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A Mathematical Model for the Biological Control of *Bactrocera dorsalis* in Tanzanian Mango Production Systems

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Abstract

Bactrocera dorsalis is a major invasive pest causing significant economic losses in mango production due to fruit infestation and damage, which can be controlled effectively by the use of their natural enemies, referred to as the biological pest control. This study presents a biologically motivated mathematical model to examine the ecological interactions between mango plants, the oriental fruit fly (*Bactrocera dorsalis*), and its parasitoid wasp (*Fopius arisanus*) in a mango agroecosystem. The model employs a system of nonlinear differential equations to represent the dynamics of mango plants, the population biology of *Bactrocera dorsalis*, and parasitism behavior of *Fopius arisanus*. The dynamics of the mango-pest-parasitoid system were analyzed using a mathematical model. The equilibrium points are proved to be locally and globally asymptotically stable under some conditions. Sensitivity analysis, reveals that the predation rate plays a crucial role in the system that has a direct impact on disease spread and the impact of ecological parameters on pest suppression. The results highlight critical thresholds for effective biological control and reveal how the efficiency of *Fopius arisanus* influences the long-term dynamics of the mango-pest-parasitoid system. Our goal is to keep the spread of Mango pests under control so that mango yield is unaffected. This work contributes to a better understanding of pest regulation mechanisms and supports the design of sustainable, biologically based pest management strategies in mango cultivation.

Keywords

Mathematical Model, *Bactrocera dorsalis*, Biological Control, *Fopius arisanus*, Parasitoid Wasps

1. Introduction

Mango (*Mangifera indica* L.) is one of the most important fruit crops in Tanzania, both economically and nutritionally. It is widely cultivated across various agroecological zones, from coastal regions to inland areas, providing livelihoods for smallholder farmers and contributing significantly to local and export markets [1]. However, mango production in Tanzania faces serious threats from insect pests, particularly the Oriental fruit fly (*Bactrocera dorsalis*), which has emerged as a major constraint to sustainable fruit production [2].

Tanzania's mango (*Mangifera indica* L.) production has experienced steady growth over the decades. Since 1966, the country's mango production has increased at an average annual rate of 1.7%. Projections indicate that by 2026, Tanzania is expected to produce approximately 501,710 metric tons of mangoes, reflecting a year-on-year growth rate of 1.2% [3]. In 2021, it was the 19th largest producer of mangoes, but Peru overtook Tanzania with 466,390 metric tons. Indonesia, China and Mexico were the top three producers [4] [5].

Bactrocera dorsalis, an invasive species native to Asia, was first detected in Tanzania in the early 2000s and has since spread rapidly throughout the country. In Tanzania, its presence was first documented in 2003 [6] [7]. The establishment and subsequent spread of *Bactrosera dorsalis* have had serious economic repercussions, negatively impacting both domestic consumption and the export potential of mangoes due to fruit damage and quarantine restrictions. Therefore, pest management is essential to prevent devastating impact on economy, food security, social life, health and biodiversity.

Farmers mainly use chemical insecticides to control the mango fruit fly. The disadvantages of the application of these insecticides led the fruit flies to develop resistance to various pesticides [8]. Also, the increased use of pesticides negatively impacts the environment and human health. Furthermore, they negatively affect beneficial pollinating insects and natural enemies of the pests [9] [10]. Thus, extensive use of pesticides is not a sustainable solution for pest control.

The Food and Agriculture Organization (FAO) advocates environmentally sustainable strategies for managing agricultural pests and diseases, emphasizing ecosystem-based approaches that minimize harm to human health and biodiversity [11]. In alignment with this framework, researchers worldwide are exploring sustainable and innovative solutions to mitigate the impacts of pests and diseases on agricultural systems. One such approach is biological control, which offers a promising and ecologically sound alternative to chemical pesticides.

The effectiveness of *Fopius arisanus* has been demonstrated across several geographic regions. Following its successful use in Hawaii, it was introduced to control and suppress tephritid fruit flies in Australia, Central America, various islands in the Pacific and Indian Oceans, and the Mediterranean basin [12]. Recent studies have further validated its potential of *Fopius arisanus* reared on guava fruits infested with *Bactrocera dorsalis*, having shown high parasitism rates, favorable emergence and sex ratios, and enhanced longevity [13] [14]. These findings affirm the suitability of *Fopius arisanus* as a reliable biological control agent, particularly in integrated pest management (IPM) programs aiming to sustainably manage *Bactrocera dorsalis* in tropical fruit production systems.

Mathematical modeling offers a powerful tool for understanding the complex interactions between mango plants, pests, and their natural enemies. By formulating a system of differential equations that captures the dynamics of mango growth, pest infestation, and biological control, researchers can predict outcomes under various ecological scenarios and guide practical interventions.

Several studies in Tanzania have focused on seasonal population of *Bactrocera dorsalis* [15] [16], invasion of *Bactrocera dorsalis* [17] eradication techniques and management [18]. To the level of our knowledge no studies have considered a Pest-Parastoid interaction between mango, pest population and natural enemies through a novel mathematical model based on ordinary differential equations, using non-linear functions which are more appropriate for estimating parameters of pest population.

In this study, we developed a mathematical model to describe the interactions within a tri-trophic system involving mango plants, the pest *Bactrocera dorsalis*, and the parasitoid wasp *Fopius arisanus* a natural enemy. Mango plants serve as the primary resource, pests as herbivores, and natural enemies as biological control agents targeting the pests. A key feature of the model is that the natural enemy affects the mango population indirectly by reducing pest pressure, distinguishing it from classical predator-prey models. Our model extends traditional models by incorporating a natural enemy class and using biologically realistic assumptions for parasitism dynamics. The model is analyzed within the Tanzanian context to offer insights into sustainable, ecologically based pest management strategies for mango production.

The remaining part of the study is organized as follows: In Section 2, we present the literature review, explaining the existing knowledge and methods related to the study. In Section 3, Mathematical Formulation of a model used in the study presented and explained. In Section 4, Mathematical analysis of the model which includes positivity and boundness of the solution, equilibrium and stability of the model has been presented and discussed. Section 5, Numerical analysis of a model has been done this includes, sensitivity analysis of the model parameters, have been discussed in this section. Finally, in Section 6, we provided concluding remarks of the study.

2. Literature Review

Mango (*Mangifera indica* L.) is a key horticultural crop in tropical and subtropical regions, including Tanzania. However, its production has been severely impacted by the invasion of the Oriental fruit fly, *Bactrocera dorsalis*, a highly invasive species first reported in Tanzania in 2003 [6]. This pest causes direct fruit damage and limits export potential due to quarantine restrictions [19].

To mitigate these effects, biological control has emerged as a sustainable and environmentally friendly alternative to chemical pesticides. This involves the introduction of natural enemies such as parasitoid wasps that target specific life stages of the pest. Among these, *Fopius arisanus* has been identified as a particularly effective agent due to its ability to parasitize fruit fly eggs and prevent emergence [13] [20].

Mathematical modeling provides a valuable framework to understand and predict the population dynamics of such Pest-natural enemy systems. Classical predator-prey models (Lotka-Volterra) have been extended to include tri-trophic interactions where the host plant (mango) is consumed by pests (*Bactrocera dorsalis*), which are in turn attacked by natural enemies (*Fopius arisanus*) [21] [22].

Recent studies have applied these models to field data. For instance, Vargas *et al.* [23] modeled the suppression of *Bactrocera dorsalis* through parasitoid releases in Hawaii, while Agboka *et al.* [24] explored the introduction of exotic parasitoids in West Africa. Zida *et al.* [25] and Cai *et al.* [13] provided empirical data on parasitism rates and development of *Fopius arisanus*, which are essential for parameterizing control models.

Models are increasingly being used to support Integrated Pest Management (IPM) strategies, which combine biological control with cultural practices, baiting, and traps and has been successfully implemented against a number of plant pests such as cassava mealybug [26], Coffee Berry Borer [27], guava borers [28] and Sugarcane borer [29]. Models simulate scenarios under different interventions and help optimize the timing and frequency of natural enemy releases [30]-[32]. These approaches enable the design of locally tailored, biologically based control programs for sustainable mango production.

3. Mathematical Formulation

This section proposes a mathematical model constituting three ordinary differential equations to study the interactions among three linked trophic levels within an ecosystem, mango plants, pests (consumers), and natural enemies (parasitoids). The model captures how energy and population dynamics flow across these levels and how changes at one level influence the others.

The proposed Pest-Parasitoids system is developed on the following assumptions:

1) The Mango population grows according to the logistic curve with carrying capacity K_M and with an intrinsic growth rate r_M, in the absence of Pest and natural enemies. The logistic equation is a mathematical model that describes the growth of a population over time. It is represented by the following differential equation:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = r_M M\left(1 - \frac{M}{K_M}\right)$$

where M is the mango population, t is time, r_M intrinsic growth rate of mango population and K_M, carrying capacity. The logistic equation incorporates the concept of density dependence, which means that the growth rate of the population decreases as it approaches the carrying capacity. The differential equation for (t) is:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = r_M\left(1 - \frac{M}{K_M}\right) - \alpha MP \tag{1}$$

2) Pests population $P(t)$ have a functional response to the availability of mango plants. We introduced the natural death of predators δ. Here we also we considered the competition between the predator and natural enemy, the term γPN, captures pest mortality due to attacks by the natural enemy N. This term is linear in prey density P, which is the defining characteristic of a Holling type I functional response, the per capita attack rate of the natural enemy increases linearly with pest density and does not saturate. Hence, from the above assumption, we have

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \beta MP - \gamma PN - \delta P \tag{2}$$

3) Natural enemies (Parastoid wasp) $N(t)$ rely solely on pests for their growth and survival. The parasitoid population grows by parasitizing mango borers, but they also die naturally by parameter μ and face a limited carrying capacity. Thus, the differential equation for $N(t)$ is;

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \gamma\eta PN - \mu N \tag{3}$$

Equations (2) and (3) describes a Pest-Parastoid interaction governed by a Holling type I functional response, which assumes a linear relationship between host density and attack rate. The functional response, the rate at which a single natural enemy attacks pests is given by $f(P) = \gamma P$ which is Holling type I.

Table 1. Variables and model parameters.

Parameter	Meaning
$M(t)$	Population density of mango plants
$P(t)$	Population density of pests (fruit flies)
$N(t)$	*Fopius arisanus* parasitoid population
K_M	Carrying capacity of mango trees
r_M	Intrinsic growth rate of mango trees,
α	Pest consumption rate of mango
β	Pest reproduction rate
γ	Parasitization rate of *Fopius arisanus*
δ	Natural pest mortality
η	Conversion efficiency of consumed pests into natural enemy
μ	Natural mortality of Fopius arisanus

Therefore, our final mathematical model describing the interactions of mango plants, pests, and natural enemies (biological control agents) is:

$$\begin{aligned}\frac{\mathrm{d}M}{\mathrm{d}t} &= r_M M\left(1-\frac{M}{K_M}\right)-\alpha MP\\ \frac{\mathrm{d}P}{\mathrm{d}t} &= \beta MP-\gamma PN-\delta P\\ \frac{\mathrm{d}N}{\mathrm{d}t} &= \gamma\eta PN-\mu N\end{aligned} \tag{4}$$

The list of the parameters used in the model are summarized in Table 1.

With initial conditions that:

$$M(0)>0, P(0)>0 \text{ and } N(0)>0 \tag{5}$$

4. Mathematical Analysis of the Model

In this section, we describe the dynamical behavior of the proposed model system. To do so, we first check the positivity of the solutions of the model and boundedness of the solution of the model.

4.1. Positivity

Theorem 1. Every solution of system (4) with initial conditions (5) exists in the interval $[0,\infty]$ and $M(0)>0$, $P(0)>0$, and $N(0)>0$ for all $t\geq 0$

Proof. Since the right-hand side of the system (4) is completely continuous and locally Lipschitzian in C the solution $M(t), P(t), N(t)$ of (4) with initial conditions (5) exists and is unique on $[0,\tau]$, where $0<\tau<\infty$. From system (5) with initial conditions (5), we have:

$$M(t)=M(0)\exp\left[\int_0^t\left\{r_M M\left(1-\frac{M}{K_M}\right)-\alpha MP\right\}\mathrm{d}\theta\right],$$

$$P(t)=P(0)\exp\left[\int_0^t\left\{\beta MP-\gamma PN-\delta P\right\}\mathrm{d}\theta\right],$$

$$N(t)=N(0)\exp\left[\int_0^t\left\{\gamma\eta PN-\mu N\right\}\mathrm{d}\theta\right],$$

Completes the proof □

4.2. Boundedness of Solution

Theorem 2. All solutions of system (4) which initiate in R_+^3 are uniformly bounded

Proof. We construct a function such as $W(t)=M(t)+P(t)+N(t)$

Differentiating both sides with respect to t, we have

$$\frac{\mathrm{d}W}{\mathrm{d}t}=\frac{\mathrm{d}M}{\mathrm{d}t}+\frac{\mathrm{d}P}{\mathrm{d}t}+\frac{\mathrm{d}N}{\mathrm{d}t},$$

Therefore,

$$\begin{aligned}\frac{\mathrm{d}W}{\mathrm{d}t} &= r_M M\left(1-\frac{M}{K_M}\right)-\alpha MP+\beta MP-\gamma PN+\eta PN-\delta P-\mu N\\ &= r_M M\left(1-\frac{M}{K_M}\right)-(\alpha-\beta)MP-(\gamma-\eta)PN-\delta P-\mu N.\end{aligned} \tag{6}$$

$$\begin{aligned}\frac{\mathrm{d}W}{\mathrm{d}t}+\epsilon W &= r_M M\left(1-\frac{M}{K_M}\right)-(\alpha-\beta)MP-(\gamma-\eta)PN-\delta P\\ &\quad -\mu N+\epsilon(M+P+N)\\ &=\left(r_M M-\frac{r_M M^2}{K_M}+\epsilon M\right)-(\alpha-\beta)MP-(\gamma-\eta)PN\\ &\quad -(\delta-\epsilon)P-(\mu-\epsilon)N.\end{aligned} \tag{7}$$

Thus, $(\alpha-\beta)MP>0$ and $(\gamma-\eta)PN>0$. **Assuming** $\epsilon<\min(\delta,\mu)$, we obtain

$$\begin{aligned}\frac{\mathrm{d}W}{\mathrm{d}t}+\epsilon W &\le r_M M-\frac{r_M M^2}{K_M}+\epsilon M\\ &\le K_M\frac{(r_M+\epsilon)^2}{4r_M}=B.\end{aligned} \tag{8}$$

Applying the differential inequality theorem, we obtain

$$0\le W\left(M(t),P(t),N(t)\right)\le\frac{B}{\epsilon}\left(1-\mathrm{e}^{-\epsilon t}\right)+W\left(M(0),P(0),N(0)\right)\mathrm{e}^{-\epsilon t}.$$

This implies that

$$0\le W\le\frac{B}{\epsilon}\text{ as }t\to\infty.$$

Hence, all solutions of system (4) are uniformly bounded.

□

4.3. Existence of Equilibria

To obtain the equilibrium points of the model (4), we set the all the derivatives equal to zero, and the following was obtained:

(a) Trivial Equilibrium $E_0=(0,0,0)$

(b) Pest-Free Equilibrium (Only Mango Exists) $E_1=(K_M,0,0)$

We set $P=0$ and $N=0$

We solve Equation (1)

$$r_M M\left(1-\frac{M}{K_M}\right)-\alpha MP=0$$

$$r_M\left(1-\frac{M}{K_M}\right)=0\Rightarrow r_M=0$$

$$1=\frac{M}{K_M}\Rightarrow M^*=K_M$$

$$\textit{Therefore:}\quad E_1=(K_M,0,0)$$

(c) Mango-Pest Equilibrium

We find the equilibrium points from Equation (4) by setting $N=0$

$\frac{\mathrm{d}M}{\mathrm{d}t}=0,\frac{\mathrm{d}P}{\mathrm{d}t}=0$ These leads to:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = r_M M\left(1-\frac{M}{K_M}\right) - \alpha MP = 0$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \beta MP - \delta P = P(\beta M - \delta) = 0 \quad \Rightarrow P(\beta M - \delta) = 0$$

$$\beta M - \delta = 0 \Rightarrow M^* = \frac{\delta}{\beta}$$

Substitute M^* Equation (1): $0 = r_M M\left(1-\frac{M}{K_M}\right) - \alpha MP$

$$\Rightarrow \alpha MP = r_M M\left(1-\frac{M}{K_M}\right) \quad \Rightarrow P = \frac{r_M}{\alpha}\left(1-\frac{M}{K_M}\right)$$

Now insert $M^* = \frac{\delta}{\beta}$ into: $\Rightarrow P = \frac{r_M}{\alpha}\left(1-\frac{\delta}{\beta K_M}\right)$

Hence: $E_2 = \left(\frac{\delta}{\beta}, \frac{r_M}{\alpha}\left(1-\frac{\delta}{\beta K_M}\right), 0\right)$

(d) Coexistence Equilibrium $E_3 = \left(M^*, P^*, N^*\right)$
Assume $M^*, P^*, N^* > 0$
From Equation (3):

$$0 = \gamma\eta PN - \mu N \quad \Rightarrow N(\gamma\eta P - \mu) = 0 \quad \Rightarrow \gamma\eta P = \mu \quad \Rightarrow P^* = \frac{\mu}{\gamma\eta}$$

Substitute P^* into (2):

$$0 = \beta MP - \gamma PN - \delta P \quad \Rightarrow \beta M - \gamma N - \delta = 0 \quad \Rightarrow \beta M = \gamma N + \delta$$

$$\Rightarrow M^* = \frac{\gamma N + \delta}{\beta}$$

$$P^* = \mu/(\gamma\eta); \quad N^* = \frac{\mu}{\gamma\eta P^*}$$

Substitute P^* and M^* into Equation (1): $0 = r_M M\left(1-\frac{M}{K_M}\right) - \alpha MP$

$$\Rightarrow r_M\left(1-\frac{M}{K_M}\right) = \alpha P \quad \Rightarrow 1-\frac{M}{K_M} = \frac{\alpha P}{r_M} \quad \Rightarrow M^* = K_M\left(1-\frac{\alpha P}{r_M}\right)$$

Substitute $P^* = \frac{\mu}{\gamma\eta}$ and $M^* = K_M\left(1-\frac{\alpha\mu}{r_M\gamma\eta}\right)$

Thus:

$$P^* = \frac{\mu}{\gamma\eta}$$

$$M^* = K_M\left(1-\frac{\alpha\mu}{r_M\gamma\eta}\right)$$

$N^* = \frac{\beta M^* - \delta}{\gamma}$. Therefore, the Coexistence equilibrium points are:

$$E^* = \left(K_M\left(1-\frac{\alpha\mu}{r_M\eta\gamma}\right), \frac{\mu}{\eta\gamma}, \frac{\beta\left(K_M\left(1-\frac{\alpha\mu}{r_M\eta\gamma}\right)\right)-\delta}{\gamma}\right)$$

4.4. Stability Analysis

To determine the stability of the equilibrium points, we performed a linear stability analysis by computing the Jacobian matrix of the system at the equilibrium points. The Jacobian matrix at J

$$\boldsymbol{J}=\begin{bmatrix} r_M\left(1-\frac{2M^*}{K_M}\right)-\alpha P^* & -\alpha M^* & 0 \\ \beta P^* & \beta M^*-\gamma N^*-\delta & -\gamma P^* \\ 0 & \eta\gamma N^* & \eta\gamma P^*-\mu \end{bmatrix}$$

We Evaluate J at each equilibrium point to determine the model stability using eigenvalues by solving the characteristic equation:

$$\det\left(J-\lambda I\right)=0$$

Theorem 3. The equilibrium point E_0 is always unstable.

Proof. $E_0=\left[0,0,0\right]$, Substitute in the Jacobian matrix J

$$\boldsymbol{J}=\begin{bmatrix} r_M\left(1-\frac{2M^*}{K_M}\right)-\alpha P^* & -\alpha M^* & 0 \\ \beta P^* & \beta M^*-\gamma N^*-\delta & -\gamma P^* \\ 0 & \eta\gamma N^* & \eta\gamma P^*-\mu \end{bmatrix}$$

The Jacobian matrix at E_0

$$\boldsymbol{E}_0=\begin{bmatrix} r_M & 0 & 0 \\ 0 & -\delta & 0 \\ 0 & 0 & -\mu \end{bmatrix}$$

Therefore the eigenvalues of the characteristic equation of E_0 are $\lambda_1=r_M>0$, $\lambda_2=-\delta<0$ and $\lambda_3=-\mu<0$. Here, one of the eigenvalues is positive and the other two are negative, so E_0 is always unstable due to positive growth rate of Mango.

Theorem 4. The equilibrium point E_1 is stable if $\beta K_M-\delta<0$ and $\beta K_M<\delta$

Proof. The Jacobian at Pest-Free Equilibrium is $E_1=\left[K_M,0,0\right]$ substituting in the Jacobian Matrix $\boldsymbol{J}$ we obtain

$$\boldsymbol{E}_1=\begin{bmatrix} -r_M & 0 & 0 \\ 0 & \beta K_M-\delta & 0 \\ 0 & 0 & -\mu \end{bmatrix}$$

Therefore the eigenvalues of the characteristic equation of E_1 are $\lambda_1=-r_M<0$, $\lambda_2=\beta K_M-\delta<0$ and $\lambda_3=-\mu<0$. Therefore E_1 is stable if $\beta K_M<\delta$ and $\beta K_M-\delta<0$. ☐

Theorem 5. The equilibrium point E_2 is locally asymptotically stable if

$$\eta\gamma\cdot\frac{r_M}{\alpha}\left[1-\frac{\delta}{\beta K_M}\right]<\mu$$

Proof. The Jacobian Matrix at E_2

$$\boldsymbol{E}_2 = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & 0 & S_{23} \\ 0 & 0 & S_{33} \end{bmatrix}$$

where: $S_{11} = \frac{-r_M \delta}{\beta K_M}$, $S_{12} = \frac{\alpha\delta}{\beta}$, $S_{21} = \frac{r_M}{\alpha}\left[\beta - \frac{\delta}{K_M}\right]$, $S_{23} = \frac{r_M \gamma}{\alpha}\left[\frac{\delta}{K_M \beta} - 1\right]$,
$S_{33} = \frac{r_M \eta\gamma}{\alpha}\left[1 - \frac{\delta}{K_M}\right] - \mu$.

$$\boldsymbol{E}_2 = \begin{bmatrix} S_{11} - \lambda & S_{12} & 0 \\ S_{21} & 0 - \lambda & S_{23} \\ 0 & 0 & S_{33} - \lambda \end{bmatrix}$$

Let $A = S_{11}$ and $C = S_{11}S_{11}$.

The equation for E_2 is:

$$E_2 = \left(S_{33} - \lambda\right)\left[\lambda^2 + A\lambda S_{11} + CS_{12}S_{21}\right]$$

Therefore, one eigenvalue of the characteristic equation above is S_{33} which is negative as $\frac{r_M \eta\gamma}{\alpha}\left[\frac{\delta}{K_M}\right] > \frac{r_M \eta\gamma}{\delta} - \mu$ and the other two eigenvalues are negative if $A > 0$ and $C > 0$.

Therefore, the second free equilibrium point $E_2\left(M^*, P^*, 0\right)$ is locally asymptotically stable if $\frac{r_M \eta\gamma}{\alpha}\left[\frac{\delta}{K_M}\right] > \frac{r_M \eta\gamma}{\delta} - \mu$, $A > 0$, $C > 0$ otherwise the system (4) will be unstable □

Theorem 6. The equilibrium point E_3 is locally asymptotically stable if $AC > 0$ and $AB - C > 0$ and unstable otherwise

Proof. The Jacobian Matrix at E_3

$$\boldsymbol{E}_3 = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & D_{23} \\ 0 & D_{32} & D_{33} \end{bmatrix}$$

where; $D_{11} = r\left(1 - \frac{2M^*}{K_M}\right) - \alpha P^*$, $D_{13} = -\alpha M^*$, $D_{21} = \beta P^*$,
$D_{22} = \beta M^* - \gamma N^* - \delta$, $D_{23} = -\gamma P^*$, $D_{32} = \eta\gamma N^*$, and $D_{33} = \eta\gamma P^* - \mu$

Therefore, the characteristic equation of the E_3 is

$$\lambda^3 + A\lambda^2 + B\lambda + C$$

where:

$A = -\left(D_{11} + D_{22} + D_{33}\right)$,
$B = D_{22}D_{22} + D_{32}D_{23} + D_{12}D_{21}$,
$C = -D_{11}D_{22}D_{33} - D_{21}D_{12}D_{33} + D_{11}D_{32}D_{23}$.

By Routh-Hurwitz criteria, E_3 will be locally asymptotically stable, if the eigenvalues will be negative real parts , thus if $AC > 0$ and $AB - C > 0$ otherwise E_3 is unstable. □

We also computed analytically the stability for E_3 using parameter values from Table 2. The eigenvalues of the Jacobian matrix at the interior (coexistence) equilibrium are using $r_M = 0.5$, $K_M = 100$, $\alpha = 0.01$, $\beta = 0.05$, $\gamma = 0.02$, $\delta = 0.1$, $\eta = 0.1$, $\mu = 0.1$:

$$P^* = \frac{\mu}{\gamma\eta} = \frac{0.1}{0.05 \times 0.1} = \frac{0.1}{0.005} = 20$$

$$M^* = K_M\left(1 - \frac{\alpha\mu}{r_M\gamma\eta}\right) = 100\left(1 - \frac{0.01 \times 0.1}{0.5 \times 0.05 \times 0.1}\right)$$

$$= 100\left(1 - \frac{0.001}{0.0025}\right) = 100(1 - 0.4) = 60$$

$$N^* = \frac{\beta M^* - \delta}{\gamma} = \frac{0.02 \times 60 - 0.1}{0.05} = \frac{1.2 - 0.1}{0.05} = \frac{1.1}{0.05} = 22$$

All equilibrium populations are positive, so coexistence is feasible.

We compute E_3 at [60, 20, 22]

- $D_{11} = r_M\left(1 - \frac{2M^*}{K_M}\right) - \alpha P^* \quad \Rightarrow 0.5 \times \left(1 - \frac{2 \times 60}{100}\right) - 0.01 \times 20 = -0.3$
- $D_{12} = 0$
- $D_{13} = -\alpha M^* \quad \Rightarrow -0.01 \times 60 = -0.6$
- $D_{21} = \beta P^* \quad \Rightarrow 0.02 \times 20 = 0.4$
- $D_{22} = \beta M^* - \gamma N^* - \delta \quad \Rightarrow 0.02 \times 60 - 0.05 \times 22 - 0.1 = 0$
- $D_{23} = -\gamma P^* \quad \Rightarrow -0.05 \times 20 = -1.0$
- $D_{31} = 0$
- $D_{32} = \eta\gamma N^* \quad \Rightarrow 0.1 \times 0.05 \times 22 = 0.11$
- $D_{33} = \eta\gamma P^* - \mu \quad \Rightarrow 0.1 \times 0.05 \times 20 - 0.1 = 0$

Jacobian matrix at equilibrium at E_3

$$\boldsymbol{E}_3 = \begin{bmatrix} -0.3 & -0.6 & 0 \\ 0.4 & 0 & -1.0 \\ 0 & 0.11 & 0 \end{bmatrix}$$

We compute Eiugenvalues λ satisfying

$$\det(E_3 - \lambda I) = 0$$

Matrix $E_3 - \lambda I \quad \Rightarrow$

$$\begin{bmatrix} -0.3-\lambda & -0.6 & 0 \\ 0.4 & 0\lambda & -1.0 \\ 0 & 0.11 & 0-\lambda \end{bmatrix}$$

Calculate determinant:

$$\det(J - \lambda I) = (-0.3 - \lambda)\begin{bmatrix} -\lambda & -1.0 \\ 0.11 & -\lambda \end{bmatrix} - (-0.6)\begin{bmatrix} 0.4 & -1.0 \\ 0 & -\lambda \end{bmatrix} + 0$$

So,

$$\det(E_3 - \lambda I) = (-0.3 - \lambda)(\lambda^2 + 0.11) + 0.6(0.4\lambda) = (-0.3 - \lambda)(\lambda^2 + 0.11) + 0.24\lambda$$

Thus,

$$-0.3\lambda^2 - 0.033 - \lambda^3 - 0.11\lambda + 0.24\lambda = -\lambda^3 - 0.3\lambda^2 + 0.13\lambda - 0.033 = 0$$

Using MATLAB solver, the roots are approximately to:

- $\lambda_1 \approx 0.07$ (positive real part)
- $\lambda_2 \approx -0.18 + 0.35i$
- $\lambda_3 \approx -0.18 - 0.35i$.

These Confirms the stability at coexistence equilibrium. The presence of imaginary parts indicates damped oscillations, due to interactions the system flactuates and exhibit spiral dynamics that stabilize over time.

5. Numerical Simulations and Discussion

In this section, numerical simulations of the model (4) were carried out using the values of the parameters shown in **Table 2**. Parameter values and ranges were adopted from the literature on fruit fly population dynamics and parasitoid–host interactions and were used to support numerical simulations and sensitivity analyses.

Table 2. Model parameters and biological interpretation.

Parameter	Description	Units	Typical range	References
$P(t)$	Density of *Bactrocera dorsalis*	individuals	50 - 80	[33] [36]
$N(t)$	Density of parasitoid (*Fopius arisanus*)	individuals	30 - 50	[29] [34]
$M(t)$	Density of Mango plants hosts	dimensionless	80 - 500	[31] [35]
r	Intrinsic growth rate of the pest	day^{-1}	0.05 - 0.30	[38] [40]
K	Carrying capacity of mango resource	individuals/tree or ha	10^2 - 10^5	[7]
β	Pest growth rate supported by host availability	day^{-1}	0.05 - 0.30	[37]
γ	Parasitoid attack rate (Holling type I)	$\text{individual}^{-1}\ \text{day}^{-1}$	0.001 - 0.05	[37] [41]
δ	Natural mortality rate of the pest	day^{-1}	0.01 - 0.10	[38]
α	Pest consumption rate of mango	pest^{-1}	0.001 - 0.05	[16] [40]
η	Conversion efficiency of parasitized hosts	dimensionless	0.1 - 0.6	[39]
μ	Natural mortality rate of the parasitoid	day^{-1}	0.02 - 0.20	[39]

To measure the model's performance and ensure that the obtained outputs are authentic. We used MATLAB ($R2019a$) to simulate the numerical results of this model by Runge-kutta method. To demonstrate the role of biological control strategy in suppressing pest outbreaks and stabilizing the agro-ecosystem. The time series plot was plotted showing the population dynamics of the pest (*Bactrocera dorsalis*) and its parasitoid (*Fopius arisanus*) over time.

The time series plot in **Figure 1** above illustrates the population dynamics of Mango $M(t)$, Pest $P(t)$ and Natural Enemy $(N(t))$. Pest populations also increase due to increased mango availability, but declines as natural enemies become more abundant, while Natural Enemy $(N(t))$ grows in response to pest availability, then stabilizes as pest numbers decreases.

Figure 2(a) and Figure 2(b) are phase portrait of the model, showing the interactions of Mango, Pest and Natural enemy and Pests versus natural enemy. The behavior of trajectories forms looped curves that converge towards a central region. This shows a damped oscillations in the population sizes, the system initially fluctuates, but over time it approaches a stable equilibrium.

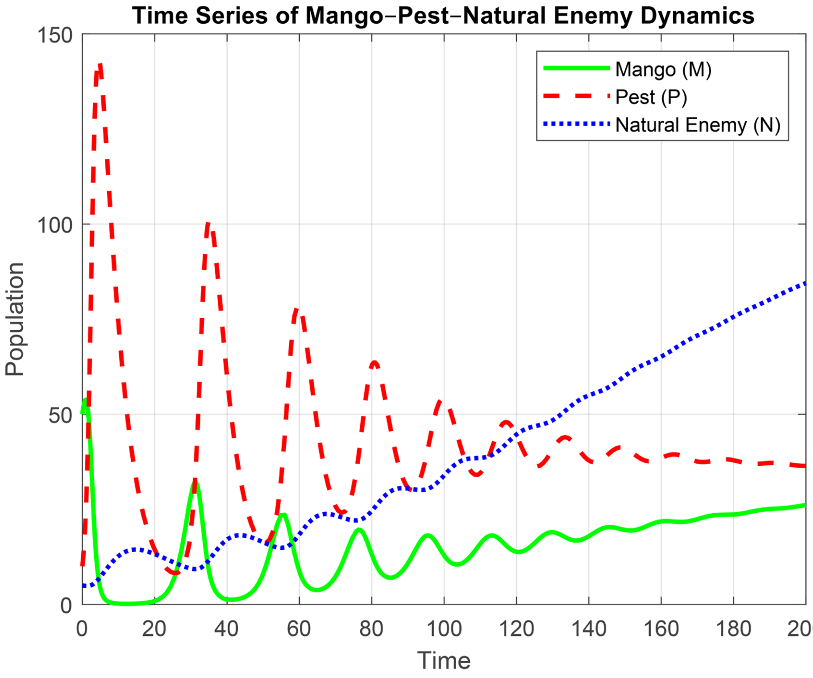
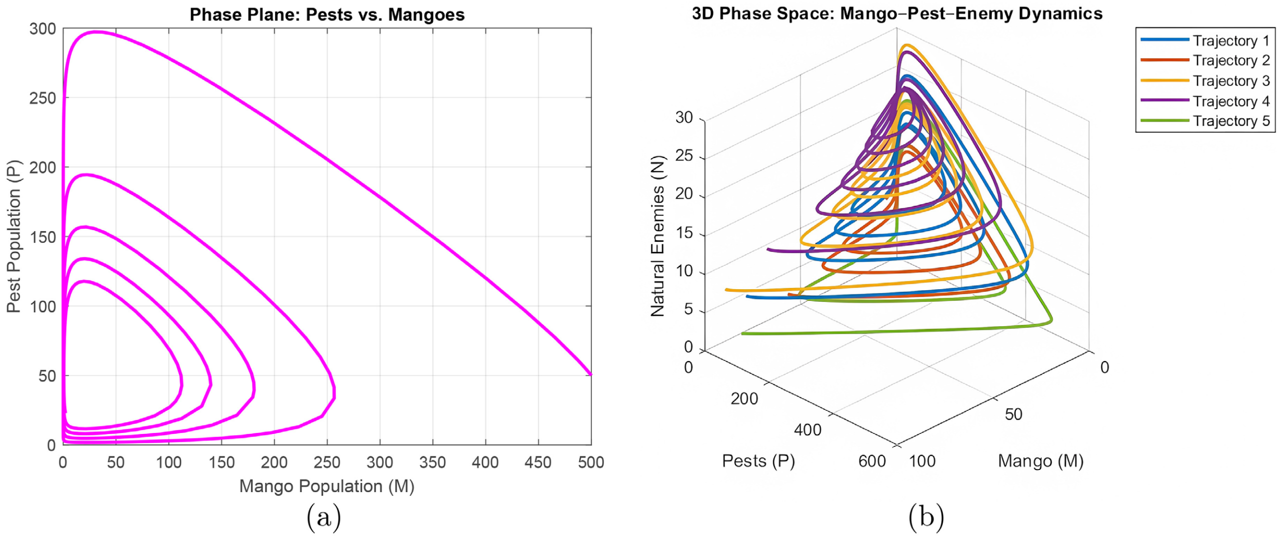


Figure 1. Time series plots displaying the dynamics of mango, pests, and natural enemies at initial point.



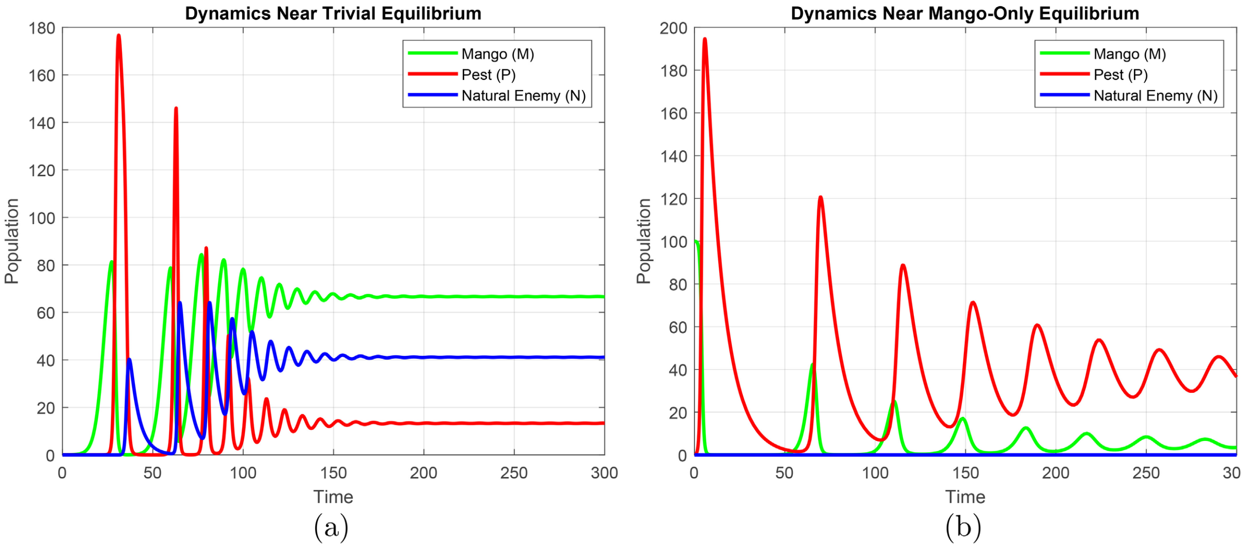
(a)

(b)

Figure 2. Phase space plots. (a) pest-enemy phase portrait; (b) 3D Phase portrait for Mango-Pest-Natural enemy.

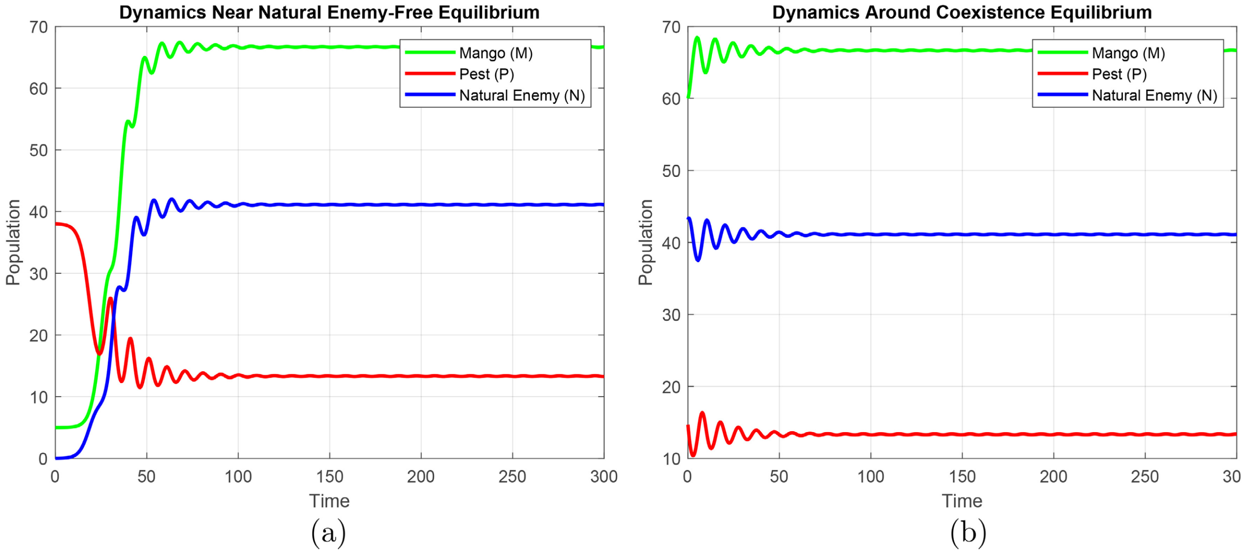
Figure 3(a) shows the stability analysis of the model at $E_0(0,0,0)$ confirming the instability predicted by the eigenvalue analysis. The second case were no pest and parastoids thus $M = K_M$, $P = 0$ and $N = 0$ Figure 3(b) shows that mango and natural enemy goes to extinction, and Figure 4(a), a stable mango-pest equilibrium implies pest invasion without control by natural enemies. Figure 4(b) shows a stable coexistence equilibrium reflecting a balanced tri-trophic interaction, representing a desirable state for biological pest control.

Figures 5-7 shows the stability of a model at equilibrium by varying some of the model parameters, below the horizontal line where $\lambda < 0$, the equilibrium is locally stable. The vertical line $\lambda > 0$. The equilibrium becomes unstable. This indicates that as β increases, at least one eigenvalue crosses zero, indicating a bifurcation point, a transition from model stability to instability. This transition reflects how pest reproductive ability influences the whole system's dynamics and control.



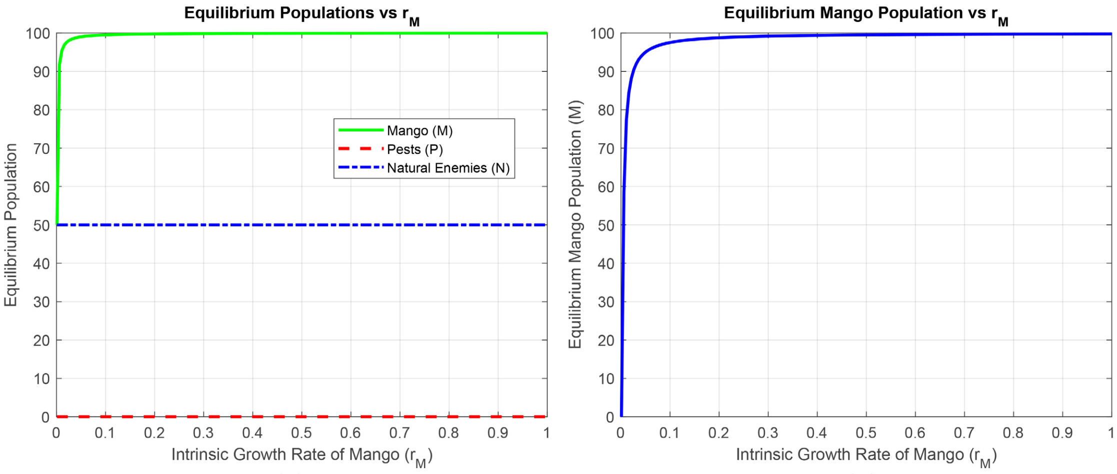
(a) (b)

Figure 3. Model system equilibrium. (a) Trivial equilibrium; (b) Mango only free equilibrium.



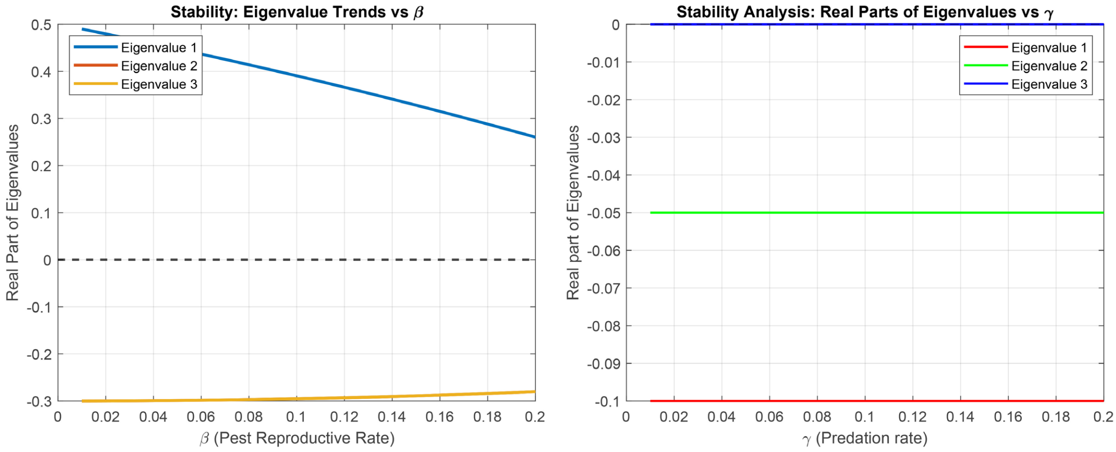
(a) (b)

Figure 4. Natural enemy and coexistence equilibrium. (a) Natural enemy free equilibrium; (b) Coexistence equilibrium.



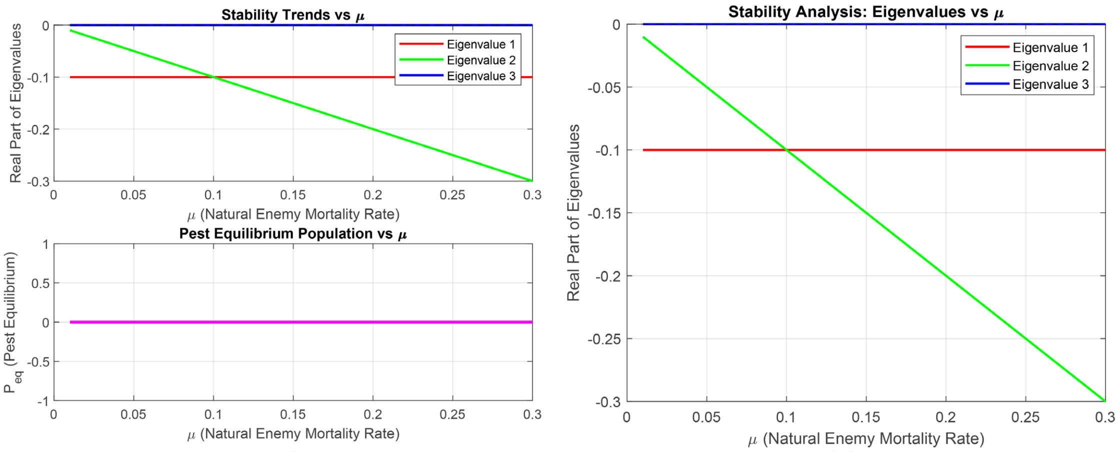
(a) (b)

Figure 5. Effect of varying R_M at equilibrium. (a) The effect of R_M at equilibrium; (b) Varying Conversion rate.



(a) (b)

Figure 6. Jacobian Eigen values. (a) Eigen values trend Vs β; (b) Eigen values trend Vs γ.



(a) (b)

Figure 7. Stability analysis plots of Eugenvalues against α, β and μ changes. (a) Sensitivity of Pest Population to β; (b) Sensitivity of Pest Population to α.

Sensitivity Analysis

The mathematical model developed for the Mango-Pest-Natural enemy (*Fopius arisanus*) system provides quantitative insights into the population dynamics of pests and their natural enemies, which are directly applicable to Integrated Pest Management (IPM) strategies in Tanzanian mango orchards. To evaluate the influence of individual parameters on the dynamics of the Mango-Pest-Natural enemy system, we performed a local sensitivity analysis on the model equations. Sensitivity analysis helps to identify the key parameters that most significantly affect model outcomes, such as pest suppression, mango recovery, or natural enemy persistence.

$r_M = 0.5$, $K_M = 100$, $\alpha = 0.01$, $\beta = 0.05$, $\gamma = 0.02$, $\delta = 0.1$, $\eta = 0.1$, and $\mu = 0.1$

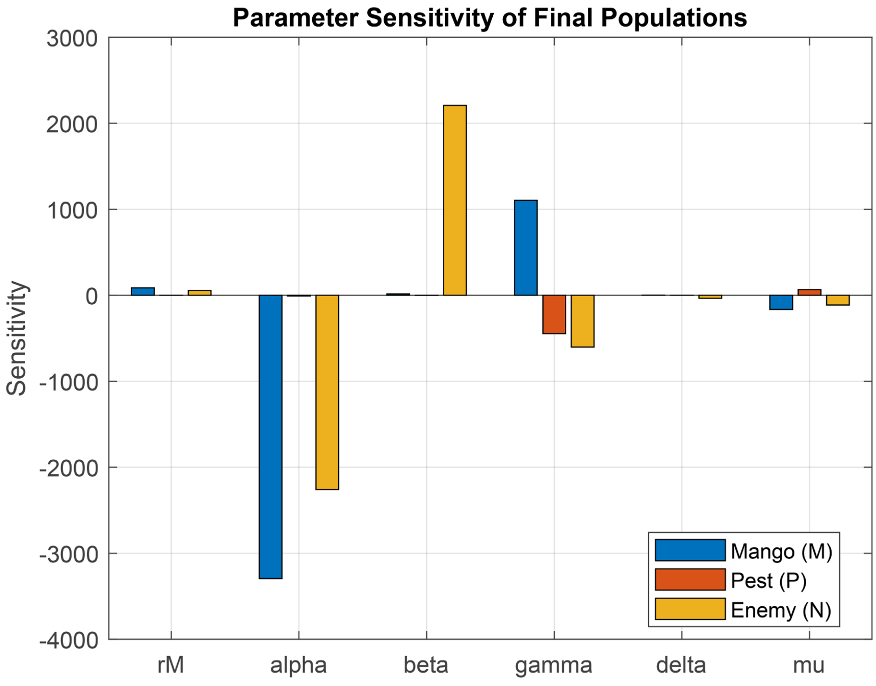


Figure 8. The bar chart for the sensitivity of mango (M), pest (P), and natural enemies (N) to small (±10%) changes in key model parameters.

Each parameter was independently varied ±10% - 20% around its baseline while keeping others fixed. The corresponding changes in state variables over time were plotted at a final simulation time where $t = 100$. We analyzed how the variation in each parameter affected final mango biomass, final pest density and final pest density. The normalized sensitivity index was computed as:

$$S_{x,p} = \frac{\partial x}{\partial P} \cdot \frac{p}{x}$$

where x is the output variable and P was the variable varied and the results were plotted in **Figure 9**. Based upon the sensitivity analysis (**Figures 8-10**), we conclude that parameters that affects the model are as follows; Pest reproduction rate (β) had a strong positive influence on pest population size and a negative effect on mango persistence. Parasitism rate (γ) was among the most influential parameters for pest suppression and natural enemy success. Carrying capacity (K_M) influenced long-term mango abundance but had nonlinear effects on pest dynamics. Natural enemy mortality rate (μ) and conversion efficiency (η) showed moderate sensitivity in determining natural enemy persistence.

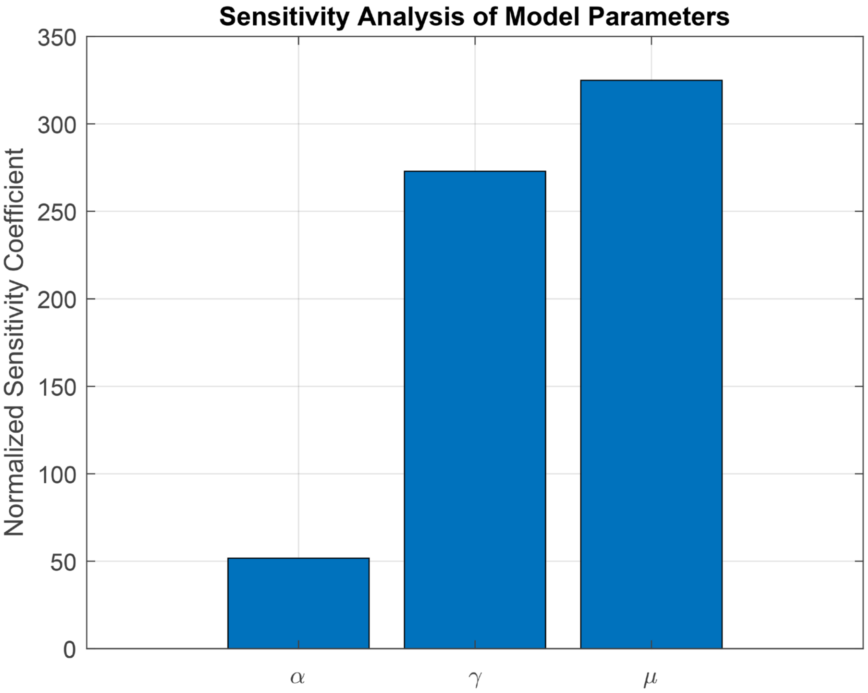
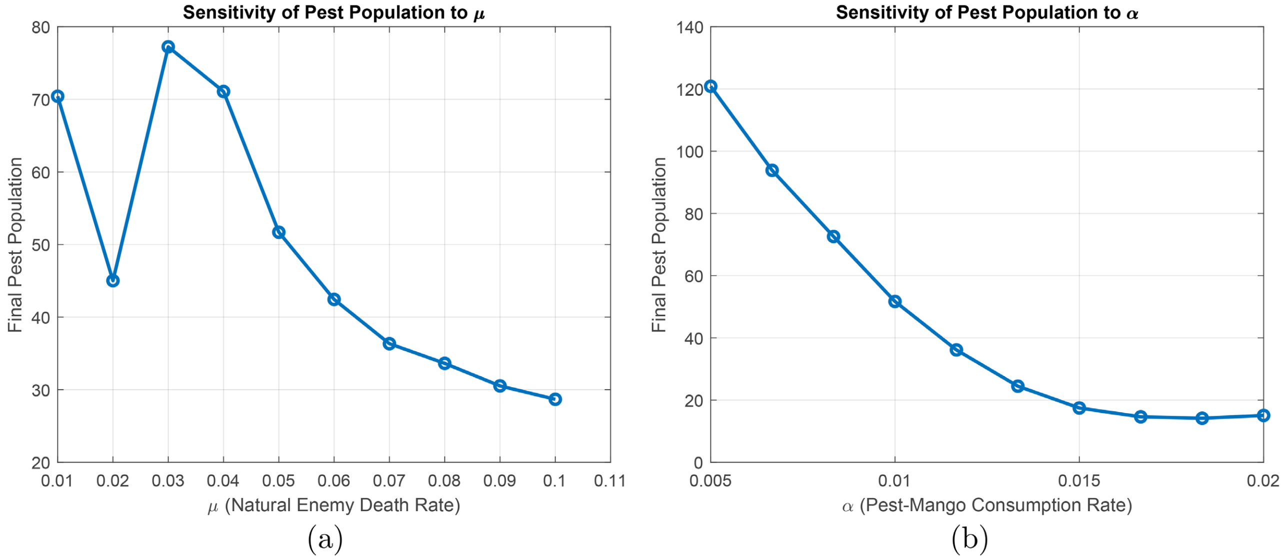


Figure 9. Normalized sensitivity coefficients pest population with varying initial values of α, γ and μ.



(a) (b)

Figure 10. Sensitivity of pest population to α and μ changes. (a) Sensitivity of Pest Population to μ; (b) Sensitivity of Pest Population to α.

Figure 8 shows that As α increases, mango population decreases significantly. This means that, pests affects mango yield hence control mechanism is inevitable.

We vary each parameter logarithmically across a wide range to explore their combined effects on the final mango population. **Figure 9** shows that (μ) Natural Enemy Mortality rate has the highest Sensitivity, which means that changes in the death rate of natural enemies have the most significant impact on the mango population. Conversion Efficiency (γ) has the second highest Sensitivity, this indicates that the efficiency of converting consumed pests to predator biomass is also critical. And lastly Pest Attack Rate (α) has the lowest Sensitivity.

Figure 11 shows as natural enemy predation rate (γ) increases, the natural enemy population decreases, at low γ, natural enemies survive in high numbers, but pest control becomes weak. **Figure 12** shows how the populations of natural enemies (N) and pests (P) interact under different values of the pest reproduction rate (β). As the pest reproduction rate (β) increases, pests also increase. Natural enemies must be increased to keep up, but this leads to volatile dynamics. A very high β 0f 0.2 can lead to a chaotic system, making pest control extremely challenging. An intermediate β 0f 0.1 shows strong but somewhat predictable oscillations, which may be manageable with careful control. A low β 0f 0.02 allows stable coexistence, making it the most desirable scenario for sustainable pest control. **Figure 10(b)** shows that increasing α typically leads to a decrease in the pest population, as pests consume mangoes more aggressively, potentially leading to resource depletion.

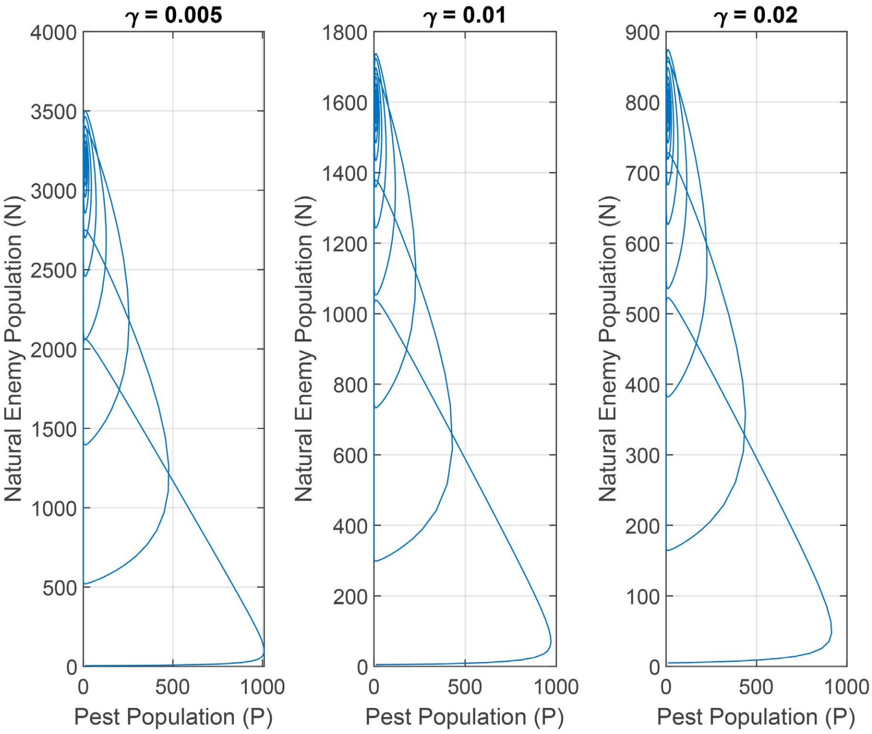


Figure 11. Predator-prey phase portraits (Natural Enemy vs Pest Population) for varying (γ).

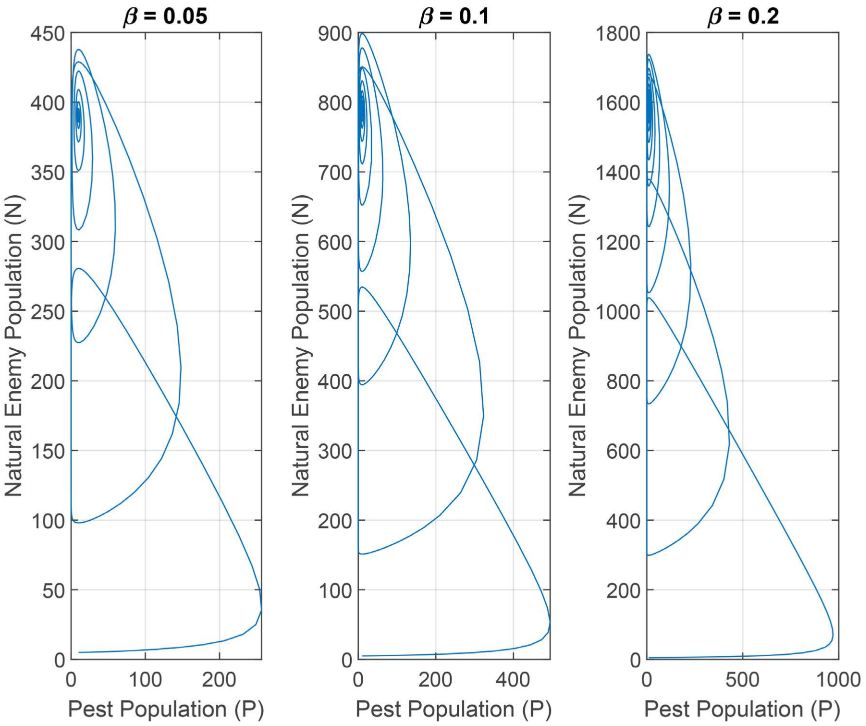


Figure 12. Predator-prey phase portraits (Natural Enemy vs Pest Population) for varying (β).

Increasing the natural enemy death rate μ often leads to higher pest populations. This occurs because a higher μ reduces the number of predators, diminishing predation pressure on pests. This phenomenon is well-documented in ecological studies [34] and [35] the "paradox of the pesticides", where interventions like pesticides accidentally boost pest populations by harming their natural enemies. Here, increasing μ directly increases the mortality rate of natural enemies (N), leading to pest decline. As N decreases, the predation term (γPN) in the pest population equation reduces, allowing the pest population (P) to grow.

Therefore, an increase in the natural enemy mortality rate (μ) reduces parasitoid persistence and weakens biological control, leading to higher pest densities and increased mango damage. Tanzanian IPM systems must consider conserving parasitoid survival through reduced pesticide use and habitat management for sustainable fruit fly suppression.

The parameters γ, η and β strongly influence pest regulation in Tanzanian mango production systems. Increased parasitism rate and conversion efficiency enhance biological control, while higher pest intrinsic growth promotes outbreaks. Effective IPM in Tanzania should focus on strengthening parasitoid performance and reducing pest reproductive potential through sanitation, early harvesting, and habitat management.

6. Conclusions and Suggestions

This study demonstrates that pest dynamics in Tanzanian mango production systems are strongly influenced by interactions among mango resources, the fruit fly (*Bactrocera dorsalis*), and natural enemies such as *Fopius arisanus*. The model results show that effective biological control, supported by favorable ecological conditions, can significantly suppress pest populations and reduce mango damage. Parameters such as pest growth rate, parasitism rate, and natural enemy survival determine whether the system stabilizes or experiences recurrent pest outbreaks.

The findings also highlight that, Integrated Pest Management (IPM) is essential for sustainable mango production in Tanzania. Reliance on a single control method is insufficient, instead, combining biological control, cultural practices, and targeted interventions leads to improved pest suppression, reduced economic losses, and protection of the agro-ecosystem.

The study also formulated and analyzed a nonlinear differential equation model describing the tri-trophic interactions between mango plants (*Mangifera indica L.*), the invasive fruit fly *Bactrocera dorsalis*, and its parasitoid *Fopius arisanus*.

We established the trivial equilibrium $(M, P, N) = (0,0,0)$ which is unstable, reflecting that once any species is introduced, the system evolves toward biologically meaningful dynamics. The pest-only equilibrium corresponding to a system without parasitoids, is locally asymptotically stable under specific conditions, when $P^* < \frac{\mu}{\gamma\eta}$, but becomes unstable upon parasitoid introduction exceeding this invasion threshold. The coexistence equilibrium (M^*, P^*, N^*) can exhibit local asymptotic stability under biologically realistic parameter regimes, especially when the parasitism rate (γ) and conversion efficiency (η) are sufficiently high to suppress pest populations below damaging thresholds. Numerical simulations illustrated a variety of dynamical outcomes including pest outbreaks, parasitoid-driven pest suppression, and damped oscillations leading to stable coexistence. Time series analyses demonstrated that the presence of *Fopius arisanus* significantly alters pest-host dynamics, enhancing mango persistence by reducing herbivore pressure. Sensitivity analysis identified β, γ and K_M. as key parameters influencing system behavior, indicating the importance of controlling pest reproductive rates and enhancing parasitoid efficacy for successful biological control.

This model provides a theoretical foundation for understanding mango, fruit fly parasitoid dynamics and inform integrated pest management (IPM) strategies in Tanzania. It also supports the deployment of *Fopius arisanus* as a promising biological control agent against *Bactrocera dorsalis*, with the potential of reducing mango yield losses in Tanzanian agro-ecosystems.

Sustainable pest management in Tanzanian mango production systems depends on the integration of ecological knowledge, mathematical modeling, and field-based IPM practices. By promoting biological control, improving orchard management, and strengthening monitoring systems, Tanzania can reduce fruit fly infestations, enhance mango productivity, and support environmentally friendly agricultural development.

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Conflicts of Interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Triphonia Ngailo reports financial support was provided by the NORHED II Project QZA-21/0159 entitled "Adaptive Environmental Monitoring Networks for East Africa" (AdEMNEA).

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