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Matter and Quantum Entanglement

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Abstract

The iteration of one-dimensional holomorphic functions allows a definition of conductivity plateaus and charge quanta which are related to nontrivial zeros of the Riemann zeta function and the Dirichlet L-function. A minimal and maximal iterated spacetime is shown to be a quadratic map of curvature. A spacetime point is defined as a congruent maximal general Riemann surface. Incongruent k-components of curvature are proven to be a bicubic bi spinor. Pseudo congruent k-components explain the low vacuum energy density whereby a small count rate of cosmic ray-like tensile forces is predicted for e.g. a conductivity plateaus and plant growth giving an enhanced air ionization.

Keywords

Charge Definition, Quantum Entanglement, Cosmic Rays, Feigenbaum Renormalization, Air Ionization

1. Introduction

Experiments concerning the cosmological constant problem (CCP), quantum entanglement (QE) and the Dirac monopole (DM) seem to prevent a unified theory of all forces [1]-[3]. A fractal zeta universe as a cosmic-ray-charge-cloud-superfluid (FZU) resolves CCP, QE and DM by treating spacetime as a bifurcating process [4]. The origin of charge and mass in FZU is explained by Feigenbaum renormalization extending Hieb's hypothesis [5] [6]. Hieb' conjecture $2\pi\delta_F^2 \simeq \alpha_f^{-1}$ already has accuracy 9.12×10^{-4} with Feigenbaum constant δ_F and fine structure constant α_f which is refinable on entropy-surface-area $4\pi R^2 \simeq g_1^{:g_n}$ for $g_1 + \cdots + g_n$ [7] [8]. Similarly, FZU introduces dimensionless coupling constants G_w on w-spherical shells of a general Riemann surface enveloped by a general sphere where g_i enter as number theoretic generators which oversee 10^3 orders of magnitude. Like a black hole a given fractal point in space consists of w-coordinate spheres encapsulating a universe of buds, shoots, leaves of a tree and of bizarre plants as k-components which are period-doublings. The enveloping sphere represents matter as Einsteinian elastic spacetime. Whereas the Feigenbaum constant δ_F is transformation degree dependent the fine structure constant α_f is energy- and time-dependent (e.g. α_f at 91GeV ≃ 127.5) [9]. This would question any algebraic approach to δ_F and α_f [9]. FZU coupling constants G_w (h_t) create a cubic van-der-Waals-like potential minimum in dependence on topological entropy h_t: At high density of chaotic k-components quadratic and quartic mass terms are higher than the linear rest mass term which creates matter from zeros of the zeta function. A pseudo-congruence $2^{2^k} = G_w^{-1}$ between k-components on the maximal general Riemann surface is called QE and defines a charge [10]. This surprising result up to $k = 10$ reproduces the static limit of α_f which is also the infinite-energy limit of α_f and solves CCP [10]. A pseudo-congruence $2^{2^{10}} \simeq 1$ of optimal k-components in $\mathbb{C}^5$ of a quadratic map around nontrivial zeros of the Riemann zeta function ζ (z) is called charge. Complex-generator-cycles of optimal k-components 2^{2^k} in $\mathbb{C}^w$ for $w = 1, 2, 3, 4, 5$ are surrounded by vanishing Gaussian periods. For a highly-composite congruent number field $2^{2^k} = g^{2^k} = g^{n^2}$ with generator g as a root of unity a vanishing series of periods can occur for components k = 2, 4, 6, 8, 10 below $k \leq 10$. If generator 2^k is a square, i.e. for k = 2, 4, 6, 8, 10 five interactions w = 1, 2, 3, 4, 5 are in accordance with the Weber-Schottky conditions [11]. Matter as spacetime cavity cycles or buds of cyclic generators is inseparably connected with k pseudo-congruence. This form of QE yields air ionisation caused by cosmic rays (≡bifurcated spacetime) not only in the exosphere but also near plants and conductivity plateaus whenever a zeta function zero is iterated. FZU sees experimental support in measurements in vegetation areas [12]. Moreover, within FZU problems with cosmological constant, Dirac monopole and quantum entanglement can be overcome. The first one consists in a 50 - 200 orders of magnitude too large vacuum energy density Λ_c (which is the factor 2^{2^9}), the second in charges as a fusilli-like cloud of magnets (a ball of k-components) and the third in a spooky action at a distance confirmed by quantum experiment (k-correlation). The paper aims to discuss these problems by k-itineraries for a bifurcated spacetime curvature. Extending the concept of vibrations of fractal strings a spacetime point is defined by congruent and incongruent k-components of quadratic maps of complex curvature [13]. Charge quanta are set as component-correlated nontrivial zeros of the Riemann zeta function and the Dirichlet L-function [10] [14]. A unified vacuum is a non-dissipative, non-radiative highly k-component correlated liquid [10] [14]. Elliptic curves as attractors are already known as an exactly solvable chaos [15] [16]. Iterates of doubly-periodic lattices as chaotic period-doublings due to lattices of algebraic units have been rarely investigated. In this paper iterated cyclic periods ν_{Sh} of intervals according to the theorem of Sharkovskii belong to pseudo-random and pseudo-congruent lattices of fundamental units. Period-doubling is discussed as a hyperelliptic-elliptic addition step supported also by the Friedmann solution for time [17]

$$ct = \int \frac{\sqrt{R_u}\,\mathrm{d}R_u}{\sqrt{\phi_3\left(R_u\right)}} \tag{1}$$

in dependence on universe radius. Scalar curvature R

$$R = \frac{c^2}{R_u^2}$$

of a spherically symmetric universe differs from R_u in Equation (1) by an arbitrary quadratic map which is extended to complex plane by $\gamma(\phi_3)$. A map $z_{k+1} \leftarrow z_k^2 + c$ is written as the quartic polynomial $\left(\boldsymbol{R}_{\mu\nu}^4 + 2\mathcal{F}\boldsymbol{R}_{\mu\nu}^2 - \mathcal{G}^2\right) = 0$ for an arbitrary matrix $\boldsymbol{R}_{\mu\upsilon} = \operatorname{Re} z_k$ and $\mathcal{F} = \frac{1}{2}\operatorname{Re}\left(c - z_{k+1}\right)$, $\mathcal{G} = \frac{1}{2}\operatorname{Im}\left(c - z_{k+1}\right)$ invariant with respect to $\gamma(\phi_3)$. Under $\gamma(\phi_3)$ $\operatorname{Re} z_k$ is viewed as complex curvature $\boldsymbol{R}_{\mu\upsilon}$ or complex universe radius R_u. The quartic $\boldsymbol{R}_{\mu\nu}$ scales $z_k \to G_w z_k$ with renormalized charge $G_w = e$ is known from quantum electrodynamics [18]. With $\mathcal{F} = \frac{1}{2}\operatorname{Re}\left(c - z_{k+1}\right)$, $\mathcal{G} = \frac{1}{2}\operatorname{Im}\left(c - z_{k+1}\right)$ on complex plane oriented in space $\boldsymbol{z} = \sqrt{\mathcal{F} + i\mathcal{G}} \simeq \boldsymbol{E} + i\boldsymbol{B}$ is a complex unified field if the normal of the complex plane of z_k is oriented in space. Similarly, $\boldsymbol{R}_{\mu\upsilon}$ is a Kepler- or Coulomb field singularity $\simeq 1/r^2$ subjected to a quadratic map. A Mandelbrot map is part of a Hermite map

$$F(t,z) = \gamma\left(\phi_3(t)\right) \circ z \simeq \phi_3(t) / (t - z) - \frac{1}{3}\phi_3'(t)$$

for subsequent cubic roots e_i of ϕ_3 from subsequent quartic roots x_q of ϕ_4. Under $\gamma(\phi_3)$ the polynomial ϕ_3 in Equation (1) gets a modular invariant where its argument can be written as an algebraic unit. Complex cubic roots $z \simeq E \to E_{i,q} \to E_{i,s}$ as algebraic units E and quartic roots x with one quartic root shifted to ±∞, ±i∞ transmit to a 3·4 degrees of freedom i, $q \simeq s$. An iterated solution for the 4·4 matrix $\boldsymbol{R}_{\mu\upsilon}$ is searched in extended μ, s space with field densities $F + \gamma_5 G = -2^{-5} F^2$ with $F = \left[\gamma_\mu, \gamma_\nu\right]_- \boldsymbol{R}_{\mu\upsilon}$, $\gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_4$ and $\gamma_5^2 = -1$. Invariants in ϕ_3 can be written in terms of complex conjugated units of the bicubic field. Under period-doubling in $\gamma(\phi_3)$ $\mathcal{F}$ and $\mathcal{G}$ get differences of $\wp(\omega)$-functions as a factor of the Legendre module λ which depends on unit $E_q = e^J$ on four possible Feigenbaum stability axes q. Rotated cardioid planes $\boldsymbol{z} \simeq e^{\boldsymbol{F}} = chz + sh(z)\boldsymbol{F}/z$ depend on the quartic polynomial for $R = R_{\mu\nu}$, $z = |\boldsymbol{z}|$ with unit determinant [19]. Iterating $\exp\left(ig_1^{g_l}\gamma_{ss'}^\mu x_\mu\right) f_{s'}$ in Equation (13) $1/2\, w(w-1)$ parameters and $3w - 3$ equations yield an underdetermined system of maximal $w = 1, \ldots, 5$ independent complex planes [20]. The importance of the linear map γ consists in identifying $\delta x_q \simeq G_{ss'}$, $\delta e_i \simeq G_{ss'} G_{ss'}$ where

$$F(t,z) \simeq A_\mu G_{ss''} \gamma_{s''s'''}^\mu G_{s'''s'} - \frac{1}{3} \frac{\delta A_\mu}{\delta\left(G_{ss''} \gamma_{s''s'''}^\mu G_{s'''s'}\right)} \tag{2}$$

A congruence $\phi_3(z) \bmod \phi_4(x) = A_\mu \simeq a_\mu$ of polynomial $\Phi_n(t) = \sum_{i=0}^n a_i t^{n-i}$ for $\delta e_i \simeq \left(\delta x_q\right)^2$ and quadruples $\delta x_q \simeq \psi_q \simeq \psi_s$ is viewed as a potential like $z \simeq \wp(u)$. The coordinate index μ = 1, 2, 3, 4 denotes a self-consistent Feigenbaum stability axis from iterates around inflection tangents. The current $j_\mu = \bar{\psi}_s \gamma_{ss'}^\mu \psi_{s'}$ is quadratic in δx_q where $q = \{k, k+1, k+2, k+3\}$ denotes a quadruple of simplest cycles of iterated intervals and ψ_s has a bicubic norm. A point as irreducible quadruple $q \simeq 1, 2, 1'2'$ permeates FZU as matter-anti-matter, tidal forces and dark non-radiative exchange scattering.

2. Quadratic Map as Iterated Curvature

A quadratic map $z_{\kappa+1} \leftarrow z_\kappa^2 + c$ is written in extensions of a pure bicubic field $\mathbb{K}$ [∂] and $\mathbb{K}$ [$\partial^{1/2}$]. Variable z is a complex algebraic unit living in doubly-periodic lattices ω. General relativity can be written as a vanishing discriminant $\Delta_F[\phi_3]$ in Equation (1)

$$\Delta_F = \frac{3^4 c_l^4}{\Lambda_c^2}\left(-\frac{4c_l^2}{3\Lambda_c} + 3A^2\right) \tag{3}$$

with velocity of light c_l, cosmological constant Λ_c, and arbitrary constant A yielding real coordinates and vacuum energy $\Lambda_c = 4c_l^2/3^2A^2$. A Minkowski-bound of Δ_F for cyclic extensions $\mathbb{K}\left[\partial, g_1^{\cdots g_l}\right]$ and rational $\Delta_F = 0$ induce a highly nonlinear L-function (4). This singular case displays elastic continua $\delta_k\mathcal{L} = 0$ as minima of action $\mathcal{L}$ for a discrete sequence of steps $k\to\infty$. Chaotic itineraries of a quadratic map of curvature R and universe radius R_u yield finite non-equilibrium values $\delta_k\mathcal{L}$, $\delta_k\delta_k\mathcal{L}$, $\delta_k R_{\mu\nu}$, $\delta_k\delta_k\boldsymbol{R}_{\mu\nu}$, $\delta_k T_{\mu\nu}$, $\delta_k\delta_k T_{\mu\nu}$ of curvature $\boldsymbol{R}_{\mu\nu}$ and stress-energy $T_{\mu\nu}$. For a Hermite-Tschirnhaus substitution $\gamma(\phi_3)$ the discriminant scales as $\Delta_F \to \phi_3^2\Delta_F$. Therefore, the complex invariant $f(\omega)$ in $\gamma(\phi_3(f(\omega)))$ is like $\boldsymbol{R}_{\mu\nu}$ and $T_{\mu\nu}$ of fundamental significance as iterates over tensile forces. The resulting dense lattice of algebraic units $\{l\}$ generates spacetime superimposed by fluctuating elliptic lattices {𝕃} with Poncelet triangles of inscribed and circumscribed cones as tidal-like forces. Discrete iterates of $\gamma(\phi_3)$ are points and segments as wave packets. Laps $l_\omega = l_{\gamma\omega}$ are orbits of an assembled shift

$$\delta_k \prod \delta_{l_\omega}$$

of k-components where modular units $g(a\omega) = g(a\gamma\omega)$ are inert for lap number $\# l_\omega \to \infty$. The map γ ($\det\gamma \neq 1$, $z_{k+1} = \gamma z_k$) forces complex multiplication (CM) of points and segments of curves by period-doubling k-component orbits which relate to Lorentz-transformations γ_L. Treating each k-component as an excited particle quantum statistics overestimates vacuum density Λ_c by a factor 2^{2^9} [10] [14]. Mathematically the Euclidean norm $\sum_{(q)} E_q^{-2} = \sum_{(s)} \psi_s \overline{\psi}_s$ in Equation (11) is formally equivalent to quantum statistics. However, $\sum_{(s)} \psi_s \overline{\psi}_s = 1$ in CCP overestimates the binary tree of k-components in QE by factor 2^{2^9} because the real algebraic unit in the cyclotomic L-function decreases.

3. Zeta Function Zeros and Poles in Mass Operator

A Riemann zeta function $\zeta(z)$ allows to start from holomorphic function $\xi(z)$ and holomorphic Dirichlet L-function [13].

$$\frac{\zeta(z,\mathbb{K})}{\zeta(z)} = \frac{\Gamma(z/2)z(z-1)\zeta(z,\mathbb{K})}{2\pi^{z/2}\xi(z)} = L(z,\chi) \tag{4}$$

which can be scanned by $\gamma(\phi_3)$ in the Dirichlet L-function with character χ of a cubic field extension $\mathbb{K}[\partial]$. The $z \to 1$ limit $L(1,\chi) \simeq H_\Delta R_\Delta$ is proportional to a regulator $R_\Delta = \ln_b E$ with fundamental unit E, for base b, class number H_Δ of a cubic normal field with discriminant Δ. The Riemann zeta function $\zeta(z)$ is related to the quantum statistical scattering amplitude $\mathbb{A}(s) = \sum_{(n)} \frac{1}{s - m_n^2}$ by the entire function

$$\xi(z) = \binom{z}{2} \pi^{-\frac{z}{2}} \Gamma\left(\frac{z}{2}\right) \zeta(z) = \mathrm{e}^{\int \mathrm{d}s \mathbb{A}(s)} \tag{5}$$

for a quadratic map between s and z. Both $\xi(z)$ and $\gamma\xi(\gamma z)$ satisfy a hyperbolic Laplace equation. A fractional substitution $\gamma(\phi_3)$ is capable to scan masses m_n in nontrivial zeros $z_{nt} = 1/2 + i m_n$ of $\xi(z)$ and $\zeta(z)$. A Mandelstam plane $s,t,u \simeq \delta_k e = \Lambda_\omega^2 (\lambda, 1, \lambda' = 1 - \lambda)$ with $s + t + u = 0$ and $m_n \simeq \sqrt{s} \simeq \lambda$ are differences of cubic roots where a scaling $\Lambda_\omega = (2K/\omega_1) \simeq 1$ (ultraviolet cutoff) is consistent with a quadratic map $\lambda_k \simeq 1/2 + i\sqrt{\lambda_{k+1}}$ for a process $s \simeq \lambda$ and $\sqrt{s} \simeq \lambda$. Iterates $\lambda = \lambda_m/m + 1/2$ are treated as Dirac-like currents where $(\lambda_m/m)^2 - 1/4 = 2^4/f^{24}$ and $\lambda_m = \bar{\psi}_q \lambda_{mqq'} \psi_{q'}$ couple to the Weber invariant $f(\omega)$. A certain quadratic map γ_n of cubic roots $s = \gamma_n \circ s$ relates ξ-zeros z_{nt} (≡ $\mathbb{A}$(s)-poles, charged excitations) to the single pole of $\zeta(z)$ at $z = 1$ (collective excitations, Kepler singularity). Equation (2) for γ_n couples the function $G_{ss'}$ to a collective potential A_μ. Then the L-function enables a Dedekind zeta function

$$(z-1)\zeta(z,\mathbb{K}) \simeq \sum \frac{z-1}{Nm^z(f(g_1))} \simeq H_\Delta R_\Delta \tag{6}$$

to relate masses $m_n \simeq z_{nt}$ to regulator index R_Δ and class number H_Δ.

4. Regulator Limits for Cyclotomic Extensions

FZU regards every point self-similarly enveloped by w-shells of a most general Riemann surface regardless of whether it is a charge, a universe or a plant. Coupling constants are derived from the L-function in particular from the regulator of a cyclic number field. A cyclic number field e.g. with periods ν_{Sh} where series of motions vanish can be multiplied by a coupling constant G_w. This Lagrange normal base with Gaussian periods is called interaction w = 1, 2, 3, 4, 5 [11]. Within a dimensionless FZU all physical fields are quadruples of simplest cycles ψ_s of algebraic units E which are information currents where the L-function is a statistical sum. Their assignment to forces, (strong weak, em, grav, dark) results from the pre-factor G_w which differs by up to 10^2 orders of magnitude. This independence on laboratory dimension allows a self-similar view to five interactions. All forces are treated uniquely by Feynman diagrams with Euclidean norm $\sum_{(q)} E_q^{-2} = \sum_{(s)} \psi_s \bar{\psi}_s$ where the bi spinor ψ_s is viewed as spacetime curvature $R_{\mu\nu} \simeq F_{\mu\nu} \simeq E, B$ of a simplest cycle quadruple of a bicubic field. A decreasing circulant regulator index $R_\Delta = \det \ln \varepsilon_{ij}(\omega_k) = \det \ln \varepsilon_i \bar{\varepsilon}_j [\nu_{Sh}]$ behaves like a minimum of an action functional. The regulator index R_Δ for a bicubic field of class number $H_k = 1$ and $R_\Delta =$

$\log E \simeq 1$ of fundamental unit $E \simeq 1$ whereas the lower limit of the regulator R_Δ for a dense lattice of units and imaginary fields with $n = r + s - 1 \to \infty$ [21] [22].

$$R_\Delta > 2^{3-n} e^{n/2} \pi^{n/2-1} n^{-2} ln^{n-1}(2)\Big|_{n\to\infty} \to \infty \tag{7}$$

can tend to infinity. In case of cyclotomic units

$$E(g, g_\infty) = \sqrt{\left(\frac{\varepsilon(g, g_\infty)}{\varepsilon(1, g_\infty)}\right)\left(\frac{\varepsilon(-g, g_\infty)}{\varepsilon(-1, g_\infty)}\right)} = \frac{1 - Z^g}{(1 - Z) Z^{(g-1)/2}} \tag{8}$$

with complex units $\varepsilon(g, g_\infty) = \left(1 - 1^{g/g_\infty}\right)$ and generator $Z = 1^{1/g_\infty}$ one has [22]

$$R_\Delta \to -\frac{1}{2}(g-1)\ln Z - \ln(1-Z) - \ln(1-Z^g) \to -\frac{1}{2}(g-1)\ln Z + Z + Z^g \tag{9}$$

Possible is $R_\Delta \to \infty$ but also for $g_\infty \to \infty$ a vanishing $R_\Delta \to \ln Z + Z \to 0$. This is because small and large generator values enter the cyclotomic norm. The lower limit in Equation (7) results from a minimum of a quadratic form $\sum(l^2 + l)$ for a given number field with $r + s - 1$ units forcing upper and lower limits of the regulator R_Δ for r real and s complex conjugated pairs of roots of unity [21] [22]. Forcing $R_\Delta \to \ln Z + Z \to 0$ requires a process for a tower of number field extension as a feasible process of optimal units $E = b^{\Omega_w - \mathcal{L}}$ leading to a tower $b^{b^{\cdots\Omega_w - \mathcal{L}}}$.

5. Optimal Regulator Process and Renormalization

An additional term in the quadratic form of logarithms $l = \ln_b E$ of fundamental units

$$\sum\left(\mu_1 l^2 + \mu_2 l + \mu_3 b^{2l} \zeta(l_s, m_s, l)\right) \tag{11}$$

ensures a feasible solution for regulator index for base b [23] [24]. An $q = r$ dimensional Euclidean norm $\sum_{(q)} E_q^{-2}$ is replaced by a norm-quadruple of finite periods of intervals multiplied by the geometric zeta function $\zeta(l_s, m_s, z) = \sum_{j\in\mathbb{N}} m_s^j l_s^{-(j+1)z}$

$$\zeta(l_s, m_s, z) = \frac{1}{m_s}\frac{1}{e^s - 1}\bigg|_{s = z\ln l_s - \ln m_s} = \frac{1}{m_s}\frac{1}{e^{\beta\nu} - 1}\bigg|_{\nu = z/H(l_s, m_s) - 1, \beta = 1/\ln m_s}. \tag{12}$$

String length $l_s = 3$ and multiplicity $m_s = 2$ describe Hausdorff dimension

$$H(l_s, m_s) = \ln_{l_s} m_s$$

of a Cantor set. Subseqent feasible solutions $E = b^{\Omega_w - \mathcal{L}}$ yield a tower $b^{b^{\cdots\Omega_w - \mathcal{L}}}$ of degree higher than 2^k of a subsequent map $\gamma(\phi_3)$. Base $b = 2$ congruences is expected at the third level for $2^{2^9} \simeq G_5^{-1}$ which supports a Fermat number transform in [25]. In this subsequent process the regulator index R_Δ can be lowered a system of equations for a power tower of generators g_i as roots of unity where cyclic periods υ_{Sh} are related to pseudo-congruences mod (g_i-1) in the tower $g_1^{\cdots g_l} \simeq g_1^{g_2^{g_3}}$. Periods υ_{Sh} are mapped to doubly-periodic lattices ω where period-doubling $\omega_k \to \omega_{k+1} + \omega_{k+2}$ is viewed as a subsequent generation of new lattices. Topological entropy $h_t(f)$ is generated by cyclic extensions $\mathbb{K}\left[\partial, g_1^{\cdots g_l}\right]$. A cyclic field for a minimum of $R_\Delta = \Omega_w - \mathcal{L}$ has the highest information densities. If the exponent of a generator g_l is a square vanishing Gaussian periods are possible where $R_\Delta \simeq g_1^{g_2^{g_3}}$. For 2^{2^k}-components with k = 2, 4, 6, 8, 10 one has $R_\Delta \simeq \Omega_w - \mathcal{L}$ for neighbouring interaction layers (w, w + 1) = (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), respectively. A Dirichlet L-function oscillating like a local Lovelock-like Lagrangian near $\mathcal{L} \simeq \mathcal{L}_0$ defines a particle. Oscillations of $z = \log E$ in $\zeta(l_s, m_s, z)$ occur on a circle of radius $|z| = H(l_s, m_s)$. A former real unit E gets complex by substituting γE which justifies to replace the quantity $l = \ln E$ by the tensorial object $\mathrm{e}^{\boldsymbol{R}} = \exp\left(\boldsymbol{R}_{\mu\upsilon}\left[\gamma_\mu, \gamma_\nu\right]_-\right)$ where the skew tensor $\boldsymbol{R}_{\mu\upsilon}$ depends on a three-component complex $\boldsymbol{z} \simeq$ l, i.e. units E and ln$E$ oscillate. Subsequent $\gamma(\phi_3)$-maps are addition steps on each iterated curve with universal covering u,v,u±v. Period-doubling $\omega_k \to \omega_{k+1} + \omega_{k+2}$ yields in case of $\omega \to 2\omega$ for the nome $\boldsymbol{q}^{i\pi\omega} = \mathrm{e}^{-\pi K'/K}$ the exact result $4\boldsymbol{q}^{2^{k-1}} = \lambda_k$. Accordingly, the Legendre module acts as a generator $4\boldsymbol{q}^{2^{k-1}} = \lambda_k$ of cyclic fields. For simplest cycle quadruples q triples of invariants f_k, f_{k+1}, f_{k+2} weave a global metrical texture that is perceived as mass relating quadruple indices q to spin indices s. Periods ν_{Sh} in $\mathbb{K}\left[\partial, g_1^{\cdot\cdot\cdot g_l}\right]$ with a tower $g_1^{\cdot\cdot\cdot g_l} \simeq g_1^{g_2^{g_3}}$ give a generator g_l e.g. for a g_∞th root of unity $g_1^{g_\infty} = 1$ on complex plane for four-component complex roots

$$i\gamma^\mu \frac{\partial f_s}{\partial x_\mu} \simeq g_1^{g_l} f_s \tag{13}$$

with symbolic solution $\exp\left(i g_1^{g_l} \gamma_{ss'}^\mu x_\mu\right) f_{s'}$ which relates congruences in $g_1^{g_l}$ to a Dirac-like mass. The bi spinor $f_s \simeq \psi_s$ is a cyclic quadruple $q \simeq s = \{1,2,3,4\}$ in a field $\mathbb{K}\left[\partial, g_1^{\cdot\cdot\cdot g_l}\right]$ projected onto complex plane. L-functions with circulant regulator determinant $\phi(g_1) = \sum_{i=0,\cdots,g_\infty - 2} a_i g_1^{g_2^i}$ yield a series expansion $\left(\sum_{(q)} E_q^{-2}\right)\zeta\left(l_s, m_s, \log E_q\right)$ in the amplitude $\mathbb{A}$. The norm $Ef'f'' = 1$ with bicubic complex conjugates f' and f'' of component $\psi_s \simeq \boldsymbol{R}_{\mu\upsilon}$ is a cyclic complex curvature pending between flat, closed and open universes. A linear-dependence between z_k, z_{k+1} and z_{k+2} gives Equation (2) as a renormalized map

$$\gamma^{(ren)} = \gamma + \gamma \circ \Gamma^{(ren)} \circ \gamma^{(ren)} \tag{14}$$

with vertex $\Gamma^{(ren)}$. In the limit $k \to \infty$ one gets the Feigenbaum equation

$$\gamma^{(ren)}(z) = -\alpha_F \gamma^{(ren)} \circ \gamma^{(ren)}\left(-z/\alpha_F\right) \tag{15}$$

with k-component generator $g^k = \alpha_F$. A particle peels out from a 2^{2^k} polar ball originating from a non-trivial zero z_{nt}. Simplest cycles of $f(\omega)$ have $\deg \gamma^{(ren)} = 2^k$ whereas $z_{nt}\left(1 - z_{nt}\right) = 2^4 / f^{24}$ has degree $(4 \cdot 3)^k$. Both congruences are consistent for a k-component-Fermat number transform which is invertible in $2^{2^{10}}$ for the first four prime number.

6. Conductivity Plateau and Leaves

Processing L-functions and regulators for various iterates the action functional $\boldsymbol{\mathcal{L}}$ in process (11) and (18) consist of a stationary bifurcating term μ_1 (outside zeros z_{nt}), a count rate μ_2 and a scattering term μ_3. These terms are called holomorphic potential or conductivity plateau, air ionization or net rate and, cosmic rays or bifurcating trees. Transitions between plateaus generate a net rate (18) with occupation number (12) due unstable orbits of bifurcation. A formerly iterated unit $z = E = \exp(l) \simeq \lambda$ undergoes additional optimizing as Feigenbaum renormalization or a regulator process $z^{(ren)} = E^{(ren)} = \exp\left(l^{(ren)}\right) \simeq \lambda^{(ren)}$ via terms μ_1, μ_2, μ_3 in Equation (18). Supposing that nontrivial zeros z_{nt} of $\zeta(z)$ describe masses m_n charge quanta are definable by entire, holomorphic $\xi(z)$ and $L(z,\chi)$ which satisfy a hyperbolic Laplacian $\Delta_h \xi(z) = 0$ with $\Delta_h = y^2 \Delta_{xy} = \operatorname{Im}\lambda^2 \Delta_{xy}$. For $\xi(z_{nt}) = 0$ and $\lambda = z_{nt}$ one gets the screened Poisson equation

$$\Delta_{xy}\left(L(z,\chi)\xi(z)\right) + \mu_s L(z,\chi)\xi(z) = \mu_c\left(\operatorname{Im}\lambda - m_n\right) \quad (16)$$

for m_n slices with $\operatorname{Im} z_{nt} = \operatorname{Im}\lambda = m_n$ with Lagrange parameter μ_s and μ_c for conditions $\Delta_h \xi(z) = 0$ and $\operatorname{Im}\lambda = \operatorname{Im} z = m_n$. Equating $z \simeq \lambda$ and $\xi(z) \simeq j$ for module λ and complex current j a plateau $\xi \simeq E(z) = \chi^{-1}(\nabla V, \nabla T)$ of conductivity χ and complex electric field $E(z)$ is equivalent to the existence of a holomorphic function $\xi(z)$. Equivalently for field ∇V and temperature gradient ∇T a plateau denotes a holomorphic global potential

$$V_{T_{global}} = \int \nabla V_{T_{cloud}} \mathrm{d}l_{xy} \quad (17)$$

which describes a non-radiative, non-dissipative superfluid of discontinuous self-similar segments $\mathrm{d}l_{xy}$. Traversed Xi-function zeros give a holomorphic current $j \simeq E$ with $\operatorname{div} j = \operatorname{div} E = 0$. Equation (16) is invariant for subsequent chaotic $\gamma(\phi_3(f(\omega))$-maps. The algorithm is completed by an optimal step for a regulator index minimum for base b of the quadratic form (11) giving

$$2\mu_1 l + 2\mu_2 E_q^{-2}\zeta(l_s, m_s, l) + \mu_3\left(\sum_q E_q^{-2}\right)\zeta'(l_s, m_s, l) = 0 \quad (18)$$

with Lagrange multiplier μ_1, μ_2, μ_3. Generators g_i in algebraic units E_{wqi} cause stable laps l_ω of orbits felt as masses. The number of stable 2^k orbits is called a lap l_ω whereas a k-component is an unstable orbit of bifurcation. Simplest cycles quadruples $q:\{k+3 \in \{k, k+1, k+2\}\}$ are stable against laps l_ω. For $z_{k+1} = F(t, z_k) = \gamma(\phi_3) \circ z_k$ entropy $H = \sum_{(i)} P_i \ln P_i$ can be calculated by $\mathrm{e}^P = \prod_{l,k} \mathrm{e}^{P_{l,k}/\left|F'(t, z_{l,k})\right|}$ in terms of the probability density $P(z)$ for finding an orbit at z. Like Lyapunov exponents highest information densities are expected at critical points $F'(t,z) = 0$. The thermodynamic variable $\Omega_w = 1/2\mu_1$ in $l = \ln_b E = \Omega_w - \mathcal{L}$ depends on topological entropy

$$h_t(f) = \lim_{n\to\infty} \frac{1}{n} \ln l_\omega\left(f^n\right) \quad (19)$$

with lap number l_ω. Then the action reads $\mathcal{L} = \mu_2 E_q^{-2}\zeta(l_s, m_s, l) + \frac{1}{2}\mu_3\left(\sum_{(q)} E_q^{-2}\right)\zeta'(l_s, m_s, l)$. Like the Weierstrass $\wp$-function the cubic $\wp^3 - g_2\wp - g_3 \simeq \phi_3(f)$ is regarded as a potential energy Ω_w. For all interactions $w = 1, 2, 3, 4, 5$ the complex logarithm of algebraic units $l = \ln E$ is viewed as curvature tensor which oscillates on a circle of radius Hausdorff dimension $H(l_s, m_s)$. A circulant regulator $l_i \to R_{k,ij} = 1^{ij/g_\infty} \cdot \operatorname{diag}\left(\ln E\left(g_\infty^i\right)\right) \cdot 1^{-ij/g_\infty}$ near

$H(l_s, m_s)$ leads to $\prod E = G_w \prod \mathrm{e}^R = G_w \prod \left(chz + sh(z) \boldsymbol{R}/z \right) \simeq G_w \boldsymbol{R}$. where $\ln G_w = w! 2^w H^w(3,2)$. The thermodynamic potential $\Omega_w = \int \mathrm{d}\sigma_5 j_w A_w$ in units $E = b^{\Omega_w - \mathcal{L}}$ gets a complex finite generation count rate. Interaction with the next neighbour in vector potential $G_{w-1} A_{w-1} + \kappa A_w$ dominates for Born-Oppenheimer parameter $\kappa = \left(G_w / G_{w-1} \right)^{1/4} > G_{w-1}$. Therefore, this interaction requires $w > 5 \ln 3 / 2 \ln 2$ or $w > 3{,}9524$ which is realized for gravity and dark matter. A physical interaction is defined as a minimum of the L-function $H_\Delta \ln_b E \simeq H_\Delta \left(\Omega_w - \mathcal{L} \right)$ in accordance with elastic spacetime. Because $\lambda \lambda' = 2^4 / f^{24}$ depends on the algebraic units E, one has $\ln_b(\lambda\lambda') = -4 \ln_b 2 + 8 H_\Delta \ln_b E = -4 \ln_b 2 + 8 H_\Delta \left(\Omega_w - \mathcal{L} \right)$ reducing action $\mathcal{L}$ by topological entropy $4\ln_b 2$. Holomorphic plateau-like information currents describe dissipation-less doubly-periodic waves of temperature and entropy. A bifurcation of curvature on a complex surface is viewed as branches and leaves of trees and bizarre plants as shown in **Figure 1**.

Figure 1. (left) Fractal zeta zeros in nature and nanostructure laboratory: Plant, green trees under blue sky [26] (right) Plateaus in quantized Hall conductivity [27].

7. Charge and Quantum Entanglement

The charge condition in Equation (16) goes via holomorphic $\xi(z)$ plateaus-like where a plateau denotes a neutral chaotic quadrupolar motion between $\pm 1/2 \pm i m_n$ with e.g. coupling constant $2\pi \delta_F^2 \simeq \alpha_f^{-1}$. A charge 'e' forms out for congruences $2^{2^k} = G_w^{-1}$ on $\mathbb{C}^w$. Then a 1meV↔10^{20}eV congruence justifies to multiply the global potential (16) by the charge 'e' and to replace fractional $\zeta(l_s, m_s, l)$-segments in Equation (17) by a differential $\mathrm{d}l_{xy}$. A maximal five- layer gyro-twist surface $\mathbb{C}^w$ contains k-component period-doubling

$$\delta_k \prod \delta_{l_\omega}$$

The tower 2^{2^k} is regarded as tree of particles on fractal length $\mathrm{d}l_{xy}$ of leaves of the tree in **Figure 1**. Beyond resolving a 10^{20}eV energy into a 1meV potential the cosmic microwave background above GZK cutoff is simply first periods ν_{Sh} at low k. It is claimed that measured microwave emission near conductivity plateaus proves this conjecture [28] [29]. Vanishing Gaussian periods in interfaces are viewed as self-contained intermediate layer between e.g. five atmospheric layer $\mathbb{C}^5$. Accordingly, the module $2^{2^{10}}$ explains CCP, the definition of quantum statistics as well QE in the universe [10]. Oscillations of L-function near vanishing Gaussian periods are particles. Topological entropy h_t is like stirring a non-dissipative highly-correlated solid-fluid-gaseous slushy with a whisk creating periods ν_{Sh}. This k-component process is universal in micro space and macro space and creates plateaus as simple zeta zeros

$$\xi\left(z \simeq z_{nt}\right) = \prod_{z'_{nt}} \left(1 - z/z'_{nt}\right) \simeq M\left(1 - z/z_{nt}\right)$$

which can be regarded as a giant mass

$$M \simeq \prod_{z'_{nt}} \left(1 - z_{nt}/z'_{nt}\right).$$

The γ-correlated k-component tree $\delta_k \xi = \gamma^{(ren)} \circ \delta_k z = \left(\gamma + \gamma \circ \Gamma^{(ren)} \circ \gamma^{(ren)}\right) \circ \delta_k z$ of bifurcating spacetime ranges from plants or conductivity plateaus in semiconductor layer laboratory up to higher atmospheric layers [12] [30].

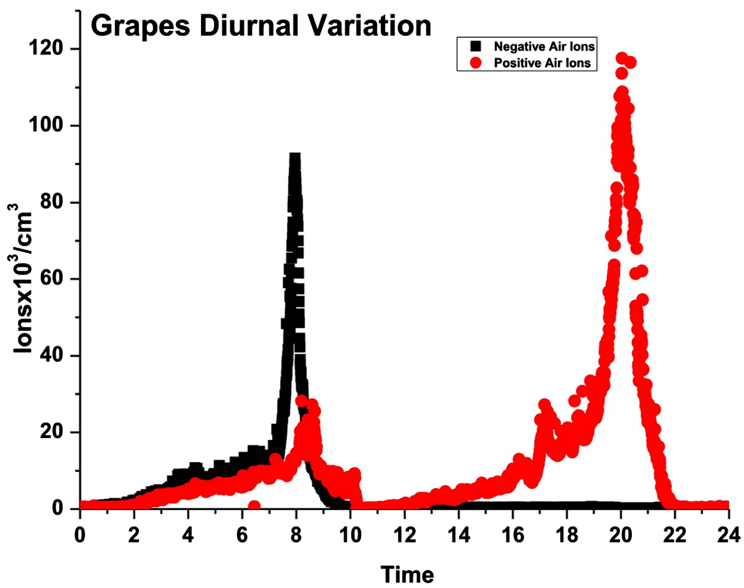

Figure 2. Diurnal variation of air ions in Grape vegetation area from [12].

Besides ergodic time-reversible laps non-ergodic time-irreversible components exist. Every point is surrounded by 2^{2^k} components which means that matter as cosmic rays is created for large M e.g. at seasonal growth of plants. Like cosmic rays enhanced air ionization rate near plants as well a seasonal behavior of cosmic rays is expected. Both predictions are measured e.g. in vegetation areas in **Figure 2** and seasonal variations of cosmic ray intensity [30].

8. Conclusion

The minimal and maximal nontrivial case is taking spacetime as discrete dynamics on elliptic curves. This iterates quadratically a complex curvature of spacetime. Complex curvature opens a spacetime bud or cavity of closed lines of complex numbers composed by roots of unity. Number-theoretically period-doubling steps k increase the lattice dimension k of algebraic units as information density which self-consistently minimizes a regulator index of cyclic extensions of number fields. Like organic growth period-doubling creates buds, shoots, leaves of a tree. Matter as correlated spacetime cavities or buds with cyclic generators is inseparably connected with *k* pseudo-congruent iterated complex curvatures. This highly correlated non radiative non-dissipative potential defines an upper velocity of quantum entanglement in spacetime which underlies causal interactions. Examples are air ionisation at plant growth due to cosmic-ray-like shower *k*-components which are highly correlated up to the exosphere and, thus, are a stabilizing environment of every matter. Like holomorphic surfaces of plants, a conductivity plateau displays a path-independent holomorphic potential between two points on a two-dimensional surface. The transition between plateaus is connected with large mass changes *M* and an emission rate of a *k*-component tree proportional to the geometric zeta function (12).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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