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H_∞ Control of Impulsive Switched Systems with Time-Varying Delays

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Abstract

This paper investigates the issue of H_∞ control for impulsive switched linear systems with time-varying delays. The effect of two types of impulses, namely stabilizing impulses and destabilizing impulses, on the systems is fully considered. Correspondingly, the Lyapunov-Razumikhin (L-R) technique and Lyapunov-Krasovskii (L-K) functional are used to deal with the time-varying delays, some conditions are given for the H_∞ control of the systems. A relationship among the ADT conditions, impulses, and continuous dynamics is established. Finally, two numerical examples are given to illustrate the effectiveness of the proposed results.

Keywords

Impulsive Switched Time-Varying Delay System, H_∞ Control, Average Dwell-Time, Lyapunov-Krasovskii Functional, Lyapunov-Razumikhin Technique

1. Introduction

With the development of modern industrial production, more and more systems encountered in practice can no longer be completely described by a single continuous-time dynamic process or discrete event process. Therefore, hybrid systems such as switched systems and impulsive systems have aroused great interest among researchers. Switched systems are a class of dynamical systems that consist of a finite number of subsystems and a logical rule that orchestrates switching between these subsystems. It is important to study switched systems because of the fact that they provide a natural and convenient unified framework for mathematical modeling of many physical phenomena [1] [2] and practical applications [3]. An impulsive system is a kind of hybrid system that shows the continuous dynamics on the continuous-time interval and the discrete dynamics at some moments which impulses occur. Due to characters of lower cost and higher confidentiality, there are many applications of impulsive systems, such as economics [4], communication security [5], and biomedical engineering [6].

It is worth noting that the subsystems in switched systems are easily affected by impulses when switching happens, a more comprehensive model, *i.e.*, impulsive switched systems are naturally formed, which have extensive applications in many fields such as biological population management [7], network control [8], flying object motions [9], frequency-modulated signal processing systems [10]. Impulsive switched systems, as is known to all, do not retain the property of constituent modes. Moreover, impulsive effects that the state of a system may undergo abrupt change at some moments can be classified into two types: stabilizing impulses and destabilizing impulses, which may affect the system dynamics in a positive and a negative way respectively. To be specific, the stabilizing impulses can promote stability and enhance prescribed system performance, while destabilizing impulses undermine system stability and lead to poor performance. Therefore, the study of impulsive switched systems is more comprehensive and challenging. In [11], the authors studied exponential stability and L_2-gain for nonlinear impulsive switched systems by the multiple Lyapunov function method. In [12], based on multiple Lyapunov functions, some conditions of finite-time stability have been derived for a class of nonlinear impulsive switched systems. By using the Lyapunov function and the average impulsive switched internal approach, the input-to-state stability of nonlinear impulsive switched systems has been investigated in [13]. In [14], under the novel Lyapunov-like function, the authors discussed the globally uniformly exponentially stable of impulsive switched linear systems.

External disturbance is inevitable in actual systems [15] [16]. The existence of external disturbance seriously affects the performance of systems and even leads to instability. H_∞ control is an effective and important approach to dealing with external disturbance. It can be successfully used to design a controller such that the output is upper bounded by limiting the external disturbance while the internal stability is ensured. Many interesting results on H_∞ control for switched systems have been reported in recent years. By utilizing Riccati-Metzler inequalities, the dynamic output feedback H_∞ control of switched linear systems has been investigated in [17]. Ref. [18] considered H_∞ control of nonlinear switched systems via introducing average dwell time (ADT) and piecewise Lyapunov functions approaches. Based on multiple Lyapunov functions and mode-dependent average dwell time, [19] studied the finite-time H_∞ control problem of switched linear systems. [20] developed the multiple Lyapunov functions to H_∞ control nonlinear switched systems. Some other interesting results on H_∞ control for switched systems refer to [21]-[23]. It is worth noting that the impulse effect has not been considered in the above contributions. In [24], based on the ADT method and Lyapunov function, the authors studied the exponential stability and L_2-gain analysis of impulsive switched linear systems, where both stabilizing impulses and destabilizing impulses were considered. In [25], from the point of impulsive control, finite-time H_∞ control for nonlinear impulsive switched systems with a dynamic output feedback controller was investigated via multiple Lyapunov functions and a mode-dependent average dwell time condition. Although the above papers take the impulse effect into account when switching happens, time-delays are excluded in their studied switched systems. Nevertheless, in practical systems, time delays are unavoidable, which are the main cause of poor performance or even instability. Therefore, it is necessary to study the H_∞ control for impulsive switched systems with time-delays. On the other hand, it is well known that the L-K functional and the L-R technique are two main distinct approaches to dealing with the stability problems of various delayed systems. Especially when the time-delays are unknown or unmeasurable, the L-R technique is more efficient and powerful. In [26], based on Razumikhin technique and Lyapunov functions, authors attempt to derive robust exponential stability of nonlinear impulsive switched time-delays systems when the impulses are stabilizing. By employing L-K functional method, [27] focused on the finite-time control for nonlinear impulsive switched with time-varying delays under destabilizing impulses. However, under the effect of the external disturbance, a basic and important issue on how to develop L-K functional method and L-R technique for H_∞ control of impulsive switched time-delays systems remains unresolved. Especially when both stabilizing impulses and destabilizing impulses are considered, it is worth deep study, which motivates this work.

In this paper, we investigate the H_∞ control problem of impulsive switched linear systems with time-varying delays. Some sufficient conditions, which rely on a relation among switchings, impulses, and external disturbance, are presented. The main contributions are threefold: 1) Compared with [24] [25], the effect of time-varying delays on the systems are taken into account in this paper; 2) Two types of impulses, *i.e.* the destabilizing impulses and stabilizing impulses, are regarded respectively 3) In the case of destabilizing impulses, utilizing L-K functional and ADT condition, the H_∞ control performance of the delayed impulsive switched systems is guaranteed. The L-R technique and reverse ADT condition is used to obtain the H_∞ control for the delayed impulsive switched systems under the stabilizing impulses.

2. Preliminaries

Notations. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of non-negative real numbers, $\mathbb{Z}_+$ the set of positive integers and $\mathbb{R}^n$ the n-dimensional real space equipped with the Euclidean norm $\|\cdot\|$. $L_2[t_0,\infty)$ denotes the space of square integrable vector functions on $[t_0,+\infty)$. $A>0\ (\geq,<,\leq 0)$ denotes that the matrix A is a symmetric positive definite (positive semi-definite, negative definite, negative semi-definite) matrix. Moreover, A^{T} and A^{-1} denote the transpose and the inverse of A, respectively. Let $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) be the maximum (minimum) eigenvalue of A. I denotes the identity matrix with appropriate dimensions. $\mathcal{M}=\{1,2,\cdots,\bar{m}\}$, $\bar{m}\in\mathbb{Z}_+$, is an index set. For any set

$U \subseteq \mathbb{R}^k (1 \le k \le n)$, interval $S \subseteq \mathbb{R}$, $PC(S,U) = \{\varphi : S \to U$ is continuous everywhere except for a finite number of points *t*, where $\varphi(t^-), \varphi(t^+)$ exist and $\varphi(t^+) = \varphi(t)\}$. Let $\|\varphi\|_\tau = \sup_{\theta \in [-\tau, 0]} \|\varphi(t_0 + \theta)\|$. In addition, the notation $\star$ always denotes the symmetric block in one symmetric matrix.

In this paper, we consider the following impulsive switched systems with time-varying delays:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)} x(t) + D_{\sigma(t)} x(t - \tau(t)) + B_{\sigma(t)} u(t) + F_{\sigma(t)} \omega(t), t \ne t_k, \\ \Delta x(t) = H_{\sigma(t)} x(t^-), t = t_k, \\ y(t) = C_{\sigma(t)} x(t) + E_{\sigma(t)} \omega(t), \\ x(t) = \varphi(t), t \in [t_0 - \tau, t_0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $y(t) \in \mathbb{R}^n$ is the output, $\omega(t) \in \mathbb{R}^q$ is the exogenous disturbance which belongs to $L_2[t_0, \infty)$, $u(t) \in \mathbb{R}^p$ is the control input; $A_{\sigma(t)}$, $B_{\sigma(t)}$, $C_{\sigma(t)}$, $D_{\sigma(t)}$, $E_{\sigma(t)}$, $F_{\sigma(t)}$, $H_{\sigma(t)}$ are constant matrices with appropriate dimensions; $\varphi(t) \in PC([t_0 - \tau, t_0], \mathbb{R}^n)$ is the initial condition; $\sigma(t) : [t_0, \infty) \to \mathcal{M} = \{1, 2, \cdots, \bar{m}\}$ is the switching signal, which can be characterized by the switching sequence $(i_0, t_0), (i_1, t_1), \cdots, (i_j, t_j), \cdots$ and $\sigma(t) = i_k$ means that the i_k th subsystem is activated when $t \in [t_k, t_{k+1})$; $\tau(t) : \mathbb{R}_+ \to \mathbb{R}_+$ denotes the time-varying delay; $\Delta x(t) = x(t^+) - x(t^-)$,
$x(t) = x(t^+) = \lim_{h \to 0^+} x(t+h)$, $x(t^-) = \lim_{h \to 0^-} x(t+h)$.

In this paper, we make the following assumptions.

Assumption 1. $\tau(t)$ is continuous and bounded

$$0 < \tau(t) \le \tau,$$

where τ is a positive constant.

Assumption 2. $\tau(t)$ is differentiable and its derivative is bounded

$$\dot{\tau}(t) \le d < 1,$$

where d is a positive constant.

This paper, in order to achieve the H_∞ performance of the system, considers a class of state feedback controllers

$$u(t) = K_{\sigma(t)} x(t), \tag{2}$$

where $K_{\sigma(t)}$ is the state feedback gain matrix. Substituting (2) into (1), one has

$$\begin{cases} \dot{x}(t) = (A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)}) x(t) + D_{\sigma(t)} x(t - \tau(t)) + F_{\sigma(t)} \omega(t), t \ne t_k, \\ x(t) = (I + H_{\sigma(t)}) x(t^-), t = t_k, \\ y(t) = C_{\sigma(t)} x(t) + E_{\sigma(t)} \omega(t), \\ x(t) = \varphi(t), t \in [t_0 - \tau, t_0]. \end{cases} \tag{3}$$

It is assumed that the switches and impulses occur at the same time. Let $\{t_k\}, k \in \mathbb{Z}_+$ be a sequence of discrete times representing the impulsive switching instants. To avoid Zeno behavior, the sequence $t_0 < t_1 < t_2 < \cdots < t_k$ satisfies that $t_k \to +\infty$ as $k \to \infty$. Given an impulsive switching time sequence $\{t_k\}$ and a switching signal $\sigma(t)$, the set $\{t_k, \sigma(t)\}$ is called the impulsive switching signal which is denoted by $\mathcal{F}$.

Definition 1. ([7]) For a class of impulsive switching signal $\mathcal{F}_1[\tau_\alpha, N_1]$ satisfying the ADT condition given by

$$N_\sigma(T,t) \le N_1 + \frac{T-t}{\tau_\alpha}, \ \forall T \ge t > 0, \tag{4}$$

where $N_\sigma(T,t)$ is the number of impulses or switches occurring on the interval $[t,T)$, $N_1 > 0$ is called the chatter bounds and τ_α is called ADT constant. Similarly, for a class of impulsive switching signal $\mathcal{F}_2[\tau_\alpha, N_2]$ satisfying the reverse ADT condition given by

$$N_\sigma(T,t) \ge -N_2 + \frac{T-t}{\tau_\alpha}, \ \forall T \ge t > 0, \tag{5}$$

where $N_\sigma(T,t)$ is the number of impulses or switches occurring over the interval $[t,T)$, $N_2 > 0$ is called the chatter bounds and τ_α is called reverse ADT constant.

Definition 2. System (3) is said to be exponentially stable (ES) over the impulsive switching signal $\mathcal{F}$, if the solution $x(t)$ of system (3) satisfies

$$\|x(t)\| \le \mathcal{K}\|\varphi\|_\tau \mathrm{e}^{-\lambda(t-t_0)}, \forall t \ge t_0,$$

where $\mathcal{K} \ge 1$ and $\lambda > 0$.

Definition 3. ([28]) For a given constant $\gamma > 0$, the system (1) is said to achieve H_∞ control performance under the impulsive switching signal $\mathcal{F}$ if there exist control input $u(t)$ and an impulsive switching signal $\mathcal{F}$ satisfying the following conditions

1) The system (3) with $\omega(t) = 0$ is ES;

2) Under the zero initial condition $x(t) = 0$, $t \in [t_0 - \tau, t_0]$, the system (3) satisfies the H_∞ performance index for any $\omega(t) \ne 0$

$$\int_{t_0}^{\infty} y^{\mathrm{T}}(s) y(s) \mathrm{d}s \le \gamma^2 \int_{t_0}^{\infty} \omega^{\mathrm{T}}(s) \omega(s) \mathrm{d}s. \tag{6}$$

Lemma 1. ([29]) Let $\mathcal{S}$ and $\mathcal{T}$ be real matrices of appropriate dimensions. Then, for any matrix $\mathcal{Q} > 0$ of appropriate dimension and any scalar $\varepsilon > 0$, the following inequality holds

$$\mathcal{S}\mathcal{T} + \mathcal{T}^{\mathrm{T}}\mathcal{S}^{\mathrm{T}} \le \varepsilon^{-1}\mathcal{S}\mathcal{Q}^{-1}\mathcal{S}^{\mathrm{T}} + \varepsilon\mathcal{T}^{\mathrm{T}}\mathcal{Q}\mathcal{T}. \tag{7}$$

3. Main Results

In this section, some sufficient conditions are established to guarantee the H_∞ performance of delayed impulsive switched systems. Both destabilizing and stabilizing impulses are taken into account. To start with, the case of destabilizing impulses is considered.

3.1. Destabilizing Impulses

In this section, Assumption 2 is required for the L-K analysis. The L-K framework relies on constructing functionals that integrate the system state over the entire delay interval, which necessitates the bound on the derivative of the time-varying delay to ensure the stability of the functional derivative. Considering the detrimental effects of destabilizing impulses on system performance, this section aims to design a state feedback controller to ensure the H_∞ control performance of system (3). Utilizing the L-K functional method and the ADT condition, we derive some sufficient conditions under which the H_∞ performance of system (3) is guaranteed.

Firstly, we study the exponential stability for the system (3) with $\omega(t)=0$, that is,

$$\begin{cases} \dot{x}(t)=\left(A_{\sigma(t)}+B_{\sigma(t)}K_{\sigma(t)}\right)x(t)+D_{\sigma(t)}x\left(t-\tau(t)\right),\ t\neq t_k, \\ x(t)=\left(I+H_{\sigma(t)}\right)x\left(t^-\right),\ t=t_k, \\ y(t)=C_{\sigma(t)}x(t), \\ x(t)=\varphi(t),\ t\in\left[t_0-\tau,t_0\right], \end{cases} \tag{8}$$

Theorem 1. Assume that Assumption 1 and 2 hold. For system (8), given constant $\alpha>0$ and $n\times n$ matrix H_i, if there exist constant $\mu\geq 1$, and $n\times n$ matrices $P_i>0$, $Q_i>0$, such that the following linear matrix inequalities (LMIs) hold

$$\begin{pmatrix} \Omega_i & P_iD_i \\ \star & -(1-d)\mathrm{e}^{-\alpha\tau}Q_i \end{pmatrix}<0,\ \forall i\in\mathcal{M}, \tag{9}$$

$$\begin{pmatrix} -\mu P_j & (I+H_i)^{\mathrm{T}}P_i \\ \star & -P_i \end{pmatrix}<0,\ \forall i,j\in\mathcal{M}, i\neq j, \tag{10}$$

$$Q_i<\mu Q_j,\ \forall i,j\in\mathcal{M}, \tag{11}$$

where $\Omega_i=P_iA_i+A_i^{\mathrm{T}}P_i+P_iB_iK_i+K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i+\alpha P_i+Q_i$, and the ADT constant τ_α satisfies

$$\tau_\alpha>\frac{\ln\mu}{\alpha}, \tag{12}$$

then, the system (8) is ES over the impulsive switching signal $\mathcal{F}_1\left[\tau_\alpha,N_1\right]$.

Proof. Choose the L-K functional candidate as

$$\begin{aligned} V_{\sigma(t)}(t)&=V_{1\sigma(t)}(t)+V_{2\sigma(t)}(t) \\ &=x^{\mathrm{T}}(t)P_{\sigma(t)}x(t)+\int_{t-\tau(t)}^{t}\mathrm{e}^{\alpha(s-t)}x^{\mathrm{T}}(s)Q_{\sigma(t)}x(s)\mathrm{d}s. \end{aligned} \tag{13}$$

The derivative of $V_i(t)$ along the trajectory of system (8) on $\left[t_k,t_{k+1}\right)$, where the i_k th subsystem is activated, is

$$\begin{aligned} D^+V_{i_k}(t)&=x^{\mathrm{T}}(t)P_{i_k}\dot{x}(t)+\dot{x}^{\mathrm{T}}(t)P_{i_k}x(t)-\alpha V_{2i_k}(t)+x^{\mathrm{T}}(t)Q_{i_k}x(t) \\ &\quad-\left(1-\dot{\tau}(t)\right)\mathrm{e}^{-\alpha\tau(t)}x^{\mathrm{T}}\left(t-\tau(t)\right)Q_{i_k}x\left(t-\tau(t)\right) \\ &\leq x^{\mathrm{T}}(t)\left(P_{i_k}A_{i_k}+A_{i_k}^{\mathrm{T}}P_{i_k}+P_{i_k}B_{i_k}K_{i_k}+K_{i_k}^{\mathrm{T}}B_{i_k}^{\mathrm{T}}P_{i_k}\right)x(t) \\ &\quad+x^{\mathrm{T}}(t)P_{i_k}D_{i_k}x\left(t-\tau(t)\right)+x^{\mathrm{T}}\left(t-\tau(t)\right)D_{i_k}^{\mathrm{T}}P_{i_k}x(t)-\alpha V_{2i_k}(t) \\ &\quad+x^{\mathrm{T}}(t)Q_{i_k}x(t)-(1-d)\mathrm{e}^{-\alpha\tau}x^{\mathrm{T}}\left(t-\tau(t)\right)Q_{i_k}x\left(t-\tau(t)\right). \end{aligned}$$

It follows from (9) that

$$\begin{aligned} D^{+}V_{i_k}(t)+\alpha V_{i_k}(t) \le\, & x^{\mathrm{T}}(t)\left(P_{i_k}A_{i_k}+A_{i_k}^{\mathrm{T}}P_{i_k}+P_{i_k}B_{i_k}K_{i_k}+K_{i_k}^{\mathrm{T}}B_{i_k}^{\mathrm{T}}P_{i_k}\right)x(t) \\ & +x^{\mathrm{T}}(t)P_{i_k}D_{i_k}x(t-\tau(t))+x^{\mathrm{T}}(t-\tau(t))D_{i_k}^{\mathrm{T}}P_{i_k}x(t) \\ & -(1-d)\mathrm{e}^{-\alpha\tau}x^{\mathrm{T}}(t-\tau(t))Q_{i_k}x(t-\tau(t)) \\ & +x^{\mathrm{T}}(t)Q_{i_k}x(t)+\alpha x^{\mathrm{T}}(t)P_{i_k}x(t) \\ & =\eta^{\mathrm{T}}(t)\begin{pmatrix}\Omega_{i_k} & P_{i_k}D_{i_k} \\ \star & -(1-d)\mathrm{e}^{-\alpha\tau}Q_{i_k}\end{pmatrix}\eta(t) \\ & <0, \end{aligned} \tag{14}$$

where $\eta(t)=\left(x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-\tau(t))\right)^{\mathrm{T}}$. Integrating both sides of (14) from t_k to t, one gets

$$V_{i_k}(t)\le \mathrm{e}^{-\alpha(t-t_k)}V_{i_k}(t_k), t\in[t_k,t_{k+1}). \tag{15}$$

In addition, let $\sigma(t)=i_{k-1}, t\in[t_{k-1},t_k)$, it follows from (11) that

$$\begin{aligned} V_{i_k}(t_k)-\mu V_{i_{k-1}}(t_k^-) =\, & x^{\mathrm{T}}(t_k)P_{i_k}x(t_k)+\int_{t_k-\tau(t_k)}^{t_k}\mathrm{e}^{\alpha(s-t_k)}x^{\mathrm{T}}(s)Q_{i_k}x(s)\mathrm{d}s \\ & -\mu x^{\mathrm{T}}(t_k^-)P_{i_{k-1}}x(t_k^-)-\mu\int_{t_k^- -\tau(t_k^-)}^{t_k^-}\mathrm{e}^{\alpha(s-t_k^-)}x^{\mathrm{T}}(s)Q_{i_{k-1}}x(s)\mathrm{d}s \\ & \le x^{\mathrm{T}}(t_k)P_{i_k}x(t_k)-\mu x^{\mathrm{T}}(t_k^-)P_{i_{k-1}}x(t_k^-) \\ & = x^{\mathrm{T}}(t_k^-)(I+H_{i_k})^{\mathrm{T}}P_{i_k}(I+H_{i_k})x(t_k^-)-\mu x^{\mathrm{T}}(t_k^-)P_{i_{k-1}}x(t_k^-) \\ & = x^{\mathrm{T}}(t_k^-)\left[(I+H_{i_k})^{\mathrm{T}}P_{i_k}(I+H_{i_k})-\mu P_{i_{k-1}}\right]x(t_k^-). \end{aligned}$$

Based on (10), it can be deduced that

$$\begin{aligned} & \begin{pmatrix}-\mu P_{i_{k-1}} & (I+H_{i_k})^{\mathrm{T}}P_{i_k} \\ \star & -P_{i_k}\end{pmatrix}<0 \\ \Leftrightarrow & \begin{pmatrix}I & (I+H_{i_k})^{\mathrm{T}} \\ 0 & I\end{pmatrix}\begin{pmatrix}-\mu P_{i_{k-1}} & (I+H_{i_k})^{\mathrm{T}}P_{i_k} \\ \star & -P_{i_k}\end{pmatrix}\begin{pmatrix}I & 0 \\ I+H_{i_k} & I\end{pmatrix}<0 \\ \Leftrightarrow & \begin{pmatrix}-\mu P_{i_{k-1}}+(I+H_{i_k})^{\mathrm{T}}P_{i_k}(I+H_{i_k}) & 0 \\ \star & -P_{i_k}\end{pmatrix}<0 \\ \Leftrightarrow & -\mu P_{i_{k-1}}+(I+H_{i_k})^{\mathrm{T}}P_{i_k}(I+H_{i_k})<0. \end{aligned}$$

Hence,

$$V_{i_k}(t_k)\le \mu V_{i_{k-1}}(t_k^-). \tag{16}$$

Combining (15) and (16), it yields

$$\begin{aligned} V_{i_k}(t) & \le \mu\mathrm{e}^{-\alpha(t-t_k)}V_{i_{k-1}}(t_k^-)\le \mu\mathrm{e}^{-\alpha(t-t_{k-1})}V_{i_{k-1}}(t_{k-1})\le\cdots \\ & \le \mu^k\mathrm{e}^{-\alpha(t-t_0)}V_{\sigma(t_0)}(t_0) \\ & = \mathrm{e}^{-\alpha(t-t_0)+N_\sigma(t,t_0)\ln\mu}V_{\sigma(t_0)}(t_0). \end{aligned}$$

It follows from (4) in definition 1 that $N_\sigma(t,t_0)\ln\mu \le N_1\ln\mu + \frac{t-t_0}{\tau_\alpha}\ln\mu$, which implies

$$a\|x(t)\|^2 \le V_{i_k}(t) \le \mathrm{e}^{\left(\frac{\ln\mu}{\tau_\alpha}-\alpha\right)(t-t_0)}\mu^{N_1}V_{\sigma(t_0)}(t_0) \le \mathrm{e}^{-\left(\alpha-\frac{\ln\mu}{\tau_\alpha}\right)(t-t_0)}b\mu^{N_1}\|\varphi\|_\tau^2.$$

where $a=\min_{i\in\mathcal{M}}\{\lambda_{\min}(P_i)\}$, $b=\max_{i\in\mathcal{M}}\{\lambda_{\max}(P_i)+\tau\lambda_{\max}(Q_i)\}$. Therefore, one obtains

$$\|x(t)\| \le \sqrt{\frac{b}{a}}\mu^{\frac{N_1}{2}}\mathrm{e}^{-\lambda(t-t_0)}\|\varphi\|_\tau,$$

where $\lambda=\frac{1}{2}\left(\alpha-\frac{\ln\mu}{\tau_\alpha}\right)$, which implies that system (8) is ES over the impulsive switching signal $\mathcal{F}_1[\tau_\alpha,N_1]$.

□

In what follows, we investigate the H_∞ performance of system (3) with zero initial condition when $\omega(t)\ne 0$.

Theorem 2. Assume that Assumption 1 and 2 hold. Given constants $\alpha>0$, $\gamma>0$ and $n\times n$ matrix H_i, if there exist constants $\mu\ge 1$, $\delta>0$, and $n\times n$ matrices $P_i>0$, $Q_i>0$, $G_i>0$, such that the (9) - (12) and the following LMI hold

$$\begin{pmatrix} C_i^{\mathrm{T}}C_i-\delta P_i & C_i^{\mathrm{T}}E_i & 0 & \sqrt{\delta\kappa}P_i & 0 \\ \star & \Phi_i & 0 & 0 & E_i^{\mathrm{T}} \\ \star & \star & -\tau\delta\mathrm{e}^{-\alpha\tau}Q_i & 0 & 0 \\ \star & \star & \star & -G_i & 0 \\ \star & \star & \star & \star & -I \end{pmatrix}<0,\ \forall i\in\mathcal{M}, \tag{17}$$

where $\Phi_i=-\gamma^2 I+\delta\kappa F_i^{\mathrm{T}}G_iF_i$, $\kappa=\frac{\tau_\alpha\mu^{N_1}}{\alpha\tau_\alpha-\ln\mu}$, then, system (1) has H_∞ control performance over the impulsive switching signal $\mathcal{F}_1[\tau_\alpha,N_1]$.

Proof. In view of the L-K functional (13), we can derive from (10) and (11) that

$$V_{i_k}(t_k)\le\mu V_{i_{k-1}}(t_k^-), \tag{18}$$

It follows from (9) that

$$\begin{aligned} D^+V_{i_k}(t)+\alpha V_{i_k}(t) &\le x^{\mathrm{T}}(t)\left(P_{i_k}A_{i_k}+A_{i_k}^{\mathrm{T}}P_{i_k}+P_{i_k}B_{i_k}K_{i_k}+K_{i_k}^{\mathrm{T}}B_{i_k}^{\mathrm{T}}P_{i_k}\right)x(t) \\ &\quad +x^{\mathrm{T}}(t)P_{i_k}D_{i_k}x(t-\tau(t))+x^{\mathrm{T}}(t)Q_{i_k}x(t) \\ &\quad +x^{\mathrm{T}}(t-\tau(t))D_{i_k}^{\mathrm{T}}P_{i_k}x(t)+2x^{\mathrm{T}}(t)P_{i_k}F_{i_k}\omega(t) \\ &\quad -(1-d)\mathrm{e}^{-\alpha\tau}x^{\mathrm{T}}(t-\tau(t))Q_{i_k}x(t-\tau(t))+\alpha x^{\mathrm{T}}(t)P_{i_k}x(t) \\ &= \eta^{\mathrm{T}}(t)\begin{pmatrix}\Omega_{i_k} & P_{i_k}D_{i_k} \\ \star & -(1-d)\mathrm{e}^{-\alpha\tau}Q_{i_k}\end{pmatrix}\eta(t)+2x^{\mathrm{T}}(t)P_{i_k}F_{i_k}\omega(t) \\ &< 2x^{\mathrm{T}}(t)P_{i_k}F_{i_k}\omega(t), \end{aligned}$$

where $\eta(t)=\left(x^{\mathrm{T}}(t)\quad x^{\mathrm{T}}(t-\tau(t))\right)^{\mathrm{T}}$. By integrating the above inequality from t_k to t, we obtain

$$V_{i_k}(t)\le \mathrm{e}^{-\alpha(t-t_k)}V_{i_k}(t_k)+\int_{t_k}^{t}\mathrm{e}^{-\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_k}F_{i_k}\omega(s)\mathrm{d}s,\ t\in[t_k,t_{k+1}). \tag{19}$$

Combining (18) and (19), it yields

$$\begin{aligned}V_{i_k}(t)&\le \mu\mathrm{e}^{-\alpha(t-t_k)}V_{i_{k-1}}(t_k^-)+\int_{t_k}^{t}\mathrm{e}^{-\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_k}F_{i_k}\omega(s)\mathrm{d}s\\&\le \mu\mathrm{e}^{-\alpha(t-t_{k-1})}V_{i_{k-1}}(t_{k-1})+\mu\int_{t_{k-1}}^{t_k}\mathrm{e}^{-\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_{k-1}}F_{i_{k-1}}\omega(s)\mathrm{d}s\\&\quad+\int_{t_k}^{t}\mathrm{e}^{-\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_k}F_{i_k}\omega(s)\mathrm{d}s\\&\le\cdots\\&\le \mu^k\mathrm{e}^{-\alpha(t-t_0)}V_{\sigma(t_0)}(t_0)+\mu^k\int_{t_0}^{t_1}\mathrm{e}^{-\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_0}F_{i_0}\omega(s)\mathrm{d}s+\cdots\\&\quad+\int_{t_k}^{t}\mathrm{e}^{-\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_k}F_{i_k}\omega(s)\mathrm{d}s\\&\le \mathrm{e}^{-\alpha(t-t_0)+N_\sigma(t,s)\ln\mu}V_{\sigma(t_0)}(t_0)+\int_{t_0}^{t}\mathrm{e}^{-\alpha(t-s)+N_\sigma(t,s)\ln\mu}2x^{\mathrm{T}}(s)P_iF_i\omega(s)\mathrm{d}s.\end{aligned}$$

Given the zero initial condition $V_{\sigma(t_0)}(t_0)=0$, one gets

$$V_{i_k}(t)\le\int_{t_0}^{t}\mathrm{e}^{-\alpha(t-s)+N_\sigma(t,s)\ln\mu}2x^{\mathrm{T}}(s)P_iF_i\omega(s)\mathrm{d}s.$$

Based on (4) in Definition 1, we obtain $N_\sigma(t,s)\ln\mu\le N_1\ln\mu+\frac{t-s}{\tau_\alpha}\ln\mu$, which implies

$$V_{i_k}(t)\le\int_{t_0}^{t}\mathrm{e}^{\left(\frac{\ln\mu}{\tau_\alpha}-\alpha\right)(t-s)}\mu^{N_1}2x^{\mathrm{T}}(s)P_iF_i\omega(s)\mathrm{d}s. \tag{20}$$

Then, we apply (20) to make an estimation of $\int_{t_0}^{t}V_{i_k}(s)\mathrm{d}s$, $t\in[t_k,t_{k+1})$. It follows from (12) that

$$\begin{aligned}\int_{t_0}^{t}V_{i_k}(s)\mathrm{d}s&\le\int_{t_0}^{t}\int_{t_0}^{s}\mathrm{e}^{\left(\frac{\ln\mu}{\tau_\alpha}-\alpha\right)(s-v)}\mu^{N_1}2x^{\mathrm{T}}(v)P_iF_i\omega(v)\mathrm{d}v\mathrm{d}s\\&=\int_{t_0}^{t}\int_{v}^{t}\mathrm{e}^{\left(\frac{\ln\mu}{\tau_\alpha}-\alpha\right)(s-v)}\mathrm{d}s\mu^{N_1}2x^{\mathrm{T}}(v)P_iF_i\omega(v)\mathrm{d}v\\&=\int_{t_0}^{t}\frac{\tau_\alpha\mu^{N_1}}{\ln\mu-\alpha\tau_\alpha}\left(\mathrm{e}^{\left(\frac{\ln\mu}{\tau_\alpha}-\alpha\right)(t-v)}-1\right)2x^{\mathrm{T}}(v)P_iF_i\omega(v)\mathrm{d}v\\&\le\kappa\int_{t_0}^{t}2x^{\mathrm{T}}(v)P_iF_i\omega(v)\mathrm{d}v.\end{aligned}$$

Letting $t\to\infty$, (7) yields

$$\begin{aligned}\int_{t_0}^{\infty}V_{i_k}(s)\mathrm{d}s&\le\kappa\int_{t_0}^{\infty}2x^{\mathrm{T}}(v)P_iF_i\omega(v)\mathrm{d}v\\&\le\kappa\int_{t_0}^{\infty}\left[x^{\mathrm{T}}(v)P_iG_i^{-1}P_ix(v)+\omega^{\mathrm{T}}(v)F_i^{\mathrm{T}}G_iF_i\omega(v)\right]\mathrm{d}v.\end{aligned} \tag{21}$$

Set $J_0=\int_{t_0}^{\infty}\left[y^{\mathrm{T}}(s)y(s)-\gamma^2\omega^{\mathrm{T}}(s)\omega(s)\right]\mathrm{d}s$. By (21), we derive

$$\begin{aligned}
J_0 &= \int_{t_0}^{\infty}\left[y^{\mathrm{T}}(s)y(s)-\gamma^2\omega^{\mathrm{T}}(s)\omega(s)-\delta V_i(s)\right]\mathrm{d}s+\delta\int_{t_0}^{\infty}V_i(s)\mathrm{d}s \\
&\le \int_{t_0}^{\infty}\Big[x^{\mathrm{T}}(s)C_i^{\mathrm{T}}C_ix(s)+x^{\mathrm{T}}(s)C_i^{\mathrm{T}}E_i\omega(s)+\omega^{\mathrm{T}}(s)E_i^{\mathrm{T}}C_ix(s)+\omega^{\mathrm{T}}(s)E_i^{\mathrm{T}}E_i\omega(s) \\
&\quad -\gamma^2\omega^{\mathrm{T}}(s)\omega(s)-\delta x^{\mathrm{T}}(s)P_ix(s)-\delta\int_{s-\tau(s)}^{s}\mathrm{e}^{\alpha(v-s)}x^{\mathrm{T}}(v)Q_ix(v)\mathrm{d}v\Big]\mathrm{d}s \\
&\quad +\delta\kappa\int_{t_0}^{\infty}\left[x^{\mathrm{T}}(v)P_iG_i^{-1}P_ix(v)+\omega^{\mathrm{T}}(v)F_i^{\mathrm{T}}G_iF_i\omega(v)\right]\mathrm{d}v \\
&\le \int_{t_0}^{\infty}\frac{1}{\tau(s)}\int_{s-\tau(s)}^{s}\zeta^{\mathrm{T}}(s,v)M_i\zeta(s,v)\mathrm{d}v\mathrm{d}s,
\end{aligned}$$

where $\zeta(s,v)=\left(x^{\mathrm{T}}(s)\quad \omega^{\mathrm{T}}(s)\quad x^{\mathrm{T}}(v)\right)^{\mathrm{T}}$,

$$M_i=\begin{pmatrix} C_i^{\mathrm{T}}C_i-\delta P_i+\delta\kappa P_iG_i^{-1}P_i & C_i^{\mathrm{T}}E_i & 0 \\ \star & E_i^{\mathrm{T}}E_i-\gamma^2I+\delta\kappa F_i^{\mathrm{T}}G_iF_i & 0 \\ \star & \star & -\tau\delta\mathrm{e}^{-\alpha\tau}Q_i \end{pmatrix}.$$ Using the Schur complement of (17), we have $J_0\le 0$, which implies

$$\int_{t_0}^{\infty}y^{\mathrm{T}}(s)y(s)\mathrm{d}s\le\gamma^2\int_{t_0}^{\infty}\omega^{\mathrm{T}}(s)\omega(s)\mathrm{d}s.$$

Hence, system (1) has H_∞ control performance over the impulsive switching signal $\mathcal{F}_1[\tau_\alpha,N_1]$.

□

It should be noted that the above results are established under the assumption that the control gain matrix $K_{\sigma(t)}$ is known. In practice, however, the matrix is usually unknown and needs be designed. Therefore, the following theorem is presented.

Theorem 3. Assume that Assumption 1 and 2 hold. Given constants $\alpha>0$, $\gamma>0$ and $n\times n$ matrix H_i, if there exist constants $\mu\ge 1$, $\delta>0$, and $n\times n$ matrices $X_i>0$, $Z_i>0$, $U_i>0$, $G_i>0$, T_i, such that the following LMIs hold

$$\begin{pmatrix} \tilde{\Omega}_i & D_iX_i \\ \star & -(1-d)\mathrm{e}^{-\alpha\tau}Z_i \end{pmatrix}<0, \tag{22}$$

$$\begin{pmatrix} -\delta X_i & X_iC_i^{\mathrm{T}}E_i & 0 & \sqrt{\delta\kappa}X_i & 0 & X_iC_i^{\mathrm{T}} \\ \star & \Phi_i & 0 & 0 & E_i^{\mathrm{T}} & 0 \\ \star & \star & -\tau\delta\mathrm{e}^{-\alpha\tau}Z_i & 0 & 0 & 0 \\ \star & \star & \star & -U_i & 0 & 0 \\ \star & \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & \star & -I \end{pmatrix}<0,\ \forall i\in\mathcal{M}, \tag{23}$$

$$\begin{pmatrix} -\mu X_j & X_j(I+H_i)^{\mathrm{T}} \\ \star & -X_i \end{pmatrix}<0,\ \ \forall i,j\in\mathcal{M}, i\ne j, \tag{24}$$

$$Z_i<\mu Z_j,\ \ \forall i,j\in\mathcal{M}, \tag{25}$$

where $\tilde{\Omega}_i=A_iX_i+X_iA_i^{\mathrm{T}}+B_iT_i+T_i^{\mathrm{T}}B_i^{\mathrm{T}}+Z_i+\alpha X_i$, $\Phi_i=-\gamma^2I+\delta\kappa F_i^{\mathrm{T}}G_iF_i$, $\kappa=\dfrac{\tau_\alpha\mu^{N_1}}{\alpha\tau_\alpha-\ln\mu}$, and the ADT constant τ_α satisfies

$$\tau_\alpha > \frac{\ln\mu}{\alpha}, \tag{26}$$

then, system (1) has H_∞ control performance over the impulsive switching signal $\mathcal{F}_1[\tau_\alpha, N_1]$. The control gain matrix is given by $K_i = T_i X_i^{-1}$.

Proof. According to Theorem 2, by pre-multiplying and post-multiplying inequality (9) with matrix $diag\{P_i^{-1}, P_i^{-1}\}$, one derives that

$$\begin{pmatrix} W_i & D_i P_i^{-1} \\ \star & -(1-d)\mathrm{e}^{-\alpha\tau} P_i^{-1} Q_i P_i^{-1} \end{pmatrix} < 0.$$

where $W_i = A_i P_i^{-1} + P_i^{-1} A_i^{\mathrm{T}} + B_i K_i P_i^{-1} + P_i^{-1} K_i^{\mathrm{T}} B_i^{\mathrm{T}} + P_i^{-1} Q_i P_i^{-1} + \alpha P_i^{-1}$. Let $X_i = P_i^{-1}$, $T_i = K_i X_i$ and $Z_i = X_i Q_i X_i$. It can be derived that (9) is equivalent to (22). Similarly, by pre-multiplying and post-multiplying inequality (10) and (17) with matrix $diag\{P_j^{-1}, P_i^{-1}\}$ and matrix $diag\{P_i^{-1}, I, P_i^{-1}, P_i^{-1}, I\}$ respectively, we derive

$$\begin{pmatrix} -\mu P_j^{-1} & P_j^{-1}(I+H_i)^{\mathrm{T}} \\ \star & -P_i^{-1} \end{pmatrix} < 0,$$

$$\begin{pmatrix} R_i^{11} & P_i^{-1} C_i^{\mathrm{T}} E_i & 0 & \sqrt{\delta\kappa} P_i^{-1} & 0 \\ \star & R_i^{22} & 0 & 0 & E_i^{\mathrm{T}} \\ \star & \star & R_i^{33} & 0 & 0 \\ \star & \star & \star & R_i^{44} & 0 \\ \star & \star & \star & \star & -I \end{pmatrix} < 0.$$

where $R_i^{11} = P_i^{-1} C_i^{\mathrm{T}} C_i^{\mathrm{T}} P_i^{-1} - \delta P_i^{-1}$, $R_i^{22} = -\gamma^2 I + E_i^{\mathrm{T}} E_i + \delta\kappa F_i^{\mathrm{T}} G_i F_i$, $R_i^{33} = -\tau\delta \mathrm{e}^{-\alpha\tau} P_i^{-1} Q_i P_i^{-1}$, $R_i^{44} = -P_i^{-1} G_i P_i^{-1}$. Define $U_i = X_i G_i X_i$, then (10) and (17) can be derived to be equivalent to (24) and (23) respectively. Therefore, the control gain is matrix designed by $K_i = T_i X_i^{-1}$. This completes the proof. □

Remark In Theorem 3, condition (24) and (25) implies that V increases ($\mu \geq 1$) at impulsive switching instants, which corresponds to characteristics of destabilizing impulses. In the meantime, the ADT τ_α is satisfied with $\tau_\alpha > \frac{\ln\mu}{\alpha}$, which indicates that the dwell-time of each subsystem can not be too short, *i.e.*, the number of impulses and switches can not be too frequent to attenuate the effect of impulsive disturbance on the state of the system.

3.2. Stabilizing Impulses

When impulses exhibit stabilizing effects, the H_∞ control performance of the system (1) is guaranteed by designing a hybrid controller, which is the state feedback controller and impulsive controller. The L-R technique and ADT condition are used to derive the sufficient conditions for the system to have H_∞ performance. Different from the L-K method, which requires the delay derivative bound to handle integral terms in the functional, the L-R technique only needs the instantaneous system state and the delayed state, making Assumption 2 unnecessary in this section. In the following, we first consider the exponential stability of system (8) with $\omega(t)=0$.

Theorem 4. Assume that Assumption 1 holds. For system (8), given constant $\alpha>0$, if there exist constants $0<\mu<1$, $\beta>0$, $\epsilon>0$, and $n\times n$ matrix $P_i>0$, such that the following LMIs hold

$$\begin{pmatrix} \Psi_i & P_iD_i \\ \star & -\epsilon e^{-\beta\tau}P_l \end{pmatrix}<0,\ \forall i,l\in\mathcal{M}, \tag{27}$$

$$\begin{pmatrix} -\mu P_j & P_i+H_i^{\mathrm{T}}P_i \\ \star & -P_i \end{pmatrix}<0,\ \forall i,j\in\mathcal{M}, i\neq j, \tag{28}$$

where $\Psi_i=P_iA_i+A_i^{\mathrm{T}}P_i+P_iB_iK_i+K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i+\left(\beta+\frac{\epsilon}{\mu^{N_2+1}}-\alpha\right)P_i$, and the reverse ADT constant τ_α satisfies

$$\tau_\alpha<-\frac{\ln\mu}{\alpha}, \tag{29}$$

then, the system (8) is ES over the impulsive switching signal $\mathcal{F}_2[\tau_\alpha,N_2]$.

Proof. Consider the Lyapunov function

$$V_{\sigma(t)}(t)=x^{\mathrm{T}}(t)P_{\sigma(t)}x(t).$$

Similar to the derivation of Theorem 1, for any $t\in[t_k,t_{k+1}),k\in\mathbb{Z}_+$, it follows from (28) that

$$\begin{aligned} V_{i_k}(t_k) &= x^{\mathrm{T}}(t_k)P_{i_k}x(t_k) \\ &= x^{\mathrm{T}}(t_k^-)(I+H_{i_k})^{\mathrm{T}}P_{i_k}(I+H_{i_k})x(t_k^-) \\ &\leq \mu x^{\mathrm{T}}(t_k^-)P_{i_{k-1}}x(t_k^-) \\ &= \mu V_{i_{k-1}}(t_k^-). \end{aligned} \tag{30}$$

Let $\lambda_0=\min_{i\in\mathcal{M}}\{\lambda_{\min}(P_i)\}$, $\lambda_1=\max_{i\in\mathcal{M}}\{\lambda_{\max}(P_i)\}$. Choose $M>0$ such that $\lambda_1<\mu^{N_2+1}\lambda_0M$. Set $W_i(t)=e^{\beta(t-t_0)}V_i(t)$, $t\geq t_0-\tau$. Then, for $t\in[t_0-\tau,t_0)$, one has

$$W_{i_0}(t)\leq V_{i_0}(t)\leq\lambda_1\|\varphi\|_\tau^2<\mu^{N_2+1}\lambda_0M\|\varphi\|_\tau^2<\lambda_0M\|\varphi\|_\tau^2. \tag{31}$$

We claim it holds for any $t\geq t_0, i_m\in\mathcal{M}$ that

$$W_{i_m}(t)<\lambda_0M\|\varphi\|_\tau^2. \tag{32}$$

First, we prove that

$$W_{i_0}(t)<\lambda_0M\|\varphi\|_\tau^2,\ t\in[t_0,t_1). \tag{33}$$

If not, there exist $t\in[t_0,t_1)$ such that

$$W_{i_0}(t)\geq\lambda_0M\|\varphi\|_\tau^2.$$

Set $t^*=\inf\left\{t\in[t_0,t_1):W_{i_0}(t)\geq\lambda_0M\|\varphi\|_\tau^2\right\}$. Then, one gets

$$W_{i_0}(t^*)=\lambda_0M\|\varphi\|_\tau^2,\ t^*\in(t_0,t_1). \tag{34}$$

Set $\bar{t}=\sup\left\{t\in\left[t_0,t^*\right):W_{i_0}\left(t\right)\le\mu^{N_2+1}\lambda_0M\left\|\varphi\right\|_\tau^2\right\}$. Then, we have

$$W_{i_0}\left(\bar{t}\right)=\mu^{N_2+1}\lambda_0M\left\|\varphi\right\|_\tau^2,\ \bar{t}\in\left(t_0,t^*\right).$$

Hence, for $t\in\left[\bar{t},t^*\right]$, it holds that

$$W_{i_0}\left(t\right)\ge\mu^{N_2+1}\lambda_0M\left\|\varphi\right\|_\tau^2\ge\mu^{N_2+1}W_{i_0}\left(t+\theta\right),\ \theta\in\left[-\tau,0\right]. \tag{35}$$

In fact,

1) for $t+\theta\in\left[t_0,t_1\right)$, (35) obviously holds;

2) for $t+\theta\in\left[t_0-\tau,t_0\right)$, it follows from (31) that (35) also holds.

In addition, consider the derivative of $W_{i_0}\left(t\right)$ on $t\in\left[\bar{t},t^*\right]$, it yields

$$\begin{aligned}
D^+W_{i_0}\left(t\right)&=\beta e^{\beta(t-t_0)}V_{i_0}\left(t\right)+e^{\beta(t-t_0)}D^+V_{i_0}\left(t\right)\\
&\le e^{\beta(t-t_0)}\left[\beta x^{\mathrm{T}}\left(t\right)P_{i_0}x\left(t\right)+D^+V_{i_0}\left(t\right)\right]+\epsilon\left[\frac{1}{\mu^{N_2+1}}W_{i_0}\left(t\right)-W_{i_0}\left(t-\tau\left(t\right)\right)\right]\\
&\le e^{\beta(t-t_0)}\Big[x^{\mathrm{T}}\left(t\right)\left(P_{i_0}A_{i_0}+A_{i_0}^{\mathrm{T}}P_{i_0}+P_{i_0}B_{i_0}K_{i_0}+K_{i_0}^{\mathrm{T}}B_{i_0}^{\mathrm{T}}P_{i_0}\right)x\left(t\right)\\
&\quad+\beta x^{\mathrm{T}}\left(t\right)P_{i_0}x\left(t\right)+x^{\mathrm{T}}\left(t\right)P_{i_0}D_{i_0}x\left(t-\tau\left(t\right)\right)+x^{\mathrm{T}}\left(t-\tau\left(t\right)\right)D_{i_0}^{\mathrm{T}}P_{i_0}x\left(t\right)\\
&\quad+\frac{\epsilon}{\mu^{N_2+1}}x^{\mathrm{T}}\left(t\right)P_{i_0}x\left(t\right)-\epsilon e^{-\beta\tau}x^{\mathrm{T}}\left(t-\tau\left(t\right)\right)P_{i_0}x\left(t-\tau\left(t\right)\right)\Big]\\
&\quad-\alpha W_{i_0}\left(t\right)+\alpha W_{i_0}\left(t\right)\\
&=e^{\beta(t-t_0)}\eta^{\mathrm{T}}\left(t\right)\begin{pmatrix}\Psi_{i_0} & P_{i_0}D_{i_0}\\ \star & -\epsilon e^{-\beta\tau}P_{i_0}\end{pmatrix}\eta\left(t\right)+\alpha W_{i_0}\left(t\right),
\end{aligned} \tag{36}$$

where $\eta\left(t\right)=\left(x^{\mathrm{T}}\left(t\right)\quad x^{\mathrm{T}}\left(t-\tau\left(t\right)\right)\right)^{\mathrm{T}}$. By (27), we have

$$D^+W_{i_0}\left(t\right)\le\alpha W_{i_0}\left(t\right),\ t\in\left[\bar{t},t^*\right], \tag{37}$$

which leads to

$$W_{i_0}\left(t^*\right)\le e^{\alpha\left(t^*-\bar{t}\right)}W_{i_0}\left(\bar{t}\right)\le e^{\alpha(t_1-t_0)}\mu^{N_2+1}\lambda_0M\left\|\varphi\right\|_\tau^2.$$

According to the reverse ADT condition (5), it is derived that $t_1-t_0\le\tau_\alpha\left(N_\sigma\left(t_1,t_0\right)+N_2\right)\le\tau_\alpha\left(1+N_2\right)$. Hence,

$$W_{i_0}\left(t^*\right)\le e^{\alpha\tau_\alpha(N_2+1)}\mu^{N_2+1}\lambda_0M\left\|\varphi\right\|_\tau^2.$$

From condition (29), $e^{\alpha\tau_\alpha(N_2+1)}\mu^{N_2+1}<1$ is deduced, which implies

$$W_{i_0}\left(t^*\right)<\lambda_0M\left\|\varphi\right\|_\tau^2. \tag{38}$$

This contradicts (34), so (33) holds.

Suppose the following inequality holds for any $k\in\mathbb{Z}_+$ that

$$W_{i_m}\left(t\right)<\lambda_0M\left\|\varphi\right\|_\tau^2,\ t\in\left[t_0-\tau,t_k\right),\ i_m\in\mathcal{M},\ m=0,1,\cdots,k-1. \tag{39}$$

We will prove that

$$W_{i_k}\left(t\right)<\lambda_0M\left\|\varphi\right\|_\tau^2,\ t\in\left[t_k,t_{k+1}\right). \tag{40}$$

Suppose not, there exists $t\in\left[t_k,t_{k+1}\right)$ such that $W_{i_k}\left(t\right)\ge\lambda_0M\left\|\varphi\right\|_\tau^2$. Combining (30) and (39), one has

$$W_{i_k}\left(t_k\right) \le \mu W_{i_{k-1}}\left(t_k^-\right) < \mu\lambda_0 M \|\varphi\|_\tau^2 < \lambda_0 M \|\varphi\|_\tau^2 .$$

Then, set $t^* = \inf\left\{t \in \left[t_k, t_{k+1}\right) : W_{i_k}\left(t\right) \ge \lambda_0 M \|\varphi\|_\tau^2\right\}$, it yields $W_{i_k}\left(t^*\right) = \lambda_0 M \|\varphi\|_\tau^2$, $t^* \in \left(t_k, t_{k+1}\right)$. Set $\bar{t} = \sup\left\{t \in \left[t_k, t^*\right) : W_{i_k}\left(t\right) \le \mu^{N_2+1}\lambda_0 M \|\varphi\|_\tau^2\right\}$. Then, $W_{i_k}\left(\bar{t}\right) = \mu^{N_2+1}\lambda_0 M \|\varphi\|_\tau^2$, $\bar{t} \in \left(t_k, t^*\right)$. Moreover, for $t \in \left[\bar{t}, t^*\right]$, we obtain

$$\mu^{N_2+1} W_{i_m}\left(t+\theta\right) \le \mu^{N_2+1}\lambda_0 M \|\varphi\|_\tau^2 \le W_{i_k}\left(t\right),\ i_m \in \mathcal{M}, m = 0,1,\cdots,k. \tag{41}$$

For $t \in \left[\bar{t}, t^*\right]$, similar to (36), it follows that

$$D^+ W_{i_k}\left(t\right) \le \mathrm{e}^{\beta(t-t_0)} \eta^{\mathrm{T}}\left(t\right) \begin{pmatrix} \Psi_{i_k} & P_{i_k} D_{i_k} \\ \star & -\epsilon \mathrm{e}^{-\beta\tau} P_{i_m} \end{pmatrix} \eta\left(t\right) + \alpha W_{i_k}\left(t\right), \tag{42}$$

where $\eta\left(t\right) = \left(x^{\mathrm{T}}\left(t\right) \quad x^{\mathrm{T}}\left(t-\tau\left(t\right)\right)\right)^{\mathrm{T}}$. Then, based on (27), we derive

$$W_{i_k}\left(t^*\right) \le \mathrm{e}^{\alpha\left(t^*-\bar{t}\right)} W_{i_k}\left(\bar{t}\right) \le \mathrm{e}^{\alpha\left(t_{k+1}-t_k\right)} \mu^{N_2+1}\lambda_0 M \|\varphi\|_\tau^2 .$$

By (5), $t_{k+1} - t_k \le \tau_\alpha\left(N_\sigma\left(t_{k+1}, t_k\right) + N_2\right) \le \tau_\alpha\left(N_2 + 1\right)$. Thus,

$$W_{i_k}\left(t^*\right) \le \mathrm{e}^{\alpha\tau_\alpha\left(N_2+1\right)} \mu^{N_2+1}\lambda_0 M \|\varphi\|_\tau^2 < \lambda_0 M \|\varphi\|_\tau^2 ,$$

which yields a contradiction. Therefore (40) holds. By mathematical induction, (32) holds. From the definition of $W_i\left(t\right)$, we have

$$V_i\left(t\right) = \mathrm{e}^{-\beta(t-t_0)} W_i\left(t\right) < \mathrm{e}^{-\beta(t-t_0)} \lambda_0 M \|\varphi\|_\tau^2 ,\ t \ge t_0 - \tau.$$

Therefore,

$$\| x\left(t\right) \| < \sqrt{M}\, \mathrm{e}^{-\frac{\beta}{2}(t-t_0)} \|\varphi\|_\tau ,$$

which implies that system (8) is ES over the class $\mathcal{F}_2\left[\tau_\alpha, N_2\right]$. This completes the proof. □

In what follows, we shall analyse the H_∞ performance of system (1) with $\omega\left(t\right) \ne 0$.

Theorem 5. Assume that Assumption 1 holds. For system (3), given constants $\alpha > 0$ and $\gamma > 0$, if there exist constants $0 < \mu < 1$, $\beta > 0$, $\epsilon > 0$, $\delta > 0$, and $n \times n$ matrices $P_i > 0$, $G_i > 0$, such that the (27) - (29) and the following LMI hold

$$\begin{pmatrix} C_i^{\mathrm{T}} C_i - \delta P_i & C_i^{\mathrm{T}} E_i & \sqrt{\delta\kappa} P_i & 0 \\ \star & \Theta_i & 0 & E_i^{\mathrm{T}} \\ \star & \star & -G_i & 0 \\ \star & \star & \star & -I \end{pmatrix} < 0,\ \forall i \in \mathcal{M}, \tag{43}$$

where $\Theta_i = -\gamma^2 I + \delta\kappa F_i^{\mathrm{T}} G_i F_i$, $\kappa = -\dfrac{\tau_\alpha \mu^{-N_2}}{\alpha\tau_\alpha + \ln\mu}$, then, system (1) has H_∞ control performance over the impulsive switching signal $\mathcal{F}_2[\tau_\alpha, N_2]$.

Proof. Set $W_i(t) = \mathrm{e}^{\beta(t-t_0)} V_i(t)$. By (30), we have

$$W_{i_k}(t_k) \le \mu W_{i_{k-1}}(t_k^-), \tag{44}$$

the derivative of $W_i(t)$ along the trajectory of system (3) on $[t_k, t_{k+1})$, which implies the i_kth subsystem is activated, that is

$$\begin{aligned} D^+W_{i_k}(t) \le{}& \mathrm{e}^{\beta(t-t_0)}\Big[x^{\mathrm{T}}(t)\big(P_{i_k}A_{i_k} + A_{i_k}^{\mathrm{T}}P_{i_k} + P_{i_k}B_{i_k}K_{i_k} + K_{i_k}^{\mathrm{T}}B_{i_k}^{\mathrm{T}}P_{i_k}\big)x(t) \\ &+ \beta x^{\mathrm{T}}(t)P_{i_k}x(t) + x^{\mathrm{T}}(t)P_{i_k}D_{i_k}x(t-\tau(t)) \\ &+ x^{\mathrm{T}}(t-\tau(t))D_{i_k}^{\mathrm{T}}P_{i_k}x(t) + 2x^{\mathrm{T}}(t)P_{i_k}F_{i_k}\omega(t)\Big] \\ &+ \epsilon\left[\frac{1}{\mu^{N_2+1}}W_{i_k}(t) - W_{i_m}(t-\tau(t))\right] - \alpha W_{i_k}(t) + \alpha W_{i_k}(t) \\ \le{}& \mathrm{e}^{\beta(t-t_0)}\eta^{\mathrm{T}}(t)\begin{pmatrix} \Psi_{i_k} & P_{i_k}D_{i_k} \\ \star & -\epsilon\mathrm{e}^{-\beta\tau}P_{i_m} \end{pmatrix}\eta(t) \\ &+ 2\mathrm{e}^{\beta(t-t_0)}x^{\mathrm{T}}(t)P_{i_k}F_{i_k}\omega(t) + \alpha W_{i_k}(t), \end{aligned}$$

where $\eta(t) = \big(x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-\tau(t))\big)^{\mathrm{T}}$, $i_m \in \mathcal{M}$, $m = 0,1,2,\cdots,k$. By (27), it holds that

$$D^+W_{i_k}(t) \le \alpha W_{i_k}(t) + 2\mathrm{e}^{\beta(t-t_0)}x^{\mathrm{T}}(t)P_{i_k}F_{i_k}\omega(t),$$

which implies

$$D^+V_{i_k}(t) \le \alpha V_{i_k}(t) + 2x^{\mathrm{T}}(t)P_{i_k}F_{i_k}\omega(t).$$

Integrating the above inequality from t_k to t, one has

$$V_{i_k}(t) \le \mathrm{e}^{\alpha(t-t_k)}V_{i_k}(t_k) + \int_{t_k}^{t}\mathrm{e}^{\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_k}F_{i_k}\omega(s)\mathrm{d}s,\ t \in [t_k, t_{k+1}). \tag{45}$$

According to (44) and (45), it follows that

$$\begin{aligned} V_{i_k}(t) \le{}& \mu\mathrm{e}^{\alpha(t-t_k)}V_{i_{k-1}}(t_k^-) + \int_{t_k}^{t}\mathrm{e}^{\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_k}F_{i_k}\omega(s)\mathrm{d}s \\ \le{}& \mu\mathrm{e}^{\alpha(t-t_{k-1})}V_{i_{k-1}}(t_{k-1}) + \mu\int_{t_{k-1}}^{t_k}\mathrm{e}^{\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_{k-1}}F_{i_{k-1}}\omega(s)\mathrm{d}s \\ &+ \int_{t_k}^{t}\mathrm{e}^{\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_k}F_{i_k}\omega(s)\mathrm{d}s \\ \le{}& \cdots \\ \le{}& \mu^k\mathrm{e}^{\alpha(t-t_0)}V_{\sigma(t_0)}(t_0) + \mu^k\int_{t_0}^{t_1}\mathrm{e}^{\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_0}F_{i_0}\omega(s)\mathrm{d}s \\ &+ \mu^{k-1}\int_{t_1}^{t_2}\mathrm{e}^{\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_1}F_{i_1}\omega(s)\mathrm{d}s + \cdots + \int_{t_k}^{t}\mathrm{e}^{\alpha(t-s)}2x^{\mathrm{T}}(s)P_{i_k}F_{i_k}\omega(s)\mathrm{d}s \\ \le{}& \mathrm{e}^{\alpha(t-t_0)+k\ln\mu}V_{\sigma(t_0)}(t_0) + \int_{t_0}^{t}\mathrm{e}^{\alpha(t-s)+N_\sigma(t,s)\ln\mu}2x^{\mathrm{T}}(s)P_iF_i\omega(s)\mathrm{d}s. \end{aligned}$$

Given the zero initial condition $V_{\sigma(t_0)}(t_0) = 0$, it yields

$$V_{i_k}(t) \le \int_{t_0}^{t}\mathrm{e}^{\alpha(t-s)+N_\sigma(t,s)\ln\mu}2x^{\mathrm{T}}(s)P_iF_i\omega(s)\mathrm{d}s.$$

From the reverse ADT condition (5), we derive

$$V_{i_k}(t) \le \int_{t_0}^{t} \mathrm{e}^{\left(\alpha+\frac{\ln\mu}{\tau_\alpha}\right)(t-s)-N_2\ln\mu} 2x^{\mathrm{T}}(s) P_i F_i \omega(s)\mathrm{d}s. \tag{46}$$

Integrating both sides of (46) from t_0 to t, it follows from (29) that

$$\begin{aligned}
\int_{t_0}^{t} V_{i_k}(s)\mathrm{d}s &\le \int_{t_0}^{t}\int_{t_0}^{s} \mathrm{e}^{\left(\alpha+\frac{\ln\mu}{\tau_\alpha}\right)(s-v)} \mu^{-N_2} 2x^{\mathrm{T}}(v) P_i F_i \omega(v)\mathrm{d}v\mathrm{d}s \\
&= \int_{t_0}^{t}\int_{v}^{t} \mathrm{e}^{\left(\alpha+\frac{\ln\mu}{\tau_\alpha}\right)(s-v)} \mathrm{d}s\mu^{-N_2} 2x^{\mathrm{T}}(v) P_i F_i \omega(v)\mathrm{d}v \\
&= \int_{t_0}^{t} \frac{\tau_\alpha}{\alpha\tau_\alpha+\ln\mu}\left(\mathrm{e}^{\left(\alpha+\frac{\ln\mu}{\tau_\alpha}\right)(t-v)}-1\right)\mu^{-N_2} 2x^{\mathrm{T}}(v) P_i F_i \omega(v)\mathrm{d}v \\
&\le \kappa\int_{t_0}^{t} 2x^{\mathrm{T}}(v) P_i F_i \omega(v)\mathrm{d}v.
\end{aligned}$$

Letting $t\to\infty$, we obtain from (7) that

$$\int_{t_0}^{\infty} V_{i_k}(s)\mathrm{d}s \le \kappa\int_{t_0}^{\infty}\left(x^{\mathrm{T}}(v) P_i G_i^{-1} P_i x(v) + \omega^{\mathrm{T}}(v) F_i^{\mathrm{T}} G_i F_i \omega(v)\right)\mathrm{d}v. \tag{47}$$

Set $J_0 = \int_{t_0}^{\infty}\left[y^{\mathrm{T}}(s)y(s)-\gamma^2\omega^{\mathrm{T}}(s)\omega(s)\right]\mathrm{d}s$. Then, together with (47), it is obtained that

$$\begin{aligned}
J_0 &= \int_{t_0}^{\infty}\left[y^{\mathrm{T}}(s)y(s)-\gamma^2\omega^{\mathrm{T}}(s)\omega(s)-\delta V_i(x(s))\right]\mathrm{d}s + \delta\int_{t_0}^{\infty} V_i(x(s))\mathrm{d}s \\
&\le \int_{t_0}^{\infty}\left[x^{\mathrm{T}}(s)C_i^{\mathrm{T}}C_i x(s) + x^{\mathrm{T}}(s)C_i^{\mathrm{T}}E_i\omega(s) + \omega^{\mathrm{T}}(s)E_i^{\mathrm{T}}C_i x(s)\right. \\
&\quad \left. + \omega^{\mathrm{T}}(s)E_i^{\mathrm{T}}E_i\omega(s) - \gamma^2\omega^{\mathrm{T}}(s)\omega(s) - \delta x^{\mathrm{T}}(s)P_i x(s)\right]\mathrm{d}s \\
&\quad + \delta\kappa\int_{t_0}^{\infty}\left[x^{\mathrm{T}}(s)P_i G_i^{-1}P_i x(s) + \omega^{\mathrm{T}}(s)F_i^{\mathrm{T}}G_iF_i\omega(s)\right]\mathrm{d}s \\
&= \int_{t_0}^{\infty}\xi^{\mathrm{T}}(s)\begin{pmatrix} C_i^{\mathrm{T}}C_i - \delta P_i + \delta\kappa P_i G_i^{-1} P_i & C_i^{\mathrm{T}}E_i \\ \star & -\gamma^2 I + E_i^{\mathrm{T}}E_i + \delta\kappa F_i^{\mathrm{T}}G_iF_i \end{pmatrix}\xi(s)\mathrm{d}s,
\end{aligned}$$

where $\xi(s) = \left(x^{\mathrm{T}}(s) \quad \omega^{\mathrm{T}}(s)\right)^{\mathrm{T}}$. By using the Schur complement of (43), one has $J_0 \le 0$, which implies

$$\int_{t_0}^{\infty} y^{\mathrm{T}}(s)y(s)\mathrm{d}s \le \gamma^2\int_{t_0}^{\infty}\omega^{\mathrm{T}}(s)\omega(s)\mathrm{d}s.$$

Hence, system (1) has H_∞ control performance over the class $\mathcal{F}_2[\tau_\alpha, N_2]$. This completes the proof. □

Theorems 4 and 5 are established under the assumption that the gain matrices $K_{\sigma(t)}$ and $H_{\sigma(t)}$ are given. Since these matrices need to be designed in practical applications, it leads to the following theorem.

Theorem 6. Assume that Assumption 1 holds. Given constants $\alpha > 0$ and $\gamma > 0$, if there exist constants $0 < \mu < 1$, $\beta > 0$, $\epsilon > 0$, $\delta > 0$, and $n \times n$ matrices $X_i > 0$, $U_i > 0$, $G_i > 0$, $S_{i,j}$, T_i, such that the following LMIs hold.

$$\begin{pmatrix} \tilde{\Psi}_i & D_i X_l \\ \star & -\epsilon \mathrm{e}^{-\beta\tau} X_l \end{pmatrix} < 0, \quad \forall i,l \in \mathcal{M}, \tag{48}$$

$$\begin{pmatrix} -\delta X_i & X_i C_i^{\mathrm{T}} E_i & \sqrt{\delta\kappa} X_i & 0 & X_i C_i^{\mathrm{T}} \\ \star & \Theta_i & 0 & E_i^{\mathrm{T}} & 0 \\ \star & \star & -U_i & 0 & 0 \\ \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & -I \end{pmatrix} < 0, \ \forall i \in \mathcal{M}, \tag{49}$$

$$\begin{pmatrix} -\mu X_j & X_j + S_{i,j}^{\mathrm{T}} \\ \star & -X_i \end{pmatrix} < 0, \ \forall i, j \in \mathcal{M}, i \neq j, \tag{50}$$

where $\tilde{\Psi}_i = A_i X_i + X_i A_i^{\mathrm{T}} + B_i T_i + T_i^{\mathrm{T}} B_i^{\mathrm{T}} + X_i \left(\beta + \frac{\epsilon}{\mu^{N_2+1}} - \alpha \right)$, $\kappa = -\frac{\tau_\alpha \mu^{-N_2}}{\alpha\tau_\alpha + \ln\mu}$, and the reverse ADT constant τ_α satisfies

$$\tau_\alpha < -\frac{\ln\mu}{\alpha}, \tag{51}$$

then, system (3) has H_∞ control performance over the impulsive switching signal $\mathcal{F}_2[\tau_\alpha, N_2]$. The control gain matrices are given by $K_i = T_i X_i^{-1}$, $H_i = S_{i,j} X_j^{-1}$.

Proof. According to Theorem 5, we perform the following congruence transformations: pre-multiply and post-multiply (27) by $diag\{P_i^{-1}, P_l^{-1}\}$, (28) by $diag\{P_j^{-1}, P_i^{-1}\}$, and (43) by $diag\{P_i^{-1}, I, P_i^{-1}, I\}$. let $X_i = P_i^{-1}$, $T_i = K_i X_i$, $U_i = X_i G_i X_i$ and $S_{i,j} = H_i X_j$, then, we derive that (27), (28), and (43) are equivalent to (48), (50), and (49), respectively. Therefore, the control gain matrices are given by $K_i = T_i X_i^{-1}$, $H_i = S_{i,j} X_j^{-1}$. The proof is completed. □

Remark. In Theorem 6, the requirement of $0 < \mu < 1$ implies that the impulses occurring at discrete moments are stabilizing impulses and the ADT τ_α is satisfied with $\tau_\alpha < -\frac{\ln\mu}{\alpha}$, which indicates that the dwell-time of each subsystem should not be too long, i.e., the number of impulses and switchings between the subsystems should occur in high frequency to enhance the positive effect of the stabilizing impulses on the state of the system.

Remark. In [24]-[27], authors have investigated the stability and stabilizability of impulsive switched delay systems. Different from these existing results, this paper considers the influence of external disturbances on system, and establishes some sufficient conditions based on ADT for the H_∞ control performance by coupling the effect of impulse with state feedback control. In addition, when the system is disturbed by external inputs, [24] investigated the L_2-gain problem of impulsive switched system under the influence of two kinds of impulses. [25] discussed the finite-time H_∞ control problem of nonlinear impulsive switched time-delay systems, but only considered the effect of stabilizing impulses. It is noteworthy that the systems studied in both [24] [25] neglected the influence of time-delays. Therefore, this paper considers a class of impulsive switched time-delay systems, which are investigated by different schemes (L-K functional method for destabilizing impulses and L-R techniques for stabilizing impulses) for different impulses, and gives a sufficient criterion for the controllability of the system. Compared with the results in [24] [25], our results are more general.

4. Examples

In this section, two numerical examples are given to illustrate the effectiveness of the obtained results.

Example 1 Consider the two-dimensional delayed impulsive switched system (1) with

$$A_1=\begin{pmatrix}-0.23 & 0\\ 1 & -0.2\end{pmatrix},\ B_1=\begin{pmatrix}1.01 & 0\\ 0 & -0.87\end{pmatrix},$$

$$C_1=\begin{pmatrix}0.1 & 0\\ 0 & 0.1\end{pmatrix},\ D_1=\begin{pmatrix}-0.1 & -0.1\\ -0.1 & 0.1\end{pmatrix},$$

$$E_1=\begin{pmatrix}0.16 & 0\\ 0 & 0.2\end{pmatrix},\ F_1=\begin{pmatrix}0.1 & 0\\ 0 & 0.1\end{pmatrix},$$

$$H_1=\begin{pmatrix}0.3 & 0.01\\ 0.02 & 0.3\end{pmatrix},\ A_2=\begin{pmatrix}-0.7 & 0\\ 0 & -0.3\end{pmatrix},$$

$$B_2=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\ C_2=\begin{pmatrix}0.01 & -0.02\\ -0.04 & -0.05\end{pmatrix},$$

$$D_2=\begin{pmatrix}-0.1 & 0\\ 0 & -0.1\end{pmatrix},\ E_2=\begin{pmatrix}0.44 & 0\\ 0 & -0.2\end{pmatrix},$$

$$F_2=\begin{pmatrix}0.1 & 0\\ 0 & 0.1\end{pmatrix},\ H_2=\begin{pmatrix}0.35 & 0\\ 0 & 0.35\end{pmatrix}.$$

Set time-delay $\tau(t)=0.22-0.03\cos(2t)$. Choose $\alpha=1.63$, $\mu=1.78$, $\delta=0.36$, $N_1=0.11$, $\tau=0.74$, $d=0.31$. Then applying Theorem 3 and using MATLAB LMI toolbox, the following feasible solutions can be obtained:

$$X_1=\begin{pmatrix}4.2278 & -0.3970\\ -0.3970 & 4.5865\end{pmatrix},\ X_2=\begin{pmatrix}4.3940 & -0.4645\\ -0.4645 & 4.7700\end{pmatrix},$$

$$Z_1=\begin{pmatrix}15.8958 & 0.1041\\ 0.1041 & 15.7911\end{pmatrix},\ Z_2=\begin{pmatrix}15.9910 & 0.0764\\ 0.0764 & 15.8876\end{pmatrix},$$

$$U_1=\begin{pmatrix}12.5517 & -0.2995\\ -0.2995 & 12.9561\end{pmatrix},\ U_2=\begin{pmatrix}12.3047 & -0.1914\\ -0.1914 & 12.6975\end{pmatrix},$$

$$T_1=\begin{pmatrix}-15.2668 & -1.4295\\ -2.0959 & 18.0985\end{pmatrix},\ T_2=\begin{pmatrix}-13.4881 & 0.0299\\ -0.2041 & 15.3917\end{pmatrix}.$$

The gain matrices of control are designed as

$$K_1=\begin{pmatrix}-3.6702 & -0.6293\\ -0.01263 & 3.9351\end{pmatrix},\ K_2=\begin{pmatrix}-3.1009 & -0.2957\\ 0.2977 & 3.2557\end{pmatrix}.$$

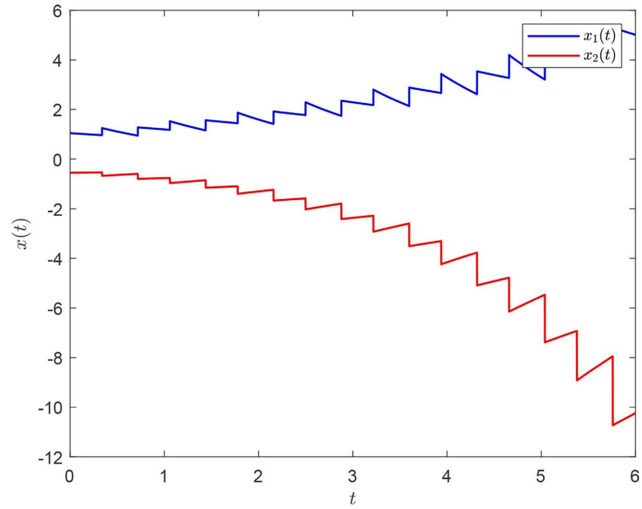

Figure 1. State trajectories $x_1(t)$ and $x_2(t)$ ($u(t)=0, \omega(t)=0$).

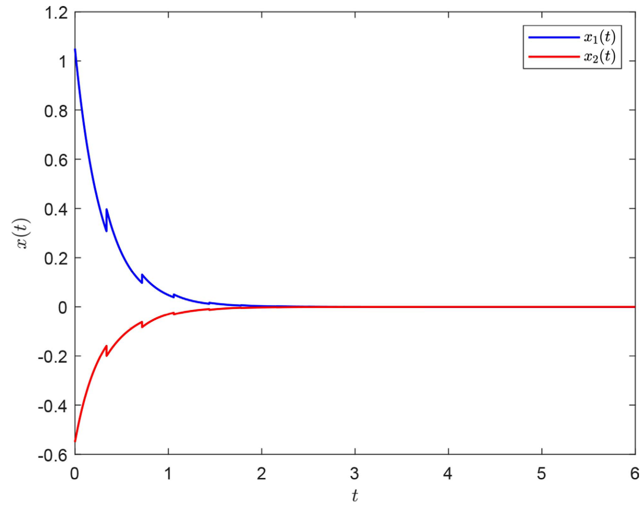

Figure 2. State trajectories $x_1(t)$ and $x_2(t)$ ($\omega(t)\neq 0$).

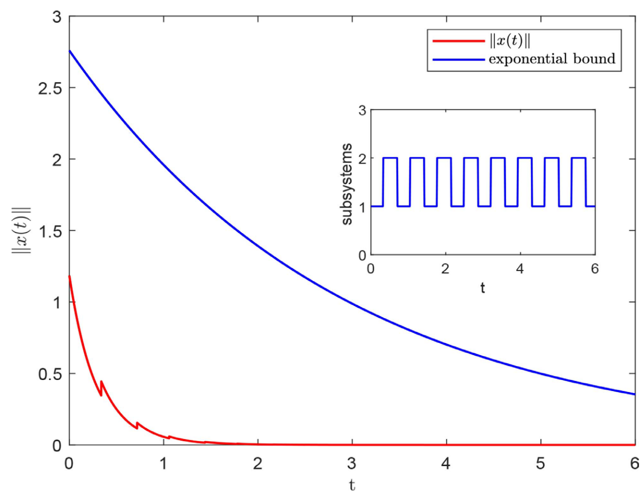

Figure 3. State norm trajectories $\|x(t)\|$ ($\omega(t)=0$).

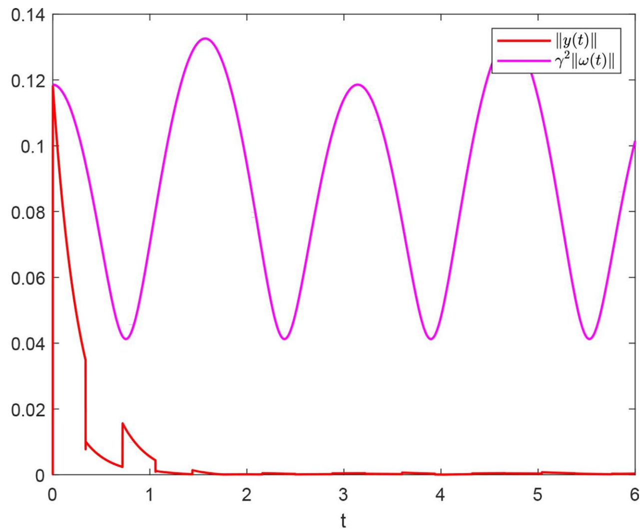

Figure 4. The system of H_∞ performance ($\omega(t) \neq 0$).

The impulsive switching sequence is given by $t_{2n-1} = 0.72n - 0.38$, $t_{2n} = 0.72n$, $n \in \mathbb{Z}_+$. Based on condition (26), letting $\tau_\alpha = 0.36$, it is satisfied that $\tau_\alpha > \frac{\ln \mu}{\alpha} = 0.3538$. The initial condition is set as $\varphi(t) = (1.05, -0.55)^{\mathrm{T}}$. It is evident that system (1) is unstable in the absence of external disturbance and the controller, see **Figure 1**. As illustrated in **Figure 2** and **Figure 3**, the state trajectories of the system under control indicates that the system (1) is ES over the class $\mathcal{F}_1[\tau_\alpha, N_1]$. Considering disturbance $\omega(t) = (0.2\cos(2t), 0.1\sin(t))^{\mathrm{T}}$, system (1) has H_∞ control performance with index $\gamma = 0.77$ under zero initial condition, which is shown in **Figure 4**.

Example 2. Consider the two-dimensional delayed impulsive switched system (1) with

$$A_1 = \begin{pmatrix} 0.3 & -0.1 \\ 0 & 0.1 \end{pmatrix},\ B_1 = \begin{pmatrix} 0.36 & 0 \\ 0 & 0.3 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} 0.2 & 0 \\ -0.04 & -0.27 \end{pmatrix},\ D_1 = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix},$$

$$E_1 = \begin{pmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{pmatrix},\ F_1 = \begin{pmatrix} -0.1 & 0 \\ 0 & 0.1 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0.5 & -0.5 \\ 0.4 & 0.2 \end{pmatrix},\ B_2 = \begin{pmatrix} 0.4 & 0.1 \\ -0.3 & 0.5 \end{pmatrix},$$

$$C_2 = \begin{pmatrix} 0.4 & -0.21 \\ 0.2 & -0.45 \end{pmatrix},\ D_2 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.9 \end{pmatrix},$$

$$E_2 = \begin{pmatrix} 0.35 & 0 \\ 0 & -0.2 \end{pmatrix},\ F_2 = \begin{pmatrix} 0.1 & 0 \\ 0 & -0.1 \end{pmatrix}.$$

The time-delay is given by $\tau(t) = 0.2 + 0.01\sin(2t)$. Applying Theorem 6 and choosing $\alpha = 1.36$, $\mu = 0.42$, $\delta = 1.19$, $\beta = 1.16$, $\epsilon = 0.5$, $N_2 = 0.7$,

$\tau = 0.36$. Then using MATLAB LMI toolbox, the following feasible solutions is obtained:

$$X_1 = \begin{pmatrix} 5.2292 & -0.5957 \\ -0.5957 & 4.1235 \end{pmatrix},\ X_2 = \begin{pmatrix} 3.2584 & 0.6468 \\ 0.6468 & 2.7781 \end{pmatrix},$$

$$T_1 = \begin{pmatrix} -63.1066 & 18.5458 \\ 14.6416 & -48.8243 \end{pmatrix},\ T_2 = \begin{pmatrix} -17.1073 & 22.3960 \\ -28.8242 & -12.7295 \end{pmatrix},$$

$$S_{12} = \begin{pmatrix} -2.9489 & 0 \\ 0 & -2.9489 \end{pmatrix},\ S_{21} = \begin{pmatrix} -4.6370 & 0 \\ 0 & -4.6370 \end{pmatrix},$$

$$U_1 = \begin{pmatrix} 11.3539 & 0.0799 \\ 0.0799 & 11.6483 \end{pmatrix},\ U_2 = \begin{pmatrix} 12.8942 & -1.5393 \\ -1.5393 & 12.6731 \end{pmatrix}.$$

The state feedback control gains and impulsive control gains are designed as follows

$$K_1 = \begin{pmatrix} -11.7492 & 2.8002 \\ 1.4754 & -11.6273 \end{pmatrix},\ K_2 = \begin{pmatrix} -7.1826 & 9.7339 \\ -8.3212 & -2.6447 \end{pmatrix},$$

$$H_1 = \begin{pmatrix} -0.9489 & 0.2209 \\ 0.2209 & -1.1129 \end{pmatrix},\ H_2 = \begin{pmatrix} -0.9016 & -0.1302 \\ -0.1302 & -1.1433 \end{pmatrix}.$$

Set the impulsive switching sequence as $t_{2n-1} = 0.42n - 0.17$, $t_{2n} = 0.42n$, $n \in \mathbb{Z}_+$ and reverse ADT $\tau\alpha = 0.21$. According to condition (51), the ADT τ_α satisfies $\tau_\alpha < -\frac{\ln \mu}{\alpha} = 0.6379$. Choose the initial condition $\varphi(t) = (0.1, -0.9)^{\mathrm{T}}$. From the simulation, it can be observed from **Figure 5** that system (1) is unstable when no external disturbance or control input is applied. And **Figure 6** shows that he system (1) is also unstable with only state feedback control input, which reveals the necessity of impulsive control. The dynamical behavior of $x(t)$ under the

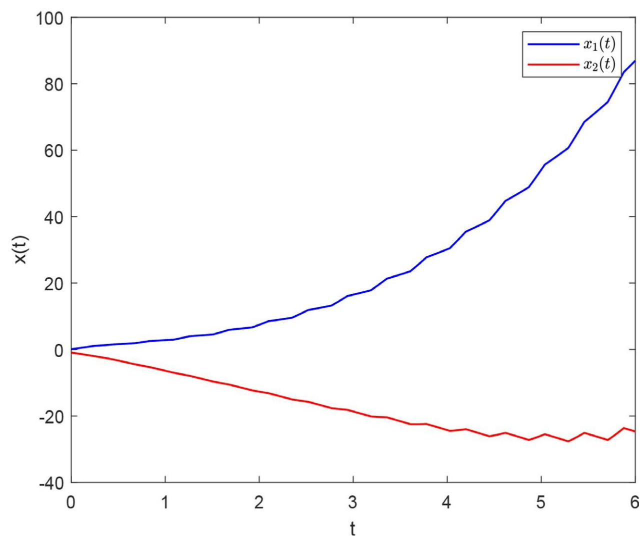

Figure 5. State trajectories $x_1(t)$ and $x_2(t)$ without state feedback control and impulsive control ($\omega(t) = 0$).

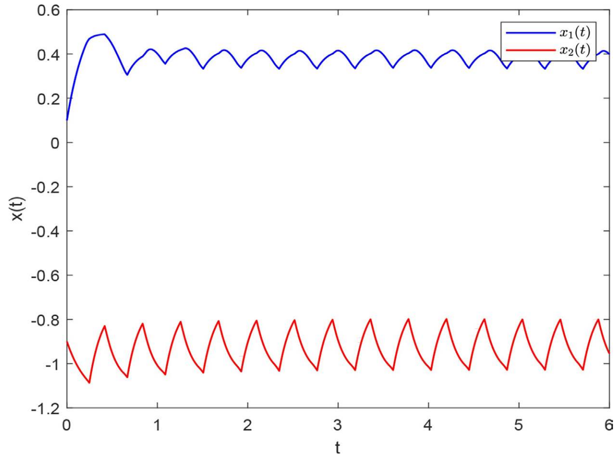

Figure 6. State trajectories $x_1(t)$ and $x_2(t)$ without impulsive control ($\omega(t)=0$).

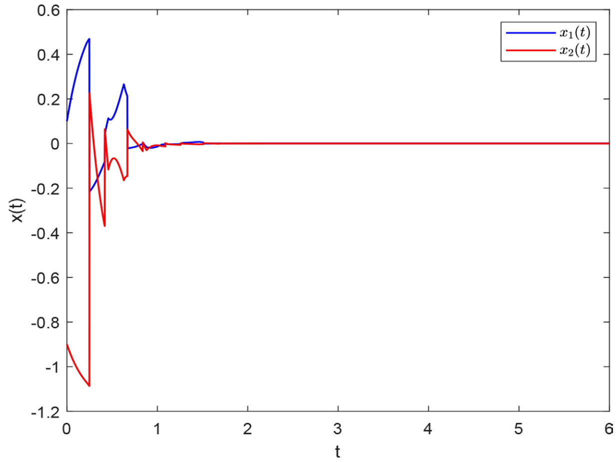

Figure 7. State trajectories $x_1(t)$ and $x_2(t)$ with mixed control ($\omega(t)=0$).

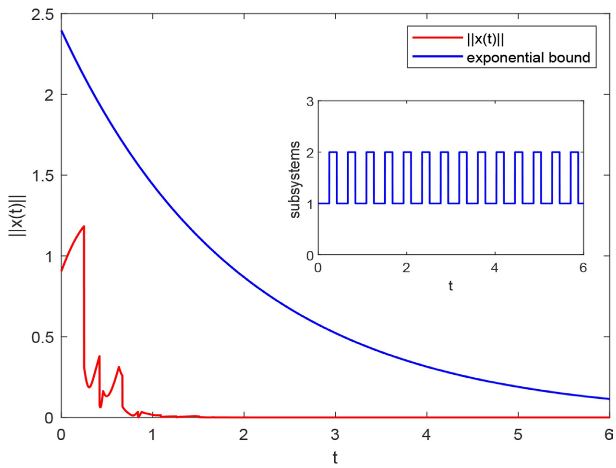

Figure 8. State norm trajectories $\|x(t)\|$ with mixed control ($\omega(t)=0$).

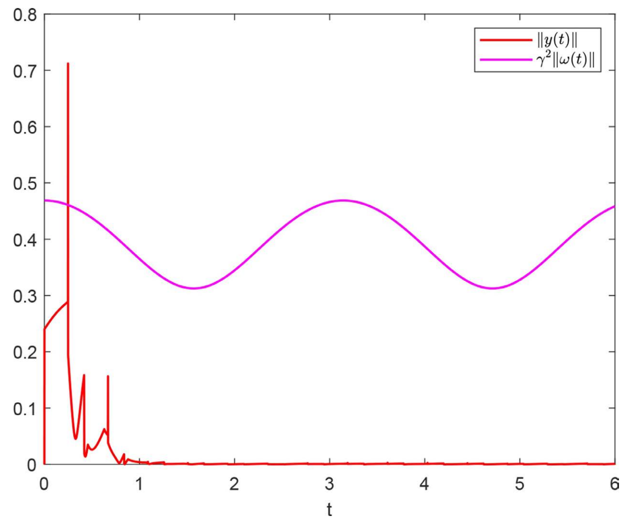

Figure 9. The system of H_∞ performance ($\omega(t) \neq 0$).

mixed action of state feedback control and impulsive control is shown in **Figure 7**. When $\omega(t)=0$, **Figure 8** implies that the system is ES over the class $\mathcal{F}_2[\tau_\alpha, N_2]$. Under zero initial conditions, let $\omega(t)=[0.2\sin(t), 0.3\cos(t)]^{\mathrm{T}}$. It is illustrated in **Figure 9** that the system achieves H_∞ control performance with index $\gamma=1.25$ under the hybrid controller.

5. Conclusion

This paper studied the H_∞ stabilization of impulsive switched systems with time-varying delays under different kinds of effects of impulses. As for the destabilizing impulses, by constructing L-K functionals and combining with the ADT condition, a state feedback controller is designed, and the sufficient conditions for the exponential stability and H_∞ performance of the system are given in the presence and absence of external disturbances. On the other hand, from the perspective of stabilizing impulses, by applying L-R method and the reverse ADT condition, the impulsive controller and state feedback controller are designed for the exponential stability of the system without $\omega(t)$ and the H_∞ performance of the system with $\omega(t) \neq 0$. The sufficient conditions obtained in two cases are both delay dependent and presented in the form of LMIs, which is easy to be effectively examined using the LMI toolbox in MATLAB. The system that we studied takes full account of the effects of time-delays and the two different kinds of impulses, which improved and extended the results in [24] [25]. Finally, two numerical simulations are presented to verify the effectiveness of the proposed results. Several limitations of the present work should be acknowledged. First, the assumption that switching and impulsive events occur at the same time may restrict the application to practical hybrid systems. Second, the proposed controllers rely on full-state feedback, which requires all system states to be measurable and available, thus limiting their implementation in scenarios where only partial state information is accessible. Third, the current framework is restricted to linear subsystems. In the future, we will extend the results to nonlinear impulsive switched systems and consider the condition that the switchings and impulses are asynchronous.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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