

A Planar Atom Model of Helium Based on Well-Defined Electron Trajectories

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Abstract

The original intention of the author's preoccupation with the quantum-mechanical behaviour of simple atoms and molecules such as Hydrogen and Helium was, on the one hand, the elegant simplicity of Niels Bohr's atom model for Hydrogen, describing the metastable states of the excited electrons by planar concentric electron orbits, and, on the other hand, the hardly intelligible wave mechanical approach of Heisenberg, Schrödinger and others, describing mainly atoms with multiple electrons by three-dimensional orbitals which were characterized by probabilities of presence. Thereby the question arose whether it would be possible to find alternative atom models with well-defined electron trajectories. Therein, Louis de Broglie's thesis of the wavy nature of electron motion implicating standing waves would have to be implemented. Nevertheless, as reviewed in the introduction, the orthodox three-dimensional concept influenced the own thinking in such a way that three-dimensional constellations for the electronic excited states were conceived. The breakthrough was achieved for the electronic ground state in the form of the spin-orbit coupling where the spin acts as a perpetuum mobile, inducing the orbital angular momentum $\hbar/2\pi$. Furthermore, the insight was gained that a circularly rotating electron intrinsically corresponds to a harmonic oscillator, thus fulfilling the condition of a standing wave. Based on this concept, a double planar model was established for the H_2 -molecule which could be empirically verified by X-ray data from literature. However, for the two electrons containing Helium a 2D-array seemed impossible since the Pauli-principle seemed to be violated. After a long stepwise succession of 3D-attempts which turned out to be impossible—not least since eccentric forces are not possible in such a system—the here presented 2D-version for Helium was found, composed by two imaginary orthogonal electron orbits. It will enable in a subsequent publication the quantum mechanical interpretation of the thermal-radiative behaviour of Helium which was reported in the author's publication nine years ago.

Keywords

Atom-Model of Niels Bohr, Questioning of the Orthodox,
Quantum-Mechanics, Spin-Orbit Coupling of Electrons,
Atomic Radius of Helium

1. Introduction

In 1913, Niels Bohr published his fundamental model for the Hydrogen atom assuming well-defined planar electronic orbits. It explained the respective discrete spectrum measured in the UV-range at very low pressures [1], attributing different electronic orbit-radii to different excited states, corresponding to different energy contents. These low pressures were required in order to avoid the recombination of the single atoms to H₂-molecules which exist at normal conditions. As a consequence, thermal effects due to interatomic collisions could be avoided.

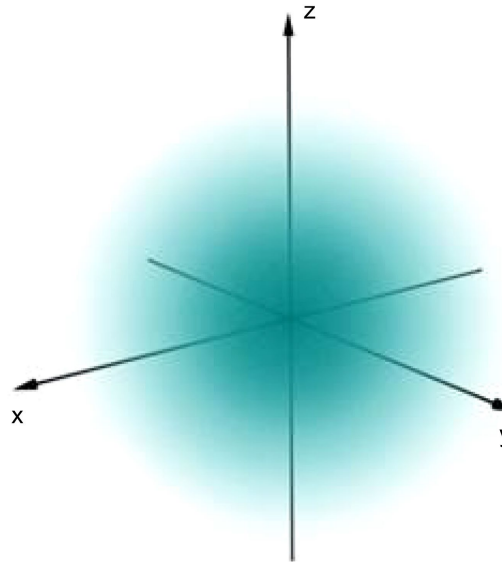
These radii can easily be computed by equalizing the Coulomb-force, which is due to the positive charged proton (=nucleus) and the negative charged electron, with the centrifugal force of the rotating electron, and by applying Einstein's formula $\Delta E = h \cdot \nu$ (h = Planck's constant) for the energy differences between the excited states, while the total energy of the electron can be computed by adding the kinetic and the potential energy. As the computation yields, the different excited states of the electron correspond to integer multiples of the angular momentum $h/2\pi$. Thus, the Planck-constant h has the character of an angular momentum with the unit Js, whereas the common indication "action quantum" appears to be misleading since the term "action", defined as the product of energy and time, is not generally used in physics. In particular, no respective conservation law exists as it is the case with respect to the angular momentum.

However, at that time no explanation could be found why the electron does not fall onto the nucleus—behaving like a Hertz dipole oscillator—instead remaining in a ground state with the angular momentum $h/2\pi$.

Ten years later, Louis de Broglie proposed a wave/particle-concept for the electronic motion, implying standing waves at metastable states. But since in Bohr's H-atom model such a wavy electronic motion was not describable, Heisenberg's "uncertainty principle" with respect of location and momentum delivered insofar an explanation as it attributed this electronic ground state to the impossibility of the electron running through a well-defined position. Based on this, so-called wave-mechanics were developed, particularly by Schrödinger, whose differential equation provided discrete energy-Eigenvalues and—instead of well-defined electron trajectories—three-dimensional electronic "orbitals" with probabilities of presence [2]. This theory is still obliging, so it may be called as orthodox.

However, considerable objections against this theory may be alleged. First of all, the theorem may be adduced that in natural science, in particular in physics, solely a unique explanation may be valid. But in the case of the ground state of

Hydrogen, two different explanations of its ground state are given in the form of different atom models, both using the Planck-constant: the planar one in the form of Bohr's model, and the orthodox one in the form of the spherical 1s-orbital (**Figure 1**).



Orbital s ($\ell = 0$, $m_\ell = 0$)

Figure 1. 1s-orbital, according to

https://de.wikipedia.org/wiki/Atomorbital#/media/Datei:Orbital_s1.png

Both are not directly observable, but both can explain observable phenomena, although various and sundry ones: Whereas Bohr's model can explain the discrete electronic spectrum of Hydrogen due to the excited metastable energy states, the orthodox model enables to explain the "Aufbau"-principle of the periodic system of the elements and the orientation of chemical bonds in the ground state. Thus, a comparison is solely reasonable in the ground state, namely with respect to the simplest elements Hydrogen and Helium.

Thereby, the most troubling aspect at the orthodox model is the absence of an angular momentum exhibiting the value $h/2\pi$ which occurs in Bohr's model and which involves a constant total energy for any stable or metastable case, being composed by potential and kinetic energy which are due to the centrifugal force and the Coulomb force between nucleus and electron. It implicates a defined electron radius as well as a defined electron velocity and thus a defined angular velocity or frequency. However, in the 1s-orbital of the orthodox model different radii and velocities—and therefore different energies—are possible whereas solely its average value is defined. That would imply an at least partly continuous electromagnetic spectrum and not a discrete one, which is due to the energy differences between the different energy states. It is characteristic for Helium, too, but it is not considered here since here solely the ground state is relevant. Moreover, it is

noteworthy that two different Schrödinger-equations exist, a time-dependent and a time-independent one—which contradicts the above theorem, too.

The fact that the orthodox theory delivers nevertheless correct results in many respects, in particular concerning real gases, may be due to the circumstance that in real gases the atoms or molecules are not fixed in an immovable position but rotate at least around several axes whereby the laws of probability are valid. This induces in the view of the observer a multiplicity of positions and may explain the uncertainty principle of Heisenberg.

An alternative, much more plausible explanation for the existence of a ground state may be delivered by the *electron spin* which was postulated in 1925/26 by Uhlenbeck and Goudsmith due to multiplett-lines in electronic beam spectra in the presence of strong magnetic fields [3] and [4]. The spin was implemented in the orthodox theory, implying the Pauli-principle—but only in the aftermath (even though the publications of Uhlenbeck and Goudsmith appeared at the same time), instead of building the entire theory on it, namely by considering the spin-orbit coupling where the spin effectively acts as a perpetuum mobile for the electron rotation.

And indeed: it turned out being possible to exactly compute the bond-length in the H_2 -molecule based on that assumption, verified by X-ray results which were available in the literature [5]. Therein, the chemical bond between the two protons (nuclei) is built by the two electrons which rotate in opposite parallel planes (**Figure 2**).

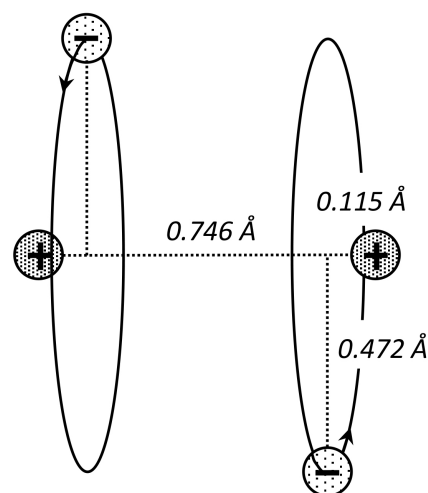


Figure 2. Model of the H_2 -molecule with well-defined electron orbits, according to [5].

The plot of the total molecular energy against the bond length yielded a potential-well, *i.e.* a minimal energy at the effective bond length of the H_2 -molecule (**Figure 3**). This finding resembles the well-known one reported by Heitler and London [6].

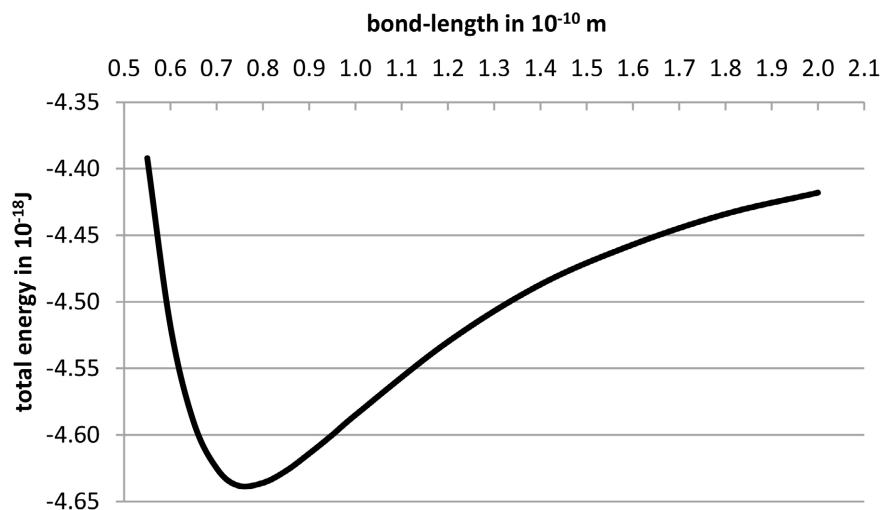


Figure 3. Total energy as a function of the bond-length at the intranuclear-antipodal array of the H_2 -molecule, according to [5].

Nevertheless, since I was accustomed as a chemist to use Kimball's charge cloud model which principally resembles the orthodox orbital model, I initially tried to describe the electronically excited states of the Hydrogen atom by electron waves fulfilling the standing wave condition (see [7] and [8]), but disregarding the fact that a three-dimensional motion of solely two particles (proton and electron) is not possible. Rather, the electronic rotation, being assumed in the Bohr-model, already corresponds to a harmonic oscillator, inherently fulfilling the standing wave condition. Thus, the planar H-atom of Bohr turned out to be accurate for any electronic state, for the ground state as well as for the excited states.

When I tried to develop an atom model with well-defined electron trajectories for the ground state of *Helium* assuming spin-orbit coupling, I was not aware of the impossibility of three-dimensional motion at two-particle systems or related ones. Since in the planar case (Figure 4) the Pauli-principle seemed to be violated, I proposed in [9] a *3D-version with eccentrically rotating electrons* (Figure 5):

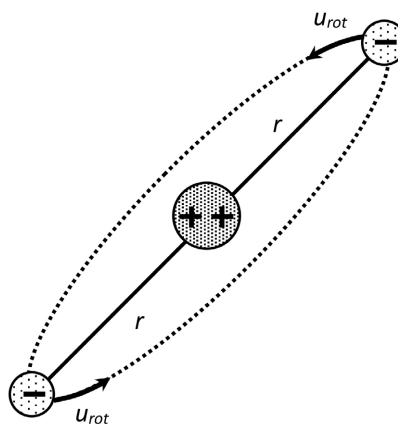


Figure 4. The concentric atom model of Helium, according to [10].

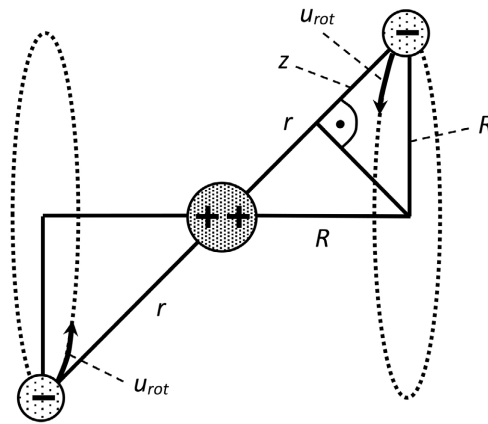


Figure 5. The eccentric atom model of Helium, according to [9].

But since this constellation turned out to be unstable, I proposed subsequently a *rotating rotor* (Figure 6 and Figure 7), which meanwhile was questioned, too, not least since it implicated variable electron velocities [11]:

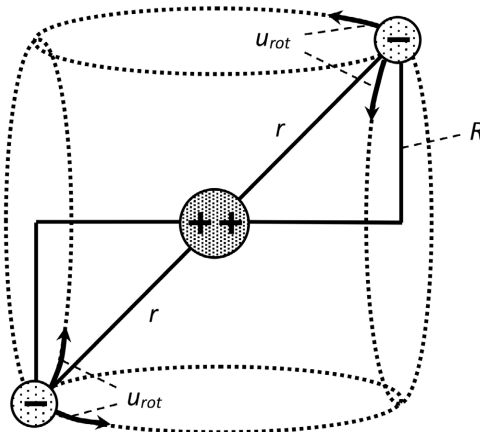


Figure 6. Spherical model of Helium (rotating rotor), according to [11].

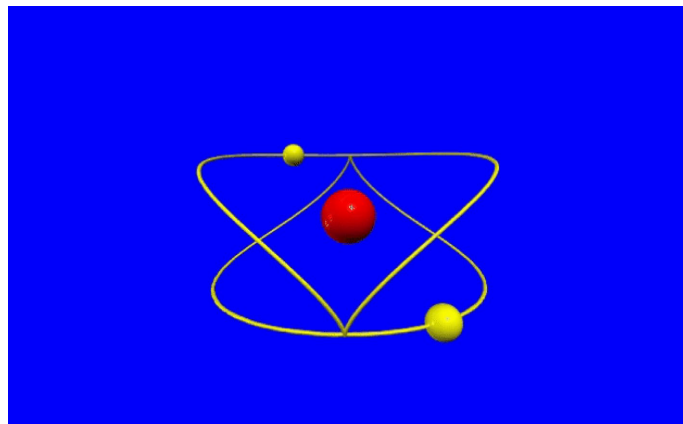


Figure 7. Freeze image of the 3D-animation according to [11].

Finally, I proposed in [12] a combination of a rotating electron with an oscillating one (Figure 8).

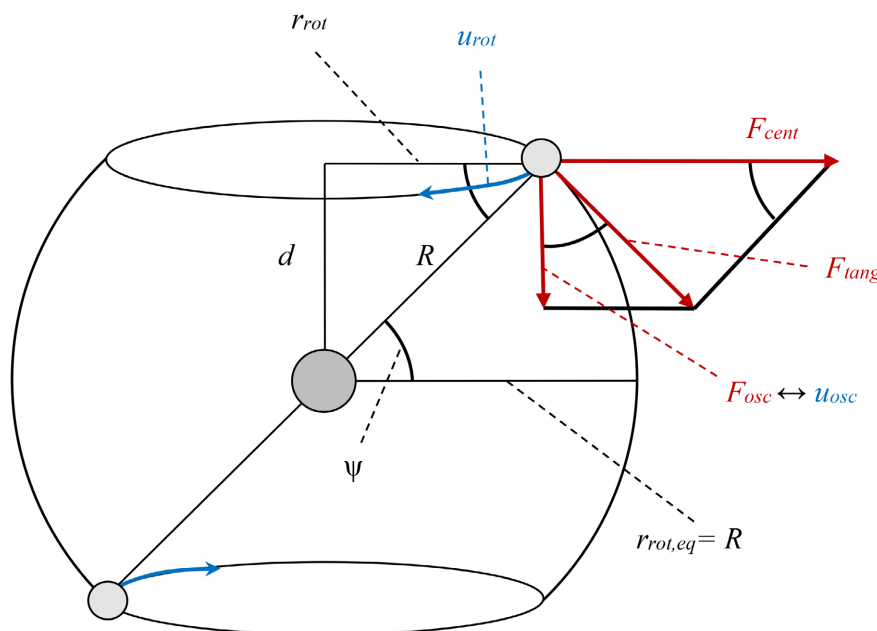


Figure 8. Another spherical model of Helium (oscillation combined with rotation), according to [12].

But as will be explained in the next chapter, this model is not correct, neither. Rather a two-dimensional planar solution is feasible, albeit differing from the initial version within an important item.

In contrast to the case of the Hydrogen-model where the above-mentioned empiric validation was possible by comparing the computed bond-length of H_2 with the measured one, in the case of noble gases such a verification is not possible since no stable bonds exist. On the other hand, the noble gases could enable to study the inter-atomic relations which are relevant in physical thermodynamics. In particular, this concerns the radiative behaviour of gases which was reported by the author already in 2016. The proposed atom model of Helium offers a quantum mechanical explanation of this phenomenon, allowing to bridge thermodynamics (which obeys the principles of probability) and quantum mechanics (which obeys exact physical laws). That application will be described in a subsequent publication in the same journal, delivering the empiric verification of this atom model.

2. Revision of the Former Atom Model for Helium

As mentioned in the introduction, already several approaches to an atom model for Helium with well-defined electron trajectories were made by the author, describing its ground state and implicating the angular momentum $h/2\pi$ for the electron motion. Thereby, the two electrons always run diametrically. In the original

planar version, depicted in **Figure 4**, both electrons exhibit the same rotating direction. If spin-orbit coupling is assumed, the Pauli-principle seems to be violated since both electrons occur in the same orbit but exhibit the same spin. Therefore, in [9] and in [10] spherical constellations of the electron orbits were proposed, depicted in the **Figure 5** and **Figure 6**, whereby in the latter case—as already mentioned and evident in **Figure 7**—the electron velocity was not constant.

At that time, the impossibility of a three-dimensional motion of such systems was not perceived. This ignorance was also predominant at the following model, described in [12] and schematically depicted in **Figure 8**.

Therein, the electron motion was provided along the surface of a sphere-zone, composed by a horizontal rotation exhibiting a continuously varying radius and a vertical oscillation, described by a harmonic oscillator. In contrast to the expected three-dimensional electron orbit, the computation yielded a nearly two-dimensional course (**Figure 9**):

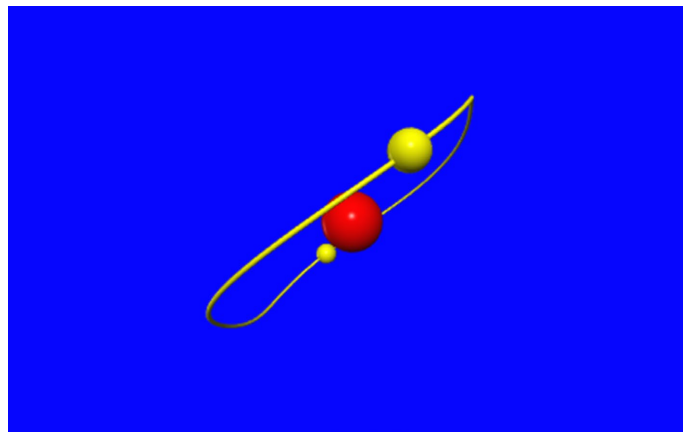


Figure 9. Freeze image of the 3D-animation according to [12].

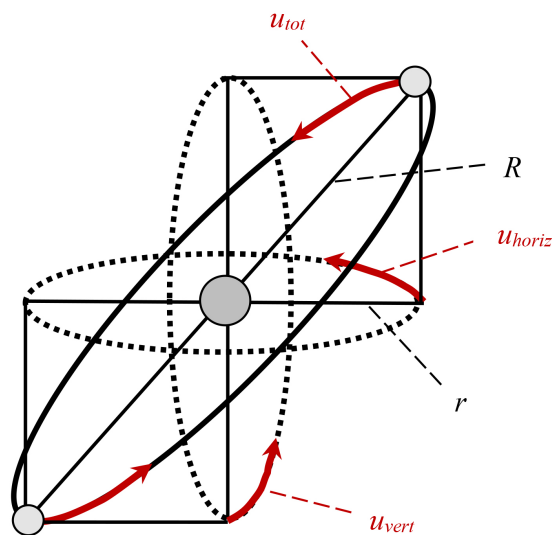


Figure 10. 2D-model of Helium composed by two imaginary orthogonal orbits of the electrons.

Meanwhile, some computation errors were found which will not be discussed in detail here. But above all, the assumption of an eccentric oscillation is questionable, so that this model has to be abandoned, too. Nevertheless, the appearance of a nearly two-dimensional electron orbit seemed to be interesting.

As a consequence, a two-dimensional model is presented here which is composed by two imaginary orthogonal electron orbits (Figure 10).

Thereby, the pivotal question arises how the two spin-induced orbital angular momenta exhibiting the value $\hbar/2\pi$ (Equation (1), related on one electron) can be implemented.

$$r \cdot u_{horiz} \cdot m_e = r \cdot u_{vert} \cdot m_e = r \cdot u_{part} \cdot m_e = \frac{\hbar}{2\pi} \tag{1}$$

whereby m_e = electron mass.

Are they additive as in the case of the excited states in Bohr’s Hydrogen model? That seems unlikely since they are not in the same plane. The answer is associated with the—so far disregarded—fact that the Planck constant \hbar intrinsically represents a *vector* since it expresses an angular momentum. As a consequence, the two orthogonal vectors must be added after squaring, yielding Equation (2):

$$R \cdot u_{tot} \cdot m_e = \sqrt{2} \cdot \frac{\hbar}{2\pi} \tag{2}$$

Moreover, the ordinary computation can be made by equating the centrifugal force of the electrons with their Coulomb-attraction.

Obeying the usual formula for the centrifugal force F_{cent} , the following relation is obtained:

$$F_{cent} = m_e \cdot (u_{tot})^2 / R \tag{3}$$

In order to compute the entire Coulomb force F_{coul} the interference of the partial Coulomb forces has to be determined. Hereto, the schema (depicted in Figure 11) can be adopted which has already been applied in [12].

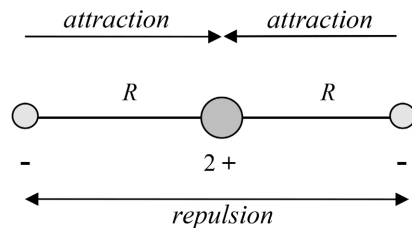


Figure 11. Interference of the Coulomb forces at the equator.

As obvious from the figure, each of the two attraction forces between the double positive nucleus and the singly negative electrons amounts to $2K/R^2$, whereas the repulsion between the two electrons amounts to $-K/4R^2$. Thereby K represents the natural constant $e^2/4\pi\epsilon_0$. When the interference is focussed onto the nucleus, the relative attraction is given by Formula (4):

$$F_{Coul} = \frac{\frac{2 \cdot 2K}{R^2} - \frac{K}{4R^2}}{2} = \frac{15K}{8R^2} \quad (4)$$

Now, the two forces F_{cent} and F_{Coul} can be equated, yielding Equation (5):

$$(u_{tot})^2 = \frac{15K}{8m_e \cdot R} \quad (5)$$

Finally, the Equations (2) and (5) can be combined, yielding the value for R :

$$R = \frac{16}{15K} \cdot \frac{1}{m_e} \cdot \left(\frac{h}{2\pi} \right)^2 \quad (6)$$

When R is known, u_{tot} can be computed using Equation (2), yielding Equation (7):

$$u_{tot} = \sqrt{2} \cdot \frac{h}{2\pi} \cdot \frac{1}{R} \cdot \frac{1}{m_e} \quad (7)$$

Moreover, the angular velocity ω can be computed using Equation (8):

$$u_{tot} = \omega \cdot R \rightarrow \omega = \frac{u_{tot}}{R} \quad (8)$$

while the frequency $\nu = \omega/2\pi$.

Using the basic data:

$$m_e = 0.9109382 \times 10^{-30} \text{ kg}$$

$$K = e^2/4\pi\epsilon_0 = 2.307 \times 10^{-28} \text{ J} \cdot \text{m}$$

$$h/2\pi = 1.0545716 \times 10^{-34} \text{ J} \cdot \text{s}$$

the following values are obtained:

$$R = 0.5644 \times 10^{-10} \text{ m}$$

$$u_{tot} = 2.901 \times 10^6 \text{ m} \cdot \text{s}^{-1}$$

$$\omega = 5.140 \times 10^{16} \text{ s}^{-1}$$

$$\nu = 0.818 \times 10^{16} \text{ s}^{-1}$$

3. Conclusion

As a result of the author's previously published attempts to create an atom model for Helium with well-defined electron trajectories, a two-dimensional solution was found, rooting on two imaginary perpendicular electron circles which are due to spin-orbit coupling. The computed rotation radius of $0.5644 \times 10^{-10} \text{ m}$ matches well the empirical value of $0.57 \times 10^{-10} \text{ m}$ which was reported in [13]. Thereby, it should be taken into consideration that the electromagnetic field, which is induced by the atomic components, is three-dimensional. Moreover, due to heat-motion, the atoms are in fact not immobile; at least they rotate around an axis. But above all, the atoms of a gas will collide, presumably inducing *oscillations of the electronic shell*. This will be subject of a subsequent publication, referring to the publication [13].

Acknowledgment

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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