Creative Education, 2020, 11, 1645-1665
https://www.scirp.org/journal/ce
ISSN Online: 2151-4771
ISSN Print: 2151-4755

# Improving the Students' Creativity in Chinese Mathematics Classrooms

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**How to cite this paper:** Zhang, C. L., Wu, J. Q., Cheng, L., Chen, X. T., Ma, X. C., & Chen, Y. R. (2020). Improving the Students' Creativity in Chinese Mathematics Classrooms. *Creative Education, 11,* 1645-1665.
https://doi.org/10.4236/ce.2020.119120

**Received:** July 20, 2020
**Accepted:** September 8, 2020
**Published:** September 11, 2020



## Abstract

This paper probes into qualities aiming at improving students' creativity in mathematics learning and attempts to share Chinese experience of mathematical teaching. Based on the "Youth Innovation Competence Model", we put forward six principles of mathematics creative teaching as follows: 1) encourage creative thinking; 2) make full use of representations and transformation of mathematical knowledge; 3) establish motivation, self-efficacy, and self-concept within students; 4) help students become self-regulated; 5) promote group interactions; and 6) implement open learning environment. With the utilization of six principles in mathematical teaching in Chinese classroom, the lesson *Understanding of Long Cuboid and Cube* of fifth grade shows that these principles can be perfectly implemented in driving problem, anchor task, and diagnostic assignment in mathematical teaching. Most importantly, students' competence in innovation is greatly improved.

## Keywords

Creativity, Creative Teaching, Chinese Mathematics Classroom

## 1. Introduction

Creativity is a person's ability to generate an idea or a product that is deemed by experts as both unique and appropriate in a certain domain (Amabile, 1996; Kaufman & Beghetto, 2009; Sternberg & Lubart, 1999). In this increasingly complex and radically changing world, creativity has been regarded as an important ability needed for the 21st century (Partnership for 21st Century Skills, 2019; World Economic Forum, 2016). It is a fundamental skill and one of the keys to success in this increasingly complex and radically changing world (Daniel &

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Laura, 2020). Creativity can be developed in educational environments (Daniel & Laura, 2020; Davies et al., 2013; Scott et al., 2004). Moreover, fostering students' creativity should be integrated into lessons and subjects taught in schools (Chan & Yuen, 2014; Kim, Roh, & Cho, 2016; Nie, 2017).

Mathematics is closely related to creative thinking (Chan, 2016). Mathematics is an important subject to cultivate students' creativity. It plays a unique and special role in students' creativity development (Wu & Wang, 2002). The fostering of students' creativity through mathematical activities is also valued (Askew, 2013; Wang, 2006). Based on the important role of classroom learning in the development of students' thinking and ability, it is worth further exploring how to foster students' creativity systematically in mathematics classes. However, few studies have systematically analyzed these questions.

The aim of this study is to explore and describe influential factors that contribute to the improvement of students' mathematical creativity. To be specific, we propose three research questions as follows: 1) by reviewing previous theories and relevant research, what are the qualities necessary for the cultivation of mathematical creativity? 2) what are the teaching principles that should be followed during mathematical teaching process? 3) how to apply these principles in mathematics teaching? With these objectives, we attempt to probe into creative teaching in mathematics.

## 2. Qualities of Creativity and Mathematical Teaching

### 2.1. Theoretical Perspective

To clarify the factors contributing to students' creativity on the basis of competence theory, learning power theory and creativity theory, we developed the "*Youth Innovation Competence Model*" (Zhang et al., 2018) (shown in **Figure 1**).

According to Onion Model and the Iceberg Model (Boyatzis, 1982; Spencer & Spencer, 1993), the Youth Innovation Competence Model includes not only implicit competencies, such as motivation and personality, but also explicit

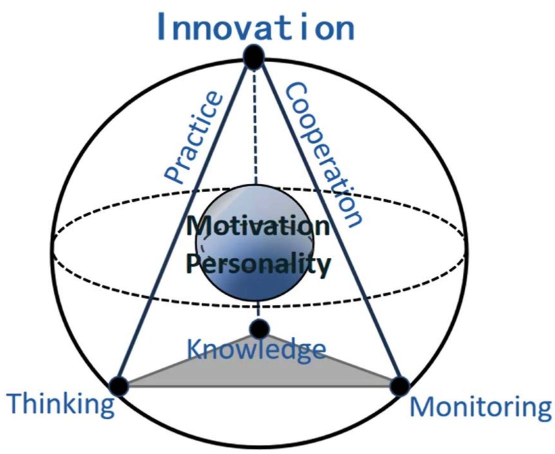


**Figure 1.** Youth innovation competence model.

competencies, such as knowledge and abilities. Moreover, as an important competence in learning, innovative competence is closely related to learning power. The learning power model developed by Pei (2016) consists of six elements in three levels. The first level includes knowledge and experience, strategies and rethinking, and willpower and gumption, which are basic competencies. The second level includes practice and activities and collaboration and communication, which are two basic paths for realizing individuals' development. The third level is criticism and innovation, and it is the highest state of individuals' development. The learning power structure described by the Effective Lifelong Learning Inventory (ELLI) also emphasizes the crucial role of mindful agency (Crick et al., 2015). Thus, the Youth Innovation Competence Model is hierarchical and regards monitoring as a basic element. Finally, the investment theory of creativity indicates that creative individuals display a combination of six resources that function interactively: intelligence, knowledge, thinking styles, personality, motivation, and environment (Sternberg & Lubart, 1993). The Youth Innovation Competence Model also presents the interactions between individual factors and environmental factors.

In summary, the Youth Innovation Competence Model has a comprehensive, inner-outer and hierarchical structure. In this model, there are seven basic elements: knowledge, thinking, monitoring, cooperation, practice, motivation and personality. To be specific, creative thinking knowledge, and monitoring abilities are individuals' basic competencies. Collaboration and practice are the channels through which individuals relate to the environment. Motivation and personality are the internal factors. It is implied that the cultivation of mathematical creativity heavily relies on factors such as motivation, knowledge, dialogue, learning environment, and so on. Therefore, we adopt the *Youth Innovation Competence Model* as theoretical basis in this study

## 2.2. Creativity and Mathematical Teaching

Mathematics is involved in most fields of scientific research and it plays a crucial role in a number of inventions. Similarly, some qualities of creativity are proven to be profoundly influential to the learning of mathematics. With the mutual relationship between mathematical learning and the development of creativity, teaching plans should be designed and implemented thoughtfully. To help improve students' mathematical creativity, we believe there are six key qualities of creativity that should be implemented in mathematical teaching and pedagogy.

***Creative Thinking in Mathematics***

Promoting mathematical creativity is increasingly acknowledged as one of the most important goals of mathematical education (Leikin & Pitta-Pantazi, 2013; Leikin & Sriraman, 2017; Mann, 2006; National Governors Association Center for Best Practices, 2010; Schoevers et al., 2019). Mathematical creativity is widely defined as the creation of something new and meaningful by breaking away from fixed mindsets (Balka, 1974; Haylock, 1978; Runco & Jaeger, 2012; Sriraman, 2005). It is also viewed as a process of combining previously known ideas and discovering new relationships among ideas in unfamiliar manners (Boden, 2004; Ervynck, 1991; Poincaré, 1948; 1956; Tammadge, 1979). In defining mathematical creativity, classification of its components is often discussed between several types of creative thinking. A large amount of research has proposed that mathematical creativity involves four types of creative thinking: divergent thinking, convergent thinking, intuition, and critical thinking (Balka, 1974; Ervynck, 1991; Guilford, 1967; Haylock, 1978; Krutetskii, 1976; Singh, 1988; Vallee, 1975).

Playing a large role in individuals' creativity, divergent thinking is the act of pursuing various solutions without fixations during problem solving (Balka, 1974; Guilford, 1967; Haylock, 1978). It not only enables students to present unusual ideas from usual perspectives and divides general problems into several sub-problems but also presents various original solutions flexibly and fluently (Balka, 1974; Haylock, 1978). Conversely, as one of the most crucial components of creative thinking in mathematics, convergent thinking is generally considered another significant attribute promoting creative thinking in mathematics. By gaining, mastering, understanding knowledge and applying skills, convergent thinking overcomes established mindsets (Balka, 1974; Haylock, 1978) and determines the only answer. Abundant research has demonstrated that both divergent thinking and convergent thinking are beneficial to mathematical creativity. Maxwell (1974) undertook an exploratory study to determine the relationship between problem solving and convergent-divergent production among geometry students in secondary schools. She observed that students who are highly convergent tend to use more generalizations in their problem solving when they are tackling a mathematical problem, revealing that convergent thinking in mathematics learning enables students to grasp the characteristics and relationships between cause and effect more easily. In addition, Kwon et al. (2006) managed to prove that programs encouraging divergent thinking are beneficial to improving flexibility, fluency and originality in mathematical creativity in students. In their research, it was indicated that students who are taught with strategies of open-ended problems performed better than students who used traditional methods from a textbook overall in each component of divergent thinking skills. In addition, encouraging students to go wrong and make mistakes and to find different solutions to the same problems is also beneficial to cultivating mathematical creative thinking in students (Grégoire, 2016).

Apart from divergent/convergent thinking, intuition in mathematics learning is equally important because intuition is closely related to the fruitful production of reasonable conjectures from formal concepts. Moreover, it serves as a driving force encouraging curiosity, imagination, and mathematical hypotheses in mathematical creativity (Ervynck, 1991; Krutetskii, 1976; Vallee, 1975). Contrarily, critical thinking is defined as a process of testing hypotheses, making modifications, and criticizing standard methods in mathematical learning (Singh, 1988). Both intuition and critical thinking allow students to make random guesses according to their instincts and later propose their hypothesis with a critical spirit later.

In conclusion, to promote creative thinking in mathematical learning, much research has demonstrated that ill-posed and open-ended problems profoundly contribute to promoting creative thinking in mathematics (Grégoire, 2016; Hiatt, 1970; Kwon et al., 2006; Sawada, 1997). Such intentional pedagogical designs and implementations enable students to discover relationships among concepts that have never been suggested, break established mindsets, overcome fixations, combine ideas in an unprecedented manner, and solve problem flexibly. Thus, teachers are suggested to encourage students to use their intuition to make random guesses so that the students can feel free to express themselves and explore the problem in their own meaningful way.

***Mathematical Knowledge—Representations and Transformations***

To find solutions to a problem, learners must represent the problem according to their previous knowledge and specific situation after they identify the problem and the goal that they hope to reach (Bransford & Schwartz, 1999; Newell & Simon, 1972; Wallas, 1926). This process requires learners to be sufficiently mentally flexible to understand the problem, acquire information that is beneficial to problem solving, transfer previously learned knowledge to new situations, and construct strategies. During this process, students must have multiple representation skills to articulate the same problem in different forms or views (Hwang et al., 2007). However, some researchers have pointed out that most students are currently trained to solve mathematical problems by memorizing knowledge that they obtain from their teachers and fail to grasp the importance of the connections between different types of representations (Ainsworth, 1999; Lesh et al., 1987). Thus, it is important for teachers to understand how multiple representations, such as formulas, graphs and pictures, help students to strengthen their creative thinking and ability (Hwang et al., 2007).

In mathematical learning, representation refers to the description of the relationship between objects and symbols, which includes external representation and internal representation. The former reveals the real world, generally including real-world representation, concrete representation, arithmetic symbol representation, spoken-language representation and pictorial or graphic representation (Lesh et al., 1987), whereas internal representation unravels mental status to build up mental images of mathematical objects and events (Janvier, 1987). Recent studies of representations have shown that students perform better in problem solving in mathematics and gain a deeper understanding of mathematical knowledge when they can interact with more than one type of appropriate representations (Ainsworth & Peevers, 2003; Mayer & Anderson, 1992). In their research on multiple representation skills and the effects of creativity on mathematical problem solving using a multimedia whiteboard system, Hwang et al. (2007) proposed that students who are able to skillfully manipulate their language representations (vocal), pictures representations (pictures and graphic) and formal representations (sentences, phrases, rules and formulas) perform better in problem solving and creating. Zhang and Norman (1994) reported that problem solving ability was significantly enhanced when learners are provided with representations that are externalized with more information.

As research of multiple representations has matured, researchers have asserted that understanding how representations relate to each other and their mutual transformation promotes knowledge comprehension and creative thinking (Cuoco & Curcio, 2001; Goldin, 1998; Kaput, 1998). There are two types of transformation between representations. The first type is transformation between internal representation and external representation. Internal representations specialize in imagination, deduction, and activities involving mathematical thinking, but they are less obviously observed since their mental structure is deeply embedded in learners' minds. In contrast, external representations are easier for learners to master because they can express their ideas through language and communication (Hiebert & Carpenter, 1992). Transformation between internal representation and external representation is the process of building connections between a mathematical idea that students attempt to understand and another mathematical idea that is easier for them to comprehend. During this process, images created in minds and forms, which are used to communicate ideas to others, exchange necessary packages of information, and thereby students' creative thinking is practiced. The second type of transformation occurs within multiple external representations (MERs). Ainsworth et al. (2002) and another study conducted in 2006 showed that different combinations of representations show different benefits for mathematical learning and problem solving. Ott et al. (2018) examined different combinations of representations (text, formula and graphic). Result showed that participants who were provided with multiple representations (text, formula and graphic, or text and formula) performed better than single representation group.

In summary, a great amount of research has revealed that mathematics learners can tremendously benefit from multiple external representations since MERs provide easier access to mathematical understanding and have a profound influence on problem solving (e.g., Debellis & Goldin, 2006; Kaput, 1998; Yerushalmy, 1997). When connecting and transforming between multiple internal and external representations, students tend to construct knowledge mentally and address current or anticipated requirements more efficiently (Dienes, 1973). Generally, students not only strengthen their understanding of mathematical concepts, but most importantly, they improve their creative thinking and ability to problem solve. Regarding pedagogical recommendations, it is suggested that teachers adopt different teaching strategies to promote students performing multiple representations in class, thereby enhancing learning performance (Cai & Hwang, 2002).

***Motivation, Self-efficacy, and Self-concept in Mathematics***

Serving as a driver of all types of learning, motivation, self-efficacy, and self-concept are widely acknowledged as one of the strongest correlates with creativity. They enable learners to concentrate on learning with deep involvement, interest, and persistence (Amabile, 1996). All these factors are correlated to one's cognitive engagement and beliefs concerning their ability to accomplish certain subject area.

Motivation and creativity are closely correlated. Self-determination theory (SDT; Deci & Ryan, 1985; Deci et al., 1991) proposes that individuals generally exhibit two types of motivation for learning: intrinsic and extrinsic motivation. Intrinsic motivation refers to engagement in a task without being subjected to external force or pressure; in contrast, extrinsic motivation is instrumental and refers to engaging in a task driven by external reasons or under pressure (Deci et al., 1991). Currently, researchers not only consider the coexistence of intrinsic and extrinsic motivations for the same task, but they also have found that the effect of intrinsic motivation often surpasses that of extrinsic motivation in terms of mathematics learning and creativity (e.g., Benware & Deci, 1984).

Serving as consistent support for creative behaviors, primary intrinsic motivation exerts a remarkable influence on deep involvement in mathematics out of curiosity and enjoyment (Amabile, 1996). Hence, motivation, especially intrinsic motivation, is of great importance to the cultivation of mathematical creativity.

Apart from intrinsic/extrinsic motivation, self-efficacy has been widely proposed to be an influential factor that is closely related to one's motivational level. According to Bandura (1997), self-efficacy refers to the subjective judgment of one's ability to accomplish. Studies have shown that students who feel more competent and motivated in school subjects tend to perform better and value these subjects more highly (Denissen et al., 2007); thereby, prior achievement has positive effects on students' perceived competence and intrinsic motivation, ultimately encouraging students to exert greater efforts in exploring the subject (Garon-Carrier et al., 2016). In other words, a student is more actively involved in a creative activity if he or she believes that he or she has a chance of succeeding.

The third well-known component is an academic self-concept in students. It is viewed as an evaluative self-perception formed through students' experience and interaction with a school environment (Marsh & Craven, 1997; Shavelson et al., 1976). Researchers have assumed that individuals only exert effort when they perceive that effort will result in fulfilment of their personal goals (e.g., Ames, 1992; Blumenfeld, 1992; Covington, 1992; Elliott & Dweck, 1988; Graham & Weiner, 1996). Thus, by telling students that they can also be creative and have control with this characteristic, they are more likely to change their perceptions of their control of creativity and be willing to develop their original thinking in mathematical problem solving (Grégoire, 2016).

With the joint effect of cognitive factors mentioned above, a large number of pedagogical approaches aimed at promoting students' self-cognition level are suggested. Recent studies have proposed that students' motivation to engage in creative work can be profoundly improved by changing their views about their abilities in mathematics learning (Dweck, 1999). Helping students to change their perceived ability from a fixation mindset to a growth mindset can encourage them to be willing to exert greater effort in mathematics learning and be more dedicated to solving challenging mathematical problems. In addition, in regard to the structure of mathematical problems, teachers are suggested to present mathematical problems that have been formulated as mathematical mindset problems (MM problems; Daly et al., 2019). Compared to problems formulated in a standard way, MM problems produce a considerable increase in reported motivation levels in students via a combination of behavioral and neurophysiological measures. All of these pedagogical methods are designed to motivate students to participate in creative activities in mathematics.

***Self-regulating in Mathematical Learning***

A major increase has been witnessed in the research on self-regulation in creativity in recent years (Butler & Winne, 1995; Csikszentmihalyi, 1997; Ivcevic & Nusbaum, 2017; Schunk, 1982; Zimmerman, 2001). Instead of being a psychological ability or an academic skill, self-regulation is rather a self-directive process by which learners transform their mental abilities into academic skills (Zimmerman, 2002). Self-regulation refers to the management of individuals' emotions to maintain and sustain their creative work until the finishing point. It is a deliberate, judgmental and adaptive process to generate individual thoughts and feelings and thereby set appropriate goals and adjust approaches according to feedback (Butler & Winne, 1995; Ivcevic & Nusbaum, 2017; Schunk, 1982; Zimmerman, 2000).

Empirical studies have shown that there is a unique correlation between self-regulated learning and qualities of creativity, such as multiple motivational beliefs, diverse knowledge, and flexible cognitive processing (Butler & Winne, 1995; Zimmerman, 2001). Research findings have demonstrated that students who are instructed to set specific and proximal goals for themselves display superior perceptions of personal efficacy and higher levels of self-satisfaction and self-motivation, which reversely influence their creativity (Zimmerman, 2001, 2002). Similarly, in mathematics learning, various psychological qualities of creativity can be provoked by self-regulation learning. For instance, simply asking students to self-record some aspects of their learning, such as the completion of assignments, often led to "spontaneous" improvements in functioning (Shapiro, 1984). Moreover, Schunk (1982) discovered that students who monitor their progress by themselves (self-monitoring) and are monitored by adults (external monitoring) show better mastery of efficacy, skills, and persistence compared to students with no monitoring. The results have suggested that, despite the monitoring agent, students will gradually acquire awareness of their capabilities and thereby evaluate their involvement and efforts for the future assignment if their progress is closely examined and monitored.

Research findings have strongly supported the importance of students' use of self-regulatory processes (Zimmerman, 2001). As perspectives on the cultivation of creativity have increasingly matured, self-regulation has been widely acknowledged to be one of the most influential factors. Currently, children are facing a new era with temptations and their ability to self-regulate is tremendously challenged. Teachers must empower students with the ability to be self-aware of their differences and constantly monitor their own behavior and therefore adjust their decisions according to changing situations. In mathematics teaching, teachers should instruct students to learn to evaluate their work or estimate their strengths and limitations regarding learning. In addition, teachers are suggested to pay close attention to students' beliefs about learning, as well as changes in their self-efficacy, motivation, or cognitive development, by implementing a learning environment in which students are free to choose their academic materials, methods, and even partners based on the problems that they encounter.

***Open Learning Environment***

Recently, there has been a significant increase in research to find an open learning environment pattern contributing to the cultivation of students' creative learning (Beghetto, 2007); fortunately, researchers have found certain evidence to demonstrate that the intentional design of the learning environment and encouragement of active interaction are profoundly important to supporting creativity in students (Beghetto & Kaufman, 2014).

With the attempt to build a classroom environment to support students' creativity, many researchers have explored the impact of creative learning environments on learners and what should be designed to create a learning-friendly environment for students (Jindal-Snape et al., 2013; Kozbelt et al., 2010). The environment for learning refers to establishing and designing physical space and the relationship one has, as well as the resources and support that are available (Beghetto & Kaufman, 2014). Richardson and Mishra (2018) reviewed the literature on learning environments for creativity and identified three scales: physical environment, learning climate, and learner engagement. These scales indicate that an ideal learning environment to support creativity in students must be designed in a way that is open and flexible.

***Active Interaction***

Regarding interaction in mathematical teaching, classroom discussion provides an ideal forum for students to develop their creative thinking skills (Beghetto, 2007). Normally, the purposes of using classroom discussion are to help students to learn academic content by encouraging verbal interactions, and also to effectively develop students' "intellectual arts" of thinking and communication (Larson, 2000). Some researchers addressed that interaction offers great opportunities for students to develop their higher-order thinking by interpreting, analyzing, and manipulating information. Rather than passively recite or recount memorized knowledge transmitted from teachers, students engage in learning as active participants, and most importantly, gain a deeper understanding and better construction of what they learn, which ultimately lead to the development of creativity.

In summary, research has demonstrated abundant empirical evidence to support how an open learning environment impacts the improvement of creativity in students. Providing a creative learning environment is as important as attaching importance to the cultivation of creative thinking, motivation, and other aspects. Teachers can implement a friendly physical environment, in which students feel safe and deeply motivated to become engaged in learning activities.

## 3. Principles for Creative Teaching in Mathematics

Based on the *The Youth Innovation Model* and literature review, we propose 6 factors in the models of cultivating creativity in mathematics. In school mathematical teaching, these factors can be fully utilized and transformed into forms beneficial to mathematical teaching. The suggestions are as follows.

- Encourage creative thinking in mathematical learning. Creative thinking in mathematics includes intuition, convergent/divergent thinking, and critical thinking. Through open-ended problems, students are encouraged to use divergent thinking to explore multiple solutions and later use convergent thinking or critical thinking to test their hypotheses. Last but not least, students should be provided with the opportunity to utilize intuition to perform random guessing.
- Make full use of representations and transformations of mathematical knowledge. To gain a better understanding of certain mathematical knowledge, students should not only know what that knowledge means but, most importantly, flexibly convert this knowledge into different forms of representations, such as text, graphics, and pictures, and also transform among them.
- Establish motivations, self-efficacy, and self-concept in students. Motivations, self-efficacy, and self-concept in mathematics tremendously impact students in mathematical learning. To help improve intrinsic motivation and positive emotions toward mathematics in students, teachers are suggested to present problems that are open ended and formulated in the mindset of mathematics, instead of standard problems.
- Help students to become self-regulated learners in mathematics. Teachers should help students to become self-regulated learners by guiding them to gain awareness of their strengths and limitations in mathematical learning, setting appropriate goals, and constantly monitoring and reflecting their behavior.
- Promote group cooperation and discussion among students themselves and teachers. Teachers should design open-ended questions so that students are afforded the opportunity to explore multiple solutions from diverse perspectives through cooperation with other students.
- Implement an open learning environment. Teachers are suggested to design and implement a learning environment in terms of three aspects: physical environment, learning climate, and learner engagement. First, students should be provided with an abundant number of learning resources, and their work should be presented so that they will feel respected and intrinsi-

cally motivated. Second, with teachers playing a role as colearner and explorer to support students' learning, a learning environment should be implemented in such a manner that students feel free to express their different ideas, and unexpected mistakes are respected and tolerated. Additionally, the atmosphere must be friendly and cooperative. Last but not least, teachers should present mathematical learning problems correlated with students' authentic lives and experience.

Principles mentioned above constitute a distinct gradation, but they can only be effective if they are carefully organized during implementation. As the most powerful drive and the elementary gradation of all elements, motivation, self-efficacy, and self-concept serve as the core of creativity. With the support of intrinsic psychological qualities, students are able to obtain and accumulate mathematical knowledge, which lays a solid foundation for the cultivation of creative thinking. In addition, while self-regulation optimizes the relationship between motivation, knowledge, and creative thinking, group operation and open learning environment offer a flexible external opportunity for the development of mathematical creativity. Conclusively, these principles guarantee the implementation of creative teaching in mathematics.

## 4. The Revolution of Mathematical Classes in China: "Teaching Process Design"

How can students be made creative while learning? As addressed above, students' creativity in mathematical learning process is influenced by multiple factors. We firmly believe that students' creative quality will be developed only if teachers design effective activities during students' learning processes. Therefore, a new teaching design scheme—"Teaching Design Based on Learning Trajectory"—has been proposed in China, reflecting achievement and experience during the revolution of the new curriculum.

"Teaching Design Based on the Learning Trajectory" was first proposed by Prof. Yunhuo Cui at East China Normal University. His research specially focuses on learning experience, and he asserts that six elements should be included in teaching design: learning topics, learning objectives, learning processes, task assessment, testing and assignments and reflection after learning. These six elements embody the process of choosing a topic as the learning unit that has large content, centers on mastering skills and guides by formative evaluation. Teachers provide "scaffolds" through the decomposition of goals for students' independent learning, assisting students in achieving the learning goals. Focusing on math classes, we simplified the teaching process and adjusted the six elements of the teaching process design into three elements: driving problems, anchor tasks and diagnostic assessments. By following these three elements, we can well implement the six principles of promoting the development of innovative quality in mathematics classrooms. We use the lesson Understanding of Long Cuboids and Cubes from fifth grade as an example to illustrate how to design a "teaching design based on learning trajectory" (as shown in the following Table 1) and put it into effect.

***Driving Problems***

As one of the elements of "Teaching Process Design", driving problems aim at driving students to solve problems creatively with challenging and open questions. By having students start from questions and change knowledge points into exploratory questions, it is helpful in guiding students to think initiatively and stimulate their motivation to learn mathematics.

For example, the knowledge point of this class is understanding the size and position relationships of the 12 edges of the cube; therefore, the teacher turns this exercise into a challenging and open driving problem: if you are given 16 sticks and imagine, can you make a cuboid? First, this question is an open and does not directly tell the students' the specific lengths of the 16 small sticks. Students are asked to assume the possible lengths of sticks and to think about whether 16 sticks are sufficient or redundant. At the same time, there are certain challenges in giving students substantial thinking space. Students must think about the relationships among the edges of the cuboid. We can see that, through these three driving questions, students can build up integrated concepts of cuboids, such as length, width, height, vertex, surface and volume, related to their life experience.

***Anchor Tasks***

The second element of "Teaching Process Design" is to turn driving problems into anchor tasks. "Anchor" refers to the original knowledge or life experience that enable students to assimilate new knowledge. The completion of the anchor task requires students to mobilize their original cognitive experience to fully

**Table 1.** A "Teaching Process Design" of understanding the long cuboid and cube.

| Driving Problem | Anchor Task | Diagnostic Assessment |
|---|---|---|
| Here are 16 small sticks for you. Imagine: Can you make a cuboid? What are the characteristics of cuboids? | Give four sets of small sticks, including three types of small sticks of 12 cm/9cm/6cm, so that students can choose and try to build them. | Ask and trace: What are the characteristics of cuboids that you found in the process of building? Feedback: The cuboid has 6 faces/12 edges/8 vertexes. |
| If you hide an edge, can you still imagine its original appearance? Hiding 2 articles ... | If you cannot imagine the structure of a cuboid until you cannot imagine it, you cannot hide one edge of the cuboid. | Ask and trace: How many edges do you with from a vertex? Feedback: The three edges starting from a vertex can be imagined as three faces and then as cuboids |
| Can you imagine what is based on these three edges (the same vertex)? | Give the data of length/width/height so that students can imagine specific objects | Ask and trace: What is the relationship between a cuboid and a cube? Feedback: The cube is a special cuboid. |

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embody the role of multiple mathematical representations and mathematical intuition by project-based learning. These processes enable students to explore and solve problems by cooperation and communication within and between groups.

For example, after addressing the driving problem "If you are given 16 sticks and imagine, can you make a cuboid?", the teacher asks students to observe objects. Students can say that a cuboid can be made up of 12 small sticks, which can be divided into 3 groups, each group being a small stick of the same length. This information is the original cognitive basis for students to learn cuboid features. The design of the anchor task requires the transformation between divergent thinking and convergent thinking through explicit operations on the basis of students' cognitive experience. When designing anchor tasks for this purpose, three points should be considered: facilitating students' inquiry learning; convenience for students to solve autonomous problems; and leaving sufficient room for students to present personalized cognition. Therefore, we design the first anchor task (shown in **Table 2**): give the students four sets of small sticks, in which each set of sticks has three different lengths (12 cm, 9 cm and 6 cm) and ask the students to think about which set of sticks can be built into a cuboid.

This anchor base task triggers project-based learning. Students must face a real situation, contact learned knowledge and past experience, choose from various information, build it up with the hands, and consider whether it can work with other teams (borrow the sticks they do not need from the other groups or exchange the sticks that they do not need with other groups for what they need) to solve the problem.

In the teaching process, we find that students can talk freely and experience a process of "insight". When students choose the number of sets, they will find that the key to successful construction lies in finding three sets of four edges of the same length in each set. In addition, the students transfer their initial thought that only the first set could be built into cuboids to that the second set could also be built into cuboids with two square faces and the third set could be built into cubes. They come to an agreement that it is the process of free discussion and attempt that enables them to experience insight, which is a sudden, profound understanding of a complex situation, often described as an "aha" moment. Because it is accompanied by a breakthrough in the traditional assumptions and the general correlations among information, it can be helpful to

**Table 2.** Anchor task 1.

| Length of sticks/set number of sticks | The first set | The second set | The third set | The fourth set |
|---|---|---|---|---|
| 12 cm | 7 | 4 | 3 | 2 |
| 9 cm | 5 | 9 | 12 | 11 |
| 6 cm | 4 | 3 | 1 | 3 |

*Note*. From "Learning Process Analysis from Learners' Perspective," by C. L. Zhang, W. Chen, and Z. Q. Zhang, 2020, Beijing Normal University Press. p.59-60. Copyright 2020 by Beijing Normal University Press. Reprinted with permission.

produce novel solutions for problems. The group that receives the fourth set of sticks will find that the sticks that are not needed by other groups are exactly what they need. Therefore, they will actively cooperate with other groups and finally build cuboids by exchanging or borrowing the sticks that are not needed by the other group, enabling them to see the importance of collaboration in creatively solving problems.

***Diagnostic Assessment***

The purpose of diagnostic assessment is to encourage students to think divergently and critically. The method can be a test with a question or questioning by teacher. The teacher judges the difficulty and key points of students' thinking according to their completion or response, including expression, movement, language and work. At the same time, the teacher uses testing and questioning, which help students to develop the ability of self-regulation. During the process of continuous mathematical reflection and questioning, students learn to think and solve problems creatively.

For example, in the process of imagining real-life objects from the length, width and height data, students will find objects that meet the requirements of length, width and height in life, including both ordinary cuboids and cubes. When the height becomes very small, such as 0.1 mm, it is a piece of paper. At this moment, the teacher asks whether it is a piece of paper or a cuboid. Through such questioning, students will be guided to distinguish and analyze the differences between cuboids and rectangles to achieve a spiral increase in knowledge. The question here helps students to develop the ability of critical thinking and to discover the essence of cuboids. We are pleasantly surprised to see that students finally realize through debate that a piece of paper, even if it is small, is a cuboid. In real life, a rectangle without thickness does not exist, and it can only be drawn on paper or on the blackboard.

After this class, the teacher asks students to complete a project-based learning assignment (as shown in **Table 3**) and asks them to judge whether the following four sets of rectangular cardboard can be assembled into a cuboid through operation, imagination and discussion in groups, and they submit a project research report.

**Table 3.** The length × width of four sets of rectangular cardboard.

| The first set | The second set | The third set | The fourth set |
|---|---|---|---|
| 6 cm × 4 cm | 6 cm × 3 cm | 3 cm × 6 cm | 3 cm × 6 cm |
| 5 cm × 4 cm | 3 cm × 5 cm | 5 cm × 3 cm | 3 cm × 5 cm |
| 3 cm × 4 cm | 3 cm × 4 cm | 3 cm × 5 cm | 5 cm × 6 cm |
| 6 cm × 4 cm | 3 cm × 6 cm | 3 cm × 6 cm | 3 cm × 6 cm |
| 5 cm × 4 cm | 3 cm × 5 cm | 3 cm × 5 cm | 3 cm × 5 cm |
| 3 cm × 4 cm | 3 cm × 4 cm | 5 cm × 3 cm | 5 cm × 6 cm |

*Note*. Adapted from "Learning Process Analysis from Learners' Perspective," by C. L. Zhang, W. Chen, and Z. Q. Zhang, 2020, Beijing Normal University Press. p.59-60. Copyright 2020 by Beijing Normal University Press. Reprinted with permission.

We can see that this homework has a certain difficulty for the students, and different from the thinking process that students conduct in the classroom, imagining the shape of 6 surfaces of cuboids by the length of the 12 edges to solve this problem; students must imagine the length, width and height of the cuboid through the shapes of 6 surfaces, which is obviously reverse thinking that is helpful to develop students' innovative qualities.

We can see that this class is guided by the driving problem, using the anchor task as a method and diagnostic assessment as teaching feedback. The three elements are interactive and integrated, providing students with the opportunity to participate in creative activities. Through this design, the teacher pays more attention to creating an open and challenging problematic environment for students, gives fuller play to students' previous cognitive experience and multiple representations, and focuses more on students' divergent thinking, critical thinking and intuitive thinking. To help students to find the truth and reach the essence and to see evidence of students' creativity in learning step by step, the teacher encourages students to develop their creativity and innovative quality in constant cognitive conflicts, interacted inspirations and collisions of thought by respecting personality and focusing on innovation.

## 5. Conclusion

In summary, innovation is one of the most important qualities in the 21st century. By reviewing the research, we conclude that among all of the qualities of innovation, creative thinking is of the greatest significance; abundant knowledge and motivation serve as powerful drivers of constant mathematical learning; self-regulation, active interaction and an open learning environment lay a solid foundation for the cultivation of creativity. Teaching that utilizes the six principles of promoting innovation tends to be beneficial to students' learning and their acquisition of creativity. Through utilization of the concept of creativity and implementation of the six principles, a large number of practices in China have shown that teaching can bring changes to students' learning styles and provide more opportunities for students to engage in creative activities. Theorists and practitioners should attach greater importance to students' creative thinking in the future by integrating ideas of creativity education in mathematical learning, the latest research findings in cognitive science, and practical strategies, which can promote the development of creativity. These processes are key to developing innovation quality.

## Funding

This work was supported by the International Joint Research Project of Faculty of Education, Beijing Normal University (ICER201904).

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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