Journal of Applied Mathematics and Physics, 2025, 13(12), 4498-4505
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

The Hubble Frequency and the Hawking Temperature

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How to cite this paper: Haug, E.G. and Tatum, E.T. (2025) The Hubble Frequency and the Hawking Temperature. *Journal of Applied Mathematics and Physics*, **13**, 4498-4505.
https://doi.org/10.4236/jamp.2025.1312247

Received: October 24, 2025
Accepted: December 21, 2025
Published: December 24, 2025

Abstract

We suggest that there should be a minimum electromagnetic radiation frequency in the Hubble sphere and that this seems to be directly linked to the Hawking Temperature and our recent discovery that the CMB temperature could simply be a geometric mean temperature between minimum and maximum Hawking temperatures in the Hubble sphere; see [1]-[3].

Keywords

CMB Temperature, Hawking Temperature, Hubble Frequency, Geometric Mean

1. Introduction

Around the circumference of the Earth, Schumann resonance waves [4] are known to travel. This fact was predicted as early as 1893 by FitzGerald [5], so they are also known as "*Schumann-FitzGerald resonances*". It took many years before the suggestion by FitzGerald was taken seriously and even longer before it was confirmed. It takes about 0.13 seconds for electromagnetic waves to circumnavigate the Earth (in the ionosphere), leading to an expected frequency of approximately 7.5 Hz. The observed Schumann resonance waves are in this frequency range. The "exact" frequency depends on how high up in the ionosphere the waves travel and other factors. The conductive ionosphere acts as a closed waveguide. The Schumann resonance is currently considered to be linked to lightning storms in the atmosphere, where some part of their electromagnetic waves circumnavigate the Earth.

We will here ask if there could be similar very low-frequency electromagnetic waves that traverse the circumference of the Hubble sphere. The Hubble sphere can be simply defined as the observable universe sphere within which all objects of the expansion recede from the central observer at speeds slower than the speed of light. That the Hubble radius is identical to the Schwarzschild radius [6] [7] of a sphere with the expected average density of our critical Friedmann universe could mean that this horizon and this circumference are very special. Some researchers [8]-[10] have even asked if we could be living inside a gigantic black hole. Others have asked if our universe was created from a black hole [11]. Therefore, that one could have special electromagnetic events linked to the Schwarzschild radius and Schwarzschild circumference of our critical density universe is a plausible consequence within this framework, or perhaps even to be expected. We entertain the distinct possibility that frequency waves related to these Schwarzschild geometric relationships could be circumnavigating the Hubble sphere and perhaps even spilling over into the interior of the Hubble sphere.

The critical Friedmann mass is given as $M_c = \frac{c^2 R_h}{2G}$; solved for the radius, this gives $R_h = \frac{2GM_c}{c^2}$. Thus, it is mathematically identical to the Schwarzschild radius $R_s = \frac{2GM}{c^2}$. So, this means that we can treat the Hubble sphere as a Schwarzschild black hole with mass equal to the critical Friedmann mass.

Bear in mind that there can be many different interpretations of a "*black hole universe*". One does not even necessarily require the theory of general relativity to imagine such a possibility. Newton's classical theory [12] [13] predicts that, for a given mass, one can get a sphere with an escape velocity of greater than c. So, in more general terms, our universe could be a sphere of our current average critical density of matter (on a cosmic scale) where the escape velocity limit of c occurs along the circumference of the sphere horizon. Different theories can lead to different behaviours in such a sphere, where general relativity theory and the Schwarzschild metric provide only one possible framework and perspective by which to construct a circumnavigating wave theory as above. Already in 1916, Einstein [14] predicted that one needed to unify his new relativistic gravity with some quantum gravity theory to understand gravity even better, or in his own words:

> "Because of the intra-atomic movement of electrons, the atom must radiate not only electromagnetic but also gravitational energy, if only in minute amounts. Since, in reality, this cannot be the case in nature, then it appears that the quantum theory must modify not only Maxwell's electrodynamics but also the new theory of gravitation."—Einstein, 1916

So, our point is that much could still be unsolved. Therefore, one should leave some room for scientific speculation concerning these topics, particularly inside a Hubble sphere that is identical to a Schwarzschild sphere with a mass density one can get from substituting the Hubble constant into the Friedmann equation:

$\rho = \frac{M_c}{\frac{4}{3}\pi R_h^3} = \frac{3H_0^2}{8\pi G}$. Naturally, this assumes that H_0 is defined as c/R_H.

2. The Hubble Frequency

The Hubble sphere circumference frequency would be expected to be approximately:

$$f_H = \frac{c}{2\pi R_h} = \frac{H_0}{2\pi} \approx 3.61 \times 10^{-19}\ \text{Hz}. \tag{1}$$

where R_h is the Hubble radius. Figure 1 illustrates this.

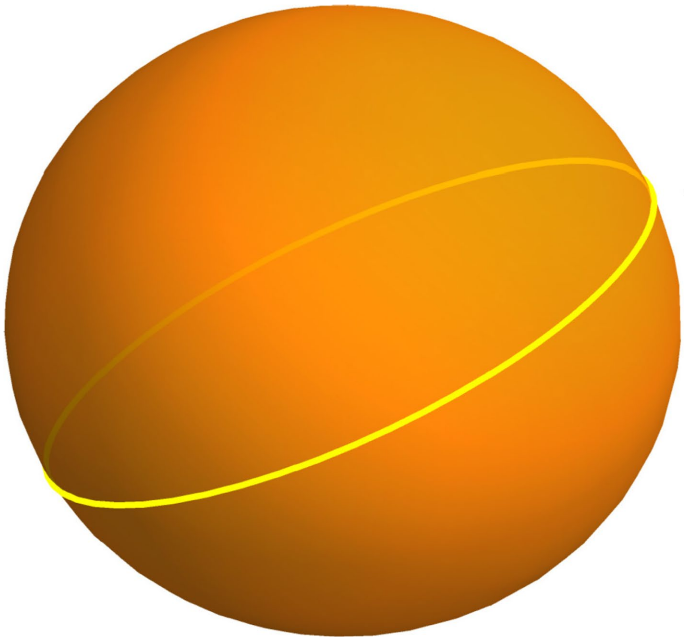

Figure 1. The figure illustrates a photon traveling the full circumference of the Hubble sphere. This is likely the lowest frequency one can possibly physically observe within such a sphere.

This calculation would perhaps put a lower bound on the frequency inside our Hubble sphere, similar to the assumptions by many physicists that the Planck frequency $f_p = \frac{c}{l_p}$ would be the upper bound on the frequency anywhere within the universe.

The Hubble frequency further corresponds to a temperature of:

$$T_H = \hbar f_H \frac{1}{k_b} \approx 2.64 \times 10^{-30}\ \text{K} \tag{2}$$

This temperature also just happens to be very close to the Hawking temperature of a black hole with the Friedmann critical mass M_c of our own Hubble sphere. This can be interpreted as the minimum possible temperature within the Hubble sphere, and it could exist at the very horizon itself.

Very little is known about what is going on precisely at the event horizon of a black hole, which is the boundary surface of the black hole. In a theory of Planck scale quantum cosmology, such a horizon would be the Hubble sphere surface. It is far from unthinkable that photons could be trapped in the horizon shell, as suggested for example by [15]. The photons trapped in the horizon shell would then be circumnavigating the Hubble sphere. And, if Hawking's black hole temperature theory and Planck scale quantum cosmology theory are indeed correct, the longest possible circumnavigating wave should have a reduced Compton wavelength connected to the Hawking temperature. So, we can think of the Hawking temperature as the minimum possible temperature of the Tatum *et al.* [16] Hubble sphere. We have called this the "Hawking Hubble temperature" see again [1].

We have shown in the paper just cited above that this temperature would be exactly the Hawking temperature of the Hubble sphere multiplied by a calibration factor of $\frac{1}{2}$, which we can also demonstrate here below. The Hawking [17] [18] temperature, as given for the critical Friedmann [19] universe, is:

$$T_{Hw} = \frac{\hbar g}{2c\pi k_b} = \frac{\hbar \frac{GM_c}{R_h^2}}{2c\pi k_b} = \frac{\hbar \frac{GM_c}{\left(\frac{2GM_c}{c^2}\right)^2}}{2c\pi k_b} = \frac{\hbar c^3}{k_b 8\pi GM_c} = \frac{\hbar c}{k_b 4\pi R_h} = \hbar f_H \frac{1}{k_b}\frac{1}{2} \tag{3}$$

In a black hole universe model which grows as a $R_h = ct$ model, the minimum temperature will fall adiabatically as the sphere increases in size. The Cosmic Microwave Background (CMB) temperature, at any point in cosmic time t, will be given by:

$$T_{CMB} = \hbar \frac{c}{\sqrt{2\pi R_H 4\pi l_p}} \frac{1}{k_b 2} = \hbar \frac{c}{4\pi\sqrt{nct_p 2l_p}} \frac{1}{k_b} = \hbar f_{gm} \frac{1}{2k_b} \tag{4}$$

where n is the number of Planck times since the beginning of the universe, and t_p is the Planck time, and the geometric mean frequency $f_{gm} = \frac{c}{\bar{\lambda}_{gm}}$, where $\bar{\lambda}_{gm} = \sqrt{\bar{\lambda}_{\max}\bar{\lambda}_{\min}}$ is simply the geometric mean reduced Compton wavelength between the maximum and minimum wavelengths ($\bar{\lambda}_{\max} = 2\pi R_h$, $\bar{\lambda}_{\min} = 2\pi R_{pl} = 4\pi l_p$), at any point in time in the universe, from the beginning of the universe to this day. This has recently been discussed in detail by Haug and Tatum [1]. The above formula is known from Tatum *et al.* [16] [20], even if written then in a slightly different form. However, it has recently been explored in much more depth from multiple angles and, for example, derived also from the Stefan-Boltzmann law by Haug and Wojnow [21] [22] as well as from a geometric mean approach by Haug and Tatum [1]-[3]. Thus, it seems that this is how one can model the CMB temperature consistently over the course of cosmic time [23]. Here it is also worth noting the great significance of the CMB radiation frequency spectrum being that of a near-perfect black body [24].

This way to express the CMB temperature seems fully consistent with at least some classes of $R_h = ct$ cosmological models. $R_h = ct$ cosmological models are actively discussed to this day, see for example [25]-[31]. But the approach above should also be carefully investigated in relation to the Λ-CDM model.

Today we have defined the Hubble time according to $t_H = \frac{1}{H_0}$ so we have $n = \frac{t_H}{t_p} \approx 8.55 \times 10^{60}$ which is the number of Planck times since the beginning of the universe, so this gives:

$$T_{CMB} = \hbar \frac{c}{4\pi\sqrt{nct_p 2l_p}} \frac{1}{k_b} \approx 2.72^{+0.082}_{-0.069}\ \text{K}$$

based upon the recent Hubble parameter value given by Kelly *et al.* [32] of $66.6^{+4.1}_{-3.3}$ km/s/Mpc. Our aim in this paper is not to predict the most accurate CMB temperature, but to highlight that there appears to be a close relationship between the Hubble frequency, the Hawking temperature and the Planck scale. Our aim is also to show how this theoretical approach is in-line with our recent geometric mean interpretation of the CMB temperature.

This discussion naturally does not exclude Hawking radiation, but it also seems likely that the Hawking radiation could be connected to a temperature within the Hubble "shell". In such a way, growing black hole type $R_h = ct$ models can be a particularly useful category of $R_h = ct$ models. In comparison to the Λ-CDM model, Planck scale quantum cosmology models referenced herein appear to model the universe in a unique and highly interesting way. Haug and Tatum [33] have recently compared this new type of $R_h = ct$ cosmological model with the ΛCDM model, and it appears that such a $R_h = ct$ model shows promise in addressing specific cosmological challenges, such as the Hubble tension [34] [35]. It even appears to offer a resolution to the Hubble tension. Naturally, it is still too early to draw firm conclusions, since such claims need to be thoroughly examined over time by multiple researchers before any consensus can be reached. Nevertheless, this paper provides new insights, particularly in how the Hawking temperature may be intimately linked to the minimum electromagnetic radiation wavelength possible within the Hubble sphere.

3. Conclusion

We conjecture that there likely exists a minimum frequency within the Hubble sphere, given by $f_H = \frac{c}{2\pi R_h} = \frac{H_0}{2\pi} \approx 3.61 \times 10^{-19}\ \text{Hz}$, where R_H is the Hubble radius. This frequency appears to be associated with a minimum temperature within the Hubble sphere. The CMB temperature seems to be related to the geometric mean of the maximum and minimum temperatures in the Hubble sphere, as pointed out by Haug and Tatum [1] [3].

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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