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Thermoelectric Stirling Engine (TEG-Stirling Engine) Based on the Analysis of Thermomechanical Dynamics (TMD)

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Abstract

The thermoelectric energy conversion technique by employing the Disk-Magnet Electromagnetic Induction (DM-EMI) is examined in detail, and possible applications to heat engines as one of the energy-harvesting technologies are discussed. The idea is induced by the analysis of thermomechanical dynamics (TMD) for a nonequilibrium irreversible thermodynamic system of heat engines, such as a drinking bird and a low temperature Stirling engine, resulting in thermoelectric energy generation different from conventional heat engines. The current thermoelectric energy conversion with DM-EMI can be applied to wide ranges of machines and temperature differences. The mechanism of DM-EMI energy converter is categorized as the axial flux generator (AFG), which is the reason why the technology is applicable to sensitive thermoelectric conversions. On the other hand, almost all the conventional turbines use the radius flux generator to extract huge electric power, which uses the radial flux generator (RFG). The axial flux generator is helpful for a low mechanoelectric energy conversion and activations of waste heat from macroscopic energy generators such as wind, geothermal, thermal, nuclear power plants and heat-dissipation lines. The technique of DM-EMI will contribute to solving environmental problems to maintain clean and sustainable energy as one of the energy harvesting technologies.

Keywords

A Low Temperature Stirling Engine, Axial Flux Generator, Thermomechanical Dynamics (TMD), Thermoelectric Energy Conversions

1. Introduction

Heat and energy are essential for the prosperity of human society and ecology on the Earth. This is the reason why human societies have advanced macroscopic energy generators (MEGs) such as turbines, motors and rotors for wind, hydroelectric, geothermal, thermal, nuclear power plants and so forth. However, the characteristic feature of MEGs is essentially directed to mass production and consumption of heat and energy, resulting in the enormous amount of abandoned waste heat and chemical substances. This has caused problems hindering sustainable developmental goals (SDGs).

On the contrary to MEGs, low temperature heat engines (a drinking bird [1] and low temperature Stirling engines [2]) are mechanical systems that can use very small amount of heat flows, and we proposed thermoelectric generators to convert the usable mechanical energy into electricity. The low temperature heat engines of a drinking bird and a Stirling engine can work at small temperature difference, producing usable electric energy [3]-[6]. It is possible to activate electric power from abandoned waste heat by the method of DM-EMI with the axial flux electromagnetic induction [3] [4].

The traditional mechanoelectric convertors are categorized as the radial flux generator (RFG) by the classification of magnetic flux lines, which is suitable for huge energy productions and requires high speed rotations of turbines. Although the radial flux generators (RFG) are qualified for producing high electric power, it is not qualified for reactivating electric power from a low temperature heat flow, such as $40^\circ\text{C} < T < 100^\circ\text{C}$ boiled water, waste heat from industries. On the other hand, the axial flux generator (AFG) is most suitable for activating sensitive boiled water, waste heat from industries. This is an important consequence derived from the analysis of thermomechanical dynamics (TMD), which is proposed by the authors for nonequilibrium irreversible states (NISs) of heat engines [5] [6].

The disk-magnet electromagnetic induction (DM-EMI) technique with a low temperature Stirling engine revealed that electric power generation from low heat flows can be possible. This is assured by the theoretical analysis of TMD, proving that an optimal speed of mechanical rotation can exist in a low rotational speed (about 30 - 60 rpm [3] [4]). Therefore, a low temperature, thermoelectric generation Stirling engine (TEG-Stirling engine) can be constructed. In the current paper, we explain and emphasize that the analysis of thermomechanical dynamics (TMD) applied to TEG-Stirling engine generates a low-speed, low-weight, optimal TEG-Stirling engine.

The method of TMD is explained in Section 2 for self-contained discussion, and the equation of motion for the flywheel of Stirling engine is discussed in Section 3. The solution to the dissipative equation of motion of TEG-Stirling engine is discussed in Section 4. The DM-EMI with TEG-Stirling engine and the relation between electric current and power are explained in Section 5. Conclusion is in Section 6.

2. The Method of TMD

The equation of motion and time-dependent physical quantities, such as internal energy $\varepsilon(t)$, work $W(t)$, entropy $S(t)$ and temperature $\tilde{T}(t)$ of heat engines are solved self-consistently by the method of thermomechanical dynamics (TMD). The method of TMD is a new classical approach proposed by the authors, along the work of Gibbs' thermodynamics which is based on fundamental thermodynamics and needs profound discussions on physical foundations. Hence, readers who are interested in theoretical discussions should be directed to references [5] [6].

The method of TMD requires three conditions.

1) The dissipative equation of motion

In the case that mechanical and thermal states coexist, such as thermomechanical states of heat engines, the dissipative equation of motion for work must be constructed by considering phenomenological effects of frictional variations, time-dependent changes of physical quantities, thermal conductivity and efficiency.

Because time-symmetry is broken in the system of heat engines, there is no Euler-Lagrange type derivation of a correct dissipative equation of motion. It would be useful to make use of Hamiltonian or Lagrangian method at the beginning to find an approximate dissipative equation of motion and then, find an appropriate dissipative equation of motion.

2) The total energy-flow conservation law

The thermodynamic work $\mathrm{d}W_{th}(t)$, the internal energy $\mathrm{d}\varepsilon(t)$ and total entropy $T(t)\mathrm{d}S(t)$, are related to one another by the energy conservation law:

$$\mathrm{d}\varepsilon(t)/\mathrm{d}t = T(t)\mathrm{d}S(t)/\mathrm{d}t + \mathrm{d}W_{th}(t)/\mathrm{d}t = \mathrm{d}Q(t)/\mathrm{d}t + \mathrm{d}W_{th}(t)/\mathrm{d}t \tag{1}$$

Thermodynamic equilibrium is defined by $\mathrm{d}W_{th}(t)/\mathrm{d}t = 0$: no thermodynamic power exists in thermodynamic equilibrium.

The expression of heat flow (entropy flow) is used

$$T(t)\mathrm{d}S(t)/\mathrm{d}t = \mathrm{d}Q(t)/\mathrm{d}t, \tag{2}$$

in the analysis of heat engines.

3) Temperature, $\tilde{T}(t) = T\tau(t)$, in a nonequilibrium irreversible state

The measure of a nonequilibrium irreversible state is defined by the ratio of entropy-flow against energy-flow:

$$\tau(t) = \frac{T(t)\mathrm{d}S(t)/\mathrm{d}t}{\mathrm{d}\varepsilon(t)/\mathrm{d}t} = \frac{\mathrm{d}Q(t)/\mathrm{d}t}{\mathrm{d}\varepsilon(t)/\mathrm{d}t}. \tag{3}$$

The value of $\tau(t)$ is a dimensionless, positive-definite function, $\tau(t) > 0$. The temperature in nonequilibrium state (NISs) is defined by,

$$\tilde{T}(t) = T_0\tau(t), \tag{4}$$

where T_0 is the initial equilibrium temperature. When $\tau(t) = 1$ holds identically with respect to time t, it defines thermodynamic equilibrium, which shows no work exists, $\mathrm{d}W_{th}(t)/\mathrm{d}t = 0$, at thermodynamic equilibrium. The conditions of near equilibrium states, local equilibrium, linearity of fluxes and forces of transport processes [7]-[9] are studied by the condition, $\tau(t)=\dfrac{T(t)\mathrm{d}S(t)/\mathrm{d}t}{\mathrm{d}\varepsilon(t)/\mathrm{d}t}\sim 1$ in the TMD method.

3. The Equation of Motion for a Stirling Engine

A theoretical and schematic low temperature Stirling engine is shown in **Figure 1**, and the device consists of following functions:

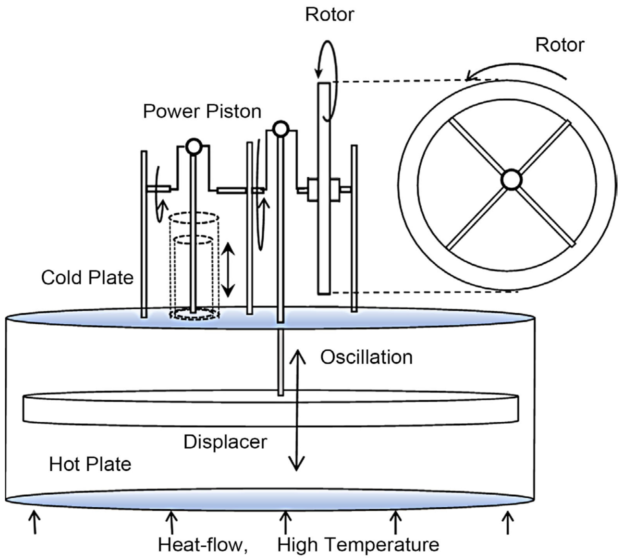

Figure 1. A theoretical and schematic structure of a low temperature Stirling engine [6].

1) Heat source: A homogeneous heat flow from boiled water (40˚C - 100˚C) and geothermal heat, etc. The heat flow coming in the system is defined by $\mathrm{d}Q(t)/\mathrm{d}t>0$.

2) Heat exchangers: The power piston is used to improve the heat flow and the flywheel rotation affected by friction losses.

3) Regenerator: The internal mechanism of heat exchangers between a hot plate and a cold plate. The thermomechanical conversion for work depends on thermal efficiency, heat transfer, viscous pumping and friction losses.

4) Heat sink: The temperature difference between a hot plate and a cold plate is needed for internal heat flows.

5) Displacer: The thermal heat flow from a hot plate to a cold plate exerts vertical oscillations of the displacer. The efficiency of displacer to maintain appropriate heat dissipations is essential for mechanical rotations of the flywheel.

It is essential to understand that heat engines in general are not in thermodynamic equilibrium, but in nonequilibrium irreversible states (NISs). Therefore, it is important to have a different theoretical approach for NISs, which is the reason why we proposed the method of TMD. The piecewise continuous driving forces produced by frictional and thermal fluctuations are assumed to couple to thermodynamic work, $Q_w(t)$, of the flywheel and power-piston with an associating dissipation of heat.

As the first requirement (1) of TMD, the *dissipative equation of motion* for a low temperature Stirling engine is proposed by:

$$I_0\theta''(t)+c\theta'(t)-\lambda_w Q_w(t)\left|\sin\theta(t)\right|=0, \tag{5}$$

where λ_w is a dimensionless coupling constant for heat and mechanical work, and the angle, $\theta(t)$, is chosen as in **Figure 2**. The term $\left|\sin\theta(t)\right|$ expresses piecewise continuous driving forces produced by frictional and nonequilibrium thermal fluctuations, and c is a friction constant.

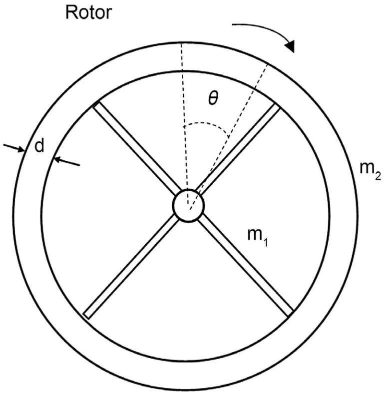

Figure 2. The rotational angle, $\theta(t)$, starting from the vertical axis.

Although the fundamental equation of motion (5) seems simple, its mathematical and physical consequences are profound. The piecewise continuous driving force in (5) immediately indicates that the acceleration is not defined as differentiable and continuous quantity as supposed in Newtonian mechanics. The acceleration cannot be determined as the second-order derivative derived from the trajectory of motion, because the driving force contains jump discontinuities in the entire domains of motion.

The jump discontinuities are not avoidable, and they naturally emerge from friction and viscosities of working fluid, sheer stress and machine structure, temperature and thermal fluctuations. The angular acceleration, $\theta''(t)$, is no longer determined by the derivative of angular velocity, $\theta'(t)$, though $\theta'(t)$ is continuous and observable. In TMD, the concept of force is physical, and force only changes directions of motion or velocities of particles, but not associated with mass × acceleration. The angular acceleration is no longer meaningful, shown numerically in the next section.

4. The Solution to the Dissipative Equation of Motion of TEG-Stirling Engine

We show computer simulations by employing the following incoming heat $Q_{in}(t)$,

$$Q_{in}(t) = Q_H\left(1.0 - \mathrm{e}^{-\xi t}\right) \tag{6}$$

and heat flow $\mathrm{d}Q_{in}(t)/\mathrm{d}t$, as shown in **Figure 3** and **Figure 4**, and Q_H and ξ are free parameters to adjust in computer simulations, e.g., $Q_H \sim 100\ \mathrm{cal}$, $\xi \sim 6.51\times10^{-3}\,(1/\mathrm{s})$ for the current simulations.

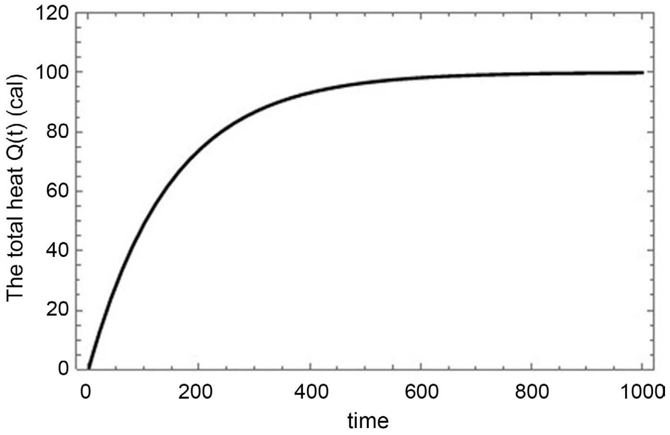

Figure 3. The total heat-in, $Q_{in}(t)$.

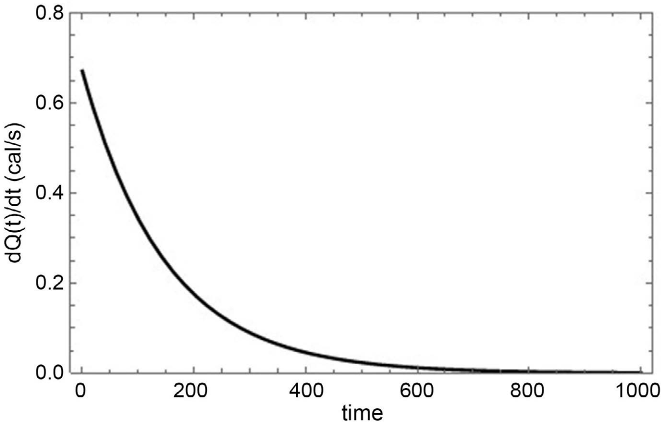

Figure 4. The heat flow, $\mathrm{d}Q_{in}(t)/\mathrm{d}t$.

The dissipative equation of motion, $Q_{in}(t)$ and $\mathrm{d}Q_{in}(t)/\mathrm{d}t$ with $Q_w(t) = \eta Q_{in}(t)$ (ξ and η are arbitrary chosen small values) and Equation (5) are used to find the heat-energy solution for kinetic work, $Q_{wk}(t)$, $\theta(t)$ and $\theta'(t)$ by maintaining the total energy-flow conservation law, (1) and (2). The computations should be repeated by taking different values of ξ and η until reasonable experimental values of angular velocity $\theta(t)$ and $\theta'(t)$ are obtained.

The number of rotations $\theta(t)/2\pi$ (revolutions) and the angular velocity $\theta'(t)/2\pi$ (revolutions/s) of the flywheel are respectively shown in **Figure 5** and **Figure 6**. One can check that the trajectory of motion, $\theta(t)/2\pi$, changes continuously. Thermal force or pressure exerted by heat flows from a cold plate to a hot plate accelerates the angular velocity of rotations in the beginning, but mechanical motions reach a plateau, a relatively stable level of the angular velocity, as shown in **Figure 6**. The maximum angular velocity seems stable and constant, but one can notice that the angular velocity in **Figure 6** has tiny fluctuations along the solution. The tiny fluctuations are caused by frictional variations and thermal fluctuations coming from the displacer and working fluid.

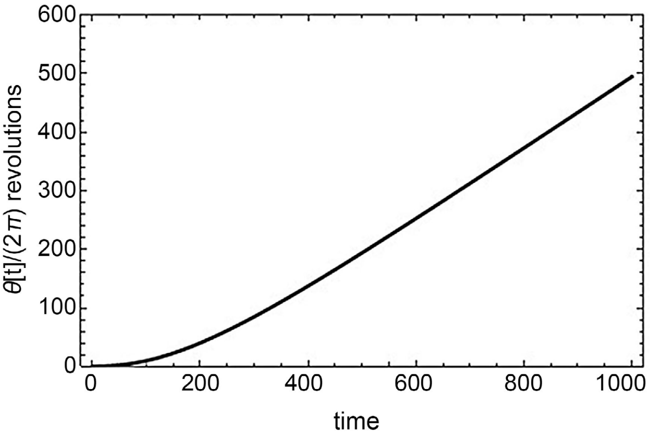

Figure 5. The number of revolutions, $\theta(t)/2\pi$, in the time range $0 < t < 1000$.

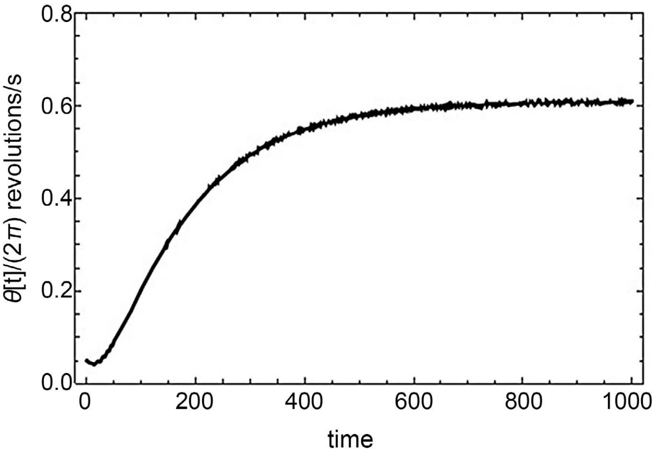

Figure 6. The angular velocity, $\theta'(t)/2\pi$ (revolutions/s). Note the tiny fluctuations along the angular velocity.

The TMD thermomechanical approach to physical phenomena demands fundamental changes regarding the concept of thermodynamic force and work, $W_{th}(t)$. Mechanical work is based on continuity and differentiability of motion, which is integrated in changes of velocity and trajectory of particles. In other words, it is essential to recognize that modifications of mechanical motion caused by friction, wear, deformation and thermal fluctuations generate the fundamental change to the concept of differentiability of physically observable quantities.

The trajectory $\theta(t)$ and angular velocity $\theta'(t)$ are continuous and differentiable, whereas the angular acceleration $\theta''(t)$ is piecewise continuous and has finite numbers of jump discontinuities in a finite interval. The whole view of acceleration results in an assembly of hedgehog-like spiny lines as shown in **Figure 7**. The realistic flywheel thermal motion is produced reasonably well by the dissipative equation of motion (5). When heat exchangers and regenerators work properly, the flywheel rotation persists in a long period of time. Numerical calculations and self-consistency relations are discussed in detail in [5] [6].

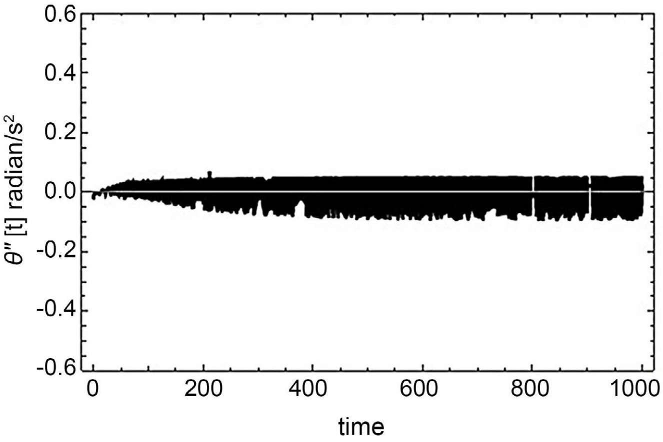

Figure 7. The piecewise continuous angular acceleration, $\theta''(t)$ (rad/s^2), $0 < t < 1000$.

Thermodynamic work,

$$Q_{wk}(t) = \frac{I_0}{2}\theta'(t)^2 \quad \text{(Joule)} \tag{7}$$

is shown in **Figure 8**. The rotational energy reaches a maximum stable value, which has a continuous, tiny-wiggly line because of $\theta'(t)$. The dissipative equation of motion is successful for producing thermomechanical flywheel rotations and applied to thermoelectric energy conversions [3] [4].

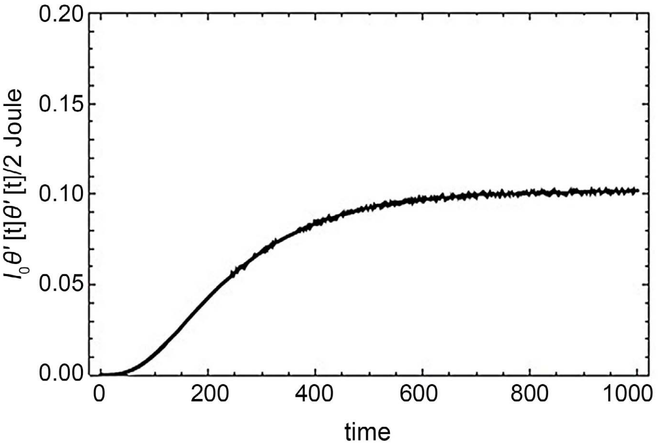

Figure 8. Thermodynamic work, $Q_{wk}(t) = \frac{I_0}{2}\theta'(t)^2$.

The thermomechanical states of the heat engine are in nonequilibrium irreversible states (NISs), and time-dependent thermodynamic work $W_{th}(t)$, internal energy $\varepsilon(t)$, energy dissipation or entropy $T(t)\mathrm{d}S(t)/\mathrm{d}t$, and temperature $\tilde{T}(t)$, are precisely obtained and computed in TMD, and physical quantities are numerically shown in [6]. We will focus on the technological method DM-EMI (disk-magnet electromagnetic induction) and its applications to TEG-Stirling engine in the following section. The detailed explanations and proposed devices are discussed in detail in the papers [3] [4].

5. The DM-EMI Applied to TEG-Stirling Engine

The computer simulations for the existence of optimal angular velocities (rpm) at low temperature and low heat flows are shown, and the fact is the proof of possibility for a low temperature TEG-Stirling engine as one of the sustainable environmental technologies (SETs). The large parts of technical as well as theoretical discussions are found in papers [3] [4]. The NS-pair disk magnet electromagnetic induction in a general schematic image is shown in **Figure 9**, and properties of electric current and power produced by the axial flux generator (AFG) are shown by changing angular velocity, ω (rpm). The numerical calculations with ω = 120 (rpm) and ω = 30 (rpm) are respectively compared, which demonstrates the character of electric current and power. The axial magnetic flux of DM-EMI method produces pulse current (PC). The higher angular velocity driven by high temperature exhibits discrete properties of pulse electric current in a very short ranges of time, whereas the lower angular velocities driven by a low temperature gradually demonstrate like a character of continuous electric currents. This also indicates one of the properties of AFG appropriate for a low temperature thermoelectric conversion.

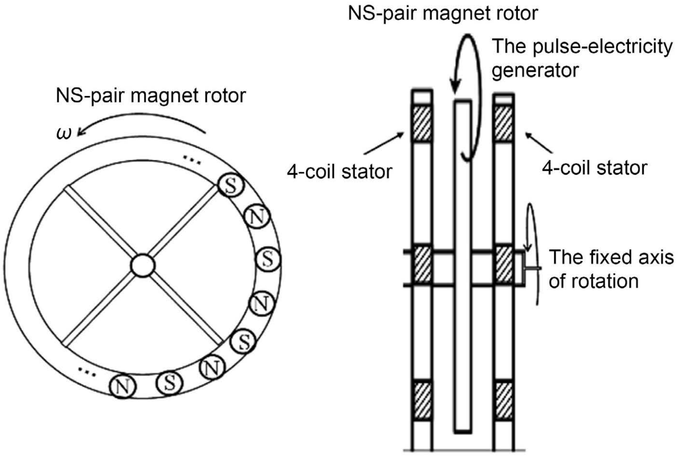

Figure 9. The image of NS-pair disk magnet electromagnetic induction.

A primitive experiment to show a pulse electric current is shown in **Figure 10**, ω~160 (rpm), which agrees with TMD theoretical calculations. Note that the pulse current direction in **Figure 10** is from down-to-up, which comes from choosing the direction of right- or left-rotations of the flywheel. The 4NS-pair disk-magnet rotor is composed of the pair of N and S magnetic poles in a rotor, and numerical simulations produce an alternating pulse current as shown in **Figure 11** ($\omega = 120$ rpm) and **Figure 12** ($\omega = 30$ rpm).

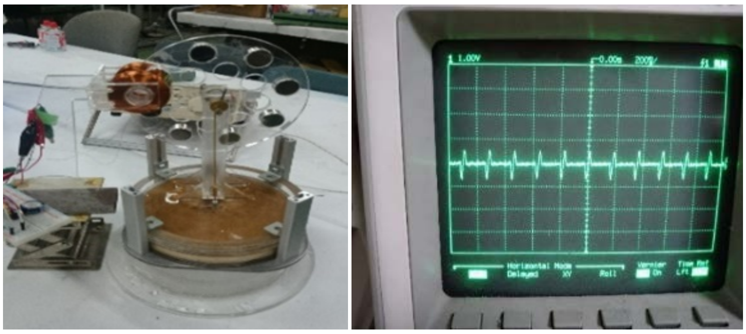

Figure 10. The primitive experiment (left) and pulse electric current (right).

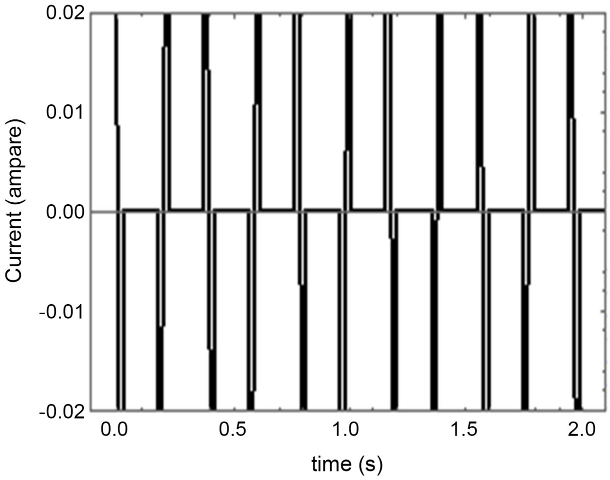

Figure 11. The produced pulse electric current: $\omega = 120$ (rpm).

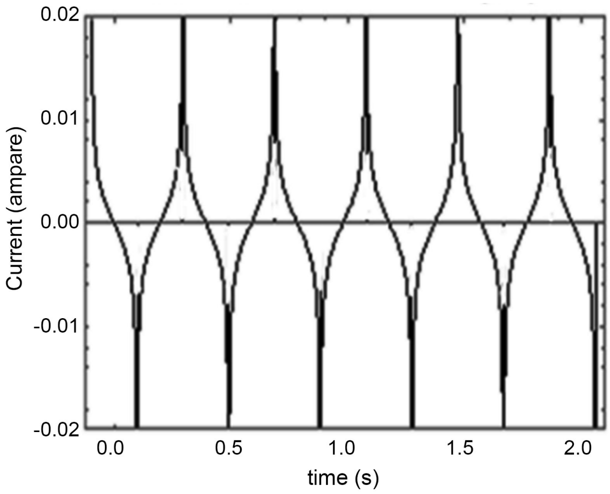

Figure 12. The produced pulse electric current: $\omega = 30$ (rpm).

The N and S poles respectively induce a reversed pulse-current, examined by the theoretical analysis of electromagnetic induction. The direction of magnetic flux induced in the coils of the stators is completely opposite to the N and S poles, resulting in reversed pulse electric current. The current and voltage produced in a coil are inverse proportional against a produced electric energy in a time interval, $\left(t_0, t_1\right)$. It is understood from the relation:

$$E_e = \int_{t_0}^{t_1} V\left(t\right) I\left(t\right) \mathrm{d}t , \tag{8}$$

where E_e is a finite amount of electric energy produced by a magnet and a coil in the mechanism of axial flux generation. The electric energy E_e is finite and a constant average value of the time interval $\left(t_0, t_1\right)$, which is the property of axial flux DM-EMI. It immediately indicates that $I\left(t\right)\mathrm{d}t$ becomes small when $V\left(t\right)$ is large, and vice versa.

The electric powers of $\omega = 120$ (rpm) and $\omega = 30$ (rpm) are specifically shown in **Figure 13** and **Figure 14**. The electric energy is shown by the area which is visible in **Figure 14**, but the time interval for $I\left(t\right)\mathrm{d}t$ becomes small in **Figure 13** (ω = 120 rpm). The time interval of **Figure 14** becomes larger than that of **Figure 13**, indicating that electric power can be better extracted in a technical sense in case of ω = 30 (rpm). The result is essential for electric-power conversions, meaning that the electric power may be better extracted from low temperature heat flows ($\omega \sim 30$ rpm) by employing the axial flux generator (AFG). The results are important for technical applications indicating that a high-speed rotation and special microscopic semiconductors are not necessarily required for extracting usable electricity, though high-speed turbines and semiconductor energy-production devices are surely important as societal infrastructures. The sensitive electric energy extractions using AF-EMI would compensate conventional power-extraction systems. We are currently investigating and proposing new electric power generators based on AF-EMI and hope that other researchers find possible applications with AF-EMI technique.

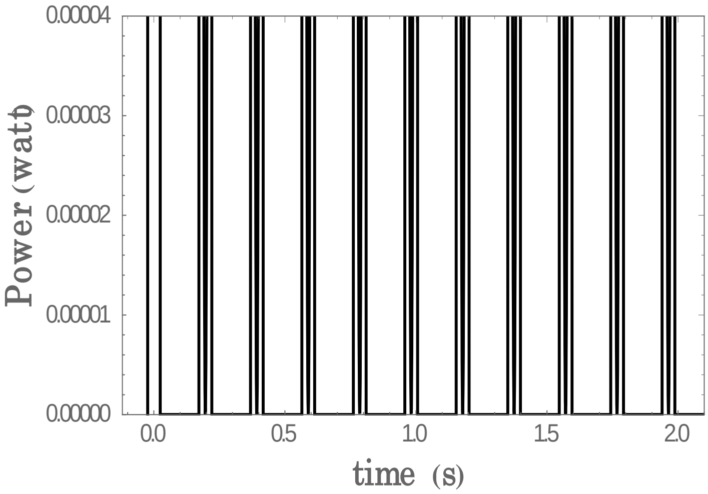

Figure 13. The produced pulse electric power: $\omega = 120$ (rpm).

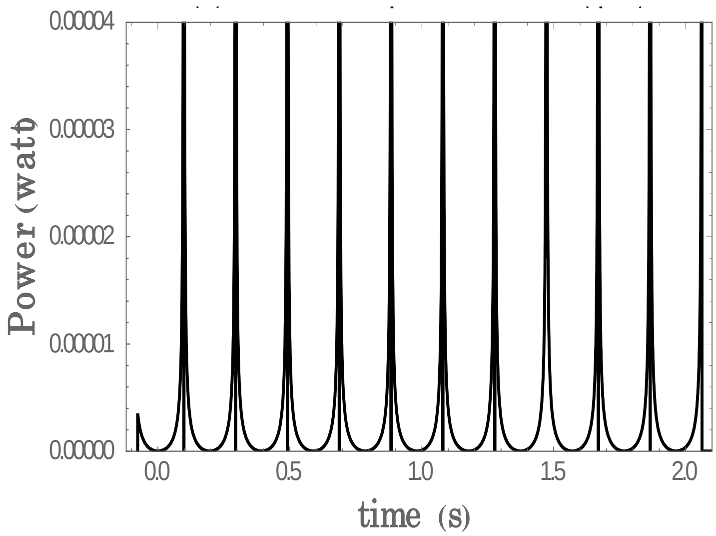

Figure 14. The produced pulse electric power: $\omega = 30$ (rpm).

The important property of the AFG concludes that an optimal angular velocity to produce electric power exists in a low angular velocity induced by a low temperature heat flow. This is one of the important results in the TMD analysis, which makes the extraction of electric power possible from 50˚C - 100˚C boiled water, which is the reason why the heat-electric power conversion device is proposed by the authors as a thermoelectric generation Stirling engine [4]-[6]. It is remarkable that an optimal thermoelectric generation device of a drinking bird is specifically constructed [10] only recently, as we discussed and expected theoretically [3] [4].

6. Conclusions

The huge power production and consumption of human societies and industries in modern world have affected ecological systems on Earth, and it is imperative to develop clean energy and energy harvesting technologies. The DM-EMI technique proves that there exists an optimal speed of rotation (rpm) to extract electric power, even in a low temperature heat flow. The property of a low-(rpm) electric-power conversion and applications of the axial flux generator are one of the new findings and should be investigated further. The DM-EMI technique is for activation of dissipated or discarded waste heat and should not be misunderstood as devices with heavy-duty front-line turbine generators. The results shown in the numerical simulations of DM-EMI technique improve overall thermal efficiency and electric power conversions of thermodynamic cycles. The detailed description of DM-EMI mechanism, including the design and operation of axial flux generators (AFG) and their role in sensitive thermoelectric conversions are shown in the papers [3] [4], and the theoretical foundations of TMD and nonequilibrium irreversible thermodynamics are discussed in [5] [6] in detail.

The new types of heat-electricity conversion devices are possible but have not been constructed nor applied sufficiently. The applications to compensate electricity productions for macroscopic energy generators (MEGs), internal combustion engines, a low-temperature TEG-rotary engine, TEG-diesel engines with hydrogen-fuel could be theoretically possible. We are planning to develop optimal devices for electric energy productions and seeking collaborations and an experimental budget to test several types of TEG engines.

The equations of motion for a low temperature Stirling engine and a drinking bird are solved self-consistently [3]-[6]. The method of TMD made us integrate very sensitive physical problems of nonequilibrium irreversible thermodynamics with technologies for thermoelectric energy conversions. It helped us understand nonequilibrium irreversible states (NISs) with the time-progress of internal energy $\mathrm{d}\varepsilon(t)$ and work $\mathrm{d}W_{th}(t)$, heat-flow or entropy-flow, $T(t)\mathrm{d}S(t)$ and nonequilibrium temperature $\tilde{T}(t)=T\tau(t)$, producing testable specific ideas for heat engines.

The TMD analysis of heat engines suggests that energy can be more efficiently produced and used so that waste of energy should be dramatically decreased. The very high-temperature pressurized steam required in a traditional RFG is not necessary in AFG for thermoelectric energy conversions. The electricity should be directly used for sustainable social infrastructure, such as electrolysis to produce basic chemicals, such as H_2, O_2, C, COOH, CH_3COOH, etc., which supports biological stability, symbiosis and ecology in nature, sustainable environmental goals (SEGs) [11].

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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