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Study of a Viscoelastic PMMA String in a Damped Pendulum

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Abstract

The viscoelastic behavior of polymer optical fibers has garnered increasing interest due to their application as fiber sensors. A common technique for determining the storage and loss modulus of optical fibers involves fitting an exponential model to damped oscillatory motion. However, few studies address the challenge of identifying and specifying the various internal and external contributions to damping. The damping of a simple pendulum is influenced by several factors, such as the friction at the pivot and air drag on the string. In this case, the bob and the string are a coupled oscillating system, for its study, we used a bare polymethyl methacrylate optical fiber as a non-ideal, extensible, and viscoelastic string. The optical fiber was attached to a quasi-punctual support to minimize friction at the pivot. We considered the contribution to the damping of the pendulum due to air drag on the bob by varying the bob's frontal area and extrapolating to the limit case where the frontal area of the bob tends to zero. This approach allowed us to calculate the damping coefficient solely due to the viscoelastic properties of the string. By conducting a dynamic analysis of the forces along the string and considering the interaction between the string and bob through the viscosity, we calculated the complex Young's modulus, a key parameter in understanding the viscoelastic properties of the system.

Keywords

Damped Pendulum, Extrapolation, Polymethyl-Methacrylate, Viscoelastic Material, Complex Young's Modulus, Loss Factor

1. Introduction

The dissipative forces in a real pendulum decrease the pendulum's oscillation amplitude until the movement ceases. There are two types of damping agents in oscillatory motion: the first is caused by external dissipative forces acting on the oscillating system, while the second arises from internal dissipative forces originating within the system. External dissipative forces include: 1) friction between the support and the fixed end of the string and 2) air drag on the bob (non-point mass) hanging from a real string. Conversely, the internal dissipative force originates from the viscoelastic properties of the string under the oscillating force. Studying the effects of the non-conservative forces can help us determine the viscoelastic properties of both the oscillating system and the medium in which it oscillates.

The sources of mechanical energy dissipation in this system have been studied both theoretically and experimentally. Theoretically, several authors have analyzed the effect of air on the string and bob [1], while others have experimentally investigated the contribution of external dissipative forces acting on the different parts of a real pendulum [2]. These experiments often involve taking multiple measurements to perform a least squares interpolation. These studies compare the energy loss due to string drag against that of bob drag. First, they determine the drag of the string by varying its length and measuring the associated damping coefficient. With these measurements, they generate a linear least square fit, whose slope represents the contribution of the string to the energy loss. Then, in a similar manner, they determine the damping by the bob varying its diameter. Mohabazzi and Shankar [2] demonstrated that the string can significantly contribute to the damping of a real pendulum.

The behavior of a real pendulum is also influenced by the viscoelastic properties of the string. In the equation of motion for the oscillating mass, the damping coefficient comprises contributions from both internal and external dissipative forces. The challenge in understanding damping measurements lies in identifying the contributions of both the internal and external dissipative forces. For example, the period of a torsion pendulum depends on the torsion constant of the string and the moment of inertia of the mass. The damping depends on the air resistance acting on the mass and the viscoelastic behavior of the string. The torsion pendulum is used as a precision instrument for measurements in various fields, such as biophysics, metrology, and gravitational physics [3]. In material science, it has been employed to measure the shear modulus of single filaments with uniform micro-sized diameters to determine the mechanical properties of materials under torsional loads. Air resistance significantly contributes to the damping in a torsion pendulum. To minimize the external energy dissipation factors, a torsion pendulum was suspended from a tungsten fiber in a vacuum vessel at 1.5×10^{-5} Pa [4]. They observed that the pendulum oscillated for four days. The oscillation amplitude exhibited typical attenuated behavior, which must stem solely from non-conservative internal forces. Similarly, a real pendulum in a vacuum is also subjected to internal dissipative forces. For this reason, a damped oscillation can be used to study viscoelastic properties of materials that are subjected to external agents such as loads and torques during their most common application as sensors or optical transmitters.

Our study aimed at theoretically and experimentally analyzing the effect of a viscoelastic string in a damped pendulum. If the string is very thin, then the most prominent source of energy dissipation in the pendulum is the air drag on the lateral surface of the bob. After an extrapolation to zero lateral surface, the results of the experiment indicate residual damping in the zero limit of lateral surface of the bob. Contrary to the proposal by Mohabazzi and Shankar [2], we attribute this to the viscoelastic behavior of the string, given the polymeric fiber nature of the string in our experiment. For the theoretical analysis, we assumed that the pendulum is suspended by a viscoelastic string with a tiny cross-section and a complex Young's modulus.

This paper is organized as follows: Section 2 presents the dynamics of the damped pendulum and the viscoelastic string. Section 3 describes the methodology and experimental setup. The results are discussed in Section 4, and conclusions are presented in Section 5.

2. Mathematical Model

2.1. The Damped Pendulum

A real pendulum is subjected to dissipative forces, which decrease their maximum amplitude over time, see **Figure 1**. The total dissipative force is proportional to the bob velocity $L\dot{\theta}$, and the damping term $\lambda L\dot{\theta}$ contains the contributions from all dissipative forces acting on the system. The equation of motion is:

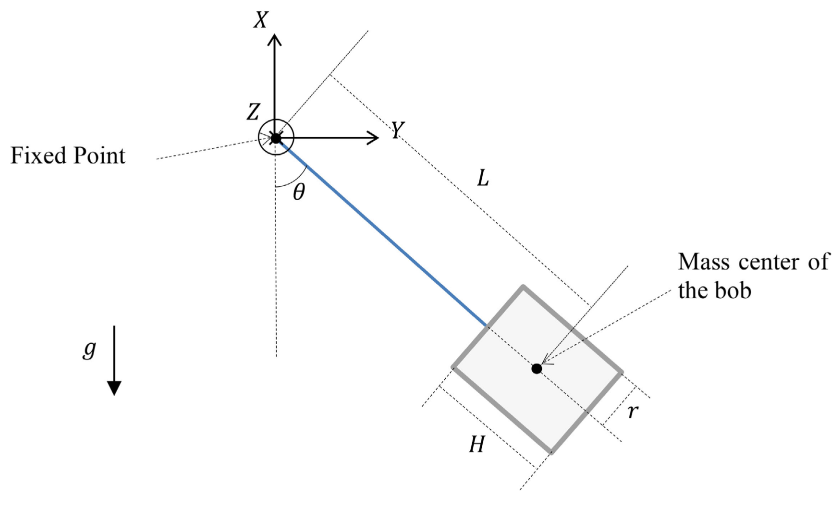

Figure 1. Scheme of the plane of oscillation of a real pendulum with cylindrical bob.

$$\ddot{\theta}+\frac{\lambda}{M}\dot{\theta}+\omega_0^2\theta=0 \tag{1}$$

where, θ is the oscillation amplitude; M is the mass of the bob; and using the Steiner's theorem for a cylindrical body moving around a fixed axis perpendicular to the oscillation plane $\omega_0^2=\frac{MgL}{I_c+ML^2}$ [5], with L being the length of the pendulum measured from the fixed point to the mass center of the bob, g being the acceleration due to gravity, and $I_c = \frac{1}{4}Mr^2 + \frac{1}{12}MH^2$ (where r and H are radius and the height of the bob, respectively).

The solution to Equation (1) is:

$$\theta = \theta_0 e^{-\gamma t}\cos(\omega t), \tag{2}$$

where θ_0 is the initial amplitude, the damping coefficient is $\gamma = \frac{\lambda}{2M}$, and the angular frequency is $\omega = \sqrt{\omega_0^2 - \gamma^2}$. The coefficient γ can be determined experimentally, and its value depends on the magnitude of all dissipative forces.

2.2. The Viscoelastic String

Viscoelastic materials are characterized by complex elastic moduli. The Young's modulus Y^* has the following general form:

$$Y^* = Y' + iY'' = Y'(1 + i\eta), \tag{3}$$

where $\eta = \frac{Y''}{Y'}$, the loss factor.

When a periodic stress σ with frequency ω_0 is applied to a viscoelastic material, a strain ε is produced. The behavior of the viscoelastic material is described by the equation:

$$\sigma^*(t) = (Y' + iY'')\varepsilon(t), \tag{4}$$

where the real stress is associated with the conservative term $\sigma'(t) = Y'\varepsilon(t)$, and the imaginary term is linked to the dissipative parameters of the system $\sigma''(t) = Y''\varepsilon(t)$. Consequently, it is possible to determine the complex Young's modulus of a material through a damping experiment [6]. However, there is an intrinsic difficulty in separating all possible energy loss agents [7]. Therefore, the experimental design must minimize losses other than dissipation in the viscoelastic medium.

The equation of motion of a pendulum with a bob of mass M suspended by a string with variable length is given by:

$$L_0 M(1+\varepsilon)\ddot{\theta} + L_0(1+\varepsilon)\lambda\dot{\theta} + Mg\sin(\theta) = 0, \tag{5}$$

where L_0 is the initial length of the pendulum and ε the strain of the string subjected to periodic stress caused by the longitudinal component of the bob's weight. In this equation, $1 \gg \varepsilon$. Therefore, its solution is known as the solution of a damped oscillator [5].

The forced oscillation of the viscoelastic string is given by the following equation:

$$mL_0\ddot{\varepsilon} + \lambda_2 L_0\dot{\varepsilon} + k_2 L_0\varepsilon = Mg\cos(\theta), \tag{6}$$

where m is the mass of the string; k_2 is the elasticity constant of the string; and $\lambda_2 L_0\dot{\varepsilon}$ is the dissipative term, with λ_2 being the damping constant of the coupled system and $Mg\cos(\theta)$ is the weight component along the string [5].

Equation (6) can be rewritten as:

$$\ddot{\varepsilon}+2\gamma_2\dot{\varepsilon}+\omega_2^2\varepsilon=\frac{Mg}{mL_0}\cos\theta\,, \tag{7}$$

where $\gamma_2=\frac{\lambda_2}{2m}$ and $\omega_2^2=\frac{k_2}{m}$. Substituting the solution of Equation (5) $\left[\theta=\theta_0\mathrm{e}^{-\gamma t}\cos(\omega t)\right]$ in Equation (7) yields the following:

$$\ddot{\varepsilon}+2\gamma_2\dot{\varepsilon}+\omega_2^2\varepsilon=\frac{Mg}{mL_0}\cos\left[\theta_0\mathrm{e}^{-\gamma t}\cos(\omega t)\right]. \tag{8}$$

Equation (5) and Equation (6) represent two oscillating systems coupled by the damping and gravitational forces. The driven frequency in Equation (8) is ω. Therefore, both systems oscillate with the coupled frequency ω.

By developing the cosine of the right side of Equation (8) in its Taylor's series and considering only up to the quadratic term, we get:

$$\ddot{\varepsilon}+2\gamma_2\dot{\varepsilon}+\omega_2^2\varepsilon=\frac{Mg}{mL_0}\cos\left[\theta_0\mathrm{e}^{-\gamma t}\cos(\omega t)\right]. \tag{9}$$

We propose the following solution for Equation (9):

$$\varepsilon(t)=A_0\left[1-A_1\mathrm{e}^{-\beta t}\cos^2(\omega t)-A_2\mathrm{e}^{-\beta t}\sin^2(\omega t)\right], \tag{10}$$

where ε_0, A_1, A_2 and β are constants; $A_1\neq A_2$; and $\omega_2\gg\omega$. To verify this as the solution, we substituted it into Equation (9) and equated the coefficients multiplying the monomials on each side of the equation as follows:

1) Constant monomials:

$$A_0=\frac{Mg}{mL_0\omega_2^2} \tag{11}$$

2) $\sin\omega t\cos\omega t$ monomials under the condition $A_1\neq A_2$:

$$\beta=\gamma_2 \tag{12}$$

3) $\sin^2(\omega t)$ monomials:

$$A_2=-\frac{\omega^2\theta_0^2}{\omega_2^2-4\omega^2}=-\frac{\omega^2}{\omega_2^2}\frac{\theta_0^2}{1-4\frac{\omega^2}{\omega_2^2}}\approx 0 \tag{13}$$

4) $\cos^2(\omega t)$ monomials:

$$A_1=\left(\frac{\theta_0^2}{2}\right)\frac{\omega_2^2-2\omega^2}{\omega_2^2-4\omega^2}=\left(\frac{\theta_0^2}{2}\right)\frac{1-2\frac{\omega^2}{\omega_2^2}}{1-4\frac{\omega^2}{\omega_2^2}}\approx\frac{\theta_0^2}{2};\gamma_2=2\gamma \tag{14}$$

On the other hand, the constitutive equation for the stress state on the viscoelastic fiber can be expressed as follows:

$$\sigma'(t)=\frac{F_k(t)}{A}=Y'\varepsilon(t) \tag{15}$$

$$\sigma''(t)=\frac{F_d(t)}{A}=Y''\varepsilon(t) \tag{16}$$

In the absence of energy dissipation, the elastic force is $F_k(t)=L_0k_2\varepsilon(t)$, where

$\varepsilon(t)$ is the solution of the undamped driven string. This means $\varepsilon(t)=\frac{M}{m}\frac{\omega^2}{\omega_2^2-\omega^2}\cos(\omega t)$, while the damping force is $F_d(t)=\lambda_0 L_o\dot{\varepsilon}(t)=2\gamma_0 m L_0\dot{\varepsilon}(t)$, where $1/\gamma_0$ is the damping time of the viscoelastic string. This property is independent of the experiment used for this calculation. The temporal change of the strain $\dot{\varepsilon}(t)$ is calculated from Equation (10). Considering that $A_2 \ll A_1$, $e^{-\gamma_2 t}\approx 1$, and $\gamma_2 \ll 2\omega$ and the ω dependence of $\dot{\varepsilon}(t)$, we get the following:

$$\frac{Y''}{Y'}=\frac{\lambda_0 L_0\dot{\varepsilon}(t)}{k_2 L_0\varepsilon(t)}\cong\frac{4\gamma_0\omega\sin(\omega t)\cos(\omega t)}{\frac{M}{m}\frac{\omega^2}{1-\frac{\omega^2}{\omega_2^2}}\cos(\omega t)}. \tag{17}$$

In Equation (17), $\omega^2/\omega_2^2 \ll 1$. In the limit case, where the damping of the pendulum is solely due to viscoelasticity in the string, we propose $\gamma_2=\gamma_0 m/M=2\gamma_{\text{lim}}$, because γ_0 is a property of the viscoelastic string and the coupling between systems is given by the ratio m/M. In this case, the damping coefficient of the pendulum is γ_{lim}. Thus, we get the following:

$$\eta=\frac{Y''}{Y'}=\frac{4\gamma_{\text{lim}}}{\omega}\sin(\omega t) \tag{18}$$

In this case, the Young's complex modulus will be as follows:

$$Y^*=Y'(1+i\eta) \tag{19}$$

3. Experimental Procedure

The amplitude of a damped simple pendulum decreases as its mechanical energy is dissipated due to frictional forces. In our analysis, we considered four sources of energy dissipation:

1) Friction at the fixed end of the pendulum;
2) String drag or air resistance on the string;
3) Air resistance on the pendulum bob;
4) Dissipation within the viscoelastic string.

To reduce friction at the fixed end of the string in the pendulum, we used quasi-punctual support to allow free oscillation around a point, as shown in Figure 2. Additionally, to minimize drag effects on the string and make them negligible compared to the drag effects on the bob, we used a string and bob with radii $r'=0.05\text{ cm}$ and $r=1.10\text{ cm}$, respectively. The pendulum length $L=0.33\text{ m}$ was measured from the fixed point to the mass center of the bob, consisting of a constant segment of 0.08 m and a variable segment $L_0(1+\varepsilon)$, where L_0 is the length of the viscoelastic string without strain. Thus, only the air resistance on the bob and the damped oscillation along the viscoelastic string are the sources of dissipation. In our experiment, the viscoelastic string is an optical fiber made of polymethyl methacrylate (PMMA) with length $L_0=0.25\text{ m}$ without strain and mass $m=0.225\text{ g}$.

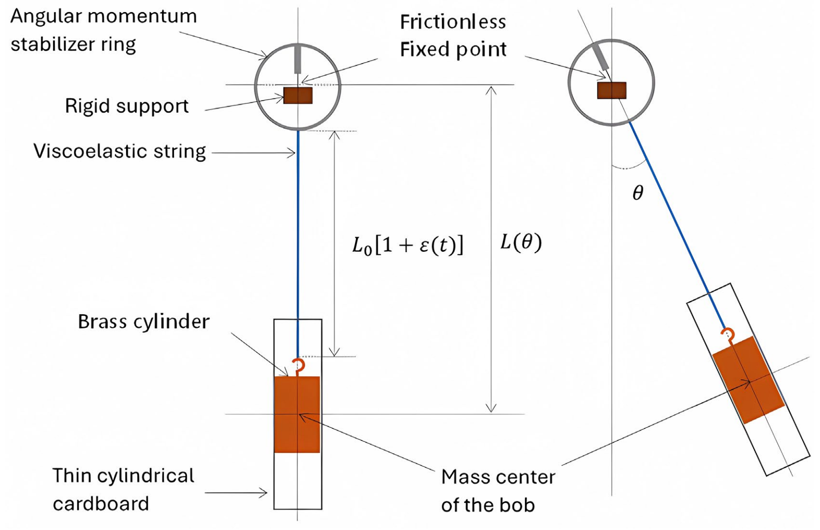

Figure 2. Experimental setup.

To determine the role of the viscoelastic string in the dissipation of mechanical energy, we conducted an experiment with zero resistance on the bob. For this purpose, we experimentally calculated the damping coefficient of the pendulum for bobs of different frontal areas. The bob is a brass cylinder with a mass of 100 g, height of 3.30 cm, and diameter of 2.20 cm. Thus, to vary the frontal area of the bob without changing either the mass center or the mass, we wrapped the bob with thin cardboard of different heights, ranging from $H' = 9.80$ to 3.80 cm, reducing the height and consequently the frontal area of the bob. The relationship between the frontal area of the bob and the damping coefficient γ of the pendulum was determined through least squares fitting. After extrapolating to zero frontal area to eliminate the air resistance on the bob, we obtained the residual damping coefficient γ_{lim}. This ensured that the only source of mechanical energy dissipation was within the viscoelastic string.

The oscillations of the bob, set at an initial amplitude of a small angle with the vertical ($\approx 10^{\circ}$) and with a fixed bob height H', were meticulously recorded with a video camera placed 0.50 m away from the device to avoid parallax errors. The motion of the pendulum while swinging back and forth coincides with the description in Equation (1), where the envelope of the oscillation is given by $\text{e}^{-\gamma t}$. Each video was carefully analyzed using Tracker, a software for video analysis, to obtain the graph of the natural logarithm of the maxima against time t and, through a linear fit of the data, we experimentally determined the value of γ. We filmed three videos per height H' to ensure the reproducibility of the results.

4. Results and Discussion

The moment of inertia of the bob $I_c \ll ML^2$. Therefore, we considered $\omega_0 = \sqrt{g/L}$ to be the frequency of the pendulum. We obtained $\omega_0 = 4.449\ \text{Hz}$, and the oscillation period $T = 1.15\ \text{s}$ coincided with the observed values in all the curves recorded and analyzed with Tracker.

Table 1 shows the pendulum damping coefficients γ for each frontal area $(\pi r H')$, determined by performing the linear fit of the natural logarithm of the oscillation maxima against time for the first video. It also shows the correlation coefficient $R^2 \approx 1$ of each fit, confirming that the pendulum movement while swinging back and forth is determined by Equation (1).

Table 1. Damping coefficients corresponding to the first video of each bob.

Frontal area, ±0.1 cm^2	*Damping coefficient* γ, s^{-1}	R^2
11.4	0.01893	0.9979
13.1	0.02414	0.9981
14.8	0.01924	0.9980
16.6	0.01827	0.9981
18.3	0.02222	0.9990
20.0	0.02077	0.9981
21.7	0.0192	0.9978
23.5	0.02386	0.9976
25.2	0.02283	0.9976
26.9	0.02133	0.9969
28.7	0.03291	0.9962
30.4	0.02425	0.9973
32.1	0.03365	0.9970
33.8	0.03340	0.9954

Table 2. Damping coefficients corresponding to the second video of each bob.

Frontal area, ±0.1 cm^2	*Damping coefficient* γ, s^{-1}	R^2
11.4	0.01879	0.9977
13.1	0.01525	0.9982
14.8	0.0209	0.9974
16.6	0.02105	0.9975
18.3	0.02144	0.9980
20.0	0.02343	0.9981
21.7	0.02206	0.9982
23.5	0.02866	0.9971
25.2	0.0218	0.9982
26.9	0.03778	0.9962
28.7	0.01836	0.9977
30.4	0.03057	0.9972
32.1	0.03307	0.9943
33.8	0.03398	0.9949

Table 2 and Table 3 show the results obtained from videos 2 and 3, respectively. The amplitude of the pendulum oscillation with $L = 0.33\ \text{m}$, as given by Equation (2) and $\gamma = 0.02343$, corresponding to a frontal area of 20 cm² from Table 2, is shown in Figure 3. The inset corresponds to the horizontal component of the oscillatory motion of the pendulum obtained through Tracker. The coincidence between the calculated frequency ω and that observed in the Tracker graph indicated that the bob can be considered a point mass, as established in Equation (2).

Table 3. Damping coefficients corresponding to the third video of each bob.

Frontal area, ±0.1 cm²	*Damping coefficient* γ, s^{-1}	R^2
11.4	0.01874	0.9979
13.1	0.01602	0.9979
14.8	0.0201	0.9973
16.6	0.01859	0.9976
18.3	0.01901	0.9987
20.0	0.02729	0.9979
21.7	0.02225	0.9982
23.5	0.02802	0.9982
25.2	0.02861	0.9976
26.9	0.02829	0.9968
28.7	0.02936	0.9967
30.4	0.02355	0.9983
32.1	0.03467	0.9956
33.8	0.04098	0.9952

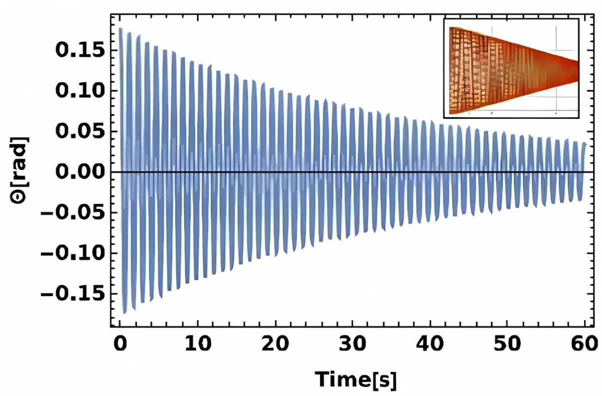

Figure 3. Plot of amplitude of pendulum oscillation with $L = 0.33\ \text{m}$ and $\gamma = 0.02343$. The inset corresponds to the horizontal component of the oscillatory motion of the pendulum obtained by Tracker.

To analyze the results for each frontal area, we considered the average value γ of the three tables. Figure 4 shows the plot of the natural logarithm of the damping coefficients γ against the bob frontal area. The equation of the line that fits the data is the following:

$$\ln\left[\gamma(x)\right] = 0.0277x - 4.3497,$$

with a correlation coefficient $R^2 = 0.8746$. In Figure 4, the fluctuations in the values shown in Tables 1-3 can be observed, however the R^2 value indicates a good correlation with an exponential relation between the bob frontal area and the pendulum damping coefficient γ. With this model a better correlation coefficient was obtained than with other nonlinear models. Other authors have also found a nonlinear relation between the bob frontal area and the damping coefficient [8].

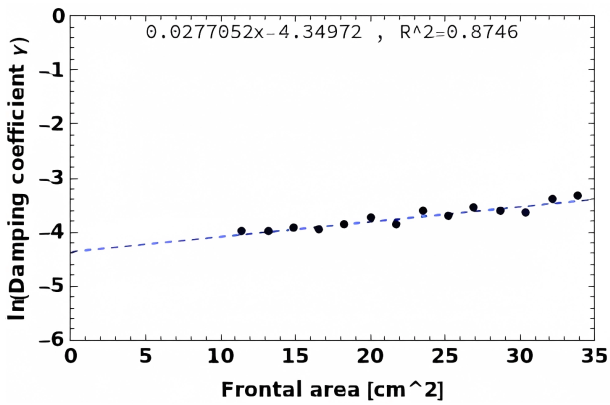

Figure 4. Fit of the natural logarithm of the damping coefficient as a function of the frontal area.

The residual damping coefficient is calculated by extrapolating the bob frontal area to zero: $\gamma_{\text{lim}} = \exp(-4.3497) = 0.0129\ \text{s}^{-1}$ (see Figure 4). This value is two orders of magnitude higher than the damping coefficient due to air drag on a string reported by Mohazzabi and Shankar [2] for a string with the same diameter, which we can attribute to the viscosity of PMMA. This result indicates that, in this specific case, assuming negligible air resistance on the string is valid. For this value:

$$\omega = \sqrt{\omega_0^2 - \gamma_{\text{lim}}^2} \cong \omega_0 = 4.449\ \text{Hz}.$$

The solution of Equation (9) with $\gamma_2 = 2\gamma_{\text{lim}}$ is illustrated in Figure 5, where it can be observed that the strain $\varepsilon(t)$ oscillates with the forcing frequency ω. The maximum strain is obtained each $T/2$, when $\theta = 0$, and the strain decreases exponentially at the extremes of the oscillation. In this solution, we assumed that both oscillatory systems, the bob and the string, are coupled through viscosity.

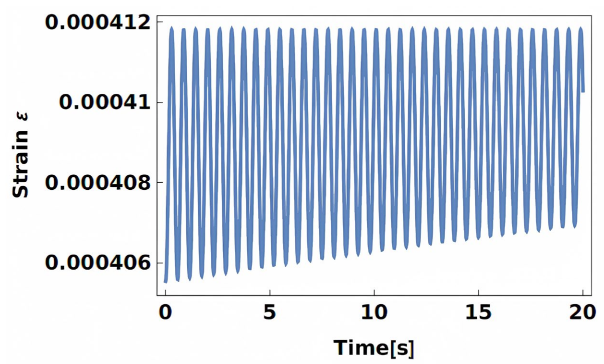

Figure 5. Plot of the strain as a function of time.

With this experiment, we can indirectly study the complex Young's modulus of the string. The damping coefficient of the string coupled to the pendulum is $\gamma_2 = \gamma_0 m/M$ where γ_0 is the property of the material (PMMA). We have used this hypothesis to calculate the maximum value of the loss factor of PMMA from Equation (18):

$$\eta = \frac{4\gamma_{\text{lim}}}{\omega} = 0.011$$

The real value of the Young's modulus $Y' = 3.030\ \text{GPa}$ was measured previously, thus $Y'' = 0.033\ \text{GPa}$. This value is within the range found for the complex Young's modulus of the PMMA in the low-frequency limit obtained through other methods [9]. Additionally, with the value of Y', we calculate k_2 and consequently the value of $\omega_2 = 6504.36\ \text{s}^{-1}$, which confirms the hypothesis $\omega_2 \gg \omega$ in Equation (13) and Equation (14) of our theoretical model.

These results demonstrate that the proposed theoretical model and experiment can be utilized to study the complex Young's modulus of a PMMA fiber under the effect of an oscillating force in the longitudinal direction.

5. Conclusions

The damping of a pendulum due to an extensible and viscoelastic string was evaluated using a procedure in which the frontal area of the mass tends to zero. Even in this case, we found that the viscosity of the string significantly affects the damping of the pendulum.

Through our theoretical model and experiment, we developed a method to evaluate the complex Young's modulus of a viscoelastic material. Regarding the coupling of the two oscillatory systems through their viscosity, we hypothesized that their damping coefficients are related by the ratio of their masses. This hypothesis was experimentally verified for a specific material (PMMA). Our findings correspond to the range of values reported in the literature for low frequencies.

Since the damping coefficient is related to the complex Young's modulus, this method is a contribution to the study of the viscoelastic properties of materials.

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In memory of our dear colleague and friend Andrés Valentín Porta Contreras.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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