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Paradoxical Cohabitation and Emergence in a Universal Structure—A Geometric Model

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Abstract

The unit circle is the universal basis for the trigonometric functions on the two-dimensional plane. It is applied to analyze the paradoxical internal structure of a universal state, the 1-*D* geometric model. The geometry includes an inner circumference to the unit circle and is analyzed by applying two mathematical frameworks to the right triangle. The first is a formal interpretation, and the second is nonformal, in which the dimensional complexity is outwardly emergent from the inner to the outer circumference. A novel format of vector structure, each with a unit identity of 1, replaces the magnitudes for the hypotenuse and its sides. Although the two formats of the unit circle are paradoxical, with the second having no rational basis in geometry, the cosine squared calculations for the right triangle agree in both frameworks. The study concludes that two paradoxical frameworks cohabitate within a universal state, and the mechanism of paradox is validated as the basis of the relationship between them. The geometric model explores the structural basis for the emergence of complexity across dimensional boundaries. Paradoxical cohabitation is conjectured not as an anomaly but to represent the static format between segments in the process of emergence. The 1-*D* geometry defines its universal state as an infinity. Beyond the two-dimensional basis of the toy model, it broadly conjectures the role of paradox as a fundamental relationship mechanism in the Universe.

Keywords

Emergent Self-Organization, Paradox, Universality, Russell's Paradox, Nonlocality, Infinity

1. Introduction

Examples of fundamental structures containing paradoxical frameworks are found across logic, mathematics and physics. The crucial distinction between paradox's

mathematical and linguistic structure is the difference between extension and circularity in their separate properties.

Mathematical symbols are fungible, having the universal property of extension. Cantor's diagonal slash argument and Gödel's incompleteness theorems, analyzed in Section 1.2, are examples of the paradoxical incompleteness introduced in the mathematical representation of universality.

Linguistic symbols have an opposing nonfungible property, and the problem identified above for mathematics does not arise. Instead, linguistic arguments with the property of self-contained universality devolve into structures containing the nonresolvable self-circularity of infinite regression. The Russell and the Liar paradoxes are two myriad examples (analyzed in Section 1.3).

1.1. Paradoxical Structures in Physics

The experimental demonstration of Bell's theorem conclusively proved the validity of the paradoxical relationship between local reality, in which the speed of light is an absolute limit to communication, and the non-local connection between quantum states, in which communication across classical space-time is instantaneously connected without limit by the speed of light [1] (pp. 211-227).

> "As in the case of the EPR paradox, it's important to realize what Bell did not do. He did not discover an experimental situation in which non-local interactions are directly observed. Instead, he invented a simple argument based on experimental results that indirectly demonstrated the necessary existence of non-local connections" [1] (p. 220).

In other words, Bell did not prove that classical relativity theory is invalid. Instead, he demonstrated that the two formal systems are paradoxical. The speed of light is the absolute classical limit for communication in the Universe, and it is violated in the quantum description. To compound the issue, both descriptions have unquestioned accuracy to the detection limit of our instrumentation. "General relativity is known to be accurate to one part in 10^7" [2] (p. 26). "Quantum field theory is known to be accurate to about one part in 10^{11}" [2] (p. 55).

1.2. Paradoxical Structures in Mathematics

1) Cantor's diagonal slash argument: Cantor's argument exposes the paradox in mathematical logic that arises when representing infinity as a unitary state. The argument constructs two theoretically infinite listings for the natural number line between zero and one. The first listing is a vertical column, and the second is reconstructed from the number on the diagonal. Paradoxically, the second listing contains numbers not included in the first, even though they represent the same infinity [3] (pp. 110-111).

2) Gödel's incompleteness theorems: Gödel proved that paradoxically, there are mathematical statements known to be true that cannot be proven true.

"Among the things that Gödel indisputably established was that no system of

sound mathematical rules of proof can ever suffice, even in principle to establish all the true propositions of ordinary arithmetic" [4] (pp. 64-65).

1.3. The Complementarity of Linguistic Paradox

Linguistic statements constructed to universally contain all logical references to an argument's property devolve into self-circular infinite regressions prohibiting conclusion.

1) Russell's paradox: Russell's paradox is the prime example of self-circular infinite regression in a logical argument [5]. The Russell set (R) is the set of all sets in which its membership property includes the logical "not" function [6].

By including the logical "not" in the argument, direct identification of the property of membership in the state is prohibited. Whatever property is named, the members of the set do not contain that property.

The paradox arises that it is impossible to determine whether the "set of all the sets" (R) shares membership in its collection of segments. If (R) is placed within itself, it is an error since it should "not" have a common property with its members. However, if it is not placed within itself, it is an error because it shares the property designation with its members.

The only resolution for the logical paradox created in the argument is to apply a restriction to the structure of sets under the Zermelo-Fraenkel (ZF) rule of set theory [7]. However, this rule does not solve the underlying paradox issue. Instead, it avoids it.

2) The liar paradox [8], "I am telling a lie": The argument is the most concise format of a universal linguistic statement devolving into entangled self-circularity. The logical "not" function is implicit because the statement and its intention are "not" consistent with truth. The speaker's statement cannot be true if it is false, and it cannot be false if it is true. The speaker's self-reference creates an entanglement between two conclusions comparable to the entanglement structure in quantum states (discussed in Section 5.1). If the statement were, "He is telling a lie", the argument would have a discrete classical format between the speaker and the subject.

2. Defining Emergence in the 1-D Geometric Model

Emergent self-organization is a complex subject because of the diversity of mechanisms and expression formats. Generally, a simple rule produces an iteration affecting a large segment domain, such as crystal growth and bird flocking group behavior.

The distinction between the above emergence formats of self-organization and the 1-D geometric model is that the model's mechanism does not have a "preexisting" rule for its iteration. Instead, it lifts the emergence of complexity in a "bootstrap" process based on the self-circular assumption for the role of paradox.

The counterintuitive claim for the model's mechanism requires strong validation and is based on analysis of the two geometries in Section 3 and their calculation

agreements in Section 4.

The Mechanism of Paradox in the Thought Experiment

The counterintuitive basis of the geometry in Section 4 (Figure 2) is based on a "thought experiment", applying a general principle of "least" change in the emergent growth of complexity across segments defined as unitary states. Although the thought experiment was instrumental in initially formatting the 1-*D* model, the geometry's validation is not dependent on that rationale. Instead, the model relies on the strength of its mathematical proof. Nevertheless, a summary of the geometry's development basis is offered.

If paradox is conjectured as a fundamental mechanism in the Universe, its initial state will have the same basis. There are two parts:

1) The mechanism of paradox prohibits observing the internal structure of such a universal state, and it cannot be formally stated to exist.

2) The Universe's initial static description is conjectured to have a dynamic phenomenological framework in which paradoxical segment structure is an outward "force" for the emergent development of complexity. An iteration cycle successively subsumes dimensional structure into its composite state as waveform complexity and particle integrity.

The one- and two-dimensional circumference surfaces represent the waveform basis of the geometry from which the hexagonal vector structure develops sequentially. The first vector projects tangentially from the intersection of the inner circumference and the *x*-axis and terminates at the outer circumference. Angular complexity then develops as a 60-degree vector reflection to the opposite side of the *x*-axis. The composite structure forms a 30 - 60 - 90-degree right triangle used in the cosine squared calculations.

3. The 1-*D* Geometric Model

The 1-*D* geometry defines the sides of the right triangle in a nonformal composite of vectors, each a unitary state with an entangled identity/value of 1. A separate paper applies the same vector structure to model the unitless values of the fine structure constant and the coupling constant (*e*) [9].

For the emergent development of dimensional complexity, the inner circumference is the lower boundary of a one-dimensional space, and the outer circumference is the boundary of a two-dimensional space. From an exterior perspective, the outer circumference is a dimensionless point to the emergence of structure and dimensionality on a classical basis.

4. Calculating the Cosine Squared Function in the Two Geometries

The 1-*D* geometry calculates the formal cosine squared identity based on Figure 1 and the nonformal (rationally nonsensical) identity based on Figure 2.

The inner, one-dimensional circumference (labelled *1d*) is preemergent to the

outer circumference's two-dimensional level (labelled 2*d*).

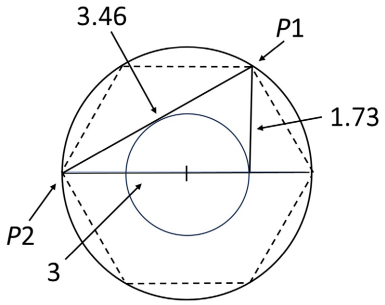

Figure 1. Formal geometry.

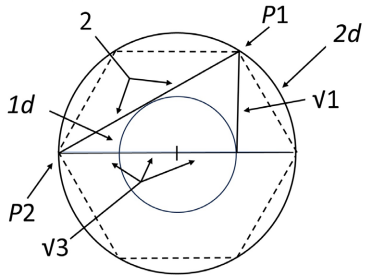

Figure 2. Nonformal geometry.

Vectors in **Figure 2** that converge eccentrically relative to the lower dimensional inner circumference have the square root attached. The hypotenuse consists of two vectors that begin and end concentrically on the outer circumference, and, in combination, the square roots cancel.

In representing the 1-*D* geometry's nonformal basis of emergence, the operational rules are counter-rational to those of the Cartesian plane's fixed dimensional framework. There is no formal linkage between the vectors, and each has the property of a closed unitary state at its boundaries. This means the outer circumference is the composite infinity of its interior structure.

The diameter of the outer circumference is assigned the value 4, and the portion that applies to the formal calculations in the geometry is 3. The sides of the 30 - 60 - 90 triangle are 3, 1.732, and 3.464 [10].

4.1. Formal Calculations

$$P1-\mathrm{Cos}^2\left(60\right)=\left(1.73205/3.4641\right)^2=0.25 \tag{1}$$

$$P2-\mathrm{Cos}^2\left(30\right)=\left(3/3.4641\right)^2=0.75 \tag{2}$$

4.2. Calculations in the Emergent Basis

$$P1-\mathrm{Cos}^2\left(60\right)=\left(\sqrt{1/2}\right)^2=0.25 \tag{3}$$

$$P2-\mathrm{Cos}^2\left(30\right)=\left(\sqrt{3/2}\right)^2=0.75 \tag{4}$$

4.3. Interpreting the Results of 4.1 and 4.2

The proof of the equivalence between Figure 1 and Figure 2, albeit in a paradoxical relationship, is the simple calculation of the cosine squared identity in both formats.

A parent/sibling relationship analogy is useful. The calculation agreement as the "parent" structure conjoins two paradoxical "sibling" geometries on a "not" function basis in a unitary common state of both.

The prohibition to observing a discrete relationship between the geometries is comparable to the prohibition created by the "not" function in the mathematical and linguistic examples above and quantum structure (see Section 5.1).

5. Conclusions

The 1-D geometric model conjectures that paradox is the fundamental precursor mechanism hidden in the diversity of the observable Universe. However, the beginning assumption of all logical formalisms discounts paradox as an anomaly open to more advanced understanding, worked around by restrictions to logic application or ignored as nonsensical.

There are two frameworks for the 1-D geometry, dynamic and stationary:

1) Emergent self-organization is a process of spontaneous dynamic growth in complexity across dimensional boundaries that each form an infinity. This differs from a formal definition of dimensional structure, in which dimensional levels have a fixed basis in mathematical operations through applying the power function.

2) The paradoxical relationship between adjacent segments in the geometry is analogous to the paradoxical structure created by the "not" function in mathematical and linguistical arguments.

5.1. The Classical and Quantum "Not"

The calculation agreements for the cosine squared function in both frameworks demonstrate that the two geometries, Figure 1 on a fixed classical basis of dimensional relationship and Figure 2 on a nonformal emergent basis, are correlated. As introduced above, a parent/sibling analogy is appropriate. The two sibling geometries are paradoxically conjoined in their parent's mathematical basis. The relationship is the logical "not" function.

The classical "not" function flips the value of two observables to the opposite for a shared property. The quantum "not" is distinguished in that the shared segments have the property of entanglement, and the "not" property becomes internally expressed on a unitary basis [11].

In general, the logical "not" function prevents the direct representation of the property represented in the segment structure of a state.

The analogy of the half-silvered mirror experiment is useful [4] (pp. 259-263). The particle description of the photon has projection as a unitary state on a classical basis. However, if projected through a half-silvered mirror, it down converts to an orthogonal structure of (x, iy).

The two vectors, x and iy (as siblings), define the space's orthogonality. Because the quantum identity (i) (the square root of minus one) is attached to the y-axis, the space has only one "real" vector even though, on a classical basis, its orthogonal representation is in two dimensions.

In transforming the vector structure to the higher classical basis (by collapsing the wavefunction), the quantum-based siblings are "hidden" within the unitary (dimensionally higher) parent state of the photon particle.

The higher dimensionality of the classical plane allows a "fracturing" of the entangled quantum state, forming the observationally classical framework of the x and y axes. In the process, the quantum "not" function transforms to its classical counterpart, and the photon occupies one of the two paths and "not" the other.

5.2. What Could a Two-Dimensional Model Reveal of the Universe's Complexity for the Creation of a Theory of Everything (TOE)?

Our theories and speculations interpreting the Universe's fundamental structure have great mathematical complexity. How could a simple two-dimensional toy model give perspective on what to expect in our search for a unitary TOE principle?

A possible answer lies in the format of the 1-D geometry. Although it has a simple two-dimensional basis, the relationship development across the structure is emergent, and further development beyond and within its basis is not restricted. Instead, that development is hidden from representation in a more complex dimensionality than available on the flat plane.

Of course, once again, this raises the question of how to interpret the limit to dimensional development as an infinity. The answer follows the thesis of the 1-D model. Regardless of the complexity of a TOE principle, it must explain the structure of the Universe in a unitary framework.

Quantum mechanics and the other examples studied above inform us that universal structures have a logical "not" basis in their construction. Any TOE principle will be subject to the restriction imposed on all frameworks of universality. The state will not be constructible in a rational argument without the formal restriction that it minimally devolves into a dualism of segments displaying a paradoxical relationship.

5.3. The Very Large and the Very Small

Quantum states are sometimes referred to in the literature as "microscopic", a misnomer. The difference between the formal and nonformal representations in the 1-D model is that dimensional structure is formatted on a consistent operational basis, and in the 1-D geometry, dimensions develop in a formally inconsistent process of emergent self-organization. The 1-D model is conjectured to have an application explaining the paradoxical relationship between quantum and classical states.

The 1-D model conjectures that its philosophical framework also applies to the

grand scale of the Universe.

From the Cosmic Background Explorer (COBE) Data:

> In principle, in an infinite universe, the waves in the cosmic fireball should appear randomly around the sky at all sizes. But, according to the new map, there seems to be a limit to the size of the waves, with none extending more than 60 degrees across the sky.
>
> The effect was first noted as a puzzle in the COBE data, according to Dr. Gary Hinshaw [12], an astronomer at the Goddard Space Flight Center and a member of the Wilkinson probe team, and now seems confirmed.
>
> If the universe were a guitar string, it would be missing its deepest notes, the ones with the longest wavelengths, perhaps because it is not big enough to sustain them.
>
> "The fact that there appears to be an angular cutoff hint at a special distance scale in the universe", Dr. Hinshaw said [13].

Dr. Hinshaw's comment is based on findings from the paper, "seven-year wilkinson microwave probe (wmap) observations: are there cosmic microwave background anomalies?" [14].

The 1-*D* model conjectures that the COBE data point to a large-scale framework of the Universe in which its dimensional structure has a primordial hexagonal basis that evolves to form our classically observable Universe. The Universe's primordial construction is maximally segmented in 60-degree rotations, and our classical Universe and our antipodal dark partner are defined across the two sides of the geometry's structure.

5.4. The 1-*D* Model's Interpretation of "Just Shut Up and Calculate"

The phrase "just shut up and calculate" attributed to David Mermin [15] epitomizes the conflicting perspectives between the theory and application of quantum formalism and the foundational attempts to understand what it philosophically tells us about reality. There is no agreement on an answer, and the phrase "just shut up and calculate" infers that it is not worth attempting to understand what is beyond understanding.

In opposition to the above viewpoint, the 1-*D* model does not give up on developing a foundational understanding. It presents a verifiable rationale explaining the difference between "what we can know" and "what we cannot know" about the Universe through rationalism.

Although the conjecture, if correct, rules out finding a mathematical TOE model of the Universe, it also offers a basis for redirecting how to understand its ultimate form as an infinity.

Supposing that the justification in the 1-*D* geometry model is correct but discounted because it introduces a nonformal basis, the alternative is an endless search for a TOE principle that does not exist. In that scenario, a deeper understanding of our place in the Universe and its fundamental beauty is hidden and lost forever.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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