Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2026, 14(4), 1746-1773
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Experimental Observation of Hyperbolic Heron Triangles in the Decays of Scalar, Strange Mesons, and *Δ*, *N*, *Λ*, *Σ*, *Ξ* Baryons, and the Investigation of New Resonances

Valeriy Pavlovich Khen, Alexei Valerevich Khen

Limited Liability Partnership "Industry 4.0", Almaty, Kazakhstan
Email: fram47@mail.ru, a@dugoba.kz

How to cite this paper: Khen, V.P. and Khen, A.V. (2026) Experimental Observation of Hyperbolic Heron Triangles in the Decays of Scalar, Strange Mesons, and *Δ*, *N*, *Λ*, *Σ*, *Ξ* Baryons, and the Investigation of New Resonances. *Journal of Applied Mathematics and Physics*, **14**, 1746-1773.
https://doi.org/10.4236/jamp.2026.144085

Received: February 20, 2026
Accepted: April 27, 2026
Published: April 30, 2026

Abstract

The endpoints of the velocity vectors of two resonance decay particles represent material velocity points in the hyperbolic Lobachevsky velocity space of curvature $k = -1/C^2$ ($C = 1$ is the speed of light; the points are assigned the masses of the decay particles). Two particle velocity points can be connected by a straight line segment and an arc of constant zero curvature, called the oricycle. Archimedes' laws of levers define a third point on the oricycle arc, to which an additive mass (the sum of the decay particle masses) is assigned. By connecting these 3 points with straight line segments, we get a triangle of resonance decay inscribed in the oricycle. The effective mass m_r of the resonance is determined by the hyperbolic cosine of the length of one side of its decay triangle and the masses of the decay particles. A Lorentz invariant function called the oricyclic cotangent of the triangle (OCT) is introduced on the triangles of resonance decays (OCT is based on the arc of the oricycle and the angle of the triangle). Using published data on the effective masses m_r of scalar, strange mesons and *Δ*, *N*, *Λ*, *Σ*, *Ξ* baryons, the function OCT_r and its nearest integer value OCT_M were calculated. For triangles with integer values of OCT_M, the hyperbolic cosines of the side lengths are also equal to integers. Therefore, triangles with integer values of OCT_M are called hyperbolic Heron triangles. The effective masses m_{her}, corresponding to Heron's triangle, differ from the masses m_r of scalar, strange mesons and *Δ*, *N*, *Λ*, *Σ*, *Ξ* baryons within the resonance widths. These results provide grounds for considering the listed resonances as a lattice structure of Heron triangles with integer OCT_M. Then, if statistically significant peaks are detected in the distribution for $OCT_M < 7$, up to 6 new resonances with masses < 1 GeV may be detected, which would explain the large variance in the measurements of the masses and widths of scalar mesons. In the discrete spectrum at $OCT_M > 500$, new resonances with masses > 10 GeV can be detected.

Keywords

Lobachevsky Velocity Space, Resonance Decay Triangles, Oricyclic Cotangent of a Triangle, Hyperbolic Heron Triangle, Lattice Structure of Hyperbolic Heron Triangle, Quantization Resonance Decay

1. Introduction

In inelastic reactions at high energies, the particle velocity vectors are measured in some frame of reference. The ends of the velocity vectors represent material points-velocities in the velocity space located inside a sphere of radius C (C is the speed of light, the points-velocities are assigned rest masses of the particles) [1]-[6]. The Lorentz group defines the Lobachevsky-Bolyai geometry of negative curvature $k = -1/C^2$ in the velocity space [1]-[4]. The material points-velocities inside a sphere of radius C represent the Lorentz invariant geometric image of inelastic reaction kinematics in hyperbolic Lobachevsky velocity space (HLVS) (further everywhere the speed of light $C = 1$) [5]-[8]. Two material points-velocities of resonance decay particles in HLVS can be connected by a straight line segment and an arc of a line of zero curvature, called an oricycle [9]. Archimedes' lever laws (3), (10) define a third point-velocity on the oricycle arc, to which an additive mass (the sum of the masses of the decay particles) is assigned. By connecting the three points by straight line segments, we obtain triangles of resonance decays inscribed in the oricycle.

Figure 1 in the Beltrami model of HLVS shows the triangles of scalar meson decays inscribed in the oricycle [7]. The circle $X^2 + Y^2 = 1$, called the **Absolute**, represents infinitely distant points of HLVS. The effective mass m_r of a resonance is determined by the hyperbolic cosine of the length of one side of its decay triangle and the masses of the decay particles (2). For resonance decay triangles, function (4) is introduced—the product of the arc length of the oricycle and the cotangent of half the angle. This function is called the oricyclic cotangent of a triangle (OCT). Using published data on the masses m_r of scalar and strange mesons and Δ, N, Λ, Σ, Ξ baryons, the function OCT_r and its nearest integer value $OCT_r = OCT_M$ are calculated. Integer values of OCT_M correspond to triangles for which the sum of the hyperbolic cosines of the side lengths and the hyperbolic cosines of the base lengths are also integers. Therefore, triangles with integer values of OCT_M are called hyperbolic Heron triangles [9]-[12]. The effective masses m_{her}, corresponding to Heron's triangle, differ from the masses m_r of the decay triangles of the listed resonances within their widths (**Tables 1-13**) [13]. These results provide grounds for considering the listed resonances as a lattice structure of Heron triangles with integer OCT_M (**Figure 4**). In this approach, many two-particle (π, π), (p, π), (n, π), (η, π), (ρ(770), π), (ω(782), π), (Δ(1232), π), (Σ, π), (Λ, π), (Ξ^0, π), (k493, π), (k892, π) decays of new resonances can be detected experimentally (**Table 14**, **Table 15**). These may be resonances with both small masses (<1 GeV) and resonances with large masses (>10 GeV).

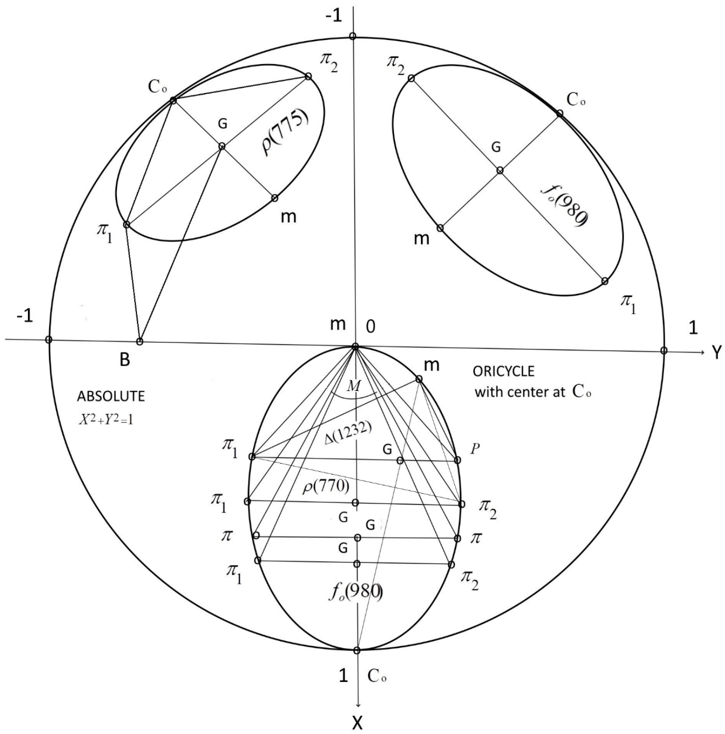

Figure 1. Decays of scalar mesons in the Beltrami model of the Lobachevsky velocity space. The separate ellipses of decay oricycles of $\rho(770)$, $f_o(980)$ scalar mesons with centers in the "C_0" points of the circle $X^2+Y^2=1$, called the *Absolute*, and $\Delta\pi_1 m \pi_2$ tringle*s* of $\rho(770)$, $f_o(980)$ scalar mesons decays, combined into one oricycle with the center at the point "C_0" (1, 0) on the *Absolute*. The point-velocity "G" represents the centers of inertia of pairs of particles (π_1, π_2), (P, π_1).

2. Decay of Scalar Mesons with Equal Masses of Decay Particles

Suppose that the velocities of particles π_1 and π_2 decay of a scalar meson in some reference frame "0" are measured. The ends of the particle velocity vectors represent material points-velocities "π_1" and "π_2" in the hyperbolic Lobachevsky velocity space (HLVS) located inside a sphere of radius C (hereinafter the speed of light C = 1, the masses $m_{\pi_1} = m_{\pi_2}$ of decay particles are attributed to the points "π_1" and "π_2") [5]-[7]. Let's draw a plane through the points "0", "π_1", "π_2". In this plane, we introduce a rectangular coordinate system $X0Y$ with the origin at the point "0" (**Figure 1**). The orthogonal projections (X_{π_1}, Y_{π_1}) of the velocity vector of the particle π_1 on the axes $0X$, $0Y$ are called the Beltrami coordinates of the point "π_1" in HLVS. The length $S_{\pi_1\pi_2}$ of the line segment ($\pi_1 - \pi_2$) with the Beltrami coordinates of its ends "π_1" (X_{π_1}, Y_{π_1}), "π_2" (X_{π_2}, Y_{π_2}) is represented by the formula [9]:

$$\operatorname{ch}\left(S_{\pi_1\pi_2}\right)=\left(1-X_{\pi_1}*X_{\pi_2}-Y_{\pi_1}*Y_{\pi_2}\right)/\left(R_{\pi_1}*R_{\pi_2}\right) \tag{1}$$

$$R_{\pi_2}=\sqrt{1-X_{\pi_2}^2-Y_{\pi_2}^2},\quad R_{\pi_1}=\sqrt{1-X_{\pi_1}^2-Y_{\pi_1}^2}$$

The length $S_{\pi_1\pi_2}$ of the line segment ($\pi_1-\pi_2$) is called the rapidity [14]. The effective mass m_r of a scalar meson is calculated using the formula [6]:

$$m_r^2 = m_{\pi_1}^2+m_{\pi_2}^2+2m_{\pi_1}m_{\pi_2}\operatorname{ch}\left(S_{\pi_1\pi_2}\right),\quad \frac{m_r^2-2m_{\pi_1}^2}{2m_{\pi_1}^2}=\operatorname{ch}\left(S_{\pi_1\pi_2}\right) \tag{2}$$

Besides the straight line ($\pi_1-\pi_2$), one pair of symmetrical arcs of zero curvature, called oricycles, passes through the velocity points "π_1" and "π_2". In **Figure 1**, in the Beltrami model of HLVS, the ellipses tangent to the circle $X^2+Y^2=1$ are oricycles with centers of rotation at the points of tangency "C_0". The straight line connecting the center of rotation "C_0" with an arbitrary point of the oricycle is called its axis. The circle $X^2+Y^2=1$, called the ***Absolute***, represents, according to formula (1), the infinitely distant points of the HLVS.

All oricycles in HLVS are congruent as straight lines of zero curvature in Euclidean space are congruent [9]. Thus, the ellipse with axis (C_0-0) in **Figure 1** represents an oricycle, which combines the oricycles of the decays of individual scalar mesons.

The point "m" on the oricycle with additive mass $m_{\pi_1\pi_2}=m_{\pi_1}+m_{\pi_2}$ are determined by Archimedes' laws of levers (3). The roles of forces in the levers are played by the masses m_{π_1} and m_{π_2}, and the arms of the levers are equal to the Euclidean lengths $l_{\pi_1\pi_2}, l_{m\pi_1}, l_{m\pi_2}$ of the arcs of the oricycle [6] [10]-[12]. For the case of equal rest masses of particles π_1, π_2 ($m_{\pi_1}=m_{\pi_2}$) the point "m" lies in the center of the arc (π_1, m, π_2) of the oricycle (**Figure 1**):

$$l_{\pi_1\pi_2}=l_{m\pi_1}+l_{m\pi_2},\quad m_{\pi_1\pi_2}=m_{\pi_1}+m_{\pi_2}=2m_{\pi_1}$$

$$m_{\pi_1}l_{m\pi_1}=m_{\pi_2}l_{m\pi_2}=m_{\pi_1}l_{\pi_1\pi_2}/\left(m_{\pi_1}+m_{\pi_2}\right)=m_{\pi_1}l_{\pi_1\pi_2}/2 \tag{3}$$

Connecting the points "π_1", "m", "π_2" with each other by straight line segments, we obtain an isosceles triangle $\Delta\pi_1 m\pi_2$ of the $f_o(980)$ meson decays inscribed in the oricycle (**Figure 1**). On the triangle $\Delta\pi_1 m\pi_2$ we introduce a dimensionless Lorentz invariant function:

$$OCT_r=l_{\pi_1\pi_2}\operatorname{ctg}\left(\frac{M}{2}\right)=\operatorname{sh}^2\left(\frac{S_{\pi_1\pi_2}}{2}\right)=\operatorname{sh}^2\left(S_{G\pi_1}\right),\quad l_{\pi_1\pi_2}=2*\operatorname{sh}\left(S_{G\pi_1}\right) \tag{4}$$

where $l_{\pi_1\pi_2}$ is the length of the oricycle arc subtending the base ($\pi_1-\pi_2$) with a rapidity $S_{\pi_1\pi_2}$, M is the angle at the vertex "m", the "G" point represents the center of inertia of the pairs (π_1, π_2) of decay particles. The function OCT_r is named oricyclic cotangent of a triangle.

In **Table 1**, the OCT_r values are calculated from published data on the effective masses of Scalar Mezon $\to\pi_1+\pi_2$ decays [13]. Columns 2 and 3 of **Table 1** present the values of the effective masses m_r and the widths Γ of the Scalar Mezon $\to \pi_1 + \pi_2$ decays [13]. According to formula (2), the mass m_r corresponds to the rapidity $S_{\pi_1\pi_2}$ of the bases ($\pi_1 - \pi_2$) of the triangles $\Delta\pi_1 m\pi_2$ of the decays of Scalar Mesons. Columns 3 and 4 of **Table 1** show the values of the function OCT_r, calculated using formula (4), and its nearest integer value OCT_M. Integer values of OCT_M correspond to triangles $\Delta\pi m\pi$ (**Figure 1**). Calculations have shown that when $OCT_M = L$, where L is an integer, then the lengths $S_{m\pi}$ and $S_{\pi\pi}$ of the lateral side and base of the triangle $\Delta\pi m\pi$ are related by the relations:

Table 1. Hyperbolic Heron triangles in decay of Scalar Mezon $\to \pi_1 + \pi_2$.

Name Scalar Mezon	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Mass in H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)	$\Delta m_r^{her} = m_r - m_{her}^{\pi,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)
1	2	3	4	5	6	7	8
Scalar Mezon $\to \pi_1 + \pi_2$, $m_{\pi_1} = m_{\pi_2} = m_\pi = 139.57001$ Mev							
$f_o(500)$	500.0 ± 100.0	300.0	$2.21^{+1.412}_{-1.155}$	2	483.5	16.5	5.5
$\rho(770)$	766.5 ± 1.1	150.0	$6.54^{+0.022}_{-0.022}$	7	789.5	23.0	15.4
$\omega(782)$	782.7 ± 0.13	8.68	$6.86^{+0.026}_{-0.026}$	7	789.5	6.8	78.6
$f_o(980)$	980.0 ± 20.0	55.0	$11.58^{+0.513}_{-0.503}$	12	1006.4	16.4	29.9
$\phi(1020)$	1019.46 ± 0.02	4.25	$12.34^{+0.0005}_{-0.0003}$	12	1006.4	13.0	307.0
$f_2(1270)$	1275.4 ± 0.80	186.6	$19.88^{+0.026}_{-0.026}$	20	1279.2	3.8	2.0
$f_o(1370)$	1350.0 ± 50.0	350.8	$22.39^{+1.765}_{-1.700}$	22	1338.7	11.3	3.2
$f_o(1500)$	1522.0 ± 25.0	108.0	$28.73^{+0.985}_{-0.968}$	29	1528.9	6.9	6.4
$\rho_3(1690)$	1686.0 ± 4.0	161.0	$35.48^{+0.173}_{-0.173}$	35	1674.8	11.2	6.9
$\rho(1700)$	1720.0 ± 20.0	250.0	$36.97^{+0.888}_{-0.877}$	37	1720.7	0.7	0.3
$f_o(1710)$	1733.0 ± 8.0	150.0	$37.54^{+0.357}_{-0.355}$	38	1743.2	10.2	6.8
$f_o(1770)$	1804.0 ± 16.0	138.0	$40.77^{+0.745}_{-0.737}$	41	1809.0	5.0	3.7
$f_2(1810)$	1815.0 ± 12.0	197.0	$41.28^{+0.561}_{-0.557}$	41	1809.0	6.0	3.0
$f_2(1950)$	1936.0 ± 12.0	197.0	$47.10^{+0.591}_{-0.594}$	47	1933.9	2.1	0.4
$f_0(2020)$	1982.0 ± 54.0	464.0	$49.42^{+2.785}_{-2.708}$	49	1973.8	8.2	1.9
$f_4(2050)$	2018.0 ± 11.0	237.0	$51.26^{+0.572}_{-0.568}$	52	2032.2	14.2	6.0
$\rho_2(2250)$	2248.0 ± 17.0	185.0	$63.86^{+0.985}_{-0.975}$	64	2250.5	2.5	1.4
$\rho_5(2350)$	2330.0 ± 35.0	400.0	$68.67^{+2.109}_{-2.077}$	69	2335.4	5.5	1.4
$f_6(2510)$	2470.0 ± 50.0	260.0	$77.30^{+3.204}_{-3.136}$	78	2481.0	11.0	4.3

$$OCT_M = l_{\pi\pi} * \text{ctg}(M/2) = L,\ l_{\pi\pi} = 2\text{sh}(S_{\pi\pi}/2)$$
$$\text{ch}S_{m\pi} = (L+2)/2,\ \text{ch}S_{\pi\pi} = 2L+1 \quad (5)$$

Therefore, triangles $\Delta\pi m\pi$ with integer values of OCT_M are called hyperbolic Heron triangles [10] [12]. The effective mass $m_{her} = m_{her}^{\pi,\pi}$ (column 6) is calculated using formula (6) (points "π", "π" of the base ($\pi - \pi$) of triangle $\Delta\pi m\pi$ are assigned the mass $m_\pi = m_{\pi_1}$):

$$m_{her} = m_{her}^{\pi,\pi} = \sqrt{2m_\pi^2\left(1+\text{ch}\left(S_{\pi\pi}\right)\right)},\ \text{ch}S_{\pi\pi} = \left(m_{her}^2 - 2m_\pi^2\right)\Big/2m_\pi^2 \quad (6)$$

where $S_{\pi\pi}$ is the rapidity of the base ($\pi - \pi$) of triangle $\Delta\pi m\pi$. Columns 7 and 8 of **Table 1** show the absolute and relative deviations of the mass $m_{her}^{\pi,\pi}$ from the experimental values m_r. From **Table 1** it can be seen that the masses m_r differ from masses $m_{her}^{\pi,\pi}$ within the resonance widths. The maximum deviation is 307% of the resonance width, the minimum is 0.3% of the resonance width, and the average deviation is 25.3% of the resonance width. Only the $\phi(1020)$ meson mass m_r differs from the $m_{her}^{\pi,\pi}$ mass by 307% widths. In the experiment, the very small value of the $\phi(1020)$ meson width (4.25 MeV) was obtained from the parameterization. Direct calculation of the OCT_M values using real data might yield an acceptable result. Or the production of the $\phi(1020)$ meson is not associated with Heron's triangles.

It should be noted that Archimedes' levers in HLVS were first used by N.A. Chernikov, who used the following expressions for the momenta $P_{G\pi_1}$ and $P_{G\pi_2}$ and kinetic energies $T_{G\pi_1}$ and $T_{G\pi_2}$ of particles π_1 and π_2 in the system of their center of mass "G" (**Figure 1**) [6]:

$$P_{G\pi_1} = m_{\pi_1}\text{sh}\left(S_{G\pi_1}\right) = P_{G\pi_2} = m_{\pi_2}\text{sh}\left(S_{G\pi_2}\right) \quad (7)$$

$$T_{G\pi_1} = m_{\pi_1}\left(\text{ch}\left(S_{G\pi_1}\right)-1\right),\ T_{G\pi_2} = m_{\pi_2}\left(\text{ch}\left(S_{G\pi_2}\right)-1\right) \quad (8)$$

$$S_{\pi_1\pi_2} = S_{G\pi_1} + S_{G\pi_2}$$

Since in the reference frame "G" the momenta $P_{G\pi_1} = P_{G\pi_2}$ are equal, then:

$$m_{\pi_1}\text{sh}\left(S_{G\pi_1}\right) = m_{\pi_2}\text{sh}\left(S_{G\pi_2}\right),\ \frac{m_{\pi_1}}{2\pi}2\pi\text{sh}\left(S_{G\pi_1}\right) = \frac{m_{\pi_2}}{2\pi}2\pi\text{sh}\left(S_{G\pi_2}\right)$$

The expression $2\pi\text{sh}\left(S_{G\pi_1}\right)$ represents the length of a circle of radius $S_{G\pi_1}$ in HLVS. Therefore, N.A. Chernikov used the lengths of circles of radii $S_{G\pi_1}$ and $S_{G\pi_2}$ as the lever arms (point "G" is assigned an effective mass m_r) (**Figure 1**). However, the expression $\text{sh}\left(S_{G\pi_1}\right)$ represents the length $l_{m\pi_1}$ of the oricycle arc and Archimedes' laws of levers can be represented in the form (3) [10]-[12].

According to (2), (7), formula (4) for OCT_r can be represented as:

$$OCT_r = \text{sh}^2\left(S_{G\pi_1}\right) = \left(P_{G\pi_1}/m_{\pi_1}\right)^2 = \left(m_r^2 - 4m_{\pi_1}^2\right)\Big/\left(4m_{\pi_1}^2\right) \quad (9)$$

The expression ($P_{G\pi_1}/m_{\pi_1}$) will be called the reduced momentum of particles π_1 in the reference frame "G".

3. Two-Particle Decays of Scalar, Strange Mesons and *Δ*, *N*, *Λ*, *Σ*, *Ξ* Baryons into Particles with Different Masses

Figure 1 shows the different sided triangles $\Delta Pm\pi_1$ of the decays of

$\Delta(1232) \to P + \pi_1$. The point "$m$" of the additive mass $m_{P\pi_1} = m_{\pi_1} + m_P$ is determined by the laws of the levers of Archimedes (10) (m_P is mass of a proton, m_{π_1} is mass of a pi meson). In the case of different masses of decay particles ($m_P > m_{\pi_1}$), the point "m" is shifted along the arc of the oricycle to the point "P" of the particle with a higher rest mass:

$$\begin{gathered} l_{P\pi_1} = l_{m\pi_1} + l_{mP},\ m_{P\pi_1} = m_P + m_{\pi_1} \\ m_P l_{mP} = m_{\pi_1} l_{m\pi_1} = m_{\pi_1}\left(l_{P\pi_1} - l_{mP}\right) \\ l_{m\pi_1} = m_P l_{P\pi_1} \Big/ \left(m_P + m_{\pi_1}\right) \\ l_{mP} = m_{\pi_1} l_{P\pi_1} \Big/ \left(m_P + m_{\pi_1}\right) \end{gathered} \tag{10}$$

The different sided triangle $\Delta P m \pi_1$ of the decay of the Δ(1232) baryon is obtained by connecting the points "π_1", "m", "P" with each other by straight line segments (**Figure 1**). The effective mass m_r of the decays Δ Barions $\to P + \pi_1$ can be calculated using formula (11) (the points "b" and "π_1" are associated with the rest mass m_b of the particle "b" and the rest mass m_{π_1} of the pi meson, $m_b \ge m_{\pi_1}$) [4]:

$$m_r^2 = m_b^2 + m_{\pi_1}^2 + 2 m_b m_{\pi_1} \operatorname{ch}\left(S_{b\pi_1}\right) \tag{11}$$

For "b" = "P" and $m_b = m_P$:

$$m_r^2 = m_P^2 + m_{\pi_1}^2 + 2 m_P m_{\pi_1} \operatorname{ch}\left(S_{P\pi_1}\right) \tag{12}$$

The mass m_r is related to the length $S_{P\pi_1}$ of the side ($P - \pi_1$) of the triangle $\Delta P m \pi_1$ (the points "P" and "π_1" are associated with the rest mass m_P of the proton and the rest mass m_{π_1} of the pi meson).

Rotate the segment ($m - \pi_1$) around the axis ($C_0 - m$) of the oricycle until the point "π_1" coincides with the point "π_2" (**Figure 1**). By connecting the points "π_1", "m", "π_2" with each other by straight line segments, we obtain isosceles rotary triangle $\Delta \pi_1 m \pi_2$ of Δ(1232) baryon decay inscribed in the oricycle (the lengths of the sides $S_{m\pi_1}$ and $S_{m\pi_2}$ are equal). The triangles $\Delta P m \pi_1$ and $\Delta \pi_1 m \pi_2$ are shown in **Figure 2** (the point "m" is placed at the origin "0" of coordinates, the different sided triangles $\Delta \pi_1 m \Delta(1232)$ represent the decay of the $N(1520) \to \Delta(1232) + \pi_1$).

In **Table 2**, the OCT_r values for rotary triangle $\Delta \pi_1 m \pi_2$ are calculated from published data on the effective masses of Δ baryon $\to P + \pi_1$ decays [13]. Columns 2 and 3 of **Table 2** give the values of the effective masses m_r and widths Γ of the decays Δ Barions. The OCT_r values for rotary triangle $\Delta \pi_1 m \pi_2$ are calculated using the formulas (4) and (12):

$$OCT_r = \left(P_{G\pi_1} / m_{\pi_1}\right)^2 = \left(m_b / m_{\pi_1}\right)\left(m_r^2 - \left(m_b + m_{\pi_1}\right)^2\right) \Big/ \left(m_b + m_{\pi_1}\right)^2 \tag{13}$$

For values $m_b = m_P$:

$$OCT_r = \left(P_{G\pi_1} / m_{\pi_1}\right)^2 = \left(m_P / m_{\pi_1}\right)\left(m_r^2 - \left(m_P + m_{\pi_1}\right)^2\right) \Big/ \left(m_P + m_{\pi_1}\right)^2 \tag{14}$$

The "G" point represents the center of inertia of the pair (π_1, π_2) of the rotary triangle $\Delta\pi_1 m\pi_2$. It is important to note that the "*G*" points of the (P, π_1) and (π_1, π_2) pairs are located on the ($C_0 - m$) axis of the oricycle. The effective mass of the corresponding particle pairs is concentrated at the "*G*" points (**Figure 2**).

Table 2. Hyperbolic Heron triangles in decays of Δ Barion $\rightarrow P + \pi_1$.

Name Δ Barion	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta P m \pi$ $m_{her}^{P,\pi}$ (Mev)	$\Delta m_r^{her} = m_r - m_{her}^{P,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
Δ Barion $\rightarrow P + \pi_1$, $m_P = 938.27209$ Mev, $m_{\pi_1} = m_\pi = 139.57001$ Mev								
Δ(1232)	1232.0 ± 0.40	117.0	$2.06^{+0.006}_{-0.006}$	2	1227.8	4.2	3.63	483.5
Δ(1600)	1570.0 ± 0.25	250.0	$7.54^{+0.005}_{-0.051}$	8	1595.1	25.1	10.0	837.4
Δ(1620)	1610.0 ± 20.0	130.0	$8.28^{+0.375}_{-0.370}$	8	1595.1	14.9	11.5	837.4
Δ(1710)	1710.0 ± 10.0	300.0	$10.20^{+0.199}_{-0.197}$	10	1700.0	10.0	3.4	925.8
Δ(1900)	1860.0± 30.0	300.0	$13.30^{+0.651}_{-0.640}$	13	1946.2	13.8	4.6	1044.5
Δ(1950)	1930.0 ± 10.0	285.0	$14.83^{+0.224}_{-0.223}$	15	1937.5	7.5	2.6	1116.6
Δ(2150)	2150.0±100.0	200.0	$20.03^{+2.547}_{-2.430}$	20	2149.0	1.1	0.5	1279.2
Δ(2300)	2300.0±100.0	350.0	$23.89^{+2.720}_{-2.604}$	24	2304.2	4.2	1.2	1395.7
Δ(2400)	2450.0±100.0	500.0	$20.01^{+2.894}_{-2.777}$	28	2449.6	0.4	0.1	1503.2
Δ(2750)	2794.0 ± 80.0	350.0	$38.35^{+2.624}_{-2.550}$	38	2780.0	14.0	4.0	1743.2
Δ(2950)	2990.0±100.0	330.0	$45.01^{+3.520}_{-3.402}$	45	2989.7	0.3	0.1	1893.2

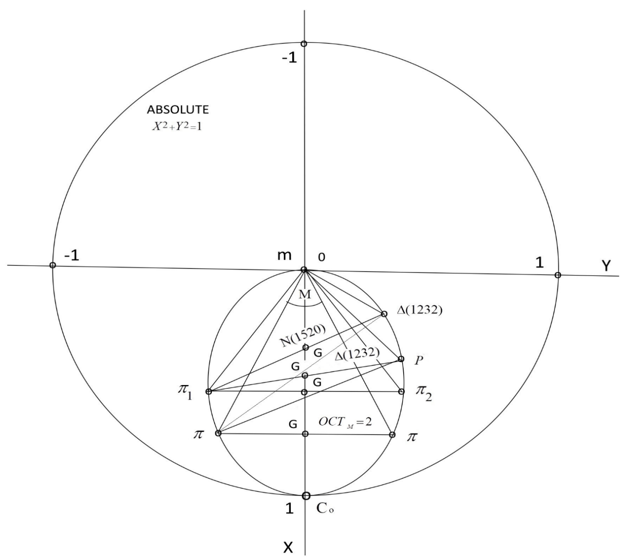

Figure 2. The different sided triangles $\Delta Pm\pi_1$ of baryon decays Δ(1232) --> $P + \pi_1$, N(1520) --> Δ(1232) + π_1, isosceles triangle $\Delta\pi_1$ m π_2 of baryon decays, isosceles Heron triangle $\Delta\pi$m π. The point-velocity "G" represents the centers of inertia of pairs of particles (P, π_1), (Δ(1232), π_1).

Columns 3 and 4 of **Table 2** show the values of the function OCT_r for rotary triangle $\Delta\pi_1 m\pi_2$ of Δ(1232) baryon decay, calculated using formula (14) and its nearest integer value OCT_M (**Figure 2**). The integer values OCT_M correspond to rotary Heron triangles $\Delta\pi m\pi$ (Rot_H triangles). In turn, the rotary Heron triangle $\Delta\pi m\pi$ corresponds to a triangle with different sides $\Delta Pm\pi$ (similar to how a triangle with different sides $\Delta Pm\pi_1$ corresponds to a rotary triangle $\Delta\pi_1 m\pi_2$). Column 6 of **Table 2** shows the effective mass $m_{her}^{P,\pi}$ of the pair proton and pi meson, calculated using formula (15) through the length $S_{\pi P}$ of the side ($\pi - P$) $\Delta Pm\pi$ (the points "P" and "π" are associated with the rest mass m_P of the proton and the rest mass m_π of the pi meson, the center of inertia "G" of the pair (P,π) lies on the axis $(C_0 - m)$ of the oricycle):

$$m_{her} = m_{her}^{b,\pi} = \sqrt{m_b^2 + m_\pi^2 + 2m_b m_\pi \text{ch} S_{\pi b}} \tag{15}$$

For values $m_b = m_P$

$$m_{her} = m_{her}^{P,\pi} = \sqrt{m_P^2 + m_\pi^2 + 2m_P m_\pi \text{ch} S_{\pi P}}$$

Columns 7 and 8 of **Table 2** show the absolute and relative deviations of the mass $m_{her}^{P,\pi}$ from the experimental values m_r. From **Table 2** it is evident that the masses $m_{her}^{P,\pi}$ differ from the masses m_r of Δ baryon decays within their widths. The maximum deviation is 10% of the resonance width, the minimum is 0.1% of the resonance width, and the average deviation is 3.8% of the resonance width. Column 9 of **Table 2** shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$ (points "π", "π" of the base ($\pi - \pi$) are associated with the rest mass m_π).

In **Table 3**, the OCT_r values for rotary triangle $\Delta\pi_1 m\pi_2$ are calculated from published data on the effective masses of 2-particle decays of N Barion $\to \Delta(1232) + \pi_1$ [13]. Columns 2 and 3 of **Table 3** give the values of the effective masses m_r and widths Γ of the N Barions decays. The mass m_r is related to the length $S_{\Delta(1232)\pi_1}$ of the side ($\Delta(1232), \pi_1$) of the triangle $\Delta\pi_1 m\Delta(1232)$ (the points "Δ(1232)" and "π_1" are associated with the mass $m_{\Delta(1232)}$ and the mass m_{π_1}) (**Figure 2**):

$$m_r^2 = m_{\Delta(1232)}^2 + m_{\pi_1}^2 + 2m_{\Delta(1232)} m_{\pi_1} \text{ch}\left(S_{\Delta(1232)\pi_1}\right) \tag{16}$$

The "G" points represent the centers of inertia of the pairs (Δ(1232), π_1) of decay particles. Columns 3 and 4 of **Table 3** show the values of the function OCT_r for rotary triangle $\Delta\pi_1 m\pi_2$ of N baryon decay, calculated using formula (13) (for values $m_b = m_{\Delta(1232)}$) and its nearest integer value OCT_M (**Figure 2**). Integer values of OCT_M correspond to rotary Heron triangles $\Delta\pi m\pi$. In turn, the rotary Heron triangle $\Delta\pi m\pi$ corresponds to the different sided triangle $\Delta\pi m\Delta(1232)$ (similarly to the fact that the different sided triangle $\Delta\pi_1 m\Delta(1232)$ corresponds to the rotary triangle $\Delta\pi_1 m\pi_2$). Column 6 of **Table 3** shows the effective mass $m_{N,her}^{\Delta(1232),\pi}$ of the pair (Δ(1232), π), calculated using formula (16) through the length $S_{\pi\Delta(1232)}$ of the side ($\Delta(1232) - \pi$) of the triangle $\Delta\pi m\Delta(1232)$ (the points "Δ(1232)" and "π" are associated with the mass $m_{\Delta(1232)}$ and the mass m_π):

Table 3. Hyperbolic Heron triangles in decays of N Barion $\to \Delta(1232)+\pi_1$.

Name N Barion	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta\pi m\Delta(1232)$ $m_{N,her}^{\Delta 232,\pi}$ (Mev)	$\Delta m_r^{her} = m_r - m_{N,her}^{\Delta 232,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
N Barion $\to \Delta(1232)+\pi_1$, $m_{\Delta 232}=1232.0$ Mev, $m_{\pi_1}=m_\pi=139.57001$ Mev								
N(1440)	1440.0 ± 30.0	350.0	$0.90^{+0.136}_{-0.135}$	1	1447.2	7.2	2.0	394.8
N(1520)	1515.0 ± 10.0	110.0	$1.94^{+0.143}_{-0.142}$	2	1519.0	4.0	3.7	483.5
N(1535)	1530.0 ± 5.0	150.0	$2.16^{+0.072}_{-0.172}$	2	1519.0	10.0	7.3	483.5
N(1650)	1650.0 ± 5.0	125.0	$3.95^{+0.077}_{-0.077}$	4	1653.4	3.4	2.7	624.8
N(1675)	1675.0 ± 5.0	150.0	$4.34^{+0.079}_{-0.078}$	4	1653.4	21.6	14.4	624.8
N(1680)	1685.0 ± 5.0	120.0	$4.50^{+0.079}_{-0.079}$	4	1653.4	31.6	26.3	624.8
N(1700)	1720.0 ± 10.0	200.0	$5.05^{+0.162}_{-0.160}$	5	1716.6	3.4	1.7	683.7
N(1710)	1710.0 ± 20.0	200.0	$4.89^{+0.323}_{-0.319}$	5	1716.6	6.6	3.3	683.7
N(1720)	1720.0 ± 10.0	250.0	$5.05^{+0.162}_{-0.161}$	5	1716.6	3.4	1.3	683.7
N(1860)	1860.0 ± 10.0	250.0	$7.41^{+0.175}_{-0.174}$	7	1836.6	23.4	9.4	789.5
N(1875)	1875.0 ± 10.0	200.0	$7.67^{+0.175}_{-0.176}$	8	1893.7	18.7	9.3	837.4
N(1880)	1880.0 ± 20.0	300.0	$7.76^{+0.355}_{-0.351}$	8	1893.7	13.7	4.6	837.4
N(1895)	1985.0 ± 10.0	200.0	$8.02^{+0.178}_{-0.177}$	8	1893.7	1.3	0.7	837.4
N(1900)	1920.0 ± 10.0	200.0	$8.47^{+0.181}_{-0.180}$	8	1893.7	26.3	13.1	837.4
N(2000)	2000.0 ± 10.0	300.0	$9.94^{+0.188}_{-0.187}$	10	2003.1	3.1	1.0	925.8
N(2060)	2100.0 ± 15.0	400.0	$11.87^{+0.297}_{-0.295}$	12	2106.8	6.8	1.7	1006.5
N(2100)	2100.0 ± 20.0	260.0	$11.87^{+0.396}_{-0.392}$	12	2106.8	6.8	1.7	1006.5
N(2190)	2180.0 ± 20.0	400.0	$13.47^{+0.412}_{-0.307}$	13	2156.8	23.2	5.8	1044.5

$$m_{her} = m_{her}^{\Delta(1232),\pi} = \sqrt{m_{\Delta(1232)}^2 + m_\pi^2 + 2m_{\Delta(1232)} m_\pi \operatorname{ch} S_{\Delta(1232)\pi}} \tag{17}$$

Columns 7 and 8 of Table 3 show the absolute and relative deviations of the mass $m_{N,her}^{\Delta(1232),\pi}$ from the experimental values m_r. From Table 3 it is evident that the masses $m_{N,her}^{\Delta(1232),\pi}$ differ from the masses m_r of Nbaryon decays within their widths. The maximum deviation is 23.4% of the resonance width, the minimum is 1.3% of the resonance width, and the average deviation is 6.1% of the resonance width. Column 9 of Table 3 shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$ (points "π", "π" of the base ($\pi-\pi$) are associated with the mass m_π).

Table 4 shows the published effective masses m_r and widths Γ of Λ Barion $\to \Sigma + \pi_1$ decays [13]. Columns 4 - 9 contain the calculated OCT_r values for the rotary triangles $\Delta\pi_1 m\pi_2$, the OCT_M values for the rotary Heron triangles $\Delta\pi m\pi$ and the absolute and relative deviations of the masses $m_{her}^{\Sigma,\pi}$ from the experimental values m_r. From **Table 4** it is evident that the masses $m_{her}^{\Sigma,\pi}$ differ from the masses m_r of Λ baryon decays within their widths. The average relative deviation is 18.4% of the resonance width. Column 9 shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$.

Table 4. Hyperbolic Heron triangles in decays of $\Lambda\ Barion \to \Sigma + \pi_1$.

Name Λ Barion	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta\Sigma m\pi$ $m_{her}^{\Sigma,\pi}$ (Mev)	$\Delta m_r^{her} = m_r - m_{her}^{\Sigma,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
$\Lambda\ Barion \to \Sigma + \pi_1$, $m_\Sigma = 1189.37$ Mev, $m_{\pi_1} = m_\pi = 139.57001$ Mev								
$\Lambda(1405)$	1405.1 ± 1.3	50.0	$1.00^{+0.018}_{-0.018}$	1	1404.8	0.4	0.7	324.8
$\Lambda(1520)$	1519.4 ± 0.2	15.7	$2.62^{+0.003}_{-0.003}$	3	1545.3	25.8	164.3	558.3
$\Lambda(1600)$	1600.0 ± 10.0	200.0	$3.83^{+0.155}_{-0.154}$	4	1610.9	10.9	5.5	624.2
$\Lambda(1670)$	1670.0 ± 5.0	30.0	$4.94^{+0.081}_{-0.080}$	5	1674.0	4.0	13.4	683.7
$\Lambda(1690)$	1690.0 ± 5.0	70.0	$5.26^{+0.082}_{-0.081}$	5	1674.0	16.0	22.8	683.7
$\Lambda(1710)$	1713.0 ± 13.0	180.0	$5.64^{+0.216}_{-0.214}$	6	1734.8	21.8	12.1	738.5
$\Lambda(1800)$	1800.0 ± 10.0	200.0	$7.11^{+0.174}_{-0.173}$	7	1793.6	6.6	3.2	789.5
$\Lambda(1810)$	1790.0 ± 10.0	110.0	$6.94^{+0.173}_{-0.172}$	7	1793.6	3.6	3.2	789.5
$\Lambda(1820)$	1820.0 ± 4.0	80.0	$7.46^{+0.070}_{-0.070}$	7	1793.6	26.4	33.0	789.5
$\Lambda(1830)$	1825.0 ± 10.0	90.0	$7.55^{+0.177}_{-0.176}$	8	1850.4	25.4	28.2	837.4
$\Lambda(1890)$	1890.0 ± 5.0	120.0	$8.71^{+0.091}_{-0.091}$	9	1905.6	15.6	13.0	882.7
$\Lambda(2050)$	2056.0 ± 22.0	493.0	$11.88^{+0.439}_{-0.434}$	12	2062.3	6.3	1.3	1006.5
$\Lambda(2070)$	2070.0 ± 24.0	370.0	$12.15^{+0.482}_{-0.477}$	12	2062.3	7.7	2.0	1006.5
$\Lambda(2080)$	2082.0 ± 13.0	181.0	$12.39^{+0.262}_{-0.260}$	12	2062.3	7.7	2.0	1006.5
$\Lambda(2100)$	2100.0 ± 22.0	200.0	$12.76^{+0.448}_{-0.443}$	13	2111.9	11.9	6.0	1044.5
$\Lambda(2110)$	2090.0 ± 22.0	250.0	$12.56^{+0.447}_{-0.441}$	13	2111.9	21.9	8.8	1044.5
$\Lambda(2250)$	2350.0 ± 22.0	150.0	$18.13^{+0.501}_{-0.497}$	18	2344.5	5.5	3.7	1216.7

Table 5 shows the published effective masses m_r and widths Γ of N Barion→ $n + \pi_1$ decays [13]. Columns 4 - 9 contain the calculated OCT_r values for the rotary triangles $\Delta\pi_1 m\pi_2$, the OCT_M values for the rotary Heron triangles $\Delta\pi m\pi$, and the absolute and relative deviations of the masses $m_{her}^{n,\pi}$ from the experimental values m_r. From **Table 5** it is evident that the masses $m_{her}^{n,\pi}$ differ

from the masses m_r of N Baryon→ $n+\ \pi_1$ decays within their widths. The average relative deviation is 5.7% of the resonance width. Column 9 shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$.

Table 5. Hyperbolic Heron triangles in decays of N Barion $\rightarrow n+\pi_1$.

Name N Barion	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta n m\pi$ $m_{her}^{n,\pi}$ (Mev)	Δm_r^{her} = m_r − $m_{her}^{n,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
N Barion $\rightarrow n+\pi_1$, $m_n = 939.56542052$ Mev, $m_{\pi_1} = m_\pi = 139.57001$ Mev								
N(1440)	1440.0 ± 30.0	350.0	$5.28^{+0.505}_{-0.495}$	5	1423.3	16.7	4.8	683.8
N(1520)	1515.0 ± 10.0	110.0	$6.56^{+0.176}_{-0.175}$	7	1539.9	24.9	22.7	789.5
N(1535)	1530.0 ± 15.0	150.0	$6.82^{+0.267}_{-0.264}$	7	1599.4	9.9	6.6	789.5
N(1650)	1650.0 ± 6.0	125.0	$9.03^{+0.115}_{-0.114}$	9	1648.3	1.6	1.3	882.7
N(1675)	1675.0 ± 5.0	145.0	$9.51^{+0.097}_{-0.097}$	10	1700.0	25.0	17.2	925.8
N(1680)	1685.0 ± 5.0	120.0	$9.71^{+0.977}_{-0.097}$	10	1700.0	15.0	12.5	925.8
N(1700)	1720.0 ± 20.0	200.0	$10.40^{+0.400}_{-0.396}$	10	1700.0	17.4	8.7	925.8
N(1875)	1875.0 ± 20.0	200.0	$13.62^{+0.436}_{-0.432}$	14	1892.4	12.4	4.1	1081.1
N(1880)	1880.0 ± 20.0	300.0	$13.73^{+0.437}_{-0.433}$	14	1892.4	0.4	0.1	1081.1
N(1895)	1985.0 ± 20.0	120.0	$14.06^{+0.441}_{-0.436}$	14	1892.4	2.6	2.2	1081.1
N(1920)	1920.0 ± 20.0	200.0	$14.61^{+0.447}_{-0.442}$	15	1937.5	17.5	8.8	1116.6
N(1990)	2020.0 ± 40.0	300.0	$16.89^{+0.945}_{-0.926}$	17	2024.7	4.7	1.6	1184.3
N(2060)	2100.0 ± 15.0	400.0	$18.80^{+0.366}_{-0.363}$	19	2108.4	8.4	2.1	1248.3
N(2220)	2250.0 ± 15.0	400.0	$22.57^{+0.392}_{-0.389}$	23	2266.3	16.4	4.1	1367.5
N(2250)	2280.0 ± 15.0	500.0	$23.36^{+0.400}_{-0.394}$	23	2266.3	13.6	2.7	1367.5
N(2600)	2600.0 ± 100.0	650.0	$32.40^{+3.067}_{-2.951}$	32	2586.9	13.2	2.0	1603.5
N(2700)	2612.0 ± 45.0	650.0	$32.76^{+1.372}_{-1.348}$	33	2620.0	8.0	1.2	1627.7
N(3000)	3000.0 ± 200.0	1650.	$45.36^{+7.177}_{-6.712}$	45	2989.7	10.3	0.6	1893.2

Table 6 shows the published effective masses m_r and widths Γ of Scalar Mezon $\rightarrow\eta+\pi_1$ decays [13]. Columns 4 - 9 contain the calculated OCT_r values for the rotary triangles $\Delta\pi_1 m\pi_2$, the OCT_M values for the rotary Heron triangle $\Delta\pi m\pi$, and the absolute and relative deviations of the masses $m_{her}^{\eta,\pi}$ from the experimental values m_r. From **Table 6** it is evident that the masses $m_{her}^{\eta,\pi}$ differ from the masses m_r of Scalar Mezon $\rightarrow\eta+\pi_1$ decays within their widths. The average relative deviation is 7.6% of the resonance width. Column 9 shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$.

Table 6. Hyperbolic Heron triangles in decays of Scalar Mezon $\to \eta + \pi_1$.

Name Scalar Mezon	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta\eta m\pi$ $m_{her}^{\eta,\pi}$ (Mev)	Δm_r^{her} = m_r – $m_{her}^{\eta,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
Scalar Mezon $\to \eta + \pi_1$, $m_\eta = 547.862$ Mev, $m_{\pi_1} = m_\pi = 139.57001$ Mev								
$a_o(980)$	980.0 ± 20.0	75.0	$4.05^{+0.330}_{-0.322}$	4	976.8	3.2	4.3	624.2
$a_2(1320)$	1317.7 ± 1.4	107.0	$10.50^{+0.031}_{-0.031}$	10	1294.8	22.9	21.4	925.8
$\pi_1(1600)$	1354.0 ± 25.0	330.0	$11.30^{+0.568}_{-0.557}$	11	1340.5	13.5	4.1	967.0
$a_o(1450)$	1439.0 ± 34.0	258.0	$13.28^{+0.822}_{-0.803}$	13	1427.5	11.6	4.5	1044.5
$a_2(1700)$	1706.0 ± 14.0	380.0	$20.25^{+0.398}_{-0.395}$	20	1697.1	8.5	2.3	1279.2
$a_0(1710)$	1713.0 ± 19.0	107.0	$20.45^{+0.544}_{-0.537}$	20	1697.1	15.9	14.8	1279.2
$a_4(1970)$	1967.0± 16.0	324.0	$28.21^{+0.525}_{-0.520}$	28	1960.5	6.5	2.0	1503.2

Table 7 shows the published masses m_r and widths Γ of Scalar Mezon $\to \rho(770) + \pi_1$ decays [13]. Columns 4 - 9 contain the calculated OCT_r values for the rotary triangles $\Delta\pi_1 m\pi_2$, the OCT_M values for the rotary Heron triangles $\Delta\pi m\pi$, and the absolute and relative deviations of the masses $m_{her}^{\rho(770),\pi}$ from the experimental values m_r. Decays $\eta'(958) \to \rho(770) + \pi_1$ with a relative deviation equal to 15,652% from **Table 7** are missing from all the calculations presented. This can only be explained by the small values of the resonance widths given (width = 0.23 Mev). Or the production of the $\eta'(958)$ meson is not associated with Heron's triangles. If we exclude the 15,652% deviation, the average relative deviation will be 4.4% of the resonance width. Column 9 shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi,\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$.

Table 7. Hyperbolic Heron triangles in decays of Scalar Mezon $\to \rho(770) + \pi_1$.

Name Scalar Mezon	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta\rho(770)m\pi$ $m_{her}^{\rho,\pi}$ (Mev)	Δm_r^{her} = m_r – $m_{her}^{\rho,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
Scalar Mezon $\to \rho(770) + \pi_1$, $m_\rho = 775.29999$ Mev, $m_{\pi_1} = m_\pi = 139.57001$ Mev								
$\eta'(958)$	957.8 ± 0.06	0.23	$0.53^{+0.001}_{-0.001}$	1	993.8	36.0	15,652	394.8
$\pi(1300)$	1300.0±100.0	400.0	$5.66^{+1.792}_{-1.659}$	6	1319.5	19.5	4.9	738.5
$a_2(1320)$	1318.2 ± 0.60	107.0	$5.98^{+0.010}_{-0.011}$	6	1319.5	1.3	1.2	738.5
$\omega(1420)$	1410.0 ± 60.0	290.0	$7.64^{+1.147}_{-1.099}$	8	1429.1	19.2	1.2	837.2
$a_2(1700)$	1706.0 ± 14.0	380.0	$20.25^{+0.398}_{-0.395}$	20	1697.1	8.5	2.3	1279.2
$a_0(1710)$	1713.0 ± 19.0	107.0	$20.45^{+0.544}_{-0.537}$	20	1697.1	15.9	14.8	1279.2
$a_4(1970)$	1967.0± 16.0	324.0	$28.21^{+0.525}_{-0.520}$	28	1960.5	6.5	2.0	1503.2

Table 8 shows the published masses m_r and widths Γ of Scalar Mezon $\to \omega(782)+\pi_1$ decays [13]. Columns 4 - 9 contain the calculated OCT_r values for the rotary triangles $\Delta\pi_1 m\pi_2$, the OCT_M values for the rotary Heron triangles $\Delta\pi m\pi$, and the absolute and relative deviations of the masses $m_{her}^{\omega(782),\pi}$ from the experimental values m_r. From **Table 8** it is evident that the masses $m_{her}^{\omega(782),\pi}$ differ from the masses m_r of Scalar Mezon $\to \omega(782)+\pi_1$ decays within their widths. The average relative deviation is 7.1% of the resonance width. Column 9 shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi,\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$.

Table 8. Hyperbolic Heron triangles in decays of Scalar Mezon $\to \omega(782)+\pi_1$.

Name Scalar Mezon	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta\omega(782)m\pi$ $m_{her}^{\omega,\pi}$ (Mev)	$\Delta m_r^{her} = m_r - m_{her}^{\omega,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
Scalar Mezon $\to \omega(782)+\pi_1$, $m_\omega = 782.65002$ Mev, $m_{\pi_1} = m_\pi = 139.57001$ Mev								
$b_1(1235)$	1229.5 ± 3.30	142.9	$4.36^{+0.052}_{-0.052}$	4	1207.1	22.4	15.6	624.2
$\rho(1450)$	1465.8 ± 25.0	400.0	$8.56^{+0.487}_{-0.479}$	9	1488.5	22.7	5.7	882.7
$\rho_3(1690)$	1688.8± 2.1	161.0	$13.20^{+0.047}_{-0.047}$	13	1679.9	8.9	5.5	1044.5
$\rho(2250)$	2150.0 ± 40.0	300.0	$24.87^{+1.145}_{-1.123}$	25	2154.6	4.6	1.5	1423.3

Table 9 shows the published masses m_r and widths Γ of Σ Barion $\to \Lambda+\pi_1$ decays [13]. Columns 4 - 9 contain the calculated OCT_r values for the rotary triangles $\Delta\pi_1 m\pi_2$, the OCT_M values for the rotary Heron triangles $\Delta\pi m\pi$, and the absolute and relative deviations of the masses $m_{her}^{\Lambda,\pi}$ from the experimental values m_r. From **Table 9** it is evident that the masses $m_{her}^{\Lambda,\pi}$ differ from the masses m_r of Σ Barion decays within their widths. The average relative deviation is 12.0% of the resonance width. Column 9 shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi,\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$.

Table 9. Hyperbolic Heron triangles in decays of Σ Barion $\to \Lambda+\pi_1$.

Name Σ Barion	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta\Lambda m\pi$ $m_{her}^{\Lambda,\pi}$ (Mev)	$\Delta m_r^{her} = m_r - m_{her}^{\Lambda,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
Σ Barion $\to \Lambda+\pi_1$, $m_\Lambda = 1115.68298$ Mev, $m_{\pi_1} = m_\pi = 139.57001$ Mev								
$\Sigma(1385)$	1383.7 ± 2.0	39.4	$1.72^{+0.028}_{-0.028}$	2	1403.5	19.8	50.3	483.5

Continued

$\Sigma(1620)$	1620.0 ± 2.0	70.0	$5.32^{+0.033}_{-0.033}$	5	1600.4	19.6	28.0	683.8
$\Sigma(1660)$	1660.0 ± 20.0	200.4	$5.99^{+0.339}_{-0.335}$	6	1660.8	0.8	0.4	738.5
$\Sigma(1670)$	1675.0 ± 2.0	70.0	$6.24^{+0.034}_{-0.034}$	6	1660.8	14.2	20.2	738.5
$\Sigma(1750)$	1750.0 ± 2.0	150.0	$7.54^{+0.036}_{-0.035}$	8	1775.5	25.5	17.0	837.4
$\Sigma(1775)$	1775.0 ± 10.0	120.0	$7.99^{+0.180}_{-0.180}$	8	1775.5	0.6	0.5	837.4
$\Sigma(1780)$	1780.0 ± 30.0	200.0	$8.08^{+0.546}_{-0.537}$	8	1775.5	4.5	2.2	837.4
$\Sigma(1900)$	1925.0 ± 20.0	165.0	$10.81^{+0.393}_{-0.389}$	11	1935.0	10.0	6.0	967.0
$\Sigma(1910)$	1910.0 ± 50.0	220.0	$10.51^{+0.982}_{-0.956}$	11	1935.0	24.9	11.3	967.0
$\Sigma(1915)$	1915.0 ± 22.0	120.0	$10.61^{+0.430}_{-0.425}$	11	1935.0	20.0	16.6	967.0
$\Sigma(1940)$	1940.0±100.0	250.0	$11.10^{+2.019}_{-1.917}$	11	1935.0	5.1	2.0	967.0
$\Sigma(2010)$	2005.0 ± 14.0	178.0	$12.40^{+0.286}_{-0.284}$	12	1985.2	19.8	11.1	1006.5
$\Sigma(2030)$	2030.0 ± 10.0	180.0	$12.91^{+0.207}_{-0.205}$	13	2034.2	4.2	2.4	1044.5
$\Sigma(2070)$	2060.0 ± 10.0	200.0	$13.54^{+0.210}_{-0.208}$	14	2082.1	22.1	11.1	1081.1
$\Sigma(2100)$	2100.0 ± 50.0	310.0	$14.38^{+0.428}_{-0.424}$	14	2082.1	18.9	6.1	1116.6
$\Sigma(2230)$	2240.0 ± 27.0	347.0	$17.46^{+0.617}_{-0.610}$	17	2119.6	20.4	5.9	1367.5
$\Sigma(2250)$	2250.0 ± 20.0	140.0	$17.69^{+0.459}_{-0.454}$	18	2263.6	13.6	13.6	1216.7
$\Sigma(2455)$	2455.0 ± 20.0	140.0	$22.58^{+0.500}_{-0.496}$	23	2471.1	16.7	11.9	1216.7

Table 10 shows the published masses m_r and widths Γ of ΞBarion $\to \Xi^0 + \pi_1$ decays [13]. Columns 4 - 9 contain the calculated OCT_r values for the rotary triangles $\Delta\pi_1 m\pi_2$, the OCT_M values for the rotary Heron triangles $\Delta\pi m\pi$, and the absolute and relative deviations of the masses $m_{her}^{\Xi^0,\pi}$ from the experimental values m_r. From **Table 10** it is evident that the masses $m_{her}^{\Xi^0,\pi}$ differ from the masses m_r of Ξ Barion $\to \Xi^0 + \pi_1$ decays within their widths. The average relative deviation is 51.7% of the resonance width. Column 9 shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi,\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$.

Table 10. Hyperbolic Heron triangles in decays of Ξ Barion $\to \Xi^0 + \pi_1$.

Name Ξ Barion	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta\Xi^0 m\pi$ $m_{her}^{\Xi^0,\pi}$ (Mev)	$\Delta m_r^{her} = m_r - m_{her}^{\Xi^0,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
Ξ Barion $\to \Xi^0 + \pi_1$, $m_{\Xi^0} = 1314.85999$ Mev, $m_{\pi_1} = m_\pi = 139.57001$ Mev								
$\Xi(1530)$	1531.8 ± 0.3	9.1	$1.03^{+0.004}_{-0.004}$	1	1529.7	2.1	23.3	394.8
$\Xi(1620)$	1620.8 ± 6.0	32.0	$2.28^{+0.087}_{-0.086}$	2	1601.4	19.4	60.7	483.5

Continued

Ξ(1690)	1690.0 ± 10.0	20.0	$3.30^{+0.151}_{-0.150}$	3	1670.0	20.0	100.0	558.3
Ξ(1820)	1823.0 ± 5.0	20.0	$5.38^{+0.081}_{-0.081}$	5	1799.5	23.5	117.7	683.7
Ξ(1950)	1950.0 ± 15.0	60.0	$7.51^{+0.261}_{-0.259}$	8	1977.8	8.8	14.6	837.4
Ξ(2030)	2025.0 ± 20.0	60.0	$8.84^{+0.363}_{-0.359}$	9	2033.8	8.8	14.6	882.7
Ξ(2500)	2500.0±150.0	60.0	$18.41^{+3.440}_{-3.240}$	18	2481.4	18.7	31.1	1216.7

Table 11 shows the published masses m_r and widths Γ of *Δ* Barion $\to \Delta(1232)+\pi_1$ decays [13]. Columns 4 - 9 contain the calculated OCT_r values for the rotary triangles $\Delta\pi_1 m\pi_2$, the OCT_M values for the rotary Heron triangles $\Delta\pi m\pi$, and the absolute and relative deviations of the masses $m_{\Delta,her}^{\Delta(1232),\pi}$ from the experimental values m_r. From **Table 11** it is evident that the masses $m_{\Delta,her}^{\Delta(1232),\pi}$ differ from the masses m_r of *Δ* Barions $\to \Delta(1232)+\pi_1$ decays within their widths. The average relative deviation is 5.1% of the resonance width. Column 9 shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi,\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$.

Table 11. Hyperbolic Heron triangles in decays of Δ Barion $\to \Delta(1232)+\pi_1$.

Name *Δ* Barion	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta\pi m\Delta(1232)$ $m_{\Delta,her}^{\Delta 232,\pi}$ (Mev)	$\Delta m_r^{her} = m_r - m_{\Delta,her}^{\Delta 232,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
Δ Barion $\to \Delta(1232)+\pi_1$, $m_{\Delta 232}=1232.0$ Mev, $m_{\pi_1}=m_\pi=139.57001$ Mev								
Δ(1600)	1570.0 ± 20.0	250.0	$2.74^{+0.300}_{-0.293}$	3	1587.6	17.6	7.0	558.3
Δ(1620)	1610.0 ± 10.0	130.0	$3.34^{+0.152}_{-0.150}$	3	1587.6	23.4	17.2	558.3
Δ(1700)	1710.0 ± 2.0	300.0	$4.89^{+0.032}_{-0.032}$	5	1716.6	6.6	2.2	683.8
Δ(1900)	1860.0± 20.0	250.0	$7.41^{+0.351}_{-0.347}$	7	1836.6	23.4	9.4	789.5
Δ(1905)	1880.0 ± 20.0	330.0	$7.76^{+0.355}_{-0.351}$	8	1893.7	13.7	4.2	837.4
Δ(1910)	1900.0 ± 20.0	300.0	$8.11^{+0.358}_{-0.355}$	8	1893.7	6.3	2.1	837.4
Δ(1950)	1930.0± 10.0	300.0	$8.65^{+0.182}_{-0.180}$	9	1949.2	19.2	6.4	882.7
Δ(1940)	2000.0± 40.0	400.0	$9.94^{+0.758}_{-0.743}$	10	2003.1	3.1	0.1	925.8
Δ(2000)	2100.0 ± 20.0	450.0	$11.87^{+0.396}_{-0.392}$	12	2106.8	6.8	1.5	1006.5
Δ(2200)	2200.0± 30.0	300.0	$13.88^{+0.624}_{-0.615}$	14	2205.6	5.6	1.2	1081.1

Table 12 shows the published masses m_r and widths Γ of Strange Mezon $\to K(493)+\pi_1$ decays [13]. Columns 4 - 9 contain the calculated OCT_r values for the rotary triangles $\Delta\pi_1 m\pi_2$, the OCT_M values for the rotary Heron triangles $\Delta\pi m\pi$, and the absolute and relative deviations of the masses $m_{her}^{K(493),\pi}$

from the experimental values m_r. From Table 12 it is evident that the masses $m_{her}^{K(493),\pi}$ differ from the masses m_r of Strange Mezon $\to K(493)+\pi_1$ decays within their widths. The average relative deviation is 12.1% of the resonance width. Column 9 shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi,\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$.

Table 13 shows the published masses m_r and widths Γ of Strange Mezon $\to K^*(892)+\pi_1$ decays [13]. Columns 4 - 9 contain the calculated OCT_r values for the rotary triangles $\Delta\pi_1 m\pi_2$, the OCT_M values for the rotary Heron triangles $\Delta\pi m\pi$, and the absolute and relative deviations of the masses $m_{her}^{K(892),\pi}$ from the experimental values m_r. From Table 13 it is evident that the masses $m_{her}^{K(892),\pi}$ differ from the masses m_r of Strange Mezon $\to K^*(892)+\pi_1$ decays within their widths. The average relative deviation is 7.1% of the resonance width. Column 9 shows the effective mass $m_{her}^{\pi,\pi}$ of the pair of pi mesons, calculated using formula (6) through the length $S_{\pi,\pi}$ of the base of the rotary Heron triangle $\Delta\pi m\pi$.

Table 12. Hyperbolic Heron triangles in decays of Strange Mezon $\to K(493)+\pi_1$.

Name Strange Mezon	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta k493m\pi$ $m_{her}^{k493,\pi}$ (Mev)	$\Delta m_{res}^{her} = m_r - m_{her}^{k493,\pi}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
Strange Mezon $\to K(493)+\pi_1$, $m_{k493}=493.677$ Mev, $m_{\pi_1}=m_\pi=139.57001$ Mev								
$K^*(700)$	838.0 ± 11.0	463.0	$2.66_{-0.161}^{+0.164}$	3	860.9	22.9	4.9	558.3
$K^*(892)$	891.8± 0.25	50.3	$3.48_{-0.004}^{+0.004}$	3	860.9	30.9	61.4	558.3
$K^*(1410)$	1421.8 ± 9.0	236.0	$14.27_{-0.225}^{+0.226}$	14	1410.0	11.0	4.6	1081.8
$K^*(1430)$	1425.6 ± 18.0	270.0	$14.39_{-0.450}^{+0.457}$	14	1410.0	15.6	5.8	1081.8
$K^*(1780)$	1718.0 ± 18.0	322.5	$22.5_{-0.542}^{+0.549}$	22	1701.5	16.5	5.1	1338.7
$K_0^*(1950)$	1957.0 ± 14.0	170.5	$30.25_{-0.481}^{+0.485}$	30	1949.9	7.1	4.2	1554.2
$K_0^*(2045)$	2045.0 ± 9.0	199.0	$33.35_{-0.324}^{+0.326}$	33	2035.2	9.8	4.9	1627.7
$K_5^*(2380)$	2382.8 ± 14.0	178.0	$46.54_{-0.586}^{+0.591}$	47	2393.6	10.8	6.1	1933.9

Table 13. Hyperbolic Heron triangles in decays of Strange Mezon $\to K^*(892)+\pi_1$.

Name Strange Mezon	Mass Resonance m_r (Mev)	Width Γ (Mev)	OCT_r	OCT_M	Heron Mass in $\Delta k892m\pi$ $m_{her}^{k892,\pi}$ (Mev)	$\Delta m_r^{her} = m_r - m_{her}^{k892,\pi}$ (Mev)	$\frac{\Delta m_r^{her}}{\Gamma}$ (%)	Mass in Rot_H $\Delta\pi m\pi$ $m_{her}^{\pi,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
Strange Mezon $\to K^*(892)+\pi_1$, $m_{k892}=891.76001$ Mev, $m_{\pi_1}=m_\pi=139.57001$ Mev								

Continued

$K_1(1270)$	1253.0 ±7.0	10.0	$3.04^{+0.107}_{-0.105}$	3	1250.2	2.8	27.8	558.3
$K_1(1400)$	1403.0± 7.0	174.0	$5.43^{+0.118}_{-0.117}$	5	1378.9	26.1	15.0	683.8
$K_1(1410)$	1414.0 ± 15.0	232.0	$5.62^{+0.256}_{-0.253}$	6	1436.1	22.1	9.5	738.5
$K_2^*(1430)$	1427.3 ± 1.5	100.0	$5.85^{+0.026}_{-0.026}$	6	1436.1	8.8	8.8	738.5
$K(1460)$	1482.4 ± 15.0	335.6	$6.81^{+0.269}_{-0.266}$	7	1493.0	10.6	3.2	789.5
$K_2^*(1580)$	1580.8 ± 5.0	110.0	$8.62^{+0.095}_{-0.095}$	9	1600.6	19.8	18.0	882.7
$K^*(1680)$	1718.8 ± 18.0	320.0	$11.36^{+0.374}_{-0.370}$	11	1701.4	17.4	5.4	967.0
$K_3^*(1780)$	1779.0 ± 8.0	161.0	$13.62^{+0.171}_{-0.170}$	13	1796.6	17.6	10.9	1044.5
$K_2(1770)$	1773.8 ± 8.0	186.0	$12.51^{+0.171}_{-0.170}$	13	1796.6	22.8	12.3	1044.5
$K_2(1820)$	1819.0 ± 12.0	264.0	$13.49^{+0.363}_{-0.261}$	13	1796.6	22.4	8.5	1044.5
$K_2^*(1980)$	1990.0 ± 50.0	348.0	$17.40^{+1.210}_{-1.180}$	17	1973.2	16.8	4.8	1184.3

4. Physics Interpretation of Heron's Triangles

As can be seen from **Tables 1-13**, the rotary triangles $\Delta\pi_1 m\pi_2$ of the decays of scalar, strange mesons and *Δ*, N, *Λ*, *Σ*, *Ξ* baryons coincide with the rotary Heron triangles $\Delta\pi m\pi$ (within the resonance widths). For the rotary Heron triangle $\Delta\pi m\pi$, formula (12) takes the form (L is an integer):

$$OCT_M = L = \left(P_{G\pi}/m_\pi\right)^2 = \left(m_b/m_\pi\right)\left(m_r^2 - \left(m_b + m_\pi\right)^2\right)\Big/\left(m_b + m_\pi\right)^2 \qquad (18)$$

Here $m_b \geq m_\pi$ and $m_b = m_\pi$, m_p, $m_{\omega(782)}$, m_η, $m_{\rho(770)}$, $m_{\Delta(1232)}$, m_{Ξ^0}, m_Λ, m_Σ, $m_{k(493)}$, $m_{k(892)}$.

Formula (18) represents the dependence of the integer L on the square of the effective mass m_r of the resonance, similar to the Regge dependence of the square of the resonance mass on its spin.

According to formulas (7) and (18) for a rotary Heron triangle $\Delta\pi m\pi$, the integer values $L = OCT_M$ are equal to the square of the reduced momentum $\left(P_{G\pi}/m_\pi\right)^2$ of the *π* meson in the reference frame "*G*" ("*G*" is the center of inertia of the pair (*π*, *π*) of the *π* mesons).

According to formulas (5) and (6) for a rotary Heron triangle $\Delta\pi m\pi$, the value $\text{ch}S_{\pi\pi}$, called the square of the reduced absorbed mass, is equal to an integer:

$$\text{ch}S_{\pi\pi} = \left(m_{her}^2 - 2m_\pi^2\right)\Big/2m_\pi^2 \qquad (19)$$

According to formula (8), the expression ($\text{ch}S_{m\pi} - 1$) is equal to the kinetic energy $T_{m\pi}$ of the *π* meson in the reference frame "*m*" of additive mass. From formulas (5) and (8), it follows that the expression ($2T_{m\pi}/m_\pi$), called the reduced kinetic energy of the pair (*π*, *π*) of *π* mesons in the reference frame "*m*", is equal to an integer.

Point "m" of the additive mass is the equilibrium point of Archimedes' levers, at the ends of which the gravitational forces of the decaying particles are applied. Therefore, discrete hyperbolic Heron triangles can reflect the self-oscillations that occur during particle decay, caused by the constant action of free-fall acceleration.

5. Looking for New Resonances Based on Heron's Triangles

For values of OCT_M from the intervals (1 - 80), (200 - 301), (398 - 498), Table 14 shows some discrete characteristics of Heron triangles and effective masses of decays of scalar mesons and *Δ*, *N* baryons. Columns 2 and 3 show the calculations of the hyperbolic cosines of the lengths of the sides and bases of Heron's triangles.

Column 4 show the calculations of the values of Kas_π^π—the generalized cosine of the angle θ between the tangents at points "π" and "π" of the base of Heron's triangles $\Delta\pi m\pi$. The values of Kas_π^π for various cases of the location of point "p" (x_p, y_p) of intersection of these tangents relative to the ***Absolute*** (Figure 3):

$$Kas_\pi^\pi = \cos(\theta) < 1,\ x_p^2 + y_p^2 < 1 \tag{20}$$

$$Kas_\pi^\pi = \mathrm{ch}(S_{cd}) > 1,\ x_p^2 + y_p^2 > 1 \tag{21}$$

Formula (20) corresponds to the case where the tangents (m – p) and (π – p) intersect inside the ***Absolute***. Formula (21) corresponds to the case where the tangents (m – p), (π – p) intersect outside the ***Absolute***, then the angle θ between them corresponds to a segment (c – d) of length S_{cd} which the tangents cut off on the line (A_1 – A_2). The straight lines (A_1 – p), (A_2 – p) are tangents to the ***Absolute***, drawn from the point "p".

As can be seen from Table 14, the values of $Kas_\pi^\pi = 2(OCT_M - 1) + 1$. From formulas (5), (6) it follows:

Table 14. Discrete characteristics and masses of Hyperbolic Heron Triangles.

OCT_M	$chS_{m\pi}$	$chS_{\pi\pi}$	Kas_π^π	Kas_m^π	$m_{her}^{\pi,\pi}$ (Mev)	$m_{\Delta,her}^{P,\pi}$ (Mev)	$m_{N,her}^{P,\pi}$ (Mev)	$m_{her}^{\eta,\pi}$ (Mev)	$m_{her}^{\rho(770),\pi}$ (Mev)	$m_{her}^{\omega(782),\pi}$ (Mev)
1	2	3	4	5	6	7	8	9	10	11
1	1.5	3	1	−0.5	394.8	1155.2	1155.9	770.0	**938.8**	1001.1
2	2.0	5	3	0.0	**485.3**	**1277.7**	1228.4	844.6	1066.9	1074.2
3	2.5	7	5	0.5	558.3	1296.2	1296.9	913.1	1135.3	1142.6
4	3.0	9	7	1.0	624.8	1361.2	1361.9	**976.8**	1199.9	**1207.1**
5	3.5	11	9	1.5	683.8	1423.3	**1423.3**	1036.6	1261.1	1268.4
6	4.0	13	10	2.0	738.5	1482.8	1484.4	1093.1	**1319.5**	1326.8
7	4.5	15	13	2.5	**789.5**	1539.9	**1599.4**	1146.9	1375.4	1382.8
8	5.0	17	14	3.0	837.4	**1595.1**	1595.7	1198.2	**1429.1**	1436.6
9	5.5	19	17	3.5	882.7	1648.3	**1648.3**	1247.4	1480.9	**1488.5**
10	6.0	21	19	4.0	925.8	**1700.0**	**1700.0**	**1294.8**	1530.9	1538.6
11	6.5	23	21	4.5	967.0	1750.1	1750.7	**1340.5**	1579.4	1587.1
12	7.0	25	23	5.0	**1006.5**	1789.7	1799.4	1384.6	1624.4	1634.2
13	7.5	27	25	5.5	1044.4	**1846.2**	1846.8	**1427.4**	1672.1	**1679.9**
14	8.0	29	26	6.0	1081.1	1892.4	**1893.1**	1469.0	1716.5	1724.5
15	8.5	31	29	6.5	1116.6	**1937.5**	**1938.2**	1509.4	1759.9	1767.9

Continued

16	9.0	33	31	7.0	1150.9	1981.6	1982.3	1548.8	1802.2	1810.3
17	9.5	35	33	7.5	1184.3	2024.7	**2025.4**	1587.2	1843.5	1851.7
18	10.0	37	34	8.0	1216.7	2067.0	2067.7	1624.7	1883.9	1892.2
19	10.5	39	37	8.5	1248.4	2108.4	**2109.1**	1661.3	192359	1931.9
20	11.0	41	39	9.0	**1279.2**	**2148.9**	2149.7	**1697.1**	**1962.3**	1970.7
21	11.5	43	41	9.5	1309.3	2188.8	2189.5	1732.3	2000.3	2008.9
22	12.0	45	43	10.0	**1338.7**	2227.9	2228.7	1766.7	2037.6	2046.3
23	12.5	47	44	10.5	1367.5	2266.4	**2267.1**	1800.4	2074.2	2083.0
24	13.0	49	47	11.0	1395.7	**2304.2**	2304.9	1833.5	2110.3	2119.1
25	13.5	51	49	11.5	1423.3	2341.4	2342.1	1866.1	2145.7	**2154.6**
26	14.0	53	51	12.0	1450.5	2378.0	2378.8	1891.1	2180.3	2189.5
27	14.5	55	53	12.5	1477.1	2414.1	2414.8	1929.5	2214.8	2223.9
28	15.0	57	55	13.0	1503.2	**2449.6**	2450.4	**1960.5**	**2248.5**	2257.7
29	15.5	59	57	13.5	**1528.9**	2484.6	2485.4	1990.9	2281.8	2291.0
30	16.0	61	59	14.0	1554.2	2519.1	2519.9	2020.9	2314.6	2323.9
31	16.5	63	61	14.5	1579.1	2553.2	2554.0	2050.5	2346.9	2356.3
32	17.0	65	63	15.0	1603.5	2586.8	**2587.7**	2079.7	2378.8	2388.3
33	17.5	67	65	15.5	1627.7	2620.2	**2620.9**	2108.4	2410.3	2419.8
34	18.0	69	67	16.0	1651.4	2252.8	2653.6	2136.8	2441.3	2451.0
35	18.5	71	69	16.5	**1674.8**	2685.2	2686.0	2164.7	2472.0	2481.7
36	19.0	73	71	17.0	1698.0	2717.2	2718.0	2192.4	2502.3	2512.1
37	19.5	75	73	17.5	**1720.7**	2748.8	2749.6	2219.7	2532.2	2542.1
38	20.0	77	74	18.0	**1743.2**	**2780.0**	2780.9	2246.6	2561.8	2571.7
39	20.5	79	77	18.5	1765.4	2811.0	2811.8	2272.3	2591.0	2601.1
40	21.0	81	79	19.0	1787.4	2841.5	2842.4	2299.6	2619.9	2630.1
41	21.5	83	81	19.5	**1809.0**	2871.8	2872.7	2325.6	2648.5	2658.7
42	22.0	85	83	20.0	1830.5	2901.7	2902.6	2351.4	2676.8	2687.1
43	22.5	87	85	20.5	1851.6	2931.3	2932.2	2376.8	2704.8	2715.2
44	23.0	89	87	21.0	1872.5	2960.7	2961.6	2402.6	2372.0	2743.0
45	23.5	91	89	21.5	1893.2	**2989.7**	**2990.6**	2426.9	2760.0	2770.5
46	24.0	93	91	22.0	1913.7	3018.5	3019.4	2451.6	2787.1	2797.7
47	24.5	95	93	22.5	**1933.9**	3047.0	3047.9	2576.0	2814.0	2824.7
48	25.0	97	95	23.0	1954.0	3075.2	3076.1	2500.2	2840.7	2851.4
49	25.5	99	97	23.5	**1973.8**	3103.2	3140.1	2524.2	2867.1	2877.9
50	26.0	101	99	24.0	1993.5	3130.9	3131.8	2547.9	2893.2	2904.1
51	26.5	103	101	24.5	**2012.9**	3158.3	3159.3	2571.4	2919.1	2930.1
52	27.0	105	103	25.0	2032.2	3185.6	3186.5	2594.7	2944.8	2955.9
53	27.5	107	105	25.5	2051.3	3212.6	3213.6	2617.9	2970.3	2981.4
54	28.0	109	107	26.0	2070.2	3239.4	3240.3	2640.7	2995.6	3006.8
56	29.0	113	110	27.0	2107.5	3292.3	3293.2	2685.9	3045.4	3056.8
60	31.0	121	119	29.0	2180.2	3395.7	3396.6	2774.1	3142.8	3154.5
63	32.5	127	125	30.5	2233.1	3471.2	3472.2	2838.5	3214.0	3225.8
64	33.0	129	127	31.0	**2250.5**	3496.0	3497.0	2859.6	3237.3	3249.2

Continued

68	35.0	137	135	33.0	2318.7	3593.5	3594.5	2942.6	3329.1	3341.2
69	35.5	139	137	33.5	**2335.5**	3617.4	3618.5	2963.0	3351.7	3363.9
70	36.0	141	139	34.0	2352.1	3641.2	3642.3	2983.2	3374.1	3383.6
77	39.5	155	153	37.5	2465.3	3803.7	3804.8	3121.3	3526.9	3539.6
78	40.0	157	154	38.0	**2481.1**	3826.4	3827.4	3140.5	3548.2	3561.0
79	40.5	159	157	38.5	2496.7	3848.9	3850.0	3159.6	3569.4	3582.2
80	41.0	161	159	39.0	2512.3	3871.3	3872.4	3178.6	3590.4	3603.3
200	101.0	401	399	99.0	3957.5	5977.0	5977.0	4954.8	5565.2	5584.3
215	108.5	431	428	106.5	4102.5	6190.0	6190.0	5138.8	5764.7	5784.4
217	109.5	435	433	107.5	4121.5	6217.9	6217.9	5157.2	5790.8	5810.5
221	111.5	443	440	109.5	4159.1	6273.2	6273.2	5203.7	5842.6	5862.5
226	114.0	453	450	112.0	4205.7	6341.7	6341.7	5261.2	5906.7	5926.8
248	125.0	497	494	123.0	4404.8	6634.7	6634.7	5507.1	6181.0	6209.9
254	128.0	509	506	126.0	4457.5	6712.4	6712.4	5572.3	6253.7	6274.9
263	132.5	527	524	130.5	4535.5	6827.2	6827.2	5668.7	6361.2	6382.7
272	137.0	545	543	135.0	4612.1	6940.2	6940.2	5763.5	6466.9	6488.8
301	151.5	603	601	149.5	4850.9	7292.3	7292.3	6058.8	6796.3	6819.3
398	200.0	797	794	198.0	5575.8	8363.1	8363.1	6956.1	7797.8	7824.0
409	205.5	819	817	203.5	5652.2	8476.0	8476.0	7050.6	7903.4	7929.9
436	219.0	873	871	217.0	5835.3	8746.9	8746.9	7277.5	8156.7	8184.9
498	250.0	997	995	248.0	6235.5	9339.3	9339.3	7773.4	8710.5	8739.6

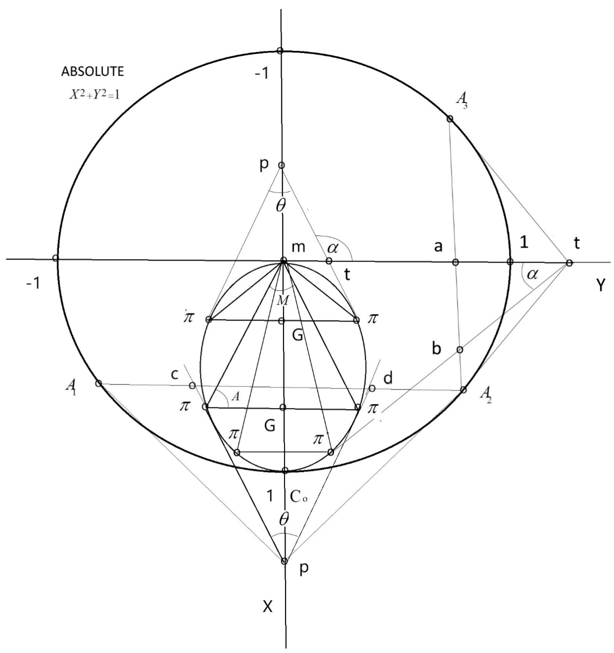

Figure 3. The values of Kas_{π}^{π} —the generalized cosine of the angle θ of decay between the tangents at the points "π", "π" of the base of Heron' s triangles $\Delta\pi m\pi$ are calculated. The values of Kas_{m}^{π} —the generalized cosine of the angle α of decay between the tangent at point "m" of the additive mass and the tangent at points "π" of the base of Heron's triangles $\Delta\pi m\pi$ are calculated.

$$\mathrm{ch}\left(S_{\pi\pi}\right) = Kas_{\pi}^{\pi} + 2 \quad (22)$$

From formula (22) it follows that the expression $\mathrm{ch}\left(S_{\pi\pi}\right)$ linearly depends on the Kas_{π}^{π}. We have a dependence similar to the Regge dependence of the square of the resonance mass on its spin.

Column 5 shows the values of Kas_{m}^{π}—the generalized cosine of the angle α between the tangent at point "*m*" of the additive mass and the tangent at points "π" of the of the side of Heron's triangles $\Delta\pi m\pi$. The values of Kas_{m}^{π} for various cases of the location of point "*t*" (x_t, y_t) of intersection of these tangents relative to the ***Absolute*** (**Figure 3**):

$$Kas_{m}^{\pi} = \cos(\alpha) < 1,\ x_t^2 + y_t^2 < 1 \quad (23)$$

$$Kas_{m}^{\pi} = \mathrm{ch}\left(S_{ab}\right) > 1,\ x_t^2 + y_t^2 > 1 \quad (24)$$

Formula (23) corresponds to the case where the tangents (m – t) and (π – t) intersect inside the ***Absolute***. Formula (24) corresponds to the case where the tangents (m – t), (π – t) intersect outside the ***Absolute***, then the angle α between them corresponds to a segment (a – b) of length S_{ab} which the tangents cut off on the line (A_3 – A_2). The straight lines (A_3 – t), (A_2 – t) are tangents to the ***Absolute***, drawn from the point "*t*". As can be seen from **Table 14**, the values of $Kas_{m}^{\pi} = \left(OCT_M - 2\right)/2$. From formula (7) it follows:

$$\mathrm{ch}\left(S_{m\pi}\right) - 1 = T_{m\pi}/m_{\pi} = Kas_{m}^{\pi} + 1 \quad (25)$$

From formula (25) it follows that expression ($\mathrm{ch}S_{m\pi} - 1$) is equal to the kinetic energy $T_{m\pi}$ π meson in the reference frame "*m*" of the additive mass, divided by the mass m_{π} of the π meson. From formula (25) it follows that the expression ($T_{m\pi}/m_{\pi}$), called the reduced kinetic energy of the π meson.

Column 6 shows the masses $m_{her}^{\pi,\pi}$ in the rotary Heron triangles $\Delta\pi m\pi$ of decays Scalar Mezon $\to \pi_1 + \pi_2$, calculated using formula (6). Column 7 shows the masses $m_{\Delta,her}^{P,\pi}$ of Δ Barions $\to P + \pi_1$ decays calculated using formula (15) for values $m_b = m_P$. Column 8 shows the masses $m_{N,her}^{P,\pi}$ of N Barions $\to P + \pi_1$ decays calculated using formula (15) for values $m_b = m_P$. Column 9 shows the masses $m_{her}^{\eta,\pi}$ of Scalar Mezon $\to \eta + \pi_1$ decays calculated using formula (15) for values $m_b = m_{\eta}$. Columns 9 and 10 show the masses $m_{her}^{\rho(770),\pi}$ and $m_{her}^{\omega(782),\pi}$ calculates using formula (15) for values $m_b = m_{\rho(770)}$, $m_{\omega(782)}$.

The mass values in **Table 14** corresponding to the resonances from **Tables 1-13** are highlighted in bold. The remaining unhighlighted mass values may correspond to the masses of new resonances in the decays of scalar mesons and Δ, N baryons. To detect these new resonances, it is necessary to investigate the distribution OCT_r calculated using formula (13) for $m_b = m_{\pi}$, m_p, $m_{\omega(782)}$, m_{η}, $m_{\rho(770)}$.

In **Table 15**, the effective masses of the decays of strange mesons and Δ, N, Λ, Σ, Ξ baryons are given for the values of OCT_M from the intervals (1 - 80), (200 - 301), (398 - 498). Columns 2 - 9 show the masses $m_{her}^{\pi,\pi}$, $m_{her}^{\Sigma,\pi}$, $m_{her}^{\Lambda,\pi}$, $m_{her}^{\Xi^0,\pi}$, $m_{\Delta,her}^{\Delta 232,\pi}$, $m_{N,her}^{\Delta 232,\pi}$, $m_{\Delta,her}^{k493,\pi}$, $m_{\Delta,her}^{k892,\pi}$ calculated using formula (15) for $m_b = m_{\pi}$,

m_{Σ}, m_{Λ}, m_{Ξ^0}, $m_{\Delta232}$, m_{k493}, m_{k892}. The mass values in **Table 15** corresponding to the resonances from **Tables 1-13** are highlighted in bold. The remaining unhighlighted mass values may correspond to the masses of new resonances in the decays of strange mesons and *Δ, N, Λ, Σ, Ξ* baryons. To detect these new resonances, it is necessary to investigate the distribution OCT_r calculated using formula (13) for $m_b = m_{\pi}$, m_{Σ}, m_{Λ}, m_{Ξ^0}, $m_{\Delta232}$, m_{k493}, m_{k892}.

Note that from **Table 2**, **Table 3**, **Tables 8-13** it follows that rotary Heron triangles with the values $OCT_M = 1,2,3,4,5,6,7$ are detected in the decays of strange mesons and *Δ, N, Λ, Ξ, Σ* baryons. **Figure 4** expands on the data from **Tables 1-13** for the first $OCT_M = 1,\cdots,7$ rotary Heron triangles. The oricycles with inscribed rotary Heron triangles are shifted upward along their axes. Each OCT_M level has Heron triangles *Δπmπ* with own lattice $\frac{\text{ch}S_{m\pi}}{0.5} \times \text{ch}S_{\pi\pi}$ (see formulas (5)).

Table 15. OCT_M and masses of Hyperbolic Heron Triangle.

OCT_M	$m_{her}^{\pi,\pi}$ (Mev)	$m_{her}^{\Sigma,\pi}$ (Mev)	$m_{her}^{\Lambda,\pi}$ (Mev)	$m_{her}^{\Xi^0,\pi}$ (Mev)	$m_{\Delta,her}^{\Delta232,\pi}$ (Mev)	$m_{N,her}^{\Delta232,\pi}$ (Mev)	$m_{her}^{k493,\pi}$ (Mev)	$m_{her}^{k892,\pi}$ (Mev)
1	2	3	4	5	6	7	8	9
1	394.8	**1404.8**	1331.5	**1579.7**	1447.2	**1447.2**	717.2	1109.1
2	483.5	1476.7	**1403.5**	**1601.4**	1519.0	**1519.0**	792.3	1181.8
3	558.3	**1545.3**	1472.1	**1670.0**	**1587.6**	**1587.6**	**860.9**	**1250.2**
4	624.2	**1610.9**	1537.6	1736.0	1653.4	**1653.4**	924.4	1315.1
5	683.8	**1674.0**	**1600.4**	**1799.5**	**1716.6**	**1716.6**	983.8	**1377.0**
6	738.5	**1734.8**	**1660.8**	1860.8	1776.6	1776.6	1039.8	**1436.1**
7	789.5	**1793.5**	1719.1	1920.2	**1836.6**	**1836.6**	1093.0	**1493.0**
8	837.4	**1850.4**	**1775.5**	**1977.8**	**1893.7**	**1893.7**	1143.7	1547.7
9	882.7	**1905.6**	1830.2	**2033.8**	1949.2	**1949.2**	1192.2	**1600.6**
10	925.8	1959.2	1883.3	2088.3	**2003.1**	**2003.1**	1238.8	1651.8
11	967.0	2011.4	**1934.9**	2141.3	2055.6	2055.6	1283.8	**1704.4**
12	1006.5	**2062.3**	**1985.2**	2193.1	**2106.8**	**2106.8**	1327.2	1749.7
13	1044.4	**2111.9**	**2034.2**	2243.8	2156.8	**2156.8**	1369.2	**1796.6**
14	1081.1	2160.4	**2082.1**	2088.3	**2205.6**	2205.6	**1410.0**	1842.3
15	1116.6	2207.9	2128.9	2541.7	2253.4	2253.4	1449.7	1887.0
16	1150.9	2254.3	2174.7	2389.2	2300.2	2300.2	1488.3	1930.6
17	1184.3	2299.8	**2219.6**	2435.7	2346.1	2346.1	1525.9	**1973.2**
18	1216.7	**2344.5**	**2263.6**	**2481.4**	2391.1	2391.1	1562.6	2015.0
19	1248.4	2388.3	2306.7	2526.2	2435.2	2435.2	1598.4	2055.9
20	1279.2	2431.3	2349.0	2570.3	2478.6	2478.6	1633.5	2096.0
21	1309.3	2473.5	2390.6	2613.6	2521.2	2521.2	1667.9	2135.3
22	1338.7	2515.1	2431.5	2656.2	2563.2	2563.2	**1701.5**	2173.9
23	1367.5	2555.9	**2471.7**	2698.1	2604.4	2604.4	1734.5	2211.9

Continued

24	1395.7	2596.1	2511.2	2379.4	2645.0	2645.0	1766.9	2249.2
25	1423.3	2635.8	2550.2	2780.1	2685.0	2685.0	1798.7	2285.9
26	1450.5	2674.8	2588.6	2820.2	2724.4	2724.4	1829.9	2322.1
27	1477.1	2713.3	2626.4	2859.7	2763.2	2763.2	1860.6	2357.6
28	1503.2	2751.2	2663.6	2898.7	2801.5	2801.5	1890.9	2392.9
29	1528.9	2788.6	2700.4	2937.2	2839.3	2839.3	1920.6	2427.2
30	1554.2	2825.2	2736.6	2975.2	2876.6	2876.6	**1949.9**	2461.3
31	1579.1	2861.9	2772.4	3012.7	2913.4	2913.4	1978.8	2494.8
32	1603.5	2897.9	2807.6	3049.7	2949.7	2949.7	2007.2	2528.0
33	1627.6	2933.5	2842.6	3086.3	2985.7	2985.7	**2035.3**	2560.7
34	1651.4	2968.6	2877.1	3122.5	3021.1	3021.1	2062.9	2593.0
35	1674.8	3003.3	2911.1	3158.2	3056.2	3056.2	2090.2	2624.9
36	1698.0	3037.6	2944.8	3193.6	3090.9	3090.9	2117.2	2656.4
37	1720.7	3071.5	2978.1	3228.5	3125.1	3125.1	2143.8	2687.6
38	1743.2	3105.1	3011.0	3263.1	3159.1	3159.1	2170.0	2718.4
39	1765.4	3138.3	3043.5	3297.4	3192.6	3192.6	2192.6	2748.8
40	1787.4	3171.1	3075.7	3331.2	3225.8	3225.8	2221.7	2778.9
41	1809.0	3203.6	3107.6	3364.8	3258.7	3258.7	2247.0	2808.7
42	1830.5	3235.8	3139.2	3398.0	3291.2	3291.2	2271.1	2838.2
43	1851.6	3267.7	3170.4	3430.9	3323.4	3323.4	2296.9	2867.4
44	1872.5	3299.2	3201.4	3463.4	3355.4	3355.4	2321.5	2896.3
45	1893.2	3330.5	3232.0	3495.7	3387.0	3387.0	2345.8	2924.9
46	1913.7	3361.5	3262.4	3527.7	3418.3	3418.3	2369.8	2953.2
47	1933.9	3392.2	3292.4	3559.4	3449.3	3449.3	**2393.6**	2981.3
48	1954.0	3422.6	3322.2	3590.7	3480.1	3480.1	2417.2	3009.0
49	1973.8	3452.7	3351.8	3621.9	3510.6	3510.6	2440.5	3036.6
50	1993.5	3482.6	3381.0	3652.8	3540.8	3540.8	2463.6	3063.9
51	2012.9	3512.2	3410.1	3683.4	3540.8	3540.8	2463.6	3063.9
52	2032.2	3541.6	3438.8	3713.7	3600.5	3600.5	2509.2	3117.7
53	2051.3	3570.8	3467.4	3743.8	3630.0	3630.0	2531.7	3144.3
54	2070.2	3599.6	3495.7	3652.8	3659.2	3659.2	2554.0	3170.7
55	2088.9	3628.3	3523.8	3803.3	3688.2	3688.2	2576.1	3196.8
56	2107.5	3656.8	3551.6	3832.8	3717.0	3717.0	2598.0	3222.8
57	2125.9	3685.0	3579.3	3861.9	3745.5	3745.5	2619.8	3248.5
58	2144.1	3713.0	3606.7	3890.9	3773.9	3773.9	2641.3	3274.0
59	2162.2	3740.3	3634.0	3919.7	3802.0	3802.0	2662.7	3299.4
60	2180.2	3768.4	3660.9	3948.2	3829.9	3829.9	2683.9	3324.4
63	2233.1	3850.0	3740.8	4032.6	3912.5	3912.5	2746.5	3398.7
64	2250.5	3876.1	3767.1	4060.3	3939.6	3939.6	2767.1	3423.1
68	2318.7	3982.3	3870.3	4169.5	4046.4	4046.4	2847.8	3519.1

Continued

69	2335.5	4008.2	3895.7	4196.3	4072.6	4072.6	2867.7	3542.6
70	2352.1	4034.1	3921.0	4223.0	4098.8	4098.8	2887.4	3566.1
77	2465.3	4210.0	4093.1	4405.2	4276.9	4276.9	3021.7	3725.9
78	2481.1	4234.5	4117.1	4430.5	4301.7	4301.7	3040.4	3748.1
79	2496.7	4259.0	4141.0	4455.8	4326.4	4326.4	3059.0	3770.3
80	2512.3	4283.2	4164.7	4481.0	4350.9	4350.9	3077.4	3792.3
200	3957.5	6573.8	6403.0	6857.4	6671.2	6671.2	4803.6	5861.6
200	3957.5	6573.8	6403.0	6857.4	6671.2	6671.2	4803.6	5861.6
215	4102.5	6806.2	6629.8	7098.7	6906.6	6906.6	4977.5	6070.8
217	4121.5	6836.6	6659.5	7130.3	6937.4	6937.4	5000.2	6098.2
221	4169.1	6896.9	6718.4	7193.0	6998.6	6998.6	5045.4	6152.5
226	4205.7	6971.6	6791.4	7270.6	7074.3	7074.3	5101.2	6219.8
248	4404.8	7291.3	7103.5	7602.8	7398.3	7398.3	5340.1	6307.6
254	4457.5	7376.1	7186.2	7690.8	7484.2	7484.2	5403.4	6583.9
263	4535.5	7501.4	7308.6	7821.1	7611.2	7611.2	5497.0	6696.7
272	4612.1	7624.8	7429.0	7949.3	7736.2	7736.2	5589.0	6807.6
301	4850.9	8009.2	7804.3	8348.8	8125.8	8125.8	5875.8	7153.4
398	5575.8	9178.8	8945.7	9564.7	9311.4	9311.4	6747.0	8204.8
409	5652.2	9302.1	9066.1	9692.9	9436.4	9436.4	6838.8	8315.7
436	5835.3	9598.2	9355.0	10000.8	9736.5	9736.5	7059.0	8581.7
498	6235.5	10245.7	9986.9	10674.1	10392.9	10392.9	7540.5	9163.3

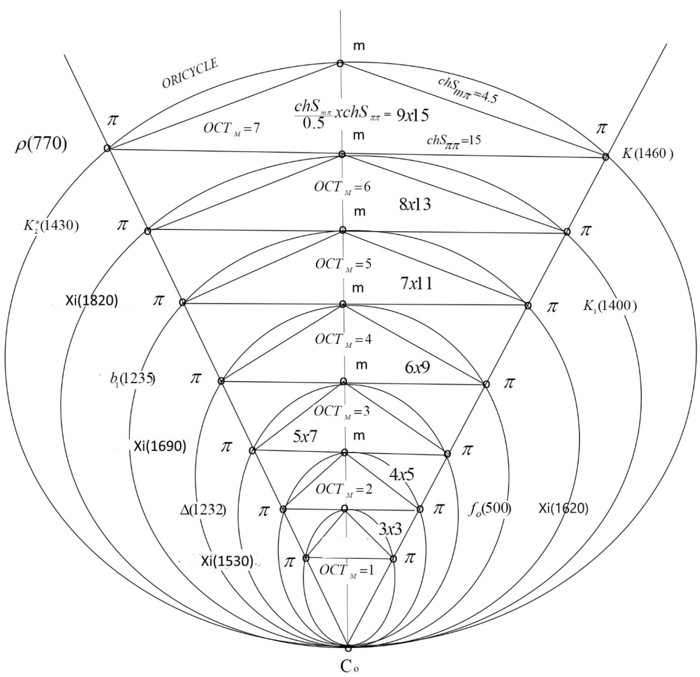

Figure 4. The data from Table 14, Table 15 for the first OCT_M = 1, …, 7 Heron triangles. The oriycles with inscribed Heron triangles are shifted upward along there axes ($C_0 - m$). Each OCT_M level has Heron triangles $\Delta\pi m\pi$ with own lattice $\frac{chS_{m\pi}}{0.5} \times chS_{\pi\pi}$.

The value $OCT_M = 2$ from **Table 1** for the scalar meson $f_o(500)$ corresponds to an average mass of 500 Mev and a width of 300 Mev. However, all measurements of the mass and width $f_o(500)$ meson have a large variance of values (mass 400 - 800 MeV, width 100 - 800 MeV) [13]. Then it is quite possible that in the region $OCT_r < 7$ (calculated using formula (9)) instead of $f_o(500)$ meson, 6 or less than 6 new scalar mesons with $OCT_M = 1,2,3,4,5,6$ and masses of (394.8, 483.5, 558.3, 624.2, 683.7, 738.5) Mev may be detected. These new scalar mesons will appear in (π^+,π^+), (π^-,π^-), (π^+,π^-), (π^+,π^0), (π^-,π^0), (π^0,π^0) decays.

Thus, based on actual data on effective masses of pairs (π_1,π_2), (P,π_1), (Ξ^0,π_1), (Σ,π_1), (η,π_1), ($\rho(770),\pi_1$), ($\omega(782),\pi_1$), (Λ,π_1), ($\Delta(1232),\pi_1$), ($K(493),\pi_1$), ($K^*(892),\pi_1$) it is sufficient to examine the distribution of OCT_r, calculated using formula (13). Statistically significant peaks in the distribution at $OCT_r < 50$ will correspond to new resonances with masses < 2.5 GeV.

Table 14, **Table 15** show that at $50 < OCT_M < 200$, new resonances with masses (3 - 4) GeV can be detected. At $200 < OCT_M < 300$, new resonances with masses (4 - 6) Gev can be detected. At $300 < OCT_M < 400$, new resonances with masses (6 - 8) Gev can be detected. At $400 < OCT_M < 500$, new resonances with masses (8 - 10) Gev can be detected. It would be interesting to detect resonances with masses > 10 Gev, corresponding to OCT_M values > 500.

Finally, we present three functions whose integer values define new types of hyperbolic Heron triangles. The first function (HSASB2) is the product of the hyperbolic sine of the altitude and the hyperbolic sine of half the base of the rotary Heron triangles $\Delta\pi_1 m\pi_2$:

$$\text{HSASB2} = \text{sh}(S_{Gm})\text{sh}(S_{\pi_1\pi_2}/2) = 0.5 * OCT_r\sqrt{OCT_r/(OCT_r+1)} \tag{26}$$

Here, S_{Gm} is the altitude dropped from vertex "m" to the base, and $S_{\pi\pi}$ is the base length of the triangle. The second function (HSP2) is the hyperbolic sine of the semi-perimeter of the $\Delta\pi_1 m\pi_2$:

$$\text{HSP2} = \text{sh}(S_{m\pi_1} + S_{\pi_1\pi_2}/2) = 0.5\sqrt{OCT_r}\left(\sqrt{(OCT_r+4)(OCT_r+1)} + OCT_r + 2\right) \tag{27}$$

Here, $S_{m\pi_1}$ is the length of the lateral side, and $S_{\pi_1\pi_2}$ is the base length of the triangle. The third function (CotHArea) is the cotangent of the hyperbolic area of triangle $\Delta\pi_1 m\pi_2$:

$$\text{CotHArea} = \text{Ctg}(\pi - M - 2A) \tag{28}$$

Here, M is the angle at vertex "m", and A is the angle at the base of the triangle $\Delta\pi_1 m\pi_2$.

6. Conclusions

The published data show that the decays of scalar, strange mesons, and *Δ*, *N*, *Λ*, *Σ*, *Ξ* baryons correspond (within the resonance width) to hyperbolic Heron triangles with integer values of OCT_r. Based on Heron's triangles, the existence of new resonances is predicted. This can be confirmed by actual measurements of effective masses in the decays of scalar, strange mesons, and *Δ*, *N*, *Λ*, *Σ*, *Ξ* baryons.

In addition, Heron's triangles have about 10 other discrete characteristics. Therefore, the detection of Heron triangles in hadron spectra can experimentally relate these discrete characteristics to quantum resonance numbers.

Further development of the described approach will consist of:

- isolation of resonances by the Heron triangle method and analysis of the angular distributions of their decays using parametrization of the dynamic axis of spin quantization by Lobachevsky straight line beams [15];
- analysis of 3 particles decays of resonances based on a 3-dimensional analogue of Heron triangles.

Funding

The work was financed by the LLP "Industry 4.0", 050020 Almaty, Kazakhstan.

Data Availability Statement

The data used in the article are taken from open sources [13].

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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