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The Balloon Model and the Mass of the Universe

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Abstract

A good example of the use of a model to describe a complex entity is the Balloon Model of the Universe. This model has long been a valuable teaching tool for explaining the observed expansion of the cosmos. By adding quantitative values to the model, we have been able to consider the red shift, z, that would be observed for a distant galaxy using two different techniques. The first way is by using the Doppler effect, which results in the Hubble distance/velocity relation. The second is the red shift predicted by Einstein's Theory of General Relativity. As the light travels from the past to the present it is also climbing from a lower gravitational potential to a higher one, *i.e.* going "uphill". The resulting values for z must be equal. However, there is an unknown value of the mass, M, needed to calculate the gravitational potentials. M becomes a free parameter that we can determine and although it is dependent on this simple model, it turns out to be quite reasonable.

Keywords

Balloon Model, Red Shift, Doppler Effect, Hubble Distance, Mass of Universe

1. Introduction

The purpose of this paper is to show how a simple model of a complex system can reveal key features of the real world. The Balloon Model captures much of the qualitative behavior of the expanding Universe, despite assuming homogeneity and isotropy. By adding a quantitative element, it can estimate the mass of the Universe in close agreement with observations. This two-dimensional model uses Newtonian gravity and does not incorporate General Relativity (e.g., Birkhoff's theorem) or dark energy.

Although astronomers are unable to determine whether the Universe is finite or infinite, the observable Universe is finite. In this paper we will use the term Universe to mean that which is available for observation, *i.e.* the observable Universe. We will use 13.8 billion years (13.8 Gyr) as the age of the observable Universe [1] [2]. Therefore, an appropriate model for the Universe should be finite. In this paper, we model the balloon radius, r, as increasing linearly with time and assign a total constant mass, M, including both luminous and dark matter. Analysis of redshift in this model yields an estimate of M that closely matches the observable Universe.

The measurement of red shift, z, was based on the Doppler effect between the frames of the Earth and the distant galaxies receding from our frame. If we consider the photon's journey from a distant galaxy to our telescope, we must conclude that it is continually red shifting as it travels along its path. The standard qualitative description is that the expansion of the universe stretches the wavelength, λ; hence a red shift. The spherical symmetry of our balloon allows us to define a gravitational potential, U, as a function of time. A red shift is predicted by Einstein's Theory of General Relativity, (GR). This gravitational red shift determination must match the measured Doppler red shift. This allows us to determine M.

2. The Balloon Model

The venerable Balloon Model[1] assumes that the Universe is the 2-D skin of a balloon as it expands. Tiny dots have been painted on the balloon, and as the expansion occurs, all the dots are moving away from each other, just as happens with galaxies in our Universe. The greater the distance between the dots, the faster they recede.

This model can be made quantitative. First let's impose spherical symmetry. This surface is expanding in analogy to the Universe's Hubble Constant rate. The interior of the sphere is empty, but very important. It represents the past. In fact, at the center point of the interior (a singularity) is when and where the Big Bang happened. The exterior of the surface is the future. It is also empty and does not exist yet. With the time axis orthogonal to the 2-D surface, we have modeled the 4-D universe with a 3-D model.

For the calculations that we intend to perform, there are two important parameters: $r(t)$, the radius of the model surface and M, the mass of the Universe, a constant. This model restricts the Universe to be finite. We believe that the Big Bang occurred 13.8 billion years (Gyr) ago [1]. We will define the age of the Universe, T = 13.8 Gyr. Since the rate of expansion has been constant for most of the time since the Big Bang, a choice for $r(t)$ at a given time should be proportional to the age, t, of the Universe at that time:

$$r(t) = ut. \tag{1}$$

[1]One of the authors of this paper, Clyde Zaidins, used this analogy of the Universe is in his astronomy classes at University of Colorado Boulder, in the 1970's. George Gamow may have incorporated this discussion in his classes at University of Colorado Boulder, but we have not found verification.

where u is the velocity of expansion.

We have chosen:

$$r(t) = ct. \tag{2}$$

This choice of velocity of expansion, $u = c$, although arbitrary, allows the model radius to be similar to the Universe scale. This means that at our present time, the model has a radius equal to 13.8 billion light years (Gly).

Later we will need this radius for the gravitational potential:

$$U = -G\frac{M}{r}. \tag{3}$$

U is necessary in calculating the gravitational red shift. (Also note that there is a gravitational-potential singularity at $r = 0$, the Big Bang.)

3. The Doppler Red Shift

In the early 20th century Edwin Hubble and Milton Humason [2] discovered a linear relationship between the distance and the velocities of recession of distant galaxies. For the next century this relationship has been extended to much greater distances. This was very strong evidence for the expanding universe that had been predicted by Georges Lemaître [3] [4] and Alexander Friedmann [5] [6] using General Relativity. The velocity was determined by measuring the redshift of known spectral lines. The distance measurement, D, requires a number of steps known as the cosmological distance ladder. We won't discuss the details of the ladder in this paper, except to say that a number of methods have been used to determine increasing distance within our galaxy. Cepheid variables have been used to get distance to the nearest galaxies [7]. Standard candles (e.g. Type la Supernovae) were used for large extragalactic distance [8]. A more detailed description is available at Wikipedia [9].

The Hubble Law is:

$$v = H_0 D. \tag{4}$$

The units of H_0 are kilometers/second per Megaparsec.

There are currently two methods to extract H_0 that yield similar but statistically different values [10]. This ranges between about $67\frac{\text{km/s}}{\text{Mpc}}$ and $73\frac{\text{km/s}}{\text{Mpc}}$ This discrepancy is known as the Hubble Tension.

Normally one chooses $H_0 = 70\frac{\text{km/s}}{\text{Mpc}}$ for calculations.

The red shift, z, is defined by:

$$z = \frac{\lambda_{Observed}}{\lambda_{Emitted}} - 1. \tag{5}$$

A positive value for z is a red shift, and a negative value is a blue shift.

The measured value for z is related to the relative velocity of the frame of the observer and the frame of the emitter. This velocity, v, is positive if the reference frames are receding from each other. The relativistic Doppler equation for light is given by:

$$z = \sqrt{\frac{c+v}{c-v}} - 1. \tag{6}$$

An excellent approximation for $\beta^2 = v/c \ll 1$ is as follows:

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \cong \left(1 + \frac{1}{2}\beta\right)^2 - 1 \cong \beta \quad \text{for} \quad \beta \ll 1, \tag{7}$$

where $\beta = \frac{v}{c} = \frac{H_0 D}{c}$. Hence, we can use $z = \frac{H_0 D}{c}$.

4. The Gravitational Red Shift

One of the predictions of Einstein's Theory of General Relativity is the effect of the gravitational potential, U, on the frequency and wavelength of electromagnetic radiation. This is a natural consequence of the equivalence principle and was first confirmed in a celebrated experiment by Pound and Rebka [11]. Due to the expansion of the Universe, the gravitational potential is continually increasing from its minimum right after the Big Bang. When we model the Universe with the Balloon Model, the spherical symmetry yields Equation (3).

$$U = -\frac{GM}{r}$$

where G is the universal gravitational constant, M is the mass of the Universe, and r is the radius of the balloon. Since the photon will be detected later than it was emitted, the gravitational potential will be greater. The photon is essentially "going uphill" and will be red shifted. The amount of this red shift is given by:

$$z = \frac{\Delta U}{c^2}. \tag{8}$$

We now have the ability to calculate the red shift of the photon between its emission and its detection at a later time due to the change in gravitational potential.

5. A Specific Example

We have chosen a specific test case to use and extract numbers for our model. This case is the detection of red-shifted light from a galaxy, Γ, one billion light-years (1 Gly) from the Earth. At the time of the photon emission, the radius of the model is at $\tau = 12.8$ Gly and when the photon is detected the model radius is at $T = 13.8$ Gly. This is shown schematically in **Figure 1**.

First, we need to find the redshift and relative velocity between the Earth and Γ.

Substituting $D = 1\,\text{Gly} = 9.47\ \text{Ym} = 9.47 \times 10^{24}\ \text{m} = 307\ \text{Mpc}$, and $H_0 = 70$ km/s per Mpc, into Equation (4) yields

$$v = H_0 D = 70 \frac{\text{km/s}}{\text{Mpc}} \times 307\ \text{Mpc} = 2.15 \times 10^4\ \text{km/s}. \tag{9}$$

By substituting $v = 2.15\times10^4$ km/s and $c = 3.00\times10^5$ km/s into Equation (7), we find, $\beta = 7.44\times10^{-2}$.

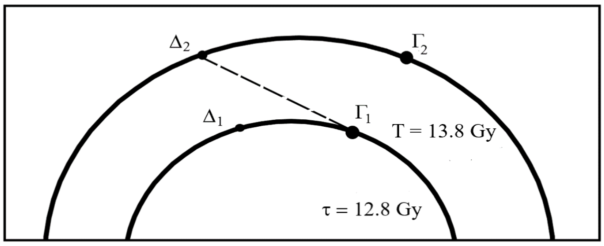

Figure 1. This figure is a 2-D slice through the balloon in the plane of the light photon emitted by the galaxy Γ and measured by a detector, Δ, on Earth a billion years later. The angle between Δ and Γ is greatly exaggerated in the figure. Γ_1 is the galaxy at the universe age of 12.8 Gy and Δ_1 is the detector's position (as if it had existed) at the same age. Δ_2 the detector position and Γ_2 is the galaxy position now, 13.8 Gy. The dotted line is the photon trajectory through space and time.

Since β is small we can use:

$$z = \beta = \frac{v}{c} = \frac{H_0 D}{c}, \tag{10}$$

$$D = c\left(T - \tau\right). \tag{11}$$

Thus

$$z = H_0\left(T - \tau\right). \tag{12}$$

Next the gravitational red shift is

$$z = \frac{\Delta U}{c^2} = \frac{GM}{c^2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right). \tag{13}$$

$$z = \frac{GM}{uc^2}\left(\frac{1}{\tau} - \frac{1}{T}\right) = \frac{GM}{uc^2}\left(\frac{T - \tau}{T\tau}\right). \tag{14}$$

When we set the Doppler redshift and the gravitational redshift to be equal to each other, we get:

$$H_0\left(T - \tau\right) = \frac{GM}{uc^2}\left(\frac{T - \tau}{T\tau}\right). \tag{15}$$

then

$$H_0 = \left(\frac{GM}{uc^2}\right)\left(\frac{1}{T\tau}\right). \tag{16}$$

Solving for M we get

$$M = \left(\frac{H_0 c^2 T}{G}\right)\left(u\tau\right). \tag{17}$$

We have separated $u\tau$ from the other factor because both u and τ were chosen by us and are not values given by the actual Universe parameters.

Define a dimensionless velocity related variable,

$$w = \frac{u\tau}{cT}. \tag{18}$$

Then the mass can be written as the product of a factor consisting of physical constants and parameters of the Universe and a factor that contains our choice of u and τ

$$M = \left(\frac{H_0 c^3 T^2}{G}\right) w. \tag{19}$$

If we define

$$K_M = \frac{H_0 c^3 T^2}{G}, \tag{20}$$

then

$$M = K_M w, \tag{21}$$

K_M has the dimensions of mass and w is dimensionless.

By substituting $H_0 = 70 \frac{\text{km/s}}{\text{Mpc}} = 2.27\times10^{18}\ \text{s}^{-1}$, $T = 13.8\ \text{Gyr} = 4.36\times10^{17}\ \text{s}$, $c = 3.00\times10^{5}\ \text{km/s}$ and $G = 6.67\times10^{11}\ \text{m}^3/\left(\text{kg}\cdot\text{s}^2\right)$ into Equation (20), we find $K_M = 1.75\times10^{53}\ \text{kg}$ by using $w = \frac{12.8}{13.8} = 0.928$. Thus, our value for $M = 1.6\times10^{53}\ \text{kg}$.

M has been calculated for $\tau = 12.8$ Gyr and $u = c$. The choice of $\tau = 12.8$ Gyr is arbitrary, but reasonable. We believe that $u = c$ is the sole choice so that the balloon and the universe are comparable in size. To remove the dependence on τ, let $M_T = \lim_{\tau\to T} M_\tau = K_M = 1.75\times10^{53}$ kg.

We believe that it's reasonable that this approach predicts M to be between 1×10^{53} and 2×10^{53} kg.

If we assume an average galaxy has a mass of about 10^{41} kg, this implies that there are roughly a trillion galaxies in the universe.

An observational calculation (following) of the mass is about 2.7×10^{52} kg which is within about an order of magnitude of our calculation: $M = 1.6\times10^{53}$ kg.

The average density of the universe is $2.8\times10^{-27}\ \frac{\text{kg}}{\text{m}^3}$ [12]. The radius of the observable universe is $13.8\times10^{9}\ \text{ly} = 1.31\times10^{26}\ \text{m}$, therefore the mass of the observable Universe is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = \left(\frac{4}{3}\pi\right)\left(2.8\times10^{-27}\ \frac{\text{kg}}{\text{m}^3}\right)\left(1.3\times10^{26}\ \text{m}\right)^3 = 2.5\times10^{52}\ \text{kg}. \tag{22}$$

Our larger extracted value for the mass may be due to our balloon model also including the region of the Universe that is not part of the Universe that we can observe.

6. Conclusions

The use of a parameterized Balloon model is useful as a teaching tool in studying cosmology and the derivation of a reasonable total mass shows it can be more than a qualitative picture of the universe. Further modifications may be useful as well.

For example, we may not know what Dark Energy is, but we can model it by altering the simple linear Equation (2) for the balloon expansion in the future.

We note, also, that our approach in using General Relativity in discussing radiation leaving a potential well (here for pedagogical purposes alone) is very similar in approach to recent models meant to do away with entirely the necessity of Dark Energy to explain the accelerating universe as implied by supernova studies; this idea has been labeled the Timescape cosmology [13].

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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