

Dynamically Tuning Away the Cosmological Constant in Effective Scalar Tensor Theories

Nimmi Singh¹, Daksh Lohiya²

¹S. G. T. B. Khalsa College, University of Delhi, Delhi, India

²Department of Physics and Astrophysics, University of Delhi, Delhi, India

Email: nimmi@sgtbkhalsa.du.ac.in, dlohiya@gmail.com

How to cite this paper: Singh, N. and Lohiya, D. (2025) Dynamically Tuning Away the Cosmological Constant in Effective Scalar Tensor Theories. *International Journal of Astronomy and Astrophysics*, 15, 293-311. <https://doi.org/10.4236/ijaa.2025.154019>

Received: June 17, 2025

Accepted: October 27, 2025

Published: October 30, 2025

Copyright © 2025 by author(s) and Scientific Research Publishing Inc.

This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).

<http://creativecommons.org/licenses/by/4.0/>



Open Access

Abstract

It is known that the cosmological constant can be dynamically tuned to an arbitrary small value in classes of scalar tensor theories. The trouble with such schemes is that effective gravity itself vanishes. We explore the possibility of avoiding this “no-go” with a spatially varying effective gravity. We demonstrate this in principle with the non-minimally coupled scalar field having an additional coupling to a fermionic field. The expectation value of the scalar field gets anchored to a non-trivial value inside compact domains. But for the non-minimal coupling to the scalar curvature, these configurations are analogous to the non-topological solutions suggested by Lee and Wick. With non-minimal coupling, this leads to a peculiar spatial variation of effective gravity. As before, one can dynamically have the long distance (global) gravitational constant G and Λ , the cosmological constant, tending to zero. However, inside compact domains, G can be held to a universal (non-vanishing) value. Long distance gravitational effects turn out to be indistinguishable from those expected of general theory of relativity (GTR). There are two ways in which the ensuing theory may lead to a viable effective gravity theory: a) the compact domains could be of microscopic (sub-nuclear) size, or b) the domains could be large enough to accommodate structures as large as a typical galaxy. Aspects of effective gravity and cosmology that follow are described. A toy Friedmann-Robertson-Walker (FRW) model free from several standard model pathologies and characteristic features emerges. Although this proposed model remained less studied but became an important aspect in initiating a viable approach in addressing the cosmological constant problem. Comparison with some additional observational constraints like quantitative constraints on gravitational constant G , from some high precision tests will be included our forthcoming paper which will address the limitations of the present paper.

Keywords

Cosmological Constant, Non-Minimal Coupling, Scalar Tensor Theories, Compact Domains, Friedman-Roberston-Walker Model

1. Introduction

It is now widely accepted that a theory of gravitation described by the Einstein-Hilbert action:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{8\pi G} R + \Lambda \right] \quad (1)$$

fares wonderfully well to classical precision tests. The success is impressive enough to rule out most alternative theories at the classical level [1]. However, the theory described by Equation (1) has severe theoretical inconsistencies. The appearance of a dimensional gravitational coupling constant G , and the smallness of the cosmological constant Λ , are two pathologies that impede any attempt to treat this action as a fundamental quantum theory. The dimensional coupling is primarily responsible for the non-renormalizability of the theory [2] while the smallness of Λ can not be justified in any reasonable theoretical model [3]. Besides these theoretical problems, a cosmological model based on the above action has to contend with dynamical inconsistencies: reasonable initial conditions can not be dynamically realized on account of the horizon and fine-tuning (flatness) problems. A resolution of these dynamical inconsistencies using inflation, without addressing the cosmological constant problem is, at best, incomplete. Inflation is founded upon the possibility that the early universe witnessed several phase transitions—each producing a large change in the effective value of the cosmological constant. Be it so, the last transition is required to exit to a state of a tiny cosmological constant. All versions of inflation require fine tuning of model parameters to get inflation going and/or to get inflation exit gracefully to the standard early hot universe [4]. The least troublesome version of inflation, the chaotic model, assigns to Λ a vanishing value by hand. Anthropic arguments are invoked to justify a small value for Λ [5]-[10] and have invited expressions of despondency and a need for a better ansatz. We share Witten's [11] reservations on the use of anthropic arguments in general.

Over the last two decades there is an emerging consensus that the Einstein-Hilbert action ought to be regarded, at most, as a correct low energy limit of a decent quantum theory. Superstring theory has emerged as a promising candidate in this respect. It is hoped that an effective gravity theory would emerge from the dynamics of an appropriate string theory. A lack of understanding of the vanishing (or the tininess) of the cosmological constant is the key obstacle to making realistic string-inspired particle physics models. A convincing mechanism of supersymmetry breaking has not emerged as yet. All known approaches generate a

large cosmological constant and must therefore be regarded as incorrect [11].

On the other hand, general considerations regarding the structure of string theory suggest that general relativity may acquire modifications, even at energies considerably lower than the Planck scale. Dilaton gravity theories in ten dimensions have recently played a prominent role in a search for a consistent effective low energy limit emerging from string inspired quantum theory of gravity [12]. This has resulted in a resurrection of interest in scalar-tensor and Brans-Dicke models. Such models have a characteristic coupling of the Ricci scalar curvature with a function of a scalar field-defining a non-minimal coupling (NMC). A search for a suitable compactification scheme that would yield the Einstein-Hilbert action in four dimensions is on within the framework of such models. To achieve this, the foremost requirement is to have a provision for an appropriate fixing of NMC to give a universal gravitational constant.

The NMC could, for example, be determined by the minimum of an effective potential of the scalar field. Such a potential may arise as a result of the compactification scheme itself or as a result of quantum corrections [13]. Determining the NMC by the minimum of an effective potential leads to a realization of a theory indistinguishable from the Einstein-Hilbert theory at low energies [14]. However, such a scheme falls short of addressing itself to the cosmological constant problem as again, there is nothing to ensure the tininess of the minimum value of the effective potential.

There have been attempts to *dynamically* tune the effective cosmological constant to a vanishingly small value in classes of NMC theories [15] [16]. All such attempts are frustrated by a “no-go theorem” [3]. In these schemes, the scalar field develops an instability, the non-minimal coupling diverges and the scalar field stress tensor evolves to neutralize the effective value of the cosmological constant to zero. The trouble with such a scheme is that with the diverging NMC, the effective gravitational constant itself vanishes. This no-go situation has not been successfully evaded. One has not found a natural way to stabilize the NMC to a bounded value that could be identified with $\approx (8\pi G)^{-1}$ and, at the same time, tune the cosmological constant to a small value.

In a sweeping review, Weinberg [3] has made out a case for the use of a judicious anthropic argument as an *explanation* for the tininess of the cosmological constant. One considers formalisms that allow an effective cosmological constant to appear dynamically in the action and appeals to some unique feature that would pick out a small value for this constant in the functional integral. The most explored principal is the imposition of a conditional probability on the functional integral that would allow for a large enough universe to exist and also allow for gravitational clustering at a sufficiently small redshift in the universe. This imposes severe constraints on the cosmological constant. Indeed the most recent attempts [5]-[10] to justify a small Λ uses an anthropic argument to have a vanishing minimum to an effective “tracker” potential for a scalar field. The peculiar profile of such potentials, and a slow asymptotic rolling of the scalar field near the

minimum, can in principle allow for an effective Λ dominated cosmology at the current epoch.

A feature that would pick out a small cosmological constant using an objective physical principle rather than having to take a recourse to an anthropic principle, is worth exploring. In this article we describe solutions in a class of non-trivial NMC theory having a *minimum* energy at a vanishing cosmological constant. The solutions arose in our search for an ansatz that would allow for a NMC with a non-trivial spatial dependence. As a result of a scalar field coupling to a fermion field, the expectation value of the scalar field can acquire a non-trivial value inside compact domains. But for the non-minimal coupling (of the scalar field) to the scalar curvature, these configurations are similar in character to the non-topological solutions suggested by Lee and Wick [17]-[19]. With NMC, one gets a peculiar spatial variation of effective gravity. We shall require G to be held to a non-trivial value inside compact domains. This is sufficient to have long distance gravitational effects indistinguishable from those expected from general theory of relativity (GTR). We point out two ways this could be possible: a) the domains could be large enough to accommodate structures as large as a typical galaxy or b) the compact domains could be of microscopic (sub-nuclear) size. This is described in the next section where we also describe some essential aspects of a generalized effective scalar tensor theory. The action is chosen to include scalar functions multiplying the ricci scalar as well as a fermionic part. Stability of large fermion number non-trivial solutions requires their interior to be near the zero of the effective potential.

In the last section we discuss features of a toy cosmological model that such a theory can support. The structure has definitive, falsifiable predictions. In standard cosmology, one has pinned down hopes on special initial conditions, motivated by anthropic considerations to account for the observed cosmological constant. Such tautologies, when applied to the universe as a whole, have little predictive power. In the structure described in this article, any fine tuning of coupling parameters of the fermion lagrangian that may be required to have stable gravity domains of required size, has testable consequences.

2. A Generalized Scalar Tensor Theory

2.1. The Action and Field Equations

Consider a classical theory of a scalar field ϕ coupled to the scalar curvature R , through an arbitrary function $U(\phi)$, in the action:

$$S = \int d^4x \sqrt{-g} \left[U(\phi)R + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) + L_m \right] \quad (2)$$

We shall treat S as a classical, effective, phenomenological action that could be good enough to do tree level (“on-shell”) physics. If need arises, we shall consider non polynomial forms for the effective potential and the NMC: $V(\phi)$ and $U(\phi)$. Such profiles have been used as “tracker-field” potentials [5] [6] [9] [10]

and can be justified in particle physics models with dynamic symmetry breaking or non-perturbative effects. In particular they can arise from non-perturbative effects that lift flat directions in supersymmetric gauge theories as well as in moduli fields in string theory [13] [20] (see also [21]). Strictly speaking, we would consider it premature to justify the forms for these functions at this formative stage on the basis of fundamental physics. For our purpose, we simply regard Equation (2) as the action of a parametrized Brans-Dicke fluid. Any arbitrary (cosmological) constant in the theory is included in $V(\phi)$. $L_m \equiv L_w + L_\psi$ is the contribution from the rest of the matter fields. It includes a contribution from a Dirac fermion field:

$$L_\psi \equiv \bar{U}(\phi)^{-1} \left[\frac{1}{2} [\bar{\psi} \bar{D}_\mu \gamma^\mu \psi - \bar{\psi} \gamma^\mu \bar{D}_\mu \psi] - m(\phi) \bar{\psi} \psi \right] \quad (3)$$

Here D_μ is the spin covariant derivative [22]. This Fermion action has an overall scaling function $\bar{U}(\phi)^{-1}$. Such an arbitrary scaling can be absorbed in a redefinition of the Dirac field: $\psi \rightarrow \psi' \equiv \psi / \bar{U}^{1/2}$ provided \bar{U} were bounded. Indeed, by such a scaling (for a bounded \bar{U}), the ϕ dependence in L_ψ can be constrained to the mass function $m(\phi)$. The anti-symmetrized derivative appearing in the fermion lagrangian ensures the cancellation of the derivative of a real, non-vanishing, bounded scaling function.¹ If $\bar{U}(\phi)^{-1}$ vanishes at any point, the Dirac action identically vanishes. There is no dynamics for the Dirac field at such a point. The invariance of the Dirac Lagrangian under arbitrary phase change: $\delta\psi = i\epsilon\psi$ implies the vanishing divergence of the current:

$$J^\mu = \bar{U}(\phi)^{-1} \bar{\psi} \gamma^\mu \psi \quad (4)$$

and the conservation of the charge:

$$Q \equiv \int_\Sigma \bar{U}(\phi)^{-1} \bar{\psi} \gamma^0 \psi \quad (5)$$

The Dirac particles would be confined to regions where \bar{U} is bounded *i.e.* $\bar{U}(\phi)^{-1}$ is non-vanishing. The only effect of a scaling function $\bar{U}(\phi)^{-1}$ that vanishes outside a compact domain is to confine the fermions within the domain. Inside the domain the scaling can be identically absorbed in a rescaling of the fermion field. In what follows we shall essentially do this. This leads to conditions similar to those that arise in non-topological soliton solutions in the Lee-Wick model [17]-[19]. *One could just as well have followed the Lee-Wick model in which confinement occurs for fermions that are not on-shell outside a domain on account of a large coupling with the Higgs field.*

Our foremost task would be to demonstrate the vanishing of the covariant divergence of the stress tensor of the rest of the matter fields described by L_w in Equation (2). Requiring the action to be stationary under variations of the metric tensor and the fields ϕ & ψ gives the equations of motion:

¹This result is not specific to the chosen form of the Dirac Lagrangian. Covariant spinor fields of arbitrary weights can be defined as geometrical objects over (direct products of) two dimensional complex spaces. An arbitrary, non vanishing, real scaling of such a spinor can be absorbed into a real part of the trace of a spinor connection and does not contribute to the covariant derivative of the spinor and therefore has no interaction with other fields [37].

$$U(\phi) \left[R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right] = -\frac{1}{2} \left[T_w^{\mu\nu} + T_\phi^{\mu\nu} + \Theta^{\mu\nu} + 2U(\phi)^{;\mu;\nu} - 2g^{\mu\nu} U(\phi)^{;\lambda}_{;\lambda} \right] \quad (6)$$

$$g^{\mu\nu} \phi_{;\mu;\nu} + \frac{\partial V}{\partial \phi} - R \frac{\partial U}{\partial \phi} + \frac{\partial m}{\partial \phi} \bar{\psi}' \psi' = 0 \quad (7)$$

$$\gamma^\mu D_\mu \psi' + m(\phi) \psi' = 0 \quad (8)$$

$$D_\mu \bar{\psi}' \gamma^\mu - m(\phi) \bar{\psi}' = 0 \quad (9)$$

Here $T_w^{\mu\nu}$, Θ_ν^μ are the energy momentum tensors constructed from L_w and L_ψ respectively, and

$$T_\phi^{\mu\nu} \equiv \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} \left[\frac{1}{2} \partial^\lambda \phi \partial_\lambda \phi - V(\phi) \right] \quad (10)$$

L_w is taken to be independent of ϕ and ψ . The two Dirac Equations (8) and (9) ensure L_ψ to be null. The scalar field equations are therefore independent of $\bar{U}(\phi)$. The Fermion stress tensor is simply:

$$\Theta_\nu^\mu \equiv -\frac{1}{2} \left[\bar{\psi}' \bar{D}_\nu \gamma^\mu \psi' - \bar{\psi}' \gamma^\mu \bar{D}_\nu \psi' \right] \quad (11)$$

The covariant divergence of (6) is easily seen to reduce to:

$$\Theta_{\nu;\mu}^\mu = \frac{\partial m}{\partial \phi} \partial_\nu \phi \bar{\psi}' \psi' \quad (12)$$

Thus there is a violation of equivalence principle as far as the Dirac field is concerned. However, in a region where the scalar field gradient vanishes or for a ϕ independent fermion mass, the covariant divergence of the fermion field stress tensor vanishes. To see how the equivalence principle strictly holds for the rest of the matter fields, *i.e.*: $T_{w;\nu}^{\mu\nu} = 0$, consider the covariant divergence of Equation (6). From the contracted Bianchi identity satisfied by the Einstein tensor, we obtain

$$U(\phi)_{;\nu} \left[R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right] = -\frac{1}{2} \left[T_{w;\nu}^{\mu\nu} + t_{;\nu}^{\mu\nu} + \Theta_{;\nu}^{\mu\nu} \right] \quad (13)$$

with

$$t^{\mu\nu} \equiv T_\phi^{\mu\nu} + 2U(\phi)^{;\mu;\nu} - 2g^{\mu\nu} U(\phi)^{;\lambda}_{;\lambda} \quad (14)$$

Using the identity:

$$U(\phi)^{;\rho} R_{\rho\alpha} = U(\phi)^{;\lambda}_{;\lambda;\alpha} - U(\phi)^{;\lambda}_{;\alpha;\lambda}$$

and Equation (12), Equation (13) reduces to

$$-\frac{1}{2} U(\phi)^{;\mu} R = -\frac{1}{2} \left[T_{w;\nu}^{\mu\nu} + T_{\phi;\nu}^{\mu\nu} + \partial^\mu m \bar{\psi}' \psi' \right] \quad (15)$$

Finally, using the equation of motion for the scalar field (7), all the ϕ dependent terms cancel the left hand side—giving the vanishing of the covariant divergence of the (w-) matter stress energy tensor. This in turn ensures that equivalence principle holds—in turn assuring the geodesic law.

2.2. The Conserved Energy

Next, we write down the expression for a conserved pseudo energy momentum

tensor [23]. Defining

$$A \equiv \sqrt{-g} g^{\sigma\rho} \left[\Gamma_{\sigma\rho}^\alpha \Gamma_{\alpha\beta}^\beta - \Gamma_{\beta\rho}^\alpha \Gamma_{\alpha\sigma}^\beta \right] \tag{16}$$

$$B \equiv \left[UA - \sqrt{-g} g^{\sigma\rho} \Gamma_{\sigma\alpha}^\alpha U_{,\rho} + \sqrt{-g} g^{\sigma\rho} \Gamma_{\sigma\rho}^\alpha U_{,\alpha} \right] \tag{17}$$

and $\hat{B} \equiv B + \sqrt{-g} L_{\phi+\psi}$, the conserved pseudo energy momentum vector defined over a hypersurface Σ is:

$$P_\mu \equiv \int_\Sigma d\Sigma \left[\sqrt{-g} T_{w\mu}^o - \hat{B} \delta_\mu^o - \frac{\partial \hat{B}}{\partial g_{,o}^{\tau\beta}} g_{,\mu}^{\tau\beta} - \frac{\partial \hat{B}}{\partial \phi_{,o}} \phi_{,\mu} - \bar{\psi}_{,\mu} \frac{\partial \hat{B}}{\partial \bar{\psi}_{,o}} - \frac{\partial \hat{B}}{\partial \psi_{,o}} \psi_{,\mu} \right] \tag{18}$$

This is also expressible as a surface integral over a 2 dimensional boundary of Σ :

$$P_\mu = - \int \left(\frac{\partial \hat{B}}{\partial g_{,j}^{\mu\nu}} g^{0\nu} \right) d\Sigma_j \tag{19}$$

Thus in a generalised Brans-Dicke theory, the generalised energy momentum is determined by the metric tensor and its derivatives on a 2-dimensional boundary.

2.3. The Dynamic Equations

We consider classical solutions to Equations (6)-(9) for a fixed number of fermions. As demonstrated in [17]-[19] and [24] [25] the thermodynamic potentials for the fermion field are assumed to be adequately described in terms of the chemical potential and the temperature. Denoting the fermion density by S_f , Equation (7) reads:

$$\square\phi + \frac{\partial V}{\partial \phi} - R \frac{\partial U}{\partial \phi} + \frac{\partial m}{\partial \phi} S_f = 0 \tag{20}$$

The trace of Equation (6) gives:

$$U(\phi)R = - \left(3U''(\phi) + \frac{1}{2} \right) \phi^\alpha \phi_{,\alpha} + 2V(\phi) + \frac{1}{2} m S_f - 3U'(\phi) \square\phi \tag{21}$$

Substituting in Equation (20) gives:

$$\square\phi + \frac{U'(\phi) \left(3U''(\phi) + \frac{1}{2} \right)}{1 + 3 \frac{U'^2}{U}} \phi^\alpha \phi_{,\alpha} + \frac{V' + m'S - \frac{U'}{U} \left(2V + \frac{1}{2} mS \right)}{1 + 3 \frac{U'^2}{U}} \tag{22}$$

2.4. An Example of Tuning of Λ

Consider a familiar NMC theory with $U(\phi) = \beta/8\pi - \xi\phi^2/2$ and $V(\phi) \equiv \Lambda_0/8\pi$ a constant. We define natural units in which β takes on a unit value. The field equations simply read:

$$\square\phi + \xi R\phi = 0 \tag{23}$$

and

$$G_{\mu\nu} + \Lambda_0 g_{\mu\nu} = -8\pi T_{\mu\nu} \tag{24}$$

with

$$T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - 1/2 g_{\mu\nu}\phi_{,\rho}\phi^{,\rho} - \xi\phi^2 (R_{\mu\nu} - 1/2 g_{\mu\nu}R) + \xi g_{\mu\nu}\square\phi^2 - \xi\phi^2_{;\mu\nu} \quad (25)$$

In terms of $\Phi \equiv \phi^2$, the scalar field Equation (23) reads

$$\square\Phi = -2\xi R\Phi + \frac{1}{2\Phi}\partial_{\mu}\Phi\partial^{\mu}\Phi \quad (26)$$

while the trace of Equation (24) is simply:

$$-R + 4\Lambda_0 = -4\pi \left[-\frac{\Phi_{;\rho}\Phi^{;\rho}}{2\Phi} + 2\Phi R\xi + 6\xi\square\Phi \right] \quad (27)$$

The wave equation for the scalar field thus reduces to:

$$\square = \frac{4\Lambda_0 - R}{4\pi(1 - 6\xi)} \quad (28)$$

We shall follow the dynamics of ϕ in a FRW metric:

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right] \quad (29)$$

For a homogeneous, time dependent scalar field, the above equations reduce to:

$$\frac{3}{8\pi} \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right] - \frac{\Lambda_0}{8\pi} = \frac{\dot{\Phi}^2}{8\Phi} + 3\xi \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} \right] \Phi + 3\xi \left(\frac{\dot{a}}{a} \right) \dot{\Phi} \quad (30)$$

and

$$\ddot{\Phi} + 3(\dot{a}/a)\dot{\Phi} = \frac{4\Lambda_0 - R}{4\pi(1 - 6\xi)} \quad (31)$$

We look for solutions with the scale factor having a power law evolution: $a(t) = ht^{\alpha}$, with h, α arbitrary constants. The equation for Φ integrates exactly for all $\alpha \neq 1/3$ in terms of integration constants K_0, C :

$$\Phi = At^2/2 + K_0t^{1-3\alpha} + C \quad (32)$$

where

$$A = \frac{\Lambda_0}{\pi(1 - 6\xi)(1 + 3\alpha)} \quad (33)$$

(For $\alpha = 1/3$ one has $K_0 \ln(t)$ instead of the second term. From what just follows, this is not feasible as a late time solution). On substituting in Equation (20) it is found that it is possible to have $k \neq 0$ solutions for large times only if $\alpha \geq 1$. For $\alpha > 1$, the solutions are independent of the curvature constant (*i.e.* hold for all k). The scale α directly relates to the parameter ξ as: $\xi = -(4\alpha - 2)^{-1}$. Similarly, for $\alpha = 1$, $\xi = -(2 + 2k/h)^{-1}$. For all these cases, scalar condensates develop to have their energy cancel the effect of Λ_0 :

$$T_{\mu\nu} \rightarrow -\frac{1}{8\pi}\Lambda_0 g_{\mu\nu} + O(t^{-2}) \quad (34)$$

The cancellation occurs over time scales large as compared to the time scales

determined by dimensional parameters in the action. If one merely uses typical particle physics time scales, this implies that the stress energy of the scalar field would compensate the cosmological constant for the entire history of the universe except the very early history. It is the coupling of the scalar field to the spacetime curvature that is primarily responsible for the above instability of the scalar field which in turn leads to an efficient damping of the cosmological constant. Unfortunately, the effective value of the gravitational constant goes as

$G_{eff} = (\beta + 8\pi|\xi|\phi^2)^{-1}$. This becomes unacceptably small at large times. This is an example of a “no-go” situation [3]. This behaviour was noted in [15] [16] for a flat $k = 0$ model. We conclude that the presence of spatial curvature does not alter the results. It is also straightforward to demonstrate that the results are independent of the presence of the Planck scale determined by $\beta^{-1} \equiv 8\pi G_0 \equiv M_{plank}^{-2}$. Indeed in the absence of β in the NMC function $U(\phi)$, the FRW scale evolves as above: $a(t) \propto t^\alpha$ with α constrained as before in the presence of β , the stress tensor rapidly cancels the cosmological constant while the effective gravitational constant again approaches zero over time scales large in comparison to the particle physics time scales.

The above behaviour of a power law behaviour of the FRW scale factor and the cancellation of the vacuum energy has been generalized to scalar tensor theories within a large class of scalar field effective potentials [21]. There have also been attempts to use the above results to constrain dimensional parameters in the theory (see eg [26]). One equates the observed gravitational constant to that calculated above using conservative age estimates of the universe. Early universe nucleosynthesis constraints and recent Viking radar experiments severely constrain the NMC in a scalar tensor theory. The particle physics parameters that follow turn out to be rather unnatural. Instead we propose and explore the possibility that the above analysis successfully accounts for a NMC diverging with time and that an effective gravitational constant over most of the universe is indeed vanishing. However, this is not the constant which ought to be equated with the Newtonian gravitational constant. Suspecting the problem to lie in the assumption of homogeneity, it has been suggested [15] [16] to look for a stabilization of G_{eff} in an inhomogeneous model.

2.5. A Strategy for Effective Gravity

Taking a cue from the above, we consider a theory described by Equations (2) & (3). We shall be interested in dynamic NMC's that diverge, as described above, outside bounded domains. It is convenient to recast the equations in terms of the effective planck length parameter which goes to zero as the NMC diverges. One such reparametrization of the scalar field is $\phi \rightarrow M^2/\phi$. The NMC becomes $U(\phi) \approx \beta - \xi M^4/\phi^{-2}$ [21]. With this reparametrization, the results of the last section translate to the vanishing of the effective gravitational and cosmological constants together with the vanishing of ϕ over time scales large in comparison to any particle physics time scales in the theory. In other words we consider an

equivalent problem that has the NMC divergent at any point that we take as $\phi = 0$ without any loss of generality. Thus $U(\phi = 0) \rightarrow \infty$. The model has a flat, Minkowski spacetime as a fixed point global solution with $\phi = 0$ for an arbitrary equation of state of matter. However, there are non-trivial solutions to the model as well [27]. We are particularly interested in solutions that have $\phi \rightarrow 0$ outside compact domains but not in the interior. This would mean that the gravitational constant would vanish in the exterior. However, this does not imply doing away with gravitation between such domains. To see this, consider the Einstein tensor that follows from Equations (2) and (6):

$$G_{\mu\nu} = [U(\phi)]^{-1} \times \text{sourceterms} \tag{35}$$

From the expansion of the metric about a flat space metric: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, in appropriate (harmonic) coordinates, one gets:

$$\square h_{\mu\nu} = [U(\phi)]^{-1} \times \text{sourceterms} \tag{36}$$

The retarded solution reads:

$$h_{\mu\nu}(x, t) = \int \frac{d^3x'}{|x-x'|} [U(\phi)]^{-1} \times \text{sourceterms}(x', t - |x-x'|) \tag{37}$$

Thus as long as the effective gravitational constant at the retarded source point $(x', t - |x-x'|)$ is bounded, the vanishing of the effective gravitational constant at the field point is of no consequence for the above solution. All one has to discover is the means of having the NMC locked to a universal value in non-trivial domains. The mechanism we invoke is to have the non-minimally coupled scalar field locked at the minimum of its effective potential [14].

2.6. Large Non Trivial Solutions

We recall studies on non-trivial configuration that have ϕ vanishing outside, and $N_f \gg 0$ ψ fermions trapped inside, compact domains in the Lee-Wick model [17]-[19]. With the fermion number conserving, the minimum energy configuration of such solutions are non-topological soliton solutions [NTS's] in the theory. We recite the essential properties of such an NTS. Lee and Wick considered a theory with a Lagrangian:

$$L = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + \bar{\psi} \gamma^\mu \partial_\mu \psi - m\bar{\psi} \left[1 - \frac{\phi}{\phi_0} \right] \psi \tag{38}$$

where

$$V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 \left[1 - \frac{\phi}{\phi_0} \right]^2 \tag{39}$$

With Fermion number conserved in this theory, solutions representing a degenerate distribution of fermions with a total fermion number N_f , at $\phi = \phi_0$, would remain confined there if the total energy of a fermion is less than its on-shell energy at $\phi = 0$. The NTS is a ball of radius r_b inside which the scalar field

is locked at $\phi \approx \phi_0$. Across the surface of the ball of thickness $\sim m_\phi^{-1}$, the scalar field transits to $\phi = 0$. As a matter of fact, the spherical shape of the distribution follows from degeneracy of the fermion ensemble. For a degenerate fermion distribution, the fermion density is determined in terms of the chemical potential that occurs as a lagrange multiplier for the conserved fermi number of the ensemble. This determines the volume for a fixed conserved number of degenerate fermions. A positive definite surface energy would break the degeneracy of volume preserving (*i.e.* fermion number conserving) deformations of the distribution—choosing a configuration of minimum surface area. This gives the minimum energy configuration to be a sphere. In terms of the chemical potential μ , the fermion density goes as $\rho_f \sim \mu^4$. The fermion number density goes as $n_f \sim \mu^3$. With the total energy and fermion number: $E_f = \rho_f v \sim \mu^4 r_b^3$, $N_f = n_f v \sim \mu^3 r_b^3$, the chemical potential eliminates to give $E_f \sim N_f^{4/3} / r_b$. For a large enough r_b , the surface tension is given by the expression: $s \sim m_\phi \phi_0^2 / 6$, the total surface energy being $E_s = 4\pi s r_b^2$. For a given N_f , one may vary the radius to seek the minimum of the volume and surface energies to give $E_f = 2E_s$. The mass of the soliton is expressed as $M = E_f + E_s = 3E_s = 3E_f / 2 = 12\pi s r_b^2$. This allows one to express:

$$N_f \sim s^{3/4} r_b^{9/4}, M \sim s^{1/3} N_f^{8/9}; r_b \sim s^{-1/3} N_f^{4/9} \tag{40}$$

Thus with a given surface tension, one could get arbitrarily massive NTS's by increasing the fermion number. However, the above flat spacetime analysis can not be extended past the Schwarzschild limit. In the Lee-Wick model, the Schwarzschild limit can be determined as the criteria for stability against gravitational collapse. As a matter of fact, gravitational instability occurs even before this limit is reached [28]. We shall use the Schwarzschild limit as a benchmark to keep sufficiently clear-off and as a convenience that ensures that the flat spacetime analysis holds. This limit is simply obtained by equating the radius of the NTS to the Schwarzschild radius for a critical mass ball M_c : equating $r_b = 2GM_c$, and with the expression for the mass of a ball, one gets $M_c \sim (48\pi G^2 s)^{-1}$. For $s \sim (30\text{Gev})^3$, M_c is roughly $10^{15} M_\odot$ with the critical radius some 10^2 Pc.

One can have NTS's as large as some 10's of kilo parsecs. This would require a class of low mass fermions that could keep the NTS from collapsing. In a hot big bang cosmology, any species of particles that decoupled from equilibrium when it was relativistic, ought to have the same relic density as say the relic photons or neutrinos: some ~ 200 per cc. To get ~ 10 to 100 Kpc NTS's, it would suffice to have $s \sim (\text{Mev})^3$. This gives $N_f \sim 10^{72}$ to 10^{75} with the mass of the ball some 12 orders of magnitude smaller than the the Schwarzschild (critical) mass for $s \sim (\text{Mev})^3$. We conclude that for such a surface tension and for NTS's even as large as hundreds of kilo parsecs, one can consistently ignore the spacetime curvature in such solutions. In the following, we do this whenever convenience demands.

In the model described by Equation (3), fermions ψ are confined to $\phi \neq 0$ domains. It is easy to follow [27] [29] to demonstrate the existence of non-trivial solutions. In the “weak (gravitational) field approximation”, that would justify re-

taining only a first order deviation from a flat metric, the metric can be expressed in terms of the spherical [Schwarzschild] coordinates:

$$ds^2 = e^{2u} dt^2 - e^{2\bar{v}} dr^2 - r^2 [d\theta^2 + \sin^2 \theta d\varphi^2] \tag{41}$$

We look for a static solution describing the scalar field trapped to a value $\phi = \phi_{in}$ in the interior of a sphere of radius r_b and making a transition across a thin surface to $\phi = 0$ outside. The fermi gas trapped inside the soliton is described, in the Thomas Fermi approximation [17]-[19], by the following distribution in momentum space: $n_k = \theta(k - k_f)$, k being the momentum measured in an appropriate local frame that depends on r , and k_f the fermi momentum. The fermion energy density is given by:

$$W = \frac{2}{8\pi^3} \int d^3k n_k \epsilon_k \tag{42}$$

with $\epsilon_k = \sqrt{k^2 + m(\phi_{in})^2}$. The fermion number density ν_f and the non-vanishing components of fermion stress energy tensor are:

$$\nu_f = \frac{2}{8\pi^3} \int d^3k n_k \tag{43}$$

$$T_t^t = W$$

$$T_r^r = T_\theta^\theta = T_\phi^\phi = T_\rho^\rho \equiv -T = -\frac{2}{8\pi^3} \int d^3k n_k \frac{k^2}{3\epsilon_k} \tag{44}$$

The trace of the stress tensor is just:

$$T_\mu^\mu = W - 3T = m(\phi_{in})S \tag{45}$$

with S the scalar density:

$$S = \frac{2}{8\pi^3} \int d^3k \frac{n_k}{\epsilon_k} m(\phi_{in}) \tag{46}$$

Defining $8\pi G_{in} \equiv U(\phi_{in})^{-1}$ as the effective interior ‘‘gravitational constant’’, the metric field equation in the interior can be expressed in the above spherical coordinates as:

$$r^2 G_t^t = e^{-2\bar{v}} - 1 - 2e^{-2\bar{v}} r \frac{d\bar{v}}{dr} = -8\pi G_{in} r^2 [W + V(in)] \tag{47}$$

$$r^2 G_r^r = e^{-2\bar{v}} - 1 + 2e^{-2\bar{v}} r \frac{du}{dr} = 8\pi G_{in} r^2 [T - V(in)] \tag{48}$$

$$r^2 G_\theta^\theta = e^{-2\bar{v}} \left[r^2 \frac{d^2 u}{dr^2} + \left[1 + r \frac{du}{dr} \right] r \frac{d}{dr} (u - \bar{v}) \right] = 8\pi G_{in} r^2 [T - V(in)] \tag{49}$$

The scalar field satisfies:

$$\phi_{;\mu}^{\mu} + V' + m'(\phi)S - U'R = 0 \tag{50}$$

Taking the trace of the Einstein tensor G_μ^μ , the Ricci scalar R substitutes in Equation (24) to give the following radial equation for the scalar field in the weak field limit:

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} + F(U) \left[\frac{d\phi}{dr} \right]^2 = \frac{dW}{d\phi} \tag{51}$$

where

$$\frac{dW}{d\phi} \equiv \left[V' + m'(\phi)S - \frac{U'}{U} \left(\frac{mS}{2} + 2V \right) \right] \sqrt{\left[1 + \frac{3U'^2}{U} \right]} \tag{52}$$

and

$$F(U) \equiv \frac{U'}{U} \left(\frac{1}{2} + 3U' \right) \sqrt{\left[1 + \frac{3U'^2}{U} \right]} \tag{53}$$

A sufficient condition for existence for a large NTS with $\phi_m \sim \text{constant}$ is the vanishing of W' at ϕ_m . A convenient way to arrange this is to have $V(\phi)$ to have a minima at ϕ_m , where U'/U is small, and to have fermions massless at ϕ_m .

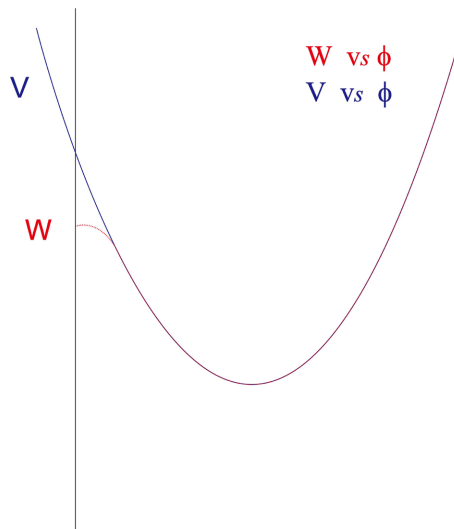


Figure 1. The profile of the potential, V and the effective potential, W . The two can be chosen to match for all $\phi > 0$ except for ϕ near zero.

The zero of W' then coincides with that for V' . For a profile of V described in **Figure 1**, and $U(\phi) = M_p^2 + M^4/\phi^2$ (with $\phi^2 \gg M^4/M_p^2$), the profile of W closely follows that of V except near $\phi \approx 0$. The divergence of the NMC at $\phi = 0$ ensures the vanishing of $W'(\phi = 0)$. The divergence of the NMC and in turn that of $F(U)$ also ensures that $\phi = 0$ is a solution with the Ricci scalar vanishing for arbitrary $V(\phi = 0)$. A typical non-trivial NTS solution has the scalar field locked to $\phi = 0$ outside a compact spherical domain of radius r_b and transits to $\phi = \phi_m$ across the boundary:

$$1 - \frac{\phi}{\phi_m} \rightarrow \exp[-V''(\phi_m)(r_b - r)]$$

The transition zone is thus related only to $V''(\phi_m)$ for large r_b . The radius

r_b is determined by the energetics of the ball and is governed by the number of trapped fermions inside the ball. The pressure of the degenerate fermions keeps the soliton from collapsing. As for the Lee-Wick NTS (see also [28]) the total energy is expressible as the sum of (i) the volume energy, (ii) the surface energy and (iii) the energy of the fermions:

$$E = \frac{4\pi}{3}V(\phi_m)r_b^3 + 4\pi sr_b^2 + \alpha \frac{N_f^{4/3}}{r_b}; \quad \alpha \equiv \frac{1}{2} \left(\frac{3}{2} \right)^{5/3} \pi^{1/3} \sim O(1) \quad (54)$$

For a vanishing $V(\phi_m)$, the minimum of (ii) and (iii) give the total energy $E \sim N_f^{8/9} (4\pi s)^{1/3}$, with an energy per fermion $\sim (4\pi s)^{1/3} / N_f^{1/9}$. For $V(\phi_m) > 0$, large r_b, N_f solutions have energy $E \sim 4(4\pi V)^{1/4} N_f / 3$, with the energy per fermion roughly independent of the number of fermions. Finally, for negative $V(\phi)$, NTS's exist only for a small fermion number (\bar{N}_f) and for small enough $|V|$. In this last case, the existence of NTS's requires the surface term to dominate over the volume term. This is possible only for small r_b, \bar{N}_f . The energy per fermion is again $\sim (4\pi s')^{1/3} / \bar{N}_f^{1/9}$. *Energy per fermion for a large fermion number configuration is smaller, the closer the minimum of $|V|$ is to zero.*

This can form the basis for a prescription to sort out the cosmological constant problem. We first recall that in standard GTR, the cosmological constant problem is sought to be *solved* by first making the cosmological constant dynamical: *i.e.* a constant of motion. This is done by using (for example) Witten's [11] prescription by Coleman, Hawking and Weinberg [3] [30] [31]. One adds to the action a term:

$$I_F = -\frac{1}{48} \int d^4x \sqrt{-g} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma}$$

with $F_{\mu\nu\rho\sigma}$ the exterior derivative of a three form gauge field $A_{\nu\rho\sigma}$:

$$F_{\mu\nu\rho\sigma} = \partial_{[\mu} A_{\nu\rho\sigma]}$$

using the fact that $F_{\mu\nu\rho\sigma}$ is totally antisymmetric, it is proportional to the permutation tensor. The field equation for $A_{\nu\rho\sigma}$ implies that the proportionality factor is a constant. This reduces the action to:

$$I_F = \frac{1}{2} c^2 \int d^4x \sqrt{-g}$$

With c a constant of integration. This makes the effective cosmological constant a constant of integration. This being the case, then in a quantum theory we expect the wave function of the universe to be a superposition of states with different values of the cosmological constant. Hawking [30] has proposed that the application of conditional probability that homosapiens would come into being and study physics in a universe then *solves* the cosmological constant. However, such an anthropic principle is not a dynamical principle in the first place. Even if the evolution of an intelligent species were related to a dynamic prescription, it would be difficult to implement it in standard GTR where every value of an effective cosmological constant corresponds to a distinct FRW solution that can not be dynamically compared with a solution with a different cosmological constant.

In the structure that we have described, the situation is simpler. One can start with the action Equation (2) and consider NTS's with a large average fermion density in the universe. Every NTS is a ball that has an effective gravitational constant vanishing in its exterior and having a unique value in its interior determined by the NMC at ϕ_m where the effective potential is minimized. The effective cosmological constant vanishes in the exterior and the value of $V(\phi_m)$ contributes to the interior volume energy of the NTS. One can now play the same game as played in standard GTR and introduce I_F as before to make $V(\phi_m)$ dynamical. Consider the wave function of the universe containing a large (conserved) number of fermions. We can expect the state vector of the universe to be a superposition of states with different $V(\phi_m)$. From the previous considerations, the minimum energy per fermion would be for the state that has $V(\phi_m)$ vanishing. The minimization of energy is a dynamic principle that may or may not have anything to do with the existence of apes!! With all NTS's having common asymptotics for an arbitrary $V(\phi_m)$, a dynamical solution that uses an energy minimization prescription makes sense and is easy to implement.

2.7. Microscopic Domains

It is interesting to note that were we to ensure bounded NMC inside domains as large as hadrons, one would again get an effective gravity model with no cosmological constant problem. Consider for example a theory described by Equation (42) in which the confined fermionic degrees of freedom were associated with baryons. With the NMC diverging at say $\phi = 0$ and the fermions confined to hadronic size microscopic $\phi \neq 0$ domains, there would be effective gravitational effects for baryons. A "cosmological constant" as large as $\Lambda_{QCD} \approx (0.2 - 0.3 \text{Gev})^4$ would merely make a small contribution to the mass of a baryon and could be absorbed in its net mass. The vanishing of the covariant divergence of the matter stress energy tensor implies the geodesic law of motion for material particles. The geodesic law in turn implies the equality of the inertial mass and the passive gravitational mass. The gravitating (active) gravitational mass, however, would depend upon the number and binding energy of non-baryonic matter of the source. Considering the relatively large error bars in the determination of the gravitational constant in a typical Cavendish experiment, the change in the active gravitational mass would not be discernible. However, it is claimed [1] that the equality of the active and passive gravitational masses is established in lunar ranging experiments to a very high accuracy. For this reason (notwithstanding our doubts on the claimed accuracy and interpretation of the above experiments) we feel that the large NTS's described earlier offer a better promise for a problem free gravity theory.

3. Features of a Toy Cosmology

In the model described above, a divergent NMC over most of the universe provides for a vanishing cosmological constant. Non-trivial solutions for the scalar

field provides for gravitating pockets inside compact domains—with the divergent NMC being restricted to their exterior. A value $N_f \approx 10^{75}$ for $s \approx (\text{Mev})^3$ would give a NTS of a size of tens of kilo parsecs. Such an N_f is of the same order as the relic background neutrinos/photons in the universe. Thus a fermion species that decouples very early in the universe when that species is still relativistic, would be sufficient to provide N_f for gravitating domains as large as a Halo of a typical large structure (galaxy/local group etc.).

With the effective gravitational constant identically vanishing outside a NTS, one could conceive of a cosmological model that starts with a hot big-bang and evolves as a Milne model [32] almost from its birth. We assume the existence of conditions conducive for the production of above NTS's with fermions confined to compact domains where the scalar field is non-vanishing. These NTS's would then evolve by mutually colliding and coalescing. As the NTS's coalesce, the fermion number adds up, the NTS becoming larger and could evolve to the current distribution of gravitating domains.

On large scales, the universe evolves with the Friedmann-Robertson-Walker scale factor increasing linearly with time: $a(t) \propto t$. This has characteristic features: 1) With $\int_0^t dt/a(t)$ unbounded for any $t > 0$, there is no horizon problem in the theory. 2) With the expansion parameter not determining any “critical-density” in the model, there is no flatness (fine tuning) problem. 3) The model is concordant with standard classical cosmological tests, viz.: the number count, angular diameter and the luminosity distance variation with redshift. The first two tests are quite sensitive to models of galactic evolution and for this reason have (of late) fallen into disfavour as reliable indicators of a viable model. However the magnitude—redshift measurements on SN IA have a great degree of concordance with $\Omega_\Lambda = \Omega_M = 0$ ([27] [33]-[35]). 4) With the scale factor evolving linearly with time, the Hubble parameter is precisely the inverse of the age t . Thus the age of the universe inferred from a measurement of the Hubble parameter is 1.5 times the age inferred by the same measurement in standard matter dominated model. Such a cosmology promises consistency with an older universe. 5) The deceleration parameter is predicted to vanish. 6) early universe nucleosynthesis is not ruled out in a Milne universe. It has been shown [36] that for a baryon entropy ratio $\eta \approx 5 \times 10^{-9}$, one gets $\sim 24\%$ of helium-4 and metallicity quite close to that observed in type II stars and low metallicity clouds. The cosmology thus comes with its characteristic predictions and may well be distinguishable by the next generation of experiments.

4. Conclusions

What we have profiled is a plausible program with an exploratory spirit. We had been looking for a framework that could provide for a spatial variation of the effective gravitational constant as a solution to the cosmological constant problem. It was this search that attracted our attention to remarkably encouraging results in the theory of non-topological solitons [NTS's] in the Lee-Wick model. Given a

(large), conserved fermion number and a given surface tension of such a soliton, the smaller the magnitude of the interior volume energy density (given by the value of the minimum of the effective potential), the smaller is the energy per fermion of the NTS. If one adds non-minimal coupling, that diverges at a point that we take (without any loss of generality) as $\phi = 0$, and makes the interior volume energy dynamical, the minimizing energy can provide a dynamic prescription to sort out the cosmological constant problem. The effective gravitational constant also approaches a universal value determined by the NMC at the minimum of the effective potential.

We have demonstrated that in a whole class of scalar tensor theories in which the non-minimal coupling diverges and for which the classical effective potential vanishes at some point, classical scalar field condensates can occur as NTS's. The effective gravitational constant inside all large domains would approach a universal value and the effective cosmological constant would drift to zero. The dynamical tuning of the effective cosmological constant to a small value and the effective gravitational constant to a universal value are compelling features—enough to explore the possibility of raising the toy model described here to the status of a viable cosmology.

The above model was a hypothetical toy model proposed in the year 2001 to resolve the cosmological constant problem which was not studied much. More viable solutions had been approached in recent time while exploring the scalar tensor theories. Davari, Marra and Malekjani [38] and Norello and Hartmann [39] examined the cosmological constraints on minimally and non-minimally coupled scalar field. Copeland, Padilla *et al.* [40] introduced models permitting self-tuning to low scale vacua while not violating local gravity constraints. Extensive researches are still going in this field of scalar tensor theories and modifying gravity to create more models in order to dynamically tune away the cosmological constant and hence, address the problem of cosmological constant in an effective manner. Although this model can make a viable contribution to the Cosmological constant problem but still has some limitations in the form of lack of comparison with the observational data. In our upcoming paper, as our future work, we will be including the quantitative observational constraints on Gravitational Constant from the high precision test—binary pulsar observations, to support our theoretical predictions.

Acknowledgements

This work has benefitted greatly from discussions with Profs. T. W. Kibble, M. Rocek, P.D. Mannheim, L. Parker and G. W. Gibbons.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

- [1] Will, C.M. (1993) Theory and Experiment in Gravitational Physics. Cambridge Uni-

- versity Press. <https://doi.org/10.1017/cbo9780511564246>
- [2] Weinberg, S. (1985) *Gravitation—Einstein Centenary Volume*. Cambridge University Press.
- [3] Weinberg, S. (1989) The Cosmological Constant Problem. *Reviews of Modern Physics*, **61**, 1-23. <https://doi.org/10.1103/revmodphys.61.1>
- [4] Linde, A.D. (1990) Particle Physics and Cosmology. In: *XXIV International Conference on High Energy Physics*, Springer, 357-386. https://doi.org/10.1007/978-3-642-74136-4_18
- [5] Weinberg, S. (2000) A Prioriprobability Distribution of the Cosmological Constant. *Physical Review D*, **61**, Article 103505. <https://doi.org/10.1103/physrevd.61.103505>
- [6] Weinberg, S. (2000) The Cosmological Constant Problems. Talk Given at Dark Matter 2000.
- [7] Vilenkin, A. (1986) Boundary Conditions in Quantum Cosmology. *Physical Review D*, **33**, 3560-3569. <https://doi.org/10.1103/physrevd.33.3560>
- [8] Vilenkin, A. (1988) Quantum Cosmology and the Initial State of the Universe. *Physical Review D*, **37**, 888-897. <https://doi.org/10.1103/physrevd.37.888>
- [9] Zlatev, I., Wang, L. and Steinhardt, P.J. (1999) Quintessence, Cosmic Coincidence, and the Cosmological Constant. *Physical Review Letters*, **82**, 896-899. <https://doi.org/10.1103/physrevlett.82.896>
- [10] Steinhardt, P.J., Wang, L. and Zlatev, I. (1999) Cosmological Tracking Solutions. *Physical Review D*, **59**, Article 123504. <https://doi.org/10.1103/physrevd.59.123504>
- [11] Witten, E. (2000) The Cosmological Constant from the Viewpoint of String Theory. In: *Sources and Detection of Dark Matter and Dark Energy in the Universe*, Springer, 27-36. https://doi.org/10.1007/978-3-662-04587-9_3
- [12] Polchinski, J. (1998) *String Theory*. Cambridge University Press. <https://doi.org/10.1017/cbo9780511618123>
- [13] Affleck, I., Dine, M. and Seiberg, N. (1985) Dynamical Supersymmetry Breaking in Four Dimensions and Its Phenomenological Implications. *Nuclear Physics B*, **256**, 557-599. [https://doi.org/10.1016/0550-3213\(85\)90408-0](https://doi.org/10.1016/0550-3213(85)90408-0)
- [14] Zee, A. (1979) Broken-Symmetric Theory of Gravity. *Physical Review Letters*, **42**, 417-421. <https://doi.org/10.1103/physrevlett.42.417>
- [15] Dolgov, A.D. (1983) *The Very Early Universe*. Cambridge University Press.
- [16] Ford, L.H. (1987) Cosmological-Constant Damping by Unstable Scalar Fields. *Physical Review D*, **35**, 2339-2344. <https://doi.org/10.1103/physrevd.35.2339>
- [17] Lee, T.D. (1987) Soliton Stars and the Critical Masses of Black Holes. *Physical Review D*, **35**, 3637-3639. <https://doi.org/10.1103/physrevd.35.3637>
- [18] Lee, T.D. and Pang, Y. (1987) Fermion Soliton Stars and Black Holes. *Physical Review D*, **35**, 3678-3694. <https://doi.org/10.1103/physrevd.35.3678>
- [19] Lee, T.D. and Wick, G.C. (1974) Vacuum Stability and Vacuum Excitation in a Spin-0 Field Theory. *Physical Review D*, **9**, 2291-2316. <https://doi.org/10.1103/physrevd.9.2291>
- [20] de Witt, B., Fayt, P. and Grisar, M. (1986) *Supersymmetry, Supergravity, Superstrings*. World Scientific.
- [21] Wetterich, C. (1988) Cosmology and the Fate of Dilatation Symmetry. *Nuclear Physics B*, **302**, Article 645.
- [22] Pagels, H. (1965) Spin and Gravitation. *Annals of Physics*, **31**, 64-87.

- [https://doi.org/10.1016/0003-4916\(65\)90232-0](https://doi.org/10.1016/0003-4916(65)90232-0)
- [23] Bose, S. and Lohiya, D. (1999) Behavior of Quasilocal Mass under Conformal Transformations. *Physical Review D*, **59**, Article 044019. <https://doi.org/10.1103/physrevd.59.044019>
- [24] Holdom, B. (1987) Cosmic Balls of Trapped Neutrinos. *Physical Review D*, **36**, 1000-1006. <https://doi.org/10.1103/physrevd.36.1000>
- [25] Dolgov, A.D. and Markin, Y.O. (1990) Cosmological Evolution of Neutrino Balls. *Journal of Experimental and Theoretical Physics*, **17**, Article 207.
- [26] Chiba, T. (1999) Quintessence, the Gravitational Constant, and Gravity. *Physical Review D*, **60**, Article 083508. <https://doi.org/10.1103/physrevd.60.083508>
- [27] Lohiya, D. and Sethi, M. (1999) ISOS-II Conference: FERMILAB-Conference 99/196A.
- [28] Cottingham, W.N. and Mau, R.V. (1991) Soliton Stars at Finite Temperature. *Physical Review D*, **44**, 1652-1660. <https://doi.org/10.1103/physrevd.44.1652>
- [29] Lohiya, D. and Sethi, M. (1999) A Programme for a Problem-Free Cosmology within the Framework of a Rich Class of Scalar-Tensor Theories. *Classical and Quantum Gravity*, **16**, 1545-1563. <https://doi.org/10.1088/0264-9381/16/5/306>
- [30] Hawking, S.W. (1987) Quantum Coherence down the Wormhole. *Physics Letters B*, **195**, 337-343. [https://doi.org/10.1016/0370-2693\(87\)90028-1](https://doi.org/10.1016/0370-2693(87)90028-1)
- [31] Coleman, S., Glaser, V. and Martin, A. (1978) Action Minima among Solutions to a Class of Euclidean Scalar Field Equations. *Communications in Mathematical Physics*, **58**, 211-221. <https://doi.org/10.1007/bf01609421>
- [32] Milne, E.A. (1935) *Relativity, Gravitation and World Structure*. The Clarendon Press.
- [33] Kaplinghat, M., Steigman, G. and Walker, T.P. (2000) Nucleosynthesis in Power-Law Cosmologies. *Physical Review D*, **61**, Article 103507. <https://doi.org/10.1103/physrevd.61.103507>
- [34] Dev, A., Sethi, M. and Lohiya, D. (2000) Linear Coasting in Cosmology and SNe Ia. astro-ph/008193.
- [35] Perlmutter, S., Aldering, G., Goldhaber, G., *et al.* (1998) Measurements of Omega and Lambda from 42 High-Redshift Supernovae. astro-ph/9812133.
- [36] Batra, A., Lohiya, D., Mahajan, S. and Mukherjee, A. (2000) Nucleosynthesis in a Universe with a Linearly Evolving Scale Factor. *International Journal of Modern Physics D*, **9**, 757-773. <https://doi.org/10.1142/s0218271800000682>
- [37] Peres, A. (1962) Spinor Fields in Generally Covariant Theories. *Il Nuovo Cimento*, **24**, 389-452. <https://doi.org/10.1007/bf02896552>
- [38] Davari, Z., Marra, V. and Malekjani, M. (2020) Cosmological Constrains on Minimally and Non-Minimally Coupled Scalar Field Models. *Monthly Notices of the Royal Astronomical Society*, **491**, 1920-1933. <https://doi.org/10.1093/mnras/stz3096>
- [39] Novello, M. and Hartmann, A.E.S. (2019) The Cosmological Constant and a Scalar Field Coupled Non Minimally to Gravity. *International Journal of Theoretical Physics*, **59**, 1-9. <https://doi.org/10.1007/s10773-019-04240-2>
- [40] Copeland, E.J., Ghataore, S., Niedermann, F. and Padilla, A. (2022) Generalised Scalar-Tensor Theories and Self-Tuning. *Journal of Cosmology and Astroparticle Physics*, **2022**, Article 4. <https://doi.org/10.1088/1475-7516/2022/03/004>