Scientific Research Publishing

Int. J. Communications, Network and System Sciences, 2025, 18(4), 39-50
https://www.scirp.org/journal/ijcns
ISSN Online: 1913-3723
ISSN Print: 1913-3715

Comparison of Two Approaches to Modeling Additive White Gaussian Noise as It Acts on Arbitrary Signals

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How to cite this paper: Rassomakhin, S., & Brifman, J. (2025) Comparison of Two Approaches to Modeling Additive White Gaussian Noise as It Acts on Arbitrary Signals. *Int. J. Communications, Network and System Sciences*, **18**, 39-50.
https://doi.org/10.4236/ijcns.2025.184004

Received: March 15, 2025
Accepted: April 27, 2025
Published: April 30, 2025

Abstract

The purpose of this paper is to substantiate the correct method for modeling additive white Gaussian noise under conditions of using analog filtering when processing a signal-noise mixture. To achieve the result, two methods of noise modeling used in signal processing theory for Gaussian channels are considered: the first is a traditional simplified discrete method, and the second is a more complex, but functionally correct analog-discrete method. As a result of the comparison, the undeniable advantages of the analog-discrete method are proven. A conclusion was made about the preference of using the proposed new method, which ensures the complete adequacy of the noise model.

Keywords

Additive White Gaussian Noise (AWGN), Normal Distribution, Sampling, Signal-to-Noise Ratio, Fourier Transformation

1. Introduction

The task of analytically describing the impact of the AWGN on the process of obtaining noisy measurements at the output of a Gaussian channel is the most important for carrying out correct modeling of digital signal processing. Currently, the most widely used is the traditional discrete method (T-method) [1] [2]. According to this method, a noisy sample of discrete measurements of the channel output is formed by simply summing two vectors: a vector of signal measurements obtained in the absence of noise, and a vector of independent Gaussian random variables with zero mathematical expectation and variance determined by a given signal-to-noise ratio (SNR). The main advantage of this method is its simplicity and high technological efficiency for Matlab modeling. Mutual independence of noise measurements means that the model corresponds to the so-called full-band noise. Such noise is present at the receiver input when there is no preliminary filtering (spectrum limitation) of the additive signal-noise mixture. If the operation of the receiver is based on the calculation of correlation integrals (or, in the case of digital implementation, discrete convolutions) of the received mixture with standards of possible signal implementations, then with ideal synchronization, a method of reception is implemented that is called Optimal Coherent Reception (OCR) [3]-[5]. Since the correlation integral itself has a frequency-selective property, the OCR method does not require any preliminary filtering. Therefore, the modeling of full-band noise using the considered T-method is initially oriented only for the further application of the OCR. The best potentially achievable result of the OCR method actually became the basis for proving the existence of a physical Capacity limit for Gaussian channels [6]-[8]. This value limits the achievable indicators of the specific efficiency of all existing systems [6].

However, there are methods for receiving noisy signals that, under certain conditions, can produce results much better than OCR. These methods are not the subject of this work, but it should be noted that they are based on the use of preliminary frequency-selective filtering. This preprocessing results in the measurements of the signal-noise mixture acquiring some mutual dependence, *i.e.*, they are correlated. Therefore, the use of the T-method of AWGS modeling becomes unacceptable.

Below we will consider a new analog-discrete method for modeling the impact of AWGN—the so-called N-method, which, due to its greater functionality, works correctly, including when performing preliminary filtering. The purpose of this analysis is to prove the complete adequacy and preference of the N-method over the traditional approach. If necessary, the proposed N-method can be generalized to other noise models. For this, it is sufficient to use the required type of distribution of the vector of random amplitude coefficients of the Fourier representation of the noise realizations. For example, to model colored noise, it is necessary to introduce the dependence of the standard deviation of the elements of the vector of amplitude coefficients of noise harmonics on the value of the harmonic frequency in the noise spectrum.

At the same time, we will keep in mind that it is impossible to build a completely adequate AWGN model. You cannot simulate a phenomenon that does not exist in nature. We think that you will agree that we can only talk about mathematical methods that, under certain conditions and under certain restrictions, with an acceptable error, are similar to the effect of AWGN.

2. Traditional Approach (T-Method)

The traditional approach to modeling noise exposure, is as follows. Let there be an arbitrary signal (harmonic or pulse) on a modulation interval with a duration T: $X(t), t \in T$. This signal is digitized at the sampling rate f_S:

$$x = \left(x_0, x_1, x_2, \cdots, x_{f_S \cdot T-1}\right). \quad (1)$$

A noise vector of similar dimension is generated:

$$n = \left(n_0, n_1, n_2, \cdots, n_{f_S \cdot T-1}\right). \tag{2}$$

Elements of the vector (2) are real, independent, and normally distributed random variables with zero mathematical expectation and standard deviation

$$\sigma_n = \left(N_0 \cdot F\right)^{1/2} = \left(N_0 \cdot K \cdot f_S/2\right)^{1/2}. \tag{3}$$

Here N_0 is a noise spectral power density, $N_0 = E_b/SNR$; E_b is a signal energy consumed to transmit one bit; $SNR = E_b/N_0$—Signal-to-Noise Ratio, specified by modeling conditions; $F = K \cdot f_S/2$—the noise frequency band; K—coefficient of expansion (narrowing) of the noise frequency band, which is absent in classically used traditional applications, when we assume $K = 1$. You will not find precedents in the literature when any other value would be used in this model. We introduce this coefficient for the generality of examples when we vary the input noise bandwidth at a fixed sampling frequency. In the Equation (3) we are taking into account that when using a sampling frequency f_S the standard deviation of noise measurements n_i increases by a factor of $\left(K \cdot f_S/2\right)$. This is done because when implementing OCR, the mixture measurements are averaged over the interval T in which the signal power per measurement increases by a such factor, while the power of random noise increases only by a factor of $\left(K \cdot f_S/2\right)^{1/2}$. Those scaling the noise power spectral density in (3) prevents false increase in SNR when implementing OCR. Due to the fixed value $K = 1$, this model takes into account only the noise components in the band $\left(0, \cdots, f_S/2\right)$ Hz. It means that the noise bandwidth in the T-model is uniquely related to the sampling frequency.

We assume that a is an informative signal parameter unknown to the receiver, then the result of noise exposure is represented by a vector of channel output measurements:

$$y = a \cdot x + n, \quad y = \left(y_0, y_1, y_2, \cdots, y_{f_S \cdot T-1}\right). \tag{4}$$

Is the considered model an adequate description of the impact of AWGN? The answer is of course not, because the frequency band taken into account by the model is fixed and limited by $f_S/2$. Exposure to real white noise would require, instead of definition (3), the value $\sigma_n = \infty$, what is not feasible. Although such a model is not correct for the properties of AWGN, it is widely used in all known applications, where it is called the discrete full-band model. The only advantage of the model is its simplicity and manufacturability for implementation, for example, in the Matlab environment, as well as its absolute protection against degeneracy. However, the fatal flaws of this model dominate.

The first major drawback is that the structure of the model is focused exclusively on the use of the OCR method, and the properties of this model are clearly aimed at proving the fact that there cannot be any reception methods better than OCR. This follows from the formula describing the implementation of OCR for this discrete case. The best estimate of the informative parameter of the signal on the interval is made by OCR according to the rule of scalar convolution:

$$a^{*} = y \times x = (a \cdot x + n) \times x, \tag{5}$$

where sign $\times$ denotes the dot product operation. Let, e.g., the signal $X(t)$ is a rectangular pulse with two possible equally probable amplitude values $a = \pm 1$. Equation (5) is equivalent to calculating the mathematical expectation of the process y on the modulation interval T. If the calculated value $a^{*} \le 0$ it is assumed that the value $a = -1$ is received, if $a^{*} > 0$, then $a = +1$. Under the described conditions, could there be a better method for finding the mathematical expectation than simple averaging over a sample y which is done by OCR calculation (5)? Of course not, otherwise there will be a contradiction with the axioms of probability theory. Thus, the first main drawback of the AWGN quasi-model under consideration leads to a conclusion that does not even allow the thought of finding something better than OCR. We have become accustomed and resigned to this, which has led to stagnation in the development of the theory and practice of signal processing.

The second drawback of the model is its inadequate behavior for values of the noise frequency band expansion (narrowing) coefficient K we introduced that are different from unity, when we want to simulate the effect of noise with a frequency band greater or less than $f_S/2$. This inadequacy is especially fatal when $K < 1$, since it is impossible to correctly take into account the narrowing of the band of the present signal-noise mixture after preliminary filtering, which our separation procedure provides. Below we will demonstrate this fatal inadequacy using examples. Using Equations (2) and (3) and assuming that the value f_S is a multiple of the target power of 2, we can construct an equivalent analogue representation of the implementation of random noise on the interval T. To do this we use a couple Fast Fourier Transforms (FFT):

- Direct conversion—To obtain a complex spectrum in the frequency band $(0, \cdots, f_S/2)$

$$G = FFT(n); \tag{6}$$

- Inverse conversion—To restore the form of the analog implementation

$$\begin{aligned} N(t) &= IFFT(G) \\ &= G_0 + 2\left(\mathrm{Re}(G_i)\cdot\cos\left(2\pi\cdot(i/T)\cdot t\right) - \mathrm{Im}(G_i)\cdot\sin\left(2\pi\cdot(i/T)\cdot t\right)\right),\ i \in (1, f_S/2). \end{aligned} \tag{7}$$

In this case, naturally, the analog implementation is periodic with a period equal to T. We will make transformations (6), (7) below in examples of the work of this T-method when comparing it with the developed by us a new N-method. Note that the block diagram of the sequence of actions of traditional T-method is presented in the form shown in **Figure 1**.

3. New Approach (N-Method)

The fundamental difference of our AWGN-like noise modeling approach is the reverse order of the stages of implementation, shown in **Figure 2**.

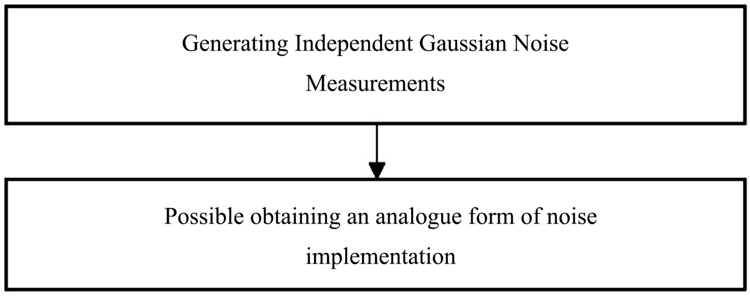

Figure 1. Stages of implementation of the T-method.

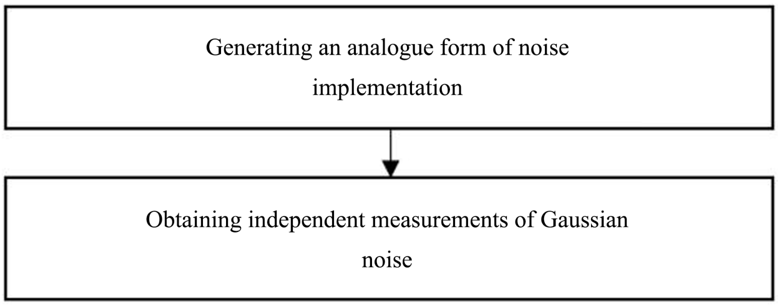

Figure 2. Stages of implementation of the N-method.

The prototype of the N-method is the mathematical model of Gaussian noise with a flat spectrum, described in [3]. We first correctly generate a non-periodic on the T an analog implementation of Gaussian noise with the right parameters, and then digitize it with frequency f_S to obtain a vector of discrete noise measurements. Our method is completely identical to that discussed earlier when using both methods under the same conditions with values of the band expansion (narrowing) coefficient $K \ge 1$. However, if preliminary filtering of noise in its mixture by a signal is applied ($K < 1$), then N-method continues to work correctly, while the T-method becomes inadequate.

The analog implementation of a non-periodic noise segment on the modulation interval T is determined based on the following Fourier expansion:

$$N(t) = \sum m_i \cdot \cos\left(2\pi\left(i/TN\right)t\right) + m_{i+TN\cdot F+1} \cdot \sin\left(2\pi\left(i/TN\right)t\right),\ i \in \left(0, TN \cdot FH\right), \quad (8)$$

here $FH = K \cdot f_S/2$ —frequency band in which noise operates at the input of the pre-filter; TN —period of the analogue implementation of noise, for the model to be non-degenerate (not periodic $N(t)$ on the interval T) it is necessary to fulfill the requirement $TN > T$, the order of this excess is not of particular importance; $m = \{m_0, m_1, \cdots, m_{2F\cdot TN+1}\}$ —initial vector of independent Gaussian random variables with zero mathematical expectation and standard deviation

$$\sigma_m = \sqrt{N_0/TN}\,. \quad (9)$$

Based on Equation (8) calculated form $N(t)$ with sampling frequency f_S the final vector of discrete noise measurements is formed:

$$n = \left(n_0, \ldots, n_{f_S\cdot T-1}\right),\quad n_i = N\left(i/f_s\right),\ i = 0, \cdots, f_s - 1. \quad (10)$$

If Equation (9) is met, the resulting vector of random measurements n in its statistical characteristics and the value of the standard deviation σ_n is completely identical to the similar vector of T-method. This is observed at the nominal value for T-method $K = 1$, as well as with an extended noise spectrum at $K > 1$. In the case when using $K < 1$, we simulate preliminary noise filtering, then with the same final values σ_n, the N-method correctly reduces the noise bandwidth, while the traditional T-method does not work correctly, *i.e.*, noise bandwidth is not reduced. This is clearly observed by the visual difference in the final analog representations of noise $N(t)$ obtained for both methods, as well as by the type of their autocorrelation functions.

4. Comparison of T-Method and N-Method

Suppose you want to obtain discrete measurements of noise acting on the modulation interval $T = 1\,\text{s}$ at sampling rate $f_S = 128\,\text{Hz}$ and $E_b = 1$, $SNR = 1$, $N_0 = 1$. Let's consider three situations of using the models of T-method and N-method. To generate independent Gaussian quantities, we use the built-in Mathcad function $rnorm(Q, m_n, \sigma_n)$, where Q—number of generated values; $m_n = 0$—mathematical expectation; σ_n—standard deviation. To implement the N-method in Equation (8), we use the value $TN = 5$. The values σ_n^* presented in the examples below obtained by averaging over 100 independent implementations of discrete noise samples. Let's consider examples illustrating the results of generating AWGN models using two compared methods.

4.1. Full Band Noise When *K* = 1

This is the normal nominal application mode of the T-method. The actual received frequency band, represented by the vector n, for both methods are 64 Hz.

T-method.

Generating a noise measurement vector $n := rnorm(128, 0, \sqrt{64}) = rnorm(128, 0, 8)$. Result is $n = (n_0, n_1, n_2, \cdots, n_{127})$. Example values for a custom implementation are $n_0 = -1.78$, $n_1 = -7.37$, $n_2 = 10.16$, $\cdots$, $n_{127} = -1.46$. Restoring the analog form for a noise segment, determined by Equations (6) and (7), gives the picture shown in **Figure 3**.

Figure 3. Analog representation of the noise realization obtained by the T-method at $K = 1$.

An estimation of the standard deviation for this realization is $\sigma_n^* \approx 7.96$.

N-method.

Generation of quadrature amplitudes based on (9):

$$m := rnorm\left(642, 0, \sqrt{1/5}\right) = rnorm\left(642, 0, 0.45\right).$$

Example values for a custom implementation are:

$$m_0 = -0.58, m_1 = 0.31, m_2 = -0.24, \cdots, m_{641} = -0.03.$$

An example of an analog implementation of a noise segment, determined by Equation (8), is shown in **Figure 4**.

Figure 4. Analog representation of the noise realization obtained by the N-method at $K = 1$.

Sampling $N(t)$ in accordance with Equation (10) gives the desired vector of noise measurements: $n = (n_0, n_1, n_2, \cdots, n_{127})$. Example values for a custom implementation are $n_0 = -1.65, n_1 = 1.43, n_2 = 11.88, \cdots, n_{127} = 4.94$. An estimation of the standard deviation for this realization is $\sigma_n^* \approx 7.94$, almost the same as previous.

Let's make the intermediate conclusion 1: when $K = 1$ (nominal T-method mode) both methods give identical results—the generation of vectors of discrete measurements in the 64 Hz noise band with almost identical values of standard deviation corresponding to the required value $\sqrt{f_s/2} = \sqrt{64} = 8$. The resulting noise realizations, which reflect the operation of the models, naturally have a different appearance, but have completely identical statistical characteristics. Both methods give the same required result.

4.2. Ultra-Full Band Noise When *K* = 2

Now let's imagine a situation where we use the same sampling rate $f_s = 128$ Hz, but the input of both methods is noise with twice the frequency band $FH = f_S = 128$ Hz. Options $E_b = 1, SNR = 1, N_0 = 1$ let's leave it the same, but use the value $K = 2$. Obviously, the following result should be expected. The noise bandwidth represented by the measurement sample will remain equal to $f_S/2 = 64$, because the value f_S hasn't changed. However, since the amplitude-frequency response of analog-to-digital conversion is periodic (with a half-period $f_S/2$), the energy contained in out-of-band frequencies $f_S/2 \cdots f_S$ will "penetrate" into the main band $0 \cdots f_S/2$ and add up with the energy of in-band frequencies. Consequently, the total noise power represented by discrete samples will increase by a factor of 2. This is equivalent to a double reduction of actual SNR. In this case, the standard deviation of noise measurements will increase by $\sqrt{2}$ times. Let's check how equally both methods under consideration work in this situation. The simulation process and its results for $K = 2$. In fact, the resulting noise bandwidth represented by the vectors for both methods are 64 Hz.

T-method.

Generating a Noise Measurement Vector:

$$n := rnorm\left(128, 0, \sqrt{128}\right) = rnorm\left(128, 0, 11.31\right), \quad n = \{n_0, n_1, n_2, \cdots, n_{127}\}.$$

An example values for a custom implementation:

$$n_0 = -4.97, n_1 = -7.69, n_2 = -5.36, \cdots, n_{127} = 5.27.$$

Restoring the analog form for a noise segment, determined by Equations (6) and (7), gives the picture shown in **Figure 5**.

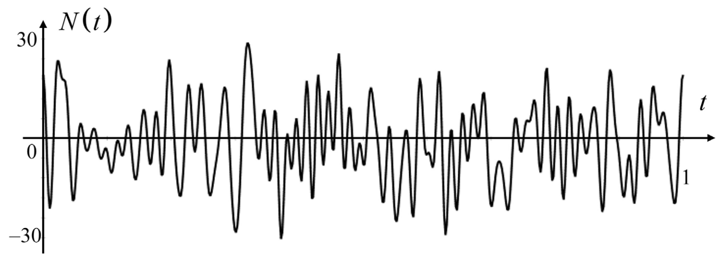

Figure 5. Analog representation of the noise realization obtained by the T-method at $K = 2$.

An estimation of the standard deviation for this realization is $\sigma_n^* \approx 11.245$.

N-method.

Generation of quadrature amplitudes based on Equation (8):

$$m := rnorm\left(1282, 0, \sqrt{1/5}\right) = rnorm\left(1282, 0, 0.45\right).$$

An example values for a custom implementation:

$$m_0 = -0.07, m_1 = -0.39, m_2 = 0.95, \cdots, m_{1281} = -0.26.$$

The form of an analog implementation of a noise segment, determined by Equation (8), is shown in **Figure 6**.

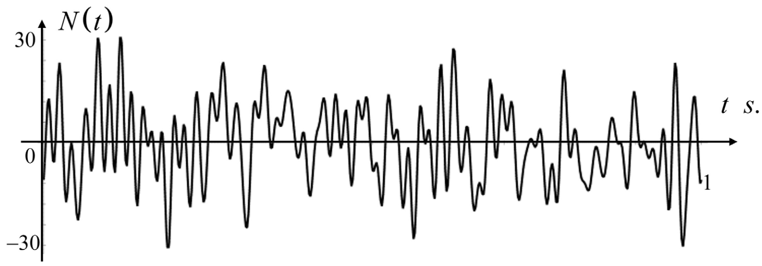

Figure 6. Analog representation of the noise realization obtained by the N-method at $K = 2$.

Sampling $N(t)$ in accordance with Equation (9) gives the desired vector of noise measurements: $n = \{n_0, n_1, n_2, \cdots, n_{127}\}$. For example:

$$n_0 = -25.55, n_1 = 14.51, n_2 = 11.62, \cdots, n_{127} = -13.12.$$

An estimation of the standard deviation for this realization is $\sigma_n^* \approx 11.26$.

Let's make the intermediate conclusion 2: when $K = 2$ (extended bandwidth abnormal mode) both methods give the same result—the generation of discrete measurement vectors, still corresponding to noise with a frequency band of 64 Hz with practically the same values of the standard deviation of discrete measurements, corresponding to the value $\sqrt{f_s} = \sqrt{128} = 11.31$. The resulting noise realizations, which reflect the operation of the models, naturally have a different appearance, but have completely identical statistical characteristics. Both methods give an incorrect result that corresponds to the expected one: the band expanded by 2 times actually transformed into a 2-fold deterioration in the channel energy $SNR = 0.5$, $N_0 = 2$.

4.3. Reduced Band Noise after Pre-Filtering *K* = 0.5

Let us now consider the situation when full-band noise with frequency band 64 [Hz] before sampling with the same frequency as in previous cases $f_S = 128$ Hz pre-filtered by low pass filter (LPF) with passband $[0 \cdots 32]$ Hz. We will use an idealized LPF, the amplitude-frequency response of which is equal to unity in the passband $[0 \cdots 32]$ Hz and equal to zero—in the suppression band $[33 \cdots 64]$ Hz, and the phase-frequency response is linear. We model a two-fold reduction in frequency band of noise using one model parameter $K = 0.5$ This idealization is used to simplify the specification of the situation using only one parameter of the models. You should not consider it as a possible error leading to a degenerate analysis. Because in a real situation, when we use N-method, it is sufficient to take into account only the amplitude-frequency characteristic of the filter. since in the noise model defined by Formula (8), the random phase of any harmonic is determined by the arctangent of the ratio of the random amplitudes of the sine and cosine quadrature components. These amplitudes are changed by the filter proportionally and the random value of the harmonic phase does not change.

We will continue to use parameters $E_b = 1, SNR = 1, N_0 = 1$. Let us show how the two considered models for generating noise measurements behave in this case. Expected Results:

- T-method, when $f_S = 128$ Hz receiving noise at the input in the 32 Hz band, by default interprets it as noise with a 64 Hz band, and since the total power of the process will be 2 times less, this will lead to an estimate $N_0 = 0.5$ and a false twofold increase in SNR;
- N-method must work absolutely correctly.

<u>T-method</u>.

Generating a Noise Measurement Vector:

$$n := rnorm\left(128, 0, \sqrt{32}\right) = rnorm(128, 0, 5.66), \quad n = \{n_0, n_1, n_2, \cdots, n_{127}\}.$$

An example values for a custom implementation:

$$n_0 = 2.52,\ n_1 = -5.46,\ n_2 = -1.73,\ \cdots,\ n_{127} = 3.69\ .$$

Restoring the analog form for a noise segment, determined by Equations (6) and (7), gives the picture shown in **Figure 7**.

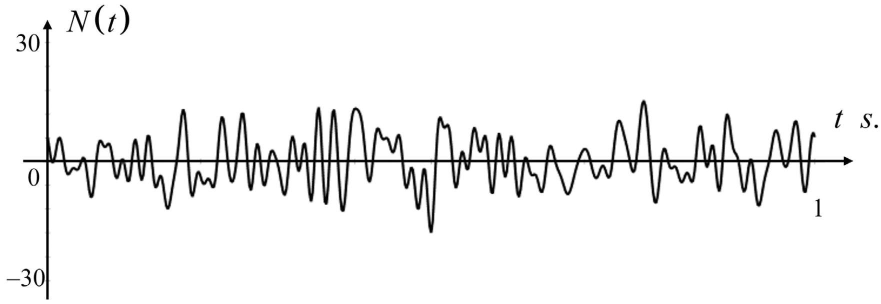

Figure 7. Analog representation of the noise realization obtained by the T-method at $K = 0.5$.

The actually obtained frequency band represented by the vector n is 64 Hz, *i.e.*, is the same as in the examples of previous cases. Standard deviation is $\sigma_n^* \approx 5.62$. Energy parameters of the obtained model $SNR = 2,\ N_0 = 0.5$, which absolutely does not correspond to the input data of the experiment.

N-method.

Generation of quadrature amplitudes based on (8):

$$m := rnorm\left(322,\ 0,\ \sqrt{1/5}\right) = rnorm\left(1282,\ 0,\ 0.45\right).$$

An example values for a custom implementation:

$$m_0 = 1.1,\ m_1 = 0.42,\ m_2 = -0.55,\ \cdots,\ m_{321} = 0.5\ .$$

An implementation example of a noise segment, determined by (8), is shown in **Figure 8**.

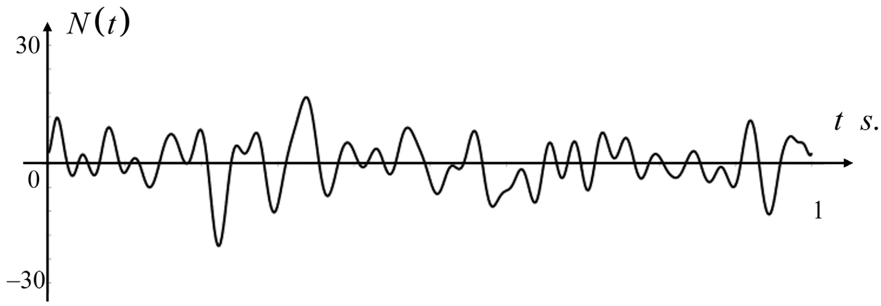

Figure 8. Analog representation of the noise realization obtained by the N-method at $K = 0.5$.

Sampling $N(t)$ in accordance with Equation (9) gives the desired vector of noise measurements: $n = \{n_0, n_1, n_2, \cdots, n_{127}\}$. For example

$$n_0 = 3.75,\ n_1 = 4.26,\ n_2 = 0.1,\ \cdots,\ n_{127} = 1.47\ .$$

The actually obtained frequency band represented by the vector n is 32 Hz. Standard deviation is $\sigma_n^* \approx 5.58$. Energy parameters of the obtained model are $SNR = 1$, $N_0 = 1$, which fully corresponds to the input data of the experiment.

The autocorrelation functions $C(\tau)$ of the process $N(t)$, calculated for one arbitrary realization of noise, in the region of the main maximum for both methods have the forms shown in **Figure 9**.

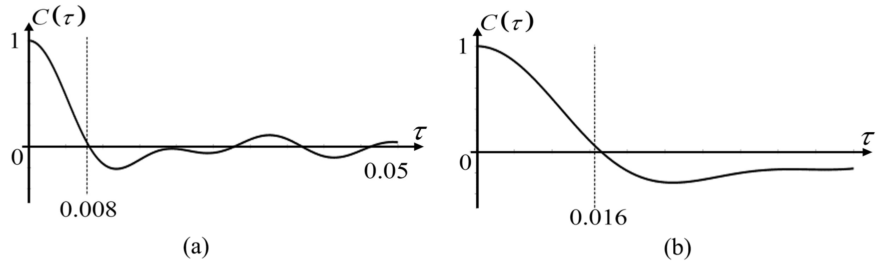

Figure 9. Autocorrelation functions of the process $N(t)$. (a) autocorrelation functions of the $N(t)$ obtained by T-method; (b) autocorrelation functions of the $N(t)$ obtained by N-method.

T-method does not correctly interpret the result of preliminary noise filtering. The noise measurement vector corresponds to the 64 Hz band. The width of the main lobe of the autocorrelation function of a discrete noise sample of N-method is 2 times larger than the similar value estimated for T-method. This means that the N-method generates a vector of discrete noise measurements that is fully compatible with the 32 Hz bandwidth. N-method is absolutely correct for generating a discrete sample of noise after its preliminary filtering.

Intermediate conclusion 3: when $K = 0.5$ (for T-method – abnormal narrowed band mode) T-method does not work correctly, giving a false predominance of signal over noise, which is equivalent to a 2-fold overestimated SNR value. In this case, the vector of discrete samples n generated by the T-method corresponds to noise with a bandwidth of 64 Hz, instead of the actual value of 32 Hz. The N-method (analog-discrete method for modeling noise measurements) works absolutely correctly in this situation. The frequency band corresponding to the resulting discrete sample of noise measurements is 32 Hz. The standard deviation of discrete noise samples, despite the difference in frequency band, for both methods is practically the same and approximately equal $\sqrt{f_S/4} = \sqrt{32} \approx 5.66$.

5. Conclusion

The traditional model (T-method) is correct for the only (standard) full-band noise situation when $K = 1$. In other cases when $K = 2$ and $K = 0.5$ T-method is not suitable for use as it gives false noise characteristics. The analog-discrete model (N-method) works correctly not only for full-band noise, but also for the case of preliminary noise filtering. As has been shown above, when a sampling rate of 128 Hz, the N-method actually generates a vector of noise measurements with a bandwidth of 32 Hz, *i.e.*, requirement $f_s > 2 \cdot FH$ fulfilled. Methods of receiving noisy signals that are capable of providing better results compared to OCR require (before discretization) preliminary filtering of the signal-noise mixture. In this case, only the N-method proposed in this work is acceptable for modeling due to its universality and correctness of operation in models of promising digital demodulators.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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