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Analysis of a New Class of Double Integrals Involving Generalized Hypergeometric Functions

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Abstract

In this study, we aim to explore a novel class of twenty-five double integrals involving generalized hypergeometric functions. These integrals take the form: $\int_0^1\int_0^1 y^c(1-x)^{c-1}(1-y)^{c-1}(1-xy)^{1-2c}\,{}_3F_2\left[\begin{matrix} a,b,c+\frac{1}{2} \\ \frac{1}{2}(a+b+i+1),2c+j \end{matrix};\frac{4y(1-x)(1-y)}{(1-xy)^2}\right]\mathrm{d}x\mathrm{d}y$ for $i,j=0,\pm1,\pm2$. The results are derived using generalized versions of Watson's summation theorem, as established in earlier work by Lavoie *et al.* Additionally, fifty integrals, split into two sets of twenty-five, are presented as special cases of our main findings, offering further insights into the structure of these integrals.

Keywords

Generalized Hypergeometric Function, Watson Theorem, Definite Integral, Beta Integral

1. Introduction

The natural generalization of Gauss's hypergeometric function ${}_2F_1$ is called the generalized hypergeometric function ${}_pF_q$, where $p,q\in\mathbb{N}_0$ is defined by [1] [2]

$$ {}_pF_q\left[\begin{matrix} a_1,\cdots,a_p \\ b_1,\cdots,b_q \end{matrix};z\right]=\sum_{n=0}^{\infty}\frac{(a_1)_n\cdots(a_p)_n}{(b_1)_n\cdots(b_q)_n}\frac{z^n}{n!} \tag{1} $$

where $(a)_n$ is the well-known Pochhammer symbol (or the raised factorial or the shifted factorial since $(1)_n = n!$) defined for any complex $a\in\mathbb{C}$ by

$$ \begin{aligned} (a)_n &= \frac{\Gamma(a+n)}{\Gamma(a)},\left(a\in\mathbb{C}\setminus\mathbb{Z}_0^-\right) \\ &= \begin{cases} a(a+1)\cdots(a+n-1), & (n\in\mathbb{N}) \\ 1, & (n=0) \end{cases} \end{aligned} \tag{2} $$

where Γ is the well-known Gamma function. For a detailed study on hypergeometric and generalized hypergeometric functions, we refer to the standard texts [1] [2].

In the theory of hypergeometric and generalized hypergeometric functions, classical summation theorems such as those of Gauss, Gauss second, Kummer, and Bailey for the series ${}_2F_1$; Watson, Dixon, Whipple and Saalschütz for the series ${}_3F_2$ play a key role.

Later, the above-mentioned classical summation theorems were generalized by Lavoie *et al.* [3]-[5].

However, in our present investigation, we are interested in the following classical Watson summation theorem [1].

$$ \begin{aligned} &{}_3F_2\left[\begin{matrix} a,b,c \\ \frac{1}{2}(a+b+1),2c \end{matrix};1\right] \\ &=\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(c+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}\right)\Gamma\left(c-\frac{1}{2}a-\frac{1}{2}b+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}a+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}b+\frac{1}{2}\right)\Gamma\left(c-\frac{1}{2}a+\frac{1}{2}\right)\Gamma\left(c-\frac{1}{2}b+\frac{1}{2}\right)} \end{aligned} \tag{3} $$

provided $\Re(2c-a-b)>-1$,
and its following generalization due to Lavoie *et al.* [3]

$$ \begin{aligned} &{}_3F_2\left[\begin{matrix} a,b,c \\ \frac{1}{2}(a+b+i+1),2c+j \end{matrix};1\right] \\ &=\mathcal{A}_{i,j}\frac{2^{a+b+i-2}\Gamma\left(\frac{1}{2}a+\frac{1}{2}b+\frac{1}{2}i+\frac{1}{2}\right)\Gamma\left(c+\left[\frac{j}{2}\right]+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma(a)\Gamma(b)} \\ &\times\Gamma\left(c-\frac{1}{2}(a+b+|i+j|-j-1)\right) \\ &\times\left\{\frac{\mathcal{B}_{i,j}\Gamma\left(\frac{1}{2}a+\frac{1}{4}\left(1-(-1)^i\right)\right)\Gamma\left(\frac{1}{2}b\right)}{\Gamma\left(c-\frac{1}{2}a+\frac{1}{2}+\left[\frac{j}{2}\right]-\frac{1}{4}(-1)^j\left(1-(-1)^i\right)\right)\Gamma\left(c-\frac{1}{2}b+\frac{1}{2}+\left[\frac{j}{2}\right]\right)}\right. \end{aligned} $$

$$
\left. + \mathcal{C}_{i,j} \frac{\Gamma\left(\frac{1}{2}a+\frac{1}{4}\left(1+(-1)^i\right)\right)\Gamma\left(\frac{1}{2}b+\frac{1}{2}\right)}{\Gamma\left(c-\frac{1}{2}a+\left[\frac{j+1}{2}\right]+\frac{1}{4}(-1)^j\left(1-(-1)^i\right)\right)\Gamma\left(c-\frac{1}{2}b+\left[\frac{j+1}{2}\right]\right)}\right\} \tag{4}
$$

$$
= \Omega\,(\text{let})
$$

for $i, j = 0, \pm 1, \pm 2$.

For $i = j = 0$, the result (4) reduces to the classical Watson summation theorem (3).

Here, $[x]$ denotes the highest integer less than or equal to x and the modulus is denoted by $|x|$. In addition, the coefficients $\mathcal{A}_{i,j}$, $\mathcal{B}_{i,j}$ and $\mathcal{C}_{i,j}$ are given in **Tables 1-3**.

In addition to this, we shall also require the following well-known and interesting double integral due to Edwards [6]

$$
\int_0^1 y^{\alpha}(1-x)^{\alpha-1}(1-y)^{\beta-1}(1-xy)^{1-\alpha-\beta}\,\mathrm{d}x\mathrm{d}y = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \tag{5}
$$

provided $\Re(\alpha) > 0$ and $\Re(\beta) > 0$.

The aim of this paper is to evaluate twenty-five double integrals involving generalized hypergeometric functions in the form of a general double integral of the form

$$
\int_0^1\int_0^1 y^{c}(1-x)^{c-1}(1-y)^{c-1}(1-xy)^{1-2c} \times {}_3F_2\left[\begin{matrix} a, b, c+\frac{1}{2} \\ \frac{1}{2}(a+b+i+1), 2c+j \end{matrix}; \frac{4y(1-x)(1-y)}{(1-xy)^2}\right]\mathrm{d}x\mathrm{d}y
$$

for $i, j = 0, \pm 1, \pm 2$.

The results are derived with the help of the generalized Watson summation theorem on the sum of a ${}_3F_2$ given by (4). Fifty interesting integrals in the form of two integrals (twenty-five each) have also been given as special cases of our main findings.

2. Main Integrals

The 25 double integrals in the form of a general double integral to be evaluated in this paper are given in the following theorem.

Theorem 1. For $\Re(c) > 0$, $\Re(2c-a-b+i+2j+1) > 0$, for $i, j = 0, \pm 1, \pm 2$, the following integral formula holds.

$$
\int_0^1\int_0^1 y^{c}(1-x)^{c-1}(1-y)^{c-1}(1-xy)^{1-2c} \times {}_3F_2\left[\begin{matrix} a, b, c+\frac{1}{2} \\ \frac{1}{2}(a+b+i+1), 2c+j \end{matrix}; \frac{4y(1-x)(1-y)}{(1-xy)^2}\right]\mathrm{d}x\mathrm{d}y = \frac{\Gamma(c)\Gamma(c)}{\Gamma(2c)}\Omega \tag{6}
$$

where Ω is the same as given in (4).

Table 1. Table for $A_{i,j}$.

i\j	−2	−1	0	1	2
	$\frac{1}{2(c-1)(a-b-1)(a-b+1)}$	$\frac{1}{2(a-b-1)(a-b+1)}$	$\frac{1}{4(a-b-1)(a-b+1)}$	$\frac{1}{4(a-b-1)(a-b+1)}$	$\frac{1}{8(c+1)(a-b-1)(a-b+1)}$
	$\frac{1}{(c-1)(a-b)}$	$\frac{1}{(a-b)}$	$\frac{1}{(a-b)}$	$\frac{1}{2(a-b)}$	$\frac{1}{2(c+1)(a-b)}$
	$\frac{1}{2(c-1)}$	1	1	1	$\frac{1}{2(c+1)}$
−1	$\frac{1}{(c-1)}$	1	2	2	$\frac{2}{(c+1)}$
−2	$\frac{1}{2(c-1)}$	1	1	2	$\frac{2}{(c+1)}$

Table 2. Table for $B_{i,j}$.

$$\mathcal{B}_{2,2} = 2c(c+1)\{(2c+1)(a+b-1) - a(a-1) - b(b-1)\} - (a-b-1)(a-b+1)\{(c+1)(2c-a-b+1) + ab\}$$

$$\mathcal{B}_{-2,-2} = 2(c-1)(c-2)\{(2c-1)(a+b-1) - a(a+1) - b(b+1) + 2\} - (a-b-1)(a-b+1)\{(c-1)(2c-a-b-3) + ab\}$$

$$\mathcal{B}_{-2,-1} = 2(c-1)(a+b-1) - (a-b)^2 + 1$$

$i \backslash j$	-2	-1	0	1	2
	$c(a+b-1)-(a+1)(b+1)+2$	$a+b-1$	$a(2c-a)+b(2c-b)-2c+1$	$2c(a+b-1)-(a-b)^2+1$	$B_{2,2}$
	$c-b-1$	1	1	$2c-a+b$	$2c(c+1)-(a-b)(c-b+1)$
	$(c-a-1)(c-b-1)+(c-1)(c-2)$	1	1	1	$(c-a+1)(c-b+1)+c(c+1)$
-1	$2(c-1)(c-2)-(a-b)(c-b-1)$	$2c-a+b-2$	1	1	$c-b+1$
-2	$B_{-2,-2}$	$B_{-2,-1}$	$a(2c-a)+b(2c-b)-2c+1$	$a+b-1$	$c(a+b-1)-(a-1)(b-1)$

Table 3. Table for $C_{i,j}$.

$$C_{-2,-1} = 8c^2 - 2(c-1)(a+b+7) - (a-b)^2 - 7$$

$i \backslash j$	-2	-1	0	1	2
	-4	$-(4c-a-b-3)$	-8	$-\left[8c^2 - 2c(a+b-1) - (a-b)^2 + 1\right]$	$-4(2c+a-b+1)(2c-a+b+1)$
	$-(c-a-1)$	-1	-1	$-(2c+a-b)$	$-\left[2c(c+1) + (a-b)(c-a+1)\right]$
	4	1	0	-1	-4
-1	$2(c-1)(c-2) + (a-b)(c-a-1)$	$2c+a-b-2$	1	1	$c-a+1$
-2	$4(2c-a+b-3)(2c+a-b-3)$	$C_{-2,-1}$	8	$4c-a-b+1$	4

Proof. The proof of our theorem is quite straightforward. For this, we proceed as follows. Denoting the left hand side of (6) by I, we have

$$I=\int_0^1\int_0^1 y^c(1-x)^{c-1}(1-y)^{c-1}(1-xy)^{1-2c}$$
$$\times{}_3F_2\left[\begin{matrix}a,b,c+\frac{1}{2}\\ \frac{1}{2}(a+b+i+1),2c+j\end{matrix};\frac{4y(1-x)(1-y)}{(1-xy)^2}\right]\mathrm{d}x\mathrm{d}y$$

Now expressing ${}_3F_2$ as a series, changing the order of integration and summation, which is easily seen to be justified due to the uniform convergence of the series in the interval (0, 1), we have

$$I=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n\left(c+\frac{1}{2}\right)_n 2^{2n}}{\left(\frac{1}{2}(a+b+i+1)\right)_n(2c+j)_n}\int_0^1\int_0^1 y^{c+n}(1-x)^{c+n-1}$$
$$\times(1-y)^{c+n-1}(1-xy)^{1-2c-2n}\mathrm{d}x\mathrm{d}y$$

Evaluating the integral using (5), we have, after some simplification

$$I=\frac{\Gamma(c)\Gamma(c)}{\Gamma(2c)}\sum_{n=0}^{\infty}\frac{(a)_n(b)_n(c)_n}{\left(\frac{1}{2}(a+b+i+1)\right)_n(2c+j)_n n!}$$

Now summing up the series, we have

$$I=\frac{\Gamma(c)\Gamma(c)}{\Gamma(2c)}{}_3F_2\left[\begin{matrix}a,b,c\\ \frac{1}{2}(a+b+i+1),2c+j\end{matrix};\frac{4y(1-x)(1-y)}{(1-xy)^2}\right] \tag{7}$$

We now observe that the ${}_3F_2$ appearing can be evaluated with the help of known result (4) and we easily arrive at the right hand side of (6).

This completes the proof of the theorem. □

3. Special Cases and Examples

In this section, we will present several fascinating special cases derived from our main results.

To achieve this, we observe that in Equation (6), by setting $b=-2n$ and replacing a with $a+2n$, or by setting $b=-2n-1$ and replacing a with $a+2n+1$, one of the two terms on the right-hand side of Equation (6) will vanish. As a result, we obtain fifty intriguing special cases (twenty-five each), which are presented below in the form of two corollaries.

Corollary 1. For $i,j=0,\pm1,\pm2$, the following twenty-five results are true.

$$\int_0^1\int_0^1 y^c(1-x)^{c-1}(1-y)^{c-1}(1-xy)^{1-2c}$$
$$\times{}_3F_2\left[\begin{matrix}-2n,a+2n,c+\frac{1}{2}\\ \frac{1}{2}(a+i+1),2c+j\end{matrix};\frac{4y(1-x)(1-y)}{(1-xy)^2}\right]\mathrm{d}x\mathrm{d}y$$

$$= D_{i,j} \frac{\Gamma(c)\Gamma(c)}{\Gamma(2c)} \frac{\left(\frac{1}{2}\right)_n \left(\frac{1}{2}a - c + \frac{3}{4} - \frac{(-1)^i}{4} - \left[\frac{1}{2}j + \frac{1}{4}\left(1+(-1)^i\right)\right]\right)_n}{\left(c + \frac{1}{2} + \left[\frac{j}{2}\right]\right)_n \left(\frac{1}{2}a + \frac{1}{4}\left(1+(-1)^i\right)\right)_n}, \tag{8}$$

where the coefficients $D_{i,j}$ are given in **Table 4**.

Corollary 2. For $i, j = 0, \pm 1, \pm 2$, the following twenty-five results are true.

$$\begin{aligned} &\int_0^1\int_0^1 y^c (1-x)^{c-1} (1-y)^{c-1} (1-xy)^{1-2c} \\ &\times {}_3F_2\left[\begin{matrix} -2n-1, a+2n+1, c+\frac{1}{2} \\ \frac{1}{2}(a+i+1), 2c+j \end{matrix}; \frac{4y(1-x)(1-y)}{(1-xy)^2}\right] \mathrm{d}x\mathrm{d}y \\ &= E_{i,j} \frac{\Gamma(c)\Gamma(c)}{\Gamma(2c)} \frac{\left(\frac{3}{2}\right)_n \left(\frac{1}{2}a - c + \frac{5}{4} + \frac{(-1)^i}{4} - \left[\frac{1}{2}j + \frac{1}{4}\left(1+(-1)^i\right)\right]\right)_n}{\left(c + \frac{1}{2} + \left[\frac{j+1}{2}\right]\right)_n \left(\frac{1}{2}a + \frac{1}{4}\left(3-(-1)^i\right)\right)_n}, \end{aligned} \tag{9}$$

where the coefficients $E_{i,j}$ are given in **Table 5**.

In particular, in (8), if we take $i = j = 0$, we get the following interesting result.

$$\begin{aligned} &\int_0^1\int_0^1 y^c (1-x)^{c-1} (1-y)^{c-1} (1-xy)^{1-2c} \\ &\times {}_3F_2\left[\begin{matrix} -2n, a+2n, c+\frac{1}{2} \\ \frac{1}{2}(a+b+1), 2c \end{matrix}; \frac{4y(1-x)(1-y)}{(1-xy)^2}\right] \mathrm{d}x\mathrm{d}y \\ &= \frac{\Gamma(c)\Gamma(c)}{\Gamma(2c)} \frac{\left(\frac{1}{2}\right)_n \left(\frac{1}{2}a - c + \frac{1}{2}\right)_n}{\left(c + \frac{1}{2}\right)_n \left(\frac{1}{2}a + \frac{1}{2}\right)_n}. \end{aligned} \tag{10}$$

Similarly, in (9), if we take $i = j = 0$, we get the following elegant result.

$$\begin{aligned} &\int_0^1\int_0^1 y^c (1-x)^{c-1} (1-y)^{c-1} (1-xy)^{1-2c} \\ &\times {}_3F_2\left[\begin{matrix} -2n-1, a+2n+1, c+\frac{1}{2} \\ \frac{1}{2}(a+b+1), 2c \end{matrix}; \frac{4y(1-x)(1-y)}{(1-xy)^2}\right] \mathrm{d}x\mathrm{d}y = 0. \end{aligned} \tag{11}$$

Similarly, we can obtain other results. However, we prefer to omit the details.

4. Conclusions

In this study, we have computed 25 noteworthy double integrals involving generalized hypergeometric functions, presented in the form of a general double integral.

The results are derived with the aid of a generalization of the classical Watson's summation theorem, as established by Lavoie *et al.*

Table 4. Table for $D_{i,j}$.

$$D_{2,2}=\frac{(a+1)\left[(a-1)(c+1)(2c-a+1)(2c-a-1)-2an(6c+a+5)(2c-a+1)+4n^2\left(5a^2+4a-5-4c(3c-a+4)\right)+64n^3(a+n)\right]}{(c+1)(2c-a+1)(2c-a-1)(a+4n+1)(a+4n-1)}$$

$$D_{-2,-2}=1-\frac{2an(6c+a-7)(2c-a-3)-4n^2\left[5a^2-4a-21-4c(3c-a-8)\right]-64n^3(a+n)}{(c-1)(a-1)(2c-a-3)(2c-a-5)}$$

$i\backslash j$	-2	-1	0	1	2
	$\frac{(a+1)\left[(c-1)(a-1)+2n(a+2n)\right]}{(c-1)(a+4n-1)(a+4n+1)}$	$\frac{(a+1)(a-1)}{(a+4n+1)(a+4n-1)}$	$\frac{(a+1)\left[(a-1)(2c-a-1)-4n(a+2n)\right]}{(2c-a-1)(a+4n+1)(a+4n-1)}$	$\frac{(a+1)\left[(a-1)(2c-a-1)-8n(a+2n)\right]}{(2c-a-1)(a+4n+1)(a+4n-1)}$	$D_{2,2}$
	$\frac{a(c+2n-1)}{(c-1)(a+4n)}$	$\frac{a}{a+4n}$	$\frac{a}{a+4n}$	$\frac{a(2c-a-4n)}{(2c-a)(a+4n)}$	$\frac{a\left[(c+1)(2c-a)-2n(2c+a+4n+2)\right]}{(c+1)(2c-a)(a+4n)}$
	$1-\frac{2n(a+2n)}{(c-1)(2c-a-3)}$	1	1	1	$1-\frac{2n(a+2n)}{(c+1)(2c-a+1)}$
-1	$1-\frac{2n(2c+a+4n-2)}{(c-1)(2c-a-4)}$	$1-\frac{4n}{(2c-a-2)}$	1	1	$1+\frac{2n}{(c+1)}$
-2	$D_{-2,-2}$	$1-\frac{8n(a+2n)}{(a-1)(2c-a-3)}$	$1-\frac{4n(a+2n)}{(a-1)(2c-a-1)}$	1	$1+\frac{2n(a+2n)}{(c+1)(a-1)}$

Table 5. Table for $E_{i,j}$.

$$E_{2,1}=\frac{(a+1)\left[(4c+a+3)(2c-a-1)-8n(a+2n+2)\right]}{(a+4n+1)(a+4n+3)(2c+1)(2c-a-1)}$$

$$E_{-2,-1}=-\frac{\left[(4c+a-1)(2c-a-3)-8n(a+2n+2)\right]}{(a-1)(2c-1)(2c-a-3)}$$

$i\backslash j$	-2	-1	0	1	2
	$\frac{(a+1)(2c-a-3)}{(c-1)(a+4n+1)(a+4n+3)}$	$\frac{(a+1)(4c-a-3)}{(a+4n+1)(a+4n+3)(2c-1)}$	$\frac{2(a+1)}{(a+4n+1)(a+4n+3)}$	$E_{2,1}$	$\frac{(a+1)(2c+a+4n+3)(2c-a-4n-1)}{(c+1)(2c-a-1)(a+4n+1)(a+4n+3)}$
	$\frac{(c-a-2n-2)}{(c-1)(a+4n+2)}$	$\frac{2c-a-2}{(a+4n+2)(2c-1)}$	$\frac{1}{a+4n+2}$	$\frac{(2c+a+4n+2)}{(2c+1)(a+4n+2)}$	$\frac{(c+a+2)(2c-a)-2n(3a-2c+4n+2)}{(c+1)(2c-a)(a+4n+2)}$
	$\frac{-1}{(c-1)}$	$\frac{-1}{(2c-1)}$	0	$\frac{1}{(2c+1)}$	$\frac{1}{(c+1)}$
-1	$E_{-1,-2}$	$\frac{-(2c+a+4n)}{a(2c-1)}$	$\frac{-1}{a}$	$\frac{-(2c-a)}{a(2c+1)}$	$\frac{-(c-a-2n)}{a(c+1)}$
-2	$\frac{-(2c+a+4n-1)(2c-a-4n-5)}{(a-1)(c-1)(2c-a-5)}$	$E_{-2,-1}$	$\frac{-2}{(a-1)}$	$\frac{-(4c-a+1)}{(a-1)(2c+1)}$	$\frac{-(2c-a+1)}{(a-1)(c+1)}$

Furthermore, fifty remarkable integrals, divided into two sets of twenty-five, have been derived as special cases of our main findings.

We conclude this work by noting that the intriguing applications of the integrals presented here are currently under investigation and will be published in the near future.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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