of Maxwell's demon mathematically and, moreover, the measure extends theoretical and technical consequences of thermodynamics more than Maxwell's demon can achieve in NISs. The constructive discussions and extensions of thermodynamic equilibrium are provided explicitly, such as the stability and instability of time-evolving thermal systems, mechanism of ignitions and detonations, existence of structured stable thermal states (homeostatic equilibrium) among solid, liquid and gas phases, and the concept of thermodynamic time and causality [28]-[31].

The definition of an open or a closed thermal system, the picture of the maximum state of entropy and metastable state are extended to homeostatic equilibrium. The homeostatic equilibrium is different from a traditional image of disordered, chaotic, static thermodynamic equilibrium. It could help a foundation of dynamics for organic cells, microbes and living beings. The stability of homeostatic equilibrium would be maintained by counterbalancing thermal fluctuations as $\mathrm{d}Q_d(t)/\mathrm{d}t = \mathrm{d}Q_\varepsilon(t)/\mathrm{d}t$ in each internal system with the law of maximum entropy $\mathrm{d}S(t)/\mathrm{d}t = 0$ as a whole. The condition clearly suggests that a thermodynamic state can integrate a structured internal system as the foundation for dynamic metastable states, which would be useful for understanding the complex dynamic structure of macroscopic world.

The time-dependent temperature $\tilde{T}(t)$ and the genesis of time are intertwined in the generation of NISs, and $\tilde{T}(t)$ becomes slowly uniform according to heat-energy flows among internal thermal systems. However, it should be noticed that tiny thermal fluctuations could be admissible as $\mathrm{d}Q_d(t)/\mathrm{d}t \sim \mathrm{d}Q_\varepsilon(t)/\mathrm{d}t$ to maintain metastable thermal states. It is imperative to distinguish an internal observer from an external observer when time is used to investigate dynamics of physical phenomena. It should be noticed that the external observer outside a metastable system can only employ the mechanical clock time to record the order of events. The time and the speed for events and actions required by internal systems are determined by structure and functionality of internal systems; this is the homeostasis in case of living beings.

Living beings have their own characteristic concept of time, and naturally, the length and the cognition concerning the thermodynamic time are mutually different. When time is recognized by an internal observer as dynamical changes of thermal structures, states of cells and organisms, the concept of time becomes more than an ordering of events and positions. Therefore, the thermodynamic time is essentially connected to causality and consciousness of living beings. On the contrary, the mechanical clock time connects data of events to the scales of time coordinates. In order to avoid logical confusions about the concept of time, one should realize the fundamental difference between the thermodynamic time and the mechanical clock time.

The special theory of relativity replaced Newton's absolute time (the mechanical clock time) with the time-axis of space-time. The time in quantum mechanics is similarly extended from $-\infty$ to $+\infty$ by maintaining time-symmetry completely; however, the time-axis is a mathematical construct for a calibration of ordering events. The causality, time-evolution and the law of maximum entropy are intertwined each other, which remains challenging fundamental problems. The method of TMD provides a physical meaning to study causality of events, time-evolution, dynamics of nature and the law of entropy in the macroscopic thermodynamic world.

The measure $\tau(t)$ in TMD is useful to interrelate the law of entropy, heat-energy flows, the thermodynamic time and metastable systems in NISs. It reminds us of a profound notion: Science has no final form and is moving away from a static geometrical picture towards a description in which evolution and history play an essential role [1], which would demand necessary extensions of equilibrium thermodynamics. The method of TMD precisely explains and enlightens the fundamental physical concept of Maxwell's demon, thermodynamic time, existence of time and nonequilibrium dynamics of the macroscopic thermodynamic world.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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