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A Link Merges Classical Mechanics to Quantum Theory (Part I)

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Abstract

Since the interest in the quantum phenomena started, there have been two main approaches to the quantum theory, the first approach investigates the motion of a wave-particle object by assuming the wave has the characteristics of a moving particle, momentum and energy, this approach was developed by physicists Max Planck, Albert Einstein, and others. The second approach investigates the motion of a particle-wave object by assuming the particle has the characteristics of a moving wave, wavelength, wave vector, and frequency, this approach was developed by physicists De Broglie, Schrodinger and others which led to the presence of quantum mechanics. This article focuses on the second approach and merging it with classical mechanics. An early mathematical formulation for the quantum theory was proposed by De Broglie's relations and later on, another formulation was given by Schrodinger's equation. A successful attempt to bridge the gap between classical mechanics with quantum theory implemented via a mathematical derivation for De Broglie's relations and Schrodinger formulation, The derivation process overcame a main obstacle was the convergence of the deterministic nature of classical mechanics with the probabilistic nature of the quantum theory by adopting an innovative method based upon replacing constant values in a deterministic relation with probabilistic variables in the same relation, and this led to switch the relation from deterministic representation to probabilistic representation. Both classical and quantum explanation are given for the deduced formulas. An energy approach is instrumented. Since classical mechanics formulas are not sufficient to explain the quantum phenomena, a new concept of complementary energy introduced in order to connect the parameters of a moving particle with the parameters of an associated wave, the value of the complementary energy for each particle adjusted the total energy equation of N moving particles without changing the magnitude of the total energy of a system of N particles. In special cases, the value of the complementary energy for some particles equals the value of the total energy for the particles. The derivation process indicates that complementary energy corresponds to the potential energy on the surface of the material which prevents the particles to escape from the boundary of the matter to the outer space and maintains its coherence, complementary energy concept also manifest the impact of the moving particles on a single particle due to the energy conveyed during direct contact in case of collision or by the remote influence through an existing field in the space, so it represents an energy of dual nature for both matter and radiation wave combined. The complementary energy is not introduced as mathematical assumption in order to manipulate the total energy equation, it's actually exists in the total energy equation and its validity has not been applied before. A general format for any potential energy is obtained regardless the nature of field it represents (a universal format for potential energy) that is by using mathematical manipulation for the equation of total energy of N moving particles. The derivation process led to a particle-wave formula. The substitution into the particle-wave formula with the speed of light, De Broglie's relations, revealed. Some additional applications for the particle-wave formula are suggested, also as a well known energy quantization formula obtained by applying all of De Broglie's relation, a general energy, and speed formulas resulting during the derivation process. Matching time independent equation called Schrodinger's amplitude equation with classical mechanics formulation demonstrated by deducing both the wave function and the magnitude of the wave vector from the classical formulation after converting it to probabilistic formulation. A dedicated section for wave function solutions for Schrodinger's time dependent equation was given in terms of the wave functions and the magnitude of the wave vectors obtained from the matching process of Schrodinger's amplitude equation with classical mechanics.

Keywords

Quantum Mechanics, Classical Mechanics, De Broglie Relations,
Complementary Energy, Schrodinger Equation

1. Introduction

In 1924, De Broglie put the hypothesis that particles should exhibit wave and particle properties simultaneously [1]. Classical mechanics applies specific mathematical equations to describe the motion of particles and wave in separate fashion, so classical mechanics equations are inadequate to implement De Broglie hypothesis, a new mathematical formulas are required to express the new proposed concept De Broglie interpreted his hypothesis by postulating two relations the first one express the energy of the moving particle in terms of the frequency of the claimed associated wave and the second relation express the momentum of a moving particle in terms of the wavelength of the claimed associated wave [1]. The postulated relations given by De Broglie based on the mathematical formulas were given to explain the interaction of light with matter which is known as photoelectric effect, the physicist Albert Einstein assumed that light wave behaves like particles called photons, he proposed mathematical formulas calculate the required amount of photon's energy in order to release the constrained electrons from the surface of the matter. De Broglie's relations have been verified experimentally more than once. These proposed mathematical formulas were not obtained from classical mechanics considerations or any mathematical derivation method. Later on, Schrodinger set up a wave equation to express the quantum dynamics of a moving particle in the form of a partial differential equation which is satisfied only by complex wave functions. In this article a mathematical derivation for De Broglie's relations is given in accordance with classical mechanics equations in addition to a new equation introduced to represents the magnitude of the complementary energy, as well as Schrodinger's amplitude equation is regarded for sake of simplification and purpose of clearness to achieve a successful convergence process of the classical approach to the quantum approach, the results obtained from convergence process of the classical approach to the quantum approach are implemented to deduce solutions for the general format of Schrodinger's time dependent equation, consequently a link between classical mechanics and quantum theory established.

2. Applied Methodology

2.1. Particle Potential Energy-Complementary Energy

The total energy of a system of N moving particles $\mathcal{E}$ is defined as the summation of the particles kinetic energy $\sum_{j=1}^{N}\frac{1}{2}m_j v_j^2$ and the total potential energy V

So that

$$V=\mathcal{E}-\sum_{j=1}^{N}\frac{1}{2}m_j v_j^2 \tag{1}$$

Equation (1) is rewritten as

$$V=\sum_{j=1}^{N}-\left(\frac{1}{2}v_j-\frac{\tilde{\alpha}_j\mathcal{E}+\tilde{\beta}_j}{m_j v_j}\right)m_j v_j \tag{2}$$

in which $\sum_{j=1}^{N}\tilde{\alpha}_j=1$, $\sum_{j=1}^{N}\tilde{\beta}_j=0$, $m_j v_j\neq 0$, $j=1,\cdots,N$

where $\tilde{\alpha}_j$ is an energy distribution coefficient and $\tilde{\beta}_j$ is a complementary energy term. The physical significance for $\tilde{\alpha}_j$ and $\tilde{\beta}_j$ is given in section 4. $\tilde{\alpha}_j$ and $\tilde{\beta}_j$ are related to the particle total energy $\mathcal{E}_j$ and the total energy of the system of particles $\mathcal{E}$ by

$$\mathcal{E}_j=\tilde{\alpha}_j\mathcal{E}+\tilde{\beta}_j,\quad j=1,\cdots,N \tag{3}$$

$\tilde{\alpha}_j$ and $\tilde{\beta}_j$ are real functions, *i.e.* $\tilde{\alpha}_j\in\mathcal{R}$ and $\tilde{\beta}_j\in\mathcal{R}$ where $\mathcal{R}$ is the set of real numbers.

Since $\sum_{j=1}^{N}\tilde{\beta}_j=0$ then

$$-\tilde{\beta}_j = \sum_{i=1}^{N-1} \tilde{\beta}_i \,, \quad j = 1, \cdots, N \tag{4}$$

Equation (4) represents the complementary energy concept which interprets the impact of a system of particles on each particle in the system. The left hand side of Equation (4) gives the value of the complementary energy for particle j whereas the right hand side is the summation of the complementary energies for the rest $N-1$ particles.

Substituting in Equation (2) from Equation (3) and writing

$$\left(\frac{1}{2} v_j - \frac{\mathcal{E}_j}{m_j v_j} \right) = \overline{v}_j \,, \quad m_j v_j \neq 0 \,, \quad j = 1, \cdots, N \tag{5}$$

in which $\overline{v}_j$ is a velocity parameter, yields the expressions for the total potential energy and the particle potential energy as

$$V = \sum_{j=1}^{N} V_j \,, \quad V_j = -m_j v_j \overline{v}_j \,, \quad j = 1, \cdots, N \tag{2.1}$$

The above relation expresses a general format for any potential energy for any moving particle independent completely of the kind of the field it represents.

2.2. Particle Velocities

Expressing Equation (5) as

$$\overline{v}_j v_j = \frac{1}{2} v_j^2 - \frac{\mathcal{E}_j}{m_j} \,, \quad j = 1, \cdots, N$$

Gives the particle velocity quadratic equation as

$$v_j^2 - 2\overline{v}_j v_j - \frac{2\mathcal{E}_j}{m_j} = 0 \,, \quad j = 1, \cdots, N \tag{6}$$

The roots of Equation (6) are $v_{j,1,2}$ given by

$$v_{j,1,2} = \frac{2\overline{v}_j \pm \sqrt{4\overline{v}_j^2 + 4\frac{2\mathcal{E}_j}{m_j}}}{2} = \overline{v}_j \pm \sqrt{\overline{v}_j^2 + \frac{2\mathcal{E}_j}{m_j}} \,, \quad j = 1, \cdots, N$$

The roots of Equation (6) result in the existence of two available velocities. Two classical interpretations are given, the first classical interpretation of the above relation is a pair particle velocities prevail along the particle path trajectory. The second classical interpretation of the above relation is one speed for transitional motion and another speed for rotational motion. The quantum interpretation of the above relation is given in detail in section 2.7.

2.3. Particle Mean Velocity

The particle mean velocity, $\tilde{v}_j$, is expressed as

$$\tilde{v}_j = \frac{v_{j,1} + v_{j,2}}{2} \,, \quad j = 1, \cdots, N$$

Expressing the roots of Equation (6) as

$$v_{j,1,2} = \tilde{v}_j \pm d_j, \quad j = 1, \cdots, N \tag{7}$$

in which

$$d_j = \sqrt{\tilde{v}_j^2 + \tilde{v}_{\perp j}^2}, \quad \tilde{v}_{\perp j} = \sqrt{\frac{2\mathcal{E}_j}{m_j}}, \quad j = 1, \cdots, N \tag{7.1}$$

gives $\bar{v}_j = \tilde{v}_j$ and d_j to be the deviation from the mean velocity; given the term deviation velocity of particle *j*.

$\bar{v}_j$ is substituted by $\tilde{v}_j$ in all further equations.

A proof that the value of the mean velocity is constant and is given as follows

Rewriting Equation (7) as

$$\left(v_j - \tilde{v}_j\right)^2 = \tilde{v}_j^2 + \frac{2\mathcal{E}_j}{m_j}, \quad j = 1, \cdots, N$$

$$\mathcal{E}_j\left(v_j\right) = \frac{1}{2} m_j \left(v_j - \tilde{v}_j\right)^2 - \frac{1}{2} m_j \tilde{v}_j^2, \quad j = 1, \cdots, N \tag{7.2}$$

Equation (7.2) is deduced by expanding $\mathcal{E}_j\left(v_j\right)$, into Taylor series around an extremal point, $\tilde{v}_j$, and substituting $\mathcal{E}_j'\left(\tilde{v}_j\right) = 0$ (the condition for extremum) in the second term. The terms above the second order are null. The condition for extremum $\mathcal{E}_j'\left(\tilde{v}_j\right) = 0$ indicates that if the velocity of the moving particle *j* reaches the value $\tilde{v}_j$ then it does not contribute to its energy $\mathcal{E}_j$, *i.e.* the moving particle is in a collision free motion or in a perfect collision motion (no change in energy due collision) or both.

2.4. Complementary Energy Value ($\tilde{\beta}_j$)-(Particle-Wave Object)

The explanation of Equation (4) whereas the summation in the right side of the equation calculates the net collective effect of $N - 1$ particles upon a single particle, based on the particle's location and particle's status whether the effect is due to direct contact as in case of collision or via remote influence through an existing field in the space. The left hand side of Equation (4) gives the value of the complementary energy for particle *j*. For a particle near the surface of the material the right hand side summation represents the required amount of work done by the rest $N - 1$ particles to pull back the particle away from the surface of the material. The complementary energy can be understood as the potential energy at the boundary of the material which prevents each particle from escaping from the boundary of the system (evaporates).

From Equation (3) in section 2.1 the total energy of particle *j* is written

$$\mathcal{E}_j = \tilde{\alpha}_j \mathcal{E} + \tilde{\beta}_j$$

In special case $\tilde{\alpha}_j = 0$ Equation (3) becomes

$$\mathcal{E}_j = \tilde{\beta}_j$$

Substituting in Equation (3) with the kinetic energy and the potential energy of particle *j* gets

$$\mathcal{E}_j = \tilde{\alpha}_j \mathcal{E} + \tilde{\beta}_j \rightarrow \tilde{\alpha}_j \mathcal{E} = \frac{1}{2} m_j v_j^2 + V_j - \tilde{\beta}_j$$

According to the argument given above the value of the complementary energy $-\tilde{\beta}_j$ is independent form the potential energy value V_j, so in case the potential energy V_j of particle *j* vanishes at certain positions. *i.e.* V_j values halt contributing to the total energy of particle *j*, the complementary energy $-\tilde{\beta}_j$ still contributing to the total energy of particle *j* then the above equation is written

$$\tilde{\alpha}_j \mathcal{E} = \frac{1}{2} m_j v_j^2 - \tilde{\beta}_j$$

where $-\tilde{\beta}_j$ can be considered as potential energy,

So the general format for any potential energy of a particle *j* given by Equation (2.1) is adequate to represent the value of the complementary energy of particle *j*. For a particle *j* its complementary energy is expressed by the equation

$$\tilde{\beta}_j = m_j v_{j,1,2} \tilde{v}_j \tag{2.2}$$

The probabilistic formulation of the quantum theory indicates an assumption of the impossibility to measure an exact value for the velocity but you can measure a probable value some time called expected value or average value, $\tilde{v}_j$ represents the measured value of the velocity for a moving particle *j*. Another aspect of the quantum formulation is an assumption of the presence of an associated wave to any moving particle [1] in other word the associated wave adheres the moving particle composing a particle-wave object. The adherence of the wave to the moving particle enables the wave to borrow one of the available speeds of the particle to be the speed of the associated wave, consequently $v_{j,1,2}$ can represent the speed of the associated wave. $\tilde{\beta}_j$ is a representation of complementary energy for the particle-wave object which is a bi-natural for both a moving particle and moving wave and Equation (2.2) is a particle-wave equation. The determination of the complementary energy $\tilde{\beta}_j$ requires two velocities values, one for a particle and another for a wave, so $\tilde{\beta}_j$ is a potential energy of a combined particle-wave object. Another application for $\tilde{\beta}_j$ is to express the motion of a moving object in a continuous medium (fluid, gas, electric field, magnetic field, gravitational field.) by considering the average velocity is the speed of the wave caused in the medium due to the motion of the object in the medium with one of its available speeds. The potential energy replaces the complementary energy in case of single object.

2.5. De Broglie's Relations-Plank's Constant (*h*)

Tiny particles such as electrons, protons, atoms, etc., their velocity value can reach the speed of light or even close to the speed of light, so *c* the speed of light could be considered an available speed for the tiny particles.

The substitution with the value of the speed of light c in the particle-wave equation for a tiny particle *j i.e.* $v_{j,1} = c$ gives

$$\tilde{\beta}_j = m_j \tilde{v}_j c$$

Since the speed of light represents a moving wave of frequency ν and wavelength λ satisfying the relation $c = \lambda \nu$ the above relation is rewritten as

$$\tilde{\beta}_j = m_j \tilde{v}_j \lambda \nu = h\nu \tag{8}$$

Where

$$h = m_j \tilde{v}_j \lambda \tag{8.1}$$

$$p_j = m_j \tilde{v}_j, \quad p_j = \frac{h}{\lambda} \tag{8.2}$$

In section 2.3 a proof is given indicating that $\tilde{v}_j$ is a constant extremal value for the energy function $\mathcal{E}_j$, since p_j and λ are both constant values this result that the value of h given by relation (8.1) is a constant value which is Plank's constant [1] [2].

Relation (8.2) is a reformulation of relation (8.1) indicating that the momentum of the wave-particle object is equal to the momentum on a perfect path *i.e.* the particle-wave object moves as same way like a free motion particle (review section 2.3) [1] [2].

Relation (8) is an expression representing the complementary energy of a moving particle *j* as a multiplication of a wave frequency ν by a constant value *h*. Relations (8) and (8.2) are obtained by De Broglies using intuitive empirical method [1] [2].

2.6. Wave Packet-Energy Quantization

In this section the value of the second velocity $v_{j,2}$ is considered to be the rotating velocity for particle *j*. Using the particle mean velocity equation given in section 2.3 the second velocity with a magnitude $v_{j,2} = v_j$ is expressed as

$$v_j = 2\tilde{v}_j - c \tag{9}$$

Since the angular velocity $\omega_j = \dot{\phi}_j$ and R_j is the distance from the rotating center then, Equation (9) can be rewritten as

$$R_j \omega_j = 2\tilde{v}_j - c \Rightarrow \dot{\phi}_j = \frac{2\pi}{R_j} \frac{\tilde{v}_j}{\pi} - \frac{\lambda_j \nu_1}{R_j}$$

where ν_1, is the first wave frequency obtained in the previous section

The above equation in terms of the wave variables is written as

$$\dot{\phi}_j = k_j v_{jp} - \nu_2 \tag{9.1}$$

The following are the conversion relations from particle variables to the wave variables:

- k, is the magnitude of the wave vector given by $k = \dfrac{2\pi}{R_j}$
- v_{jp}, is the phase velocity given by $v_{jp} = \dfrac{\tilde{v}_j}{\pi}$
- λ_j, is the wave length of the associated wave

In case $2R_j \geq \lambda_j$, $2R_j = n_j \lambda_j$, where n_j is a natural number.

In case $n_j = 2$, $R_j = \lambda_j$ the magnitude of the wave vector k becomes $\frac{2\pi}{\lambda_j}$ which is well known result obtained by quantum method [1].

In case the rotating center inside the particle or on its boundary the rotation is considered spinning, if the rotating center outside the particle the rotation is considered revolving. Reflecting and refracting (bending) of particle-wave object are considered a kind of rotation.

- ν_2, is the second wave frequency of the associate wave given by $\nu_2 = \frac{2\nu_1}{n_j}$

Integrating Equation (9.1) gives

$$\phi_j = \phi_{0j} + ks_j - \nu_2 t \tag{9.2}$$

Equation (9.2) is valid in the interval, $0 \le s_j \le v_{pj}\tau_j$, $0 \le t \le \tau_j$, and describing the phase of a plane wave propagating along the particle displacement direction s_j.

Substituting from (8.1) into the phase velocity relation gives, $v_{jp} = \frac{h}{\pi m_j \lambda_j}$, so a dispersion takes place, each velocity generates a corresponding wave with the same wave length λ and two frequencies ν_1, ν_2 related by the relation $\nu_2 = \frac{2\nu_1}{n_j}$ the addition of the two waves creates a wave packet [1].

Substituting in Equation (8.2) with the wave length in terms of the distance from the rotating center gives

$$p_j = n_j \frac{h}{2R_j}$$

From Equation (7.2), Equation (8.2), and the above relation the energy of a particle *j* moving with speed $\tilde{v}_j$ is

$$\mathcal{E}_j\left(\tilde{v}_j\right) = -\frac{1}{2} m_j \tilde{v}_j^2 = -\frac{p_j^2}{2m_j} = -n_j^2 \frac{h^2}{8m_j R_j^2}$$

The above formula is known as the energy quantization formula obtained by the methods of quantum mechanics [1] [2].

2.7. Schrodinger's Formulation

Schrodinger's formulation describes the motion of moving particles using a wave equation which is a partial differential equation its solution is a wave function [1]. The square value of the wave function results the corresponding probability value for the parameters of the moving particle such as speed, energy, and displacement [1]. In order to switch the deterministic relations to be expressed in probabilistic format the following procedure adopted to achieve successful convergence process.

From section 2.3 the particle mean velocity, $\tilde{v}_j$, is expressed as

$$\tilde{v}_j = \frac{v_{j,1} + v_{j,2}}{2} = \frac{1}{2} v_{j,1} + \frac{1}{2} v_{j,2}, \quad j = 1, \cdots, N$$

The particle mean velocity relation can be interpreted as follows the probability of the first speed $v_{j,1}$ is $\frac{1}{2}$ and the probability of the second speed $v_{j,2}$ is $\frac{1}{2}$ then $\tilde{v}_j$ is the value of expected speed and that is the quantum interpretation of the particle mean velocity relation. Instead of using the equal fixed probabilities value $\frac{1}{2}$ for each speed which is a special case, a variable functions can replace the equal fixed probabilities value $\frac{1}{2}$. There are two available sets of functions can fully fill the replacing purpose explained previously. The first set is trigonometric functions $\sin\theta, \cos\theta$ where $\sin^2\theta + \cos^2\theta = 1$. The second set is hyperbolic functions

$-\sinh^2\theta, \cosh^2\theta$ where $-\sinh^2\theta + \cosh^2\theta = 1$ in this article the trigonometric functions used to represent the required probabilities.

From section 2.3

$$v_{j,1,2} = \tilde{v}_j \pm d_j, \quad j = 1, \cdots, N \tag{7}$$

in which

$$d_j = \sqrt{\tilde{v}_j^2 + \tilde{v}_{\perp j}^2}, \quad \tilde{v}_{\perp j} = \pm\sqrt{\frac{2\mathcal{E}_j}{m_j}}, \quad j = 1, \cdots, N \tag{7.1}$$

Figure 1 is the geometrical representation of Equation (7.1) is a right triangle its sides $\tilde{v}_j$, $\tilde{v}_{\perp j}$, and d_j. It will be shown the fundamental contribution of the three sides of the triangle as well as the angel $\tilde{\theta}_j$ in determining and finding the appropriate structure of the wave function and the wave vector as a solution for Schrodinger's equation.

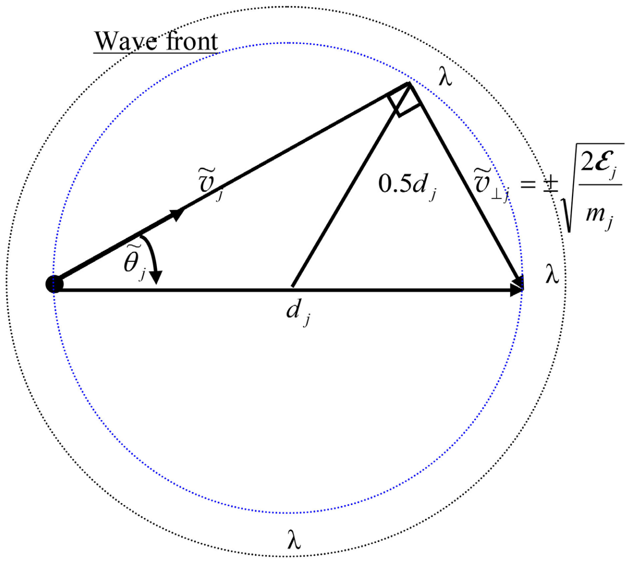

Figure 1. Geometrical representation of Equation (7.1).

From **Figure 1** the geometrical representation of Equation (7.1) $\sin\theta_j$ is written as follows:

$$\pm\sin\theta_j = \pm\frac{\sqrt{\dfrac{2\mathcal{E}_j}{m_j}}}{d_j}$$

Substituting $\theta_j = k_{0j}x_j$ where x_j is the displacement of particle *j*. Specifying the value of k_{0j} requires three steps, first step is expanding $\sin k_j x_j$ in power series and second step is equating the expansion with the above relation and third step is adjusting the $\sin k_{0j}x_j$ ratio to match the value of k_{0j}. The previous steps will be implemented as follows:

$$\sin k_{0j}x_j = k_{0j}x_j - \frac{\left(k_{0j}x_j\right)^3}{3!} + \frac{\left(k_{0j}x_j\right)^5}{5!} - \frac{\left(k_{0j}x_j\right)^7}{7!} + \frac{\left(k_{0j}x_j\right)^9}{9!} - \cdots$$

$$\frac{\sqrt{\dfrac{2\mathcal{E}_j}{m_j}}}{d_j} = \frac{1}{\hbar}\sqrt{2m_j\mathcal{E}_j}\left(\frac{\hbar}{m_j d_j}\right) = k_{0j}\left(x_j - \frac{k_{0j}^2\left(x_j\right)^3}{3!} + \frac{k_{0j}^4\left(x_j\right)^5}{5!} - \frac{k_{0j}^6\left(x_j\right)^7}{7!} + \frac{k_{0j}^8\left(x_j\right)^9}{9!} - \cdots\right)$$

Comparing the left side with right side of the above equation results.

$$k_{0j} = \frac{1}{\hbar}\sqrt{2m_j\mathcal{E}_j}$$

Substituting with k_{0j} value into $\sin k_{0j}x_j$ gives.

A wave function ψ_j for a particle *j* is written as:

$$\psi_j = \sin k_{0j}x_j$$

The same wave function ψ_j for a particle *j* can be obtained also by solving the Schrodinger's amplitude equation given as [1] [3]:

$$\frac{\mathrm{d}^2\psi_j}{\mathrm{d}x_j^2} + \frac{2m_j}{\hbar^2}\mathcal{E}_j\psi_j = 0 \quad \text{or} \quad \frac{\mathrm{d}^2\psi_j}{\mathrm{d}x_j^2} + k_{0j}^2\psi_j = 0 \quad \text{where} \quad k_{0j} = \frac{1}{\hbar}\sqrt{2m_j\mathcal{E}_j}$$

The wave function ψ_j is the solution of the above Schrodinger's equation after applying the boundary conditions, to describe the motion of a particle *j* enclosed in a potential well where the potential energy vanishes inside the well, Also the wave function ψ_j is the solution of the problem of passage of particles through the potential barrier. Tunnel effect [1] [3].

Substituting in $\cos\theta_j$ with $\theta_j = k_{1j}x_j$ and From Figure1 the geometrical representation of Equation (7.1) $\cos\theta_j$ is written as follows

$$\cos\theta_j = \cos k_{1j}x_j = \frac{\tilde{v}_j}{d_j}$$

From section (2.3) the relation between the total energy of particle *j* $\mathcal{E}_j$ and its potential energy V_j is given as

$$\mathcal{E}_j\left(v_j\right) = \frac{1}{2}m_j\left(v_j - \tilde{v}_j\right)^2 - \frac{1}{2}m_j\tilde{v}_j^2 = -\frac{1}{2}m_j\tilde{v}_j^2 + V_j, \quad V_j = \frac{1}{2}m_j\left(v_j - \tilde{v}_j\right)^2, \quad j = 1,\cdots,N \quad (7.2)$$

The above formula is valid for values of v_j and $\tilde{v}_j^2$ satisfying the relation $\frac{1}{2}m_j\left(v_j^2 + \tilde{v}_j^2\right) = 0$.

$\tilde{v}_j = \frac{v_{j,1}+v_{j,2}}{2} \rightarrow v_{j,1}^2 + \tilde{v}_j^2 = v_{j,1}^2 + \frac{\left(v_{j,1}+v_{j,2}\right)^2}{4} = 0$ which leads to the quadratic equation $5v_{j,1}^2 + 2v_{j,2}v_{j,1} + v_{j,2}^2 = 0$ the same for $v_{j,2}$ we get the quadratic equation $5v_{j,2}^2 + 2v_{j,1}v_{j,2} + v_{j,1}^2 = 0$.

From the expansion of $\cos k_{1j}x_j$ in power series and the above formula and adjusting the $\cos k_{1j}x_j$ ratio obtained from **Figure 1** the geometrical representation of Equation (7.1) to match k_{1j} value we get

$$\frac{\tilde{v}_j}{d_j} = \tilde{v}_j \frac{\hbar}{\hbar d_j} = \pm\frac{1}{\hbar}\sqrt{2m_j\left(V_j-\mathcal{E}_j\right)}\left(\frac{\hbar}{m_j d_j}\right)$$

$$= \pm k_{1j}\left(\frac{1}{k_{1j}} - \frac{k_{1j}\left(x_j\right)^2}{2!} + \frac{k_{1j}^3\left(x_j\right)^4}{4!} - \frac{k_{1j}^5\left(x_j\right)^6}{6!} + \frac{k_{1j}^7\left(x_j\right)^8}{8!} - \cdots\right)$$

Comparing the left side with right side of the above equation results

$$\pm k_{1j} = \pm\frac{1}{\hbar}\sqrt{2m_j\left(V_j-\mathcal{E}_j\right)}$$

A wave function ψ_j for a particle *j* is written as

$$\psi_j = \cos k_{1j}x_j$$

The same wave function ψ_j for a particle *j* can be obtained also by solving the Schrodinger's amplitude equation given as [1].

$$\frac{\mathrm{d}^2\psi_j}{\mathrm{d}x_j^2} + k_{1j}^2\psi_j = 0 \quad \text{where} \quad \pm k_{1j} = \pm\frac{1}{\hbar}\sqrt{2m_j\left(V_j-\mathcal{E}_j\right)}$$

The wave function ψ_j is the solution of the above Schrodinger's equation after applying the boundary conditions, to describe the motion of a particle *j* in a region where the potential energy has a value V_j [1].

2.8. Solutions for Time Dependent Schrodinger's Equation

In this section proofs are given for some wave functions to be solutions for time dependent Schrodinger's equation based on the results obtained in the previous section.

Some relations required to derive the proposed solutions for time dependent Schrodinger's equation. The purpose of these relations is to transform the particle parameters to the wave parameters to be adequate to Schrodinger's equation.

- First we write d_j in terms of k_{0j} and k_{1j}

We have the deviation d_j is given as follows

$$d_j = \sqrt{\tilde{v}_j^2 + \tilde{v}_{\perp j}^2}, \quad \tilde{v}_{\perp j} = \pm\sqrt{\frac{2\mathcal{E}_j}{m_j}}, \quad j = 1,\cdots,N \tag{7.1}$$

$$d_j^2 = \tilde{v}_j^2 + \frac{2\mathcal{E}_j}{m_j} \rightarrow m_j^2 d_j^2 = m_j^2\tilde{v}_j^2 + 2m_j\mathcal{E}_j$$

From the previous section we have

$$k_{0j} = \frac{1}{\hbar}\sqrt{2m_j\boldsymbol{\mathcal{E}}_j} \to \hbar^2 k_{0j}^2 = 2m_j\boldsymbol{\mathcal{E}}_j$$

$$\pm k_{1j} = \pm\frac{1}{\hbar}\sqrt{2m_j\left(V_j - \boldsymbol{\mathcal{E}}_j\right)} \to \hbar^2 k_{1j}^2 = 2m_j\left(V_j - \boldsymbol{\mathcal{E}}_j\right)$$

Using the relation (7.2) and the above relation we get

$$\boldsymbol{\mathcal{E}}_j\left(v_j\right) = -\frac{1}{2}m_j\tilde{v}_j^2 + V_j \to m_j^2\tilde{v}_j^2 = 2m_j\left(V_j - \boldsymbol{\mathcal{E}}_j\right) = \hbar^2 k_{1j}^2$$

From the above relations $m_j^2 d_j^2$ is rewritten as follows

$$m_j^2 d_j^2 = \hbar^2 k_{0j}^2 + \hbar^2 k_{1j}^2$$

Now we write the functions $\sin\theta_j$ and $\cos\theta_j$ in terms of k_{0j} and k_{1j} from the previous section and the substitution with value of $m_j^2 d_j^2$ given above we get

$$\sin^2\theta_{1j} = \frac{k_{0j}^2\hbar^2}{m_j^2 d_j^2} = \frac{k_{0j}^2\hbar^2}{\hbar^2 k_{0j}^2 + \hbar^2 k_{1j}^2} = \frac{k_{0j}^2}{k_{0j}^2 + k_{1j}^2},$$

$$\cos^2\theta_{2j} = \frac{k_{1j}^2\hbar^2}{m_j^2 d_j^2} = \frac{k_{1j}^2\hbar^2}{\hbar^2 k_{0j}^2 + \hbar^2 k_{1j}^2} = \frac{k_{1j}^2}{k_{0j}^2 + k_{1j}^2}$$

Using the equality $\sin^2\theta + \cos^2\theta = 1$ and multiplying by i we get

$$i\cos\theta_{1j} = \sin\theta_{1j}\sqrt{1 - \frac{1}{\sin^2\theta_{1j}}}, \quad i\sin\theta_{2j} = \cos\theta_{2j}\sqrt{1 - \frac{1}{\cos^2\theta_{2j}}}$$

$$i\cos\theta_{1j} = \sin\theta_{1j}\sqrt{\frac{k_{0j}^2 - \left(k_{0j}^2 + k_{1j}^2\right)}{k_{0j}^2}} = \frac{ik_{1j}}{k_{0j}}\sin\theta_{1j}$$

$$i\sin\theta_{2j} = \cos\theta_{2j}\sqrt{\frac{k_{1j}^2 - \left(k_{0j}^2 + k_{1j}^2\right)}{k_{1j}^2}} = \frac{ik_{0j}}{k_{1j}}\cos\theta_{2j}$$

One dimension functions to be solutions for time dependent Schrodinger's equation

- $\psi_j = \sin\theta_{1j}$ where $\theta_{1j} = k_{0j}x + \frac{k_{0j}}{i\hbar k_{1j}}k_{2j}^2 t + \theta_{0j}$, $k_{2j} = \sqrt{V_j + \boldsymbol{\mathcal{E}}_j}$

Is a solution for time dependent Schrodinger's equation given as [1]

$$-i\hbar\frac{\partial\psi_j(x,t)}{\partial t} = \frac{\hbar^2}{2m_j}\frac{\partial^2\psi_j(x,t)}{\partial x^2} - V_j(x)\psi_j(x,t)$$

Proof

The first term on the right hand side

$$\frac{\hbar^2}{2m_j}\frac{\partial^2\psi_j(x,t)}{\partial x^2} = -\frac{\hbar^2}{2m_j}k_{0j}^2\sin\theta_{1j} = -\boldsymbol{\mathcal{E}}_j\sin\theta_{1j}$$

$$\text{R.H.S} = -\boldsymbol{\mathcal{E}}_j\sin\theta_{1j} - V_j(x)\sin\theta_{1j} = -\left(\boldsymbol{\mathcal{E}}_j + V_j\right)\sin\theta_{1j} = -k_{2j}^2\sin\theta_{1j}$$

The left hand side

$$-i\hbar\frac{\partial\psi_j(x,t)}{\partial t} = -\hbar\frac{k_{0j}}{i\hbar k_{1j}}k_{2j}^2 i\cos\theta_{1j} = -\hbar\frac{k_{0j}}{i\hbar k_{1j}}k_{2j}^2\frac{ik_{1j}}{k_{0j}}\sin\theta_{1j} = -k_{2j}^2\sin\theta_{1j} = \text{R.H.S}$$

Let the frequency $v=\frac{k_{0j}}{ihk_{1j}}k_{2j}^2$ in case the potential energy $V_j=0$ the frequency becomes $v=\frac{k_{0j}}{ihk_{1j}}k_{2j}^2=\frac{\frac{1}{\hbar}\sqrt{2m_j\mathcal{E}_j}}{i\hbar\frac{1}{\hbar}\sqrt{-2m_j\mathcal{E}_j}}\mathcal{E}_j=\frac{\mathcal{E}_j}{\hbar}\to\mathcal{E}_j=\hbar v$ comparing this result with De Broglie's relations obtained in section (2.5) $\tilde{\beta}_j=hv$ using the relation $\mathcal{E}_j=\tilde{\alpha}_j\mathcal{E}+\tilde{\beta}_j$ we get

$$\tilde{\alpha}_j\mathcal{E}=\hbar v-hv=v\left(\hbar-h\right)=hv\left(\frac{1}{2\pi}-1\right)$$

- $\psi_j=\cos\theta_{2j}$ where $\theta_{2j}=k_{1j}x+\frac{k_{1j}}{i\hbar k_{0j}}k_{3j}^2t+\theta_{0j}$, $k_{3j}=\sqrt{\mathcal{E}_j-2V_j}$

is a solution for time dependent Schrodinger's equation.

Proof

The first term on the right hand side

$$\frac{\hbar^2}{2m_j}\frac{\partial^2\psi_j\left(x,t\right)}{\partial x^2}=-\frac{\hbar^2}{2m_j}k_{1j}^2\cos\theta_{2j}=-\left(V_j-\mathcal{E}_j\right)\cos\theta_{2j}$$

$$\text{R.H.S}=-\left(V_j-\mathcal{E}_j\right)\cos\theta_{2j}-V_j\cos\theta_{2j}=\left(\mathcal{E}_j-2V_j\right)\cos\theta_{2j}=k_{3j}^2\cos\theta_{2j}$$

The left hand side

$$-i\hbar\frac{\partial\psi_j\left(x,t\right)}{\partial t}=\hbar\frac{k_{1j}}{i\hbar k_{0j}}k_{3j}^2 i\sin\theta_{2j}=\hbar\frac{k_{1j}}{i\hbar k_{0j}}k_{3j}^2\frac{ik_{0j}}{k_{1j}}\cos\theta_{2j}=k_{3j}^2\cos\theta_{2j}=\text{R.H.S}$$

- $\psi_j=\mathrm{e}^{\theta_{3j}}$ where $\theta_{3j}=k_{0j}x+\frac{\hbar}{2m_j}\frac{k_{1j}^2}{i}t+\theta_{0j}$

is a solution for time dependent Schrodinger's equation.

Proof

The first term on the right hand side

$$\frac{\hbar^2}{2m_j}\frac{\partial^2\psi_j\left(x,t\right)}{\partial x^2}=\frac{\hbar^2}{2m_j}k_{0j}^2\mathrm{e}^{\theta_{3j}}=\mathcal{E}_j\mathrm{e}^{\theta_{3j}}$$

$$\text{R.H.S}=\mathcal{E}_j\mathrm{e}^{\theta_{3j}}-V_j\mathrm{e}^{\theta_{3j}}=-\left(V_j-\mathcal{E}_j\right)\mathrm{e}^{\theta_{3j}}=-\frac{\hbar^2}{2m_j}k_{1j}^2\mathrm{e}^{\theta_{3j}}$$

The left hand side

$$-i\hbar\frac{\partial\psi_j\left(x,t\right)}{\partial t}=-i\hbar\frac{\hbar}{2m_j}\frac{k_{1j}^2}{i}\mathrm{e}^{\theta_{3j}}=-\frac{\hbar^2}{2m_j}k_{1j}^2\mathrm{e}^{\theta_{3j}}=\text{R.H.S}$$

- $\psi_j=\mathrm{e}^{\theta_{4j}}$ where $\theta_{4j}=k_{1j}x+\frac{\hbar}{2m_j}\frac{k_{0j}^2}{i}t+\theta_{0j}$

is a solution for time dependent Schrodinger's equation.

Proof

The first term on the right hand side

$$\frac{\hbar^2}{2m_j}\frac{\partial^2\psi_j\left(x,t\right)}{\partial x^2}=\frac{\hbar^2}{2m_j}k_{4j}^2\mathrm{e}^{\theta_{4j}}=\left(V_j-\mathcal{E}_j\right)\mathrm{e}^{\theta_{4j}}$$

$$\text{R.H.S}=\left(V_j-\mathcal{E}_j\right)\mathrm{e}^{\theta_{4j}}-V_j\mathrm{e}^{\theta_{4j}}=-\mathcal{E}_j\mathrm{e}^{\theta_{4j}}=-\frac{\hbar^2}{2m_j}k_{0j}^2\mathrm{e}^{\theta_{4j}}$$

The left hand side

$$-i\hbar\frac{\partial\psi_j\left(x,t\right)}{\partial t}=-i\hbar\frac{\hbar}{2m_j}\frac{k_{0j}^2}{i}\mathrm{e}^{\theta_{4j}}=-\frac{\hbar^2}{2m_j}k_{0j}^2\mathrm{e}^{\theta_{4j}}=\text{R.H.S}$$

Three dimensions functions to be solutions for time dependent Schrodinger's equation

- $\psi_j=\sin\theta_{1j}$ where $\theta_{1j}=k_{x0j}x+k_{y0j}y+k_{z0j}z+\frac{k_{0j}}{ihk_{1j}}k_{2j}^2t+\theta_{0j}$, $k_{2j}=\sqrt{V_j+\mathcal{E}_j}$

$$k_{x0j}=c_1k_{0j},k_{y0j}=c_2k_{0j},k_{z0j}=c_3k_{0j}\quad\text{where}\quad c_1^2+c_2^2+c_3^2=1$$

Suggested values for c_1,c_2,c_3 $c_1=\frac{1}{\sqrt{3}},c_2=\frac{1}{\sqrt{3}},c_3=\frac{1}{\sqrt{3}}$ or $c_1=\frac{1}{\sqrt{2}},c_2=\frac{1}{2},c_3=\frac{1}{2}$ or $c_1=\sqrt{\frac{1}{5}},c_2=\sqrt{\frac{2}{5}},c_3=\sqrt{\frac{2}{5}}$

$$\sqrt{k_{x0j}^2+k_{y0j}^2+k_{z0j}^2}=\sqrt{k_{0j}^2\left(c_1^2+c_2^2+c_3^2\right)}=k_{0j}\to k_{0j}^2=k_{x0j}^2+k_{y0j}^2+k_{z0j}^2$$

is a solution for time dependent Schrodinger's equation given as [1] [4].

$$-i\hbar\frac{\partial\psi_j}{\partial t}=\frac{\hbar^2}{2m_j}\left(\frac{\partial^2\psi_j}{\partial x^2}+\frac{\partial^2\psi_j}{\partial y^2}+\frac{\partial^2\psi_j}{\partial z^2}\right)-V_j\psi_j$$

Proof

The first term on the right hand side

$$\frac{\hbar^2}{2m_j}\left(\frac{\partial^2\psi_j}{\partial x^2}+\frac{\partial^2\psi_j}{\partial y^2}+\frac{\partial^2\psi_j}{\partial z^2}\right)=-\frac{\hbar^2}{2m_j}\left(k_{x0j}^2+k_{y0j}^2+k_{z0j}^2\right)\sin\theta_{1j}$$
$$=-\frac{\hbar^2}{2m_j}k_{0j}^2\sin\theta_{1j}=-\mathcal{E}_j\sin\theta_{1j}$$

Continue the proof the same steps as the proof in one dimension function.

The argument θ_{1j} can be rewritten

$$\theta_{1j}=k_{x0j}x+k_{y0j}y+k_{z0j}z+\frac{k_{0j}}{ihk_{1j}}k_{2j}^2t+\theta_{0j}=k_{0j}r_j\cos\varphi_j+\frac{k_{0j}}{ihk_{1j}}k_{2j}^2t+\theta_{0j}$$

where $\boldsymbol{r}_j$ is the radius vector with components $r_j\left(x,y,z\right)$ and $\boldsymbol{k}_{0j}$ is the wave vector with components $\boldsymbol{k}_{0j}\left(k_{x0j},k_{y0j},k_{z0j}\right)$ their dot product is given by $k_{0j}r_j\cos\varphi_j$, φ_j is the angel between the two vectors $\boldsymbol{r}_j$ and $\boldsymbol{k}_{0j}$. $\cos\varphi_j$ is a wave function could be used to interpret the spinning motion of the particle.

The following functions are solutions for time dependent Schrodinger's equation the proof steps as the same proofs given above

$$\psi_j = \cos\theta_{2j} \quad \text{where} \quad \theta_{2j} = k_{x1j}x + k_{y1j}y + k_{z1j}z + \frac{k_{1j}}{ihk_{0j}}k_{3j}^2 t + \theta_{0j}\,, \quad k_{3j} = \sqrt{\mathcal{E}_j - 2V_j}$$

$$\psi_j = \mathrm{e}^{\theta_{3j}} \quad \text{where} \quad \theta_{3j} = k_{x0j}x + k_{y0j}y + k_{z0j}z + \frac{\hbar}{2m_j}\frac{k_{1j}^2}{i}t + \theta_{0j}$$

$$\psi_j = \mathrm{e}^{\theta_{4j}} \quad \text{where} \quad \theta_{4j} = k_{x1j}x + k_{y1j}y + k_{z1j}z + \frac{\hbar}{2m_j}\frac{k_{0j}^2}{i}t + \theta_{0j}$$

3. Conclusions

- The derivation for De Broglie's relations, the energy quantization formula, and the wave functions of a solution of Schrodinger's equation implies the consistent accommodation of the deterministic approach and the probabilistic approach, which led to a successful merge of the classical mechanics and quantum theory and erased completely the boundaries between the two approaches which is the basic purpose for this article.
- The particle- wave equation indicates the possibility of the existence of an associated wave to any moving object, A necessary proportional constant is required to be substituted in the particle- wave equation in order to link the particle's parameters to the wave parameters, Planck's constant is the adequate constant in case of tiny particles its substitution in the particle- wave equation results De Broglie's relations. Further investigation is required to apply the particle-wave equation to all moving masses with different sizes, not only tiny particles.
- The De Broglie's wave is a physical wave and the wave function solution of Schrodinger's equation is a mathematical wave to be used to calculate the probabilities of particle's parameters. The relation between the two waves can be obtained from the deduced formulas in this article.
- The solution for time dependent Schrodinger's equation in three dimensions shows a wave function could be used to interpret the spinning motion of the particles.
- The merging process between particle's parameters (displacement, velocity, momentum, energy) and wave parameters (frequency, wave length, wave vector, speed) required intermediate parameters (complementary energy, mean velocity, deviation from mean velocity) to successfully achieve the convergence process. The intermediate parameters could be used to make convenient calculations for both particle's parameters and wave parameters.
- The wave nature of the complementary energy is a required necessity to enable the interaction between particles to occur and transmission of energy to be achieved. The communication of a single particle with the rest of the universe is impossible without an associated wave to each particle.
- This is the last conclusion and the most interesting which is the motion of the particle-wave object can be represented by the smallest geometric unit a triangle, whose sides represent the particle's parameters and its angels represent the wave parameters. The analogy between the motion of the particle-wave object

and the triangle gives a new deeper understanding of the nature of the quantum phenomena from a geometrical perspective, as well as the correlation between the particle parameters and the wave parameters. The geometrical formulation of the quantum phenomena leads to a fact that solving Schrodinger's equation is like solving a triangle in trigonometry or plane geometry. Further investigations are required in order to unveil the geometrical foundation of the quantum theory.

4. Discussion

- The physical interpretation of the complementary energy in Section 2.1.

If there is an object that does not radiate thermal energy, magnetic field or electric field it does not mean on the macroscopic and microscopic scale there is no activity for that reason the summation of (β_j) is zero at the same time each particle has value for(β_j). It is a mathematical method to express equilibrium of the object without neglecting its internal reactions. It is analogous to some extent to the concept of internal energy in thermodynamics but varies.

- The physical interpretation of energy distribution coefficient" (α_j) in Section 2.1.

The energy distribution coefficient it is the magnitude of the share (quota, portion) of the total energy of the particle in the total energy of the system. which means that the total energy of the system is distributed among the particles.

- A definition for the complementary energy is the energy connecting the matter with radiation. The radiation is a wave natured.
- The title of the article "A link merges classical mechanics to quantum theory (Part I)", the link is a mathematical derivation process, starts with expressing the total energy of N moving particles and ends with derivation of De Broglie's relations and wave functions solution for Schrodinger's equation which shows the acquaintance with classical mechanics is incomplete, there still unknown aspects, future parts will shed the light on those aspects with collaboration with quantum theory.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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