

Student Error Analysis in Learning Algebraic Expression: A Study in Secondary School Putrajaya

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Abstract

Algebraic expressions are an essential topic in the mathematics curriculum as they are interrelated in the application of problem-solving in science and mathematics. Previous studies showed that most students have a low level of understanding of algebraic expressions. This study was conducted to identify the type of student error in learning Algebra Expression topics among Form 4 students. A total of 67 students from Secondary School Putrajaya, comprising 31 males and 36 females, were involved in the study. The instrument used in this study was the Algebraic Expression Diagnostic Test with a reliability value of $\alpha = 0.819$. This diagnostic test covers two critical concepts in algebraic expressions, namely the development and functioning of algebraic expressions. The types of errors are classified according to Newman's Error Hierarchical Model, such as the reading error, comprehension, transformation, process skills, encoding or negligence. Data were analyzed using descriptive statistics, namely mean, frequency, and percentage. Inferential statistics, namely the Pearson r correlation. The findings show that students often make the wrong kind of process skills. No students made any reading mistakes. The number of students who form the wrong kind of comprehension, transformation, encoding, and negligence is small. Student's error in the topic of algebraic expressions is due to students' weaknesses in finding the most significant common factor, cross-sectional processes, and weaknesses in mastering fractions and negative numbers. It is hoped that the information obtained in this study will help students and teachers to solve learning problems related to the topic of algebraic expressions.

Keywords

Algebraic Expressions, Error Analysis, Transformation, Process Skills, Encoding

1. Introduction

The Mathematics Curriculum in Secondary School has been formulated, refined, and rearranged under the National Education Philosophy to provide students with mathematical knowledge and skills aimed at developing systematic and competent individuals who apply mathematical knowledge effectively and are responsible for solving problems or making decisions. This motivates them to address the challenges of everyday life in keeping with the latest science and technology developments (MOE, 2013). Mathematics is an essential subject in the curriculum of our country's schools. These subjects are taught from all levels, from kindergarten to higher education. Even in colleges and universities, mathematics is still an important subject in most courses. Mastery in mathematics requires students to understand and master basic concepts of computation. Through conceptual understanding, directly train students to think constantly in finding solutions to the problems they are facing. Through mastery of mathematics, students' thinking will grow and develop. Every problem encountered will be investigated from various angles to find a solution. Therefore, a student needs to develop an understanding of learning mathematics skills and concepts to increase his/her desire and interest in learning mathematics and to improve his/her ability to solve a problem especially in the 21st-century learning practice (Saliza & Siti Mistima, 2019).

2. Background of the Study

Mathematics is a core subject at the primary or secondary level. However, student achievement and interest in Mathematics is not very encouraging (Mazlan, 2002). Students still think that Mathematics is a difficult subject (Bed Raj, 2017). Various changes in the educational world, including the teaching and learning approach in the classroom, the use of computer technology and calculators, the application of creative and critical thinking skills, and mastery learning have not been able to erase students' negative perceptions towards Mathematics. Mathematics is an abstract subject. Therefore, the construction of a mathematical concept will not be successful by memorization alone. Students often encounter such problems with low levels of ability. They think that mathematics is a difficult subject to master and boring. This negative perception causes them to lose interest in learning mathematics. As a result, mathematical achievement relatively low and not satisfied by a particular individual.

Among the factors that influence students' weaknesses in mathematics are from the knowledge base or basic concepts and skills from previous learning (Aini Haziah & Zanaton, 2018; Petterson, 1991). Students who fail to master the necessary skills and ideas at any level of learning, can influence their achievement at the next level. Hence, it is vital for a teacher to make an assessment of teaching and learning in the classroom and try to identify the difficulties and mistakes that are often experienced by students.

2.1. Problems Statements

Algebraic expressions are one of the most important topics in the school mathematics curriculum. In general, algebraic applications can be found in all fields of mathematics and science covering one-third of the secondary school mathematics curriculum (Kementerian Pendidikan Malaysia, 2003). It is considered as one of the important topics in the examination, namely the Lower Secondary Assessment (PMR) and the Malaysian Certificate of Education (MCE). Therefore, proficiency in solving algebra problems is very important to the overall achievement of students' mathematics in the national examination. Studies on the development of understanding of topics in difficult mathematics have been well documented (Warren, 2003). However, most of the studies emphasized the topic of linear equations and students' difficulty in solving linear equations. However, studies related to the topic of algebraic expressions are still lacking and need to be explored.

Algebra involves variables, while algebraic expressions contain variables, constants, and operations symbols such as add, minus, multiply and divide. Therefore, it is necessary for students to understand the concepts of the variables and the meanings of algebraic terms in order to master algebra correctly (Filloy & Rojano, 1989). Many studies have been able to identify some of the errors and misconceptions among students in understanding algebra (Ling et al., 2016). Many of the students do not understand the idea of a letter being used as a variable (Booth, 1981). They tend to interpret a letter as just a specific number, and different letters necessarily represent a varying number (Kuchemann, 1981). Terms and regulations in algebra may also be a source of confusion for students. Many of them find it difficult to follow abstract terms and to manipulate symbols and numbers at the same time. Many laws or rules in algebra seem insignificant to students and this often causes them to create their own laws (Demby, 1997).

The majority of students in secondary schools still have a low level of understanding in algebra. This condition should be taken seriously by teachers who teach mathematics. This is because mathematics teaching in secondary school is at the highest level (Aida Suraya, 1991). For example, students will study the title of Algebraic Expressions I during Form One. While in Form Two, students will learn the topic of Algebraic Expression II where in this topic, the content of the lesson will be deeper than the previous Algebraic Expressions I. Later, in Form Three, students will also learn the topic of Algebraic Expressions III. If students are not able to fully master the content of the lesson in the Algebraic Expression I topic, it will be difficult for them to master the Algebraic Expressions II and Algebraic expressions III.

In the topic of algebraic expressions, Saripah Latipah (2000) found that students did not understand the basic concepts of algebraic expressions well, which led to misconceptions in basic algebraic operations. Rosli (2000) also found that students make mistakes in certain aspects of algebraic expressions such as sim-

plifying algebraic fractures, factoring and developing two expressions. [Azrul Fahmi and Marlina \(2007\)](#) point out that algebraic expressions are one of the topics in mathematics that students often make mistakes. If students cannot master the basic concepts of mathematics in primary school, students would face problems in the study of mathematics in secondary school and subsequently at the tertiary level ([Wong, 1987](#)). Among the main factors that cause a low level of understanding of the topics in Algebra expression among students is a poor mastery of fundamental concepts and abstract algebraic expressions.

Based on the problem statement discussed, the researchers would like to conduct a study to identify and determine the extent to which the basic concepts of mathematics in the Algebraic Expression topic among Form 4 students by conducting the analysis of the types of errors made by the students in the Algebraic Expressions topic since this topic is relevant and closely related to other topics such as Functions, Expressions and Quadratic Equations and Concurrent Equations.

2.2. Research Questions

Based on the objectives of the study, this study was conducted to answer the following research questions:

- 1) What are the types of student errors in solving algebraic expressions?
- 2) What are the types of student errors in solving the problem of algebraic expressions factorization?
- 3) To what extent there is a relationship between student achievement in the topic of algebraic expression and student mathematics achievement in the Lower Secondary Assessment (PMR) examination?

2.3. Research Design

This study was a descriptive study conducted to identify the mistakes made by Form 4 students in solving problems related to algebraic topics. The sample consisted of 67 Form 4 students (science stream), different gender from Secondary School Putrajaya. The sampling method used in this study was a simple random sample method. This research instrument consists of a set of test questions that focus on the topic of Algebraic Expressions, namely the Algebraic Expression Diagnostic Test. The items contained in this instrument were modified from the research instruments of [Azrul Fahmi and Marlina \(2007\)](#). Meanwhile, scoring was based on modifications of [Charles and Lester's \(1987\)](#) analytical scoring scheme to assess respondents' answers. The instrument was divided into two sections, Part A and Part B. Part A is the demographic information of the respondents. Meanwhile, Part B is an Algebraic Expression Diagnostic Test that contains 20 subjective questions.

This diagnostic test question was given to respondents to be answered within an hour. **Table 1** is the subtopic table tested and item order in the Algebraic Expression Diagnostic.

Table 1. Algebraic expression diagnostic test subtopic.

Algebraic Expression Diagnostic Test Subtopic	Item No.	Item Order No.
Development of Algebraic Expression:		
Determining the development of the product for an expression with a term	5	1, 2, 3, 4, 5
Determining the development of the product of two expressions	5	6, 7, 8, 9, 10
Algebraic Expressions Factorisation:		
Converting the algebraic expressions that contain two terms to the product of the term with an expression	5	11, 12, 13, 14, 15
Converting the algebraic expressions that contain two terms to the product of two expressions	2	16, 17
Converting the algebraic expressions that contain three terms to the product of two expressions	3	18, 19, 20

In this study, two field experts were consulted to determine the validity of the study tool. Subsequently, a pilot study was conducted on 32 Form 4 students who had similar characteristics to the sample in the actual study. The purpose of this pilot study was to test the suitability of items used in terms of validity and reliability. The Cronbach's alpha value obtained was 0.819. According to [Majid Konting \(2000\)](#), the alpha coefficient value exceeds 0.60 indicates that the instrument has high reliability.

3. Finding and Discussion

The discussion and findings of this study will focus on aspects related to the respondents' demographics and analysis of error types in the topic of Algebraic Expressions based on Frequency of Errors. Newman's Error Hierarchy Model consists of six aspects, namely: 1) Reading, 2) Comprehension, 3) Transformation, 4) Process Skills, 5) Encoding and 6) Negligence.

3.1. Respondent Demographics

The respondent demographics consisted of 31 (46.3%) male students and 36 (53.7%) female students. In terms of Mathematics achievement at the Lower Secondary Assessment (PMR) level, 43 (64.2%) of students received grade A, 19 (28.3%) of them received grade B, and 5 (7.5%) of students received grade C as shown in [Table 2](#).

3.2. Type of Error Analysis in Algebraic Expression Topics

The types of errors were classified based on student written work analysis. The types of errors were identified based on the first breakdown point performed by students. This type of error diagnosis was based on Newman's Error Hierarchy Model. The types of errors were classified either from reading type, comprehension, transformation, process skills, encoding, and negligence. This type of error was verified through interviews conducted after the analysis of written work done.

Table 2. Respondent demographics of the study.

Statement	Frequency	Percentage (%)
Gender:		
Male	31	46.3
Female	36	53.7
Total	67	100.0
Mathematics Achievement (PMR):		
A	43	64.2
B	19	28.3
C	5	7.5
Total	67	100.0

The findings of the students' error analysis of the subject of Algebraic expressions as a whole indicate that the student's achievement of the subtopics tested was satisfactory, especially in the subtopics involved in the concept of algebraic expressions. This can be seen by the percentage of respondents who answered correctly for each subtopic tested above 70%. However, for the conceptual solution involving algebraic expressions factorization, there is one subtopic that indicates that the percentage of respondents who answer correctly is less than 70%, namely, the subtopic that converts algebraic expressions contains three terms to the product of two expressions.

Table 3 shows the percentage of students who responded correctly to the sub-topics tested based on the development of algebraic expressions and algebraic expressions factorization. Further description is based on error analysis based on development factors and algebraic expressions factorization.

3.3. Types of Errors in the Development of Algebraic Expressions

The analysis of the types of errors in the subtopics of Algebraic Expression Development is described in detail as follows:

1) Type of error in determining the expansion of the product of an expression with a term

The most common types of mistakes made in this sub-topic are the types of negligence and the error types of process skills. For the first item, which is $2(x + 5y)$, 4 (6.0%) of the students made mistakes in the process skills in developing operations involving algebraic expressions. For example, they solve $2(x + 5y)$ as $2x + 5xy$ and $2x + 5y$. There was also one (1.5%) student who made a mistake by giving the final answer as $2x + 10xy$. The student can answer correctly when asked to answer for the second time.

For the second item, $m(m + 8)$, the most common type of error is negligence. There were 19 (29.4%) students who made the mistake of negligence. Of these, 13 (19.4%) of the students answered $m^2 + 4$. They got the right answer when asked for a second time. Other errors made for this item were that 9 (13.4%) of

Table 3. Percentage of correct answers according to Sub-Topic.

Algebraic Expression Diagnostic Test Subtopic	Percentage of Correct Answers
Development of Algebraic Expression:	
Determining the development of the product for an expression with a term	86.9%
Determining the development of the product of two expressions	76.1%
Algebraic Expressions Factorisation:	
Converting algebraic expressions that contain two terms to a product of one pronunciation with one expression	74.6%
Converting algebraic expressions that contain two terms to the product of two expressions	72.4%
Converting algebraic expressions that contain three terms to the product of two expressions	66.2%

the students made mistakes in the process skills. Students know how to use the correct operations, but fail to develop algebraic fragments and mostly give the final answer $m^2 + 4$. Also, 2 (3.0%) students made the wrong type of encoding and transformation for the item.

The most common type of error for the next item, $uv(v + w)$, is also the negligence type. A total of 4 (6.0%) students answered $uv^2 + uw$. Students were careless when multiplying the uv pronunciation with the second pronunciation in the expression $v + w$. When asked for the second time, they can respond to the item correctly. In addition, one (1.5%) of the students made the wrong kind of process skills and gave the final answer as $u^2v + uw$. Students know how to use proper operations and methods, but fail to do the calculations correctly.

The most common mistake for items $-3x(2y - z)$ is the type of process skills. There were 2 (3.0%) of the students made this type of error. The most common error for items $-3x(2y - z)$ is the type of process skills. 2 (3.0%) of the students made this type of error. This error occurs when the student fails to handle the negative sign when multiplying algebraic expressions and giving the final answer as $-6xy - 3xz$. There was one (1.5%) student who made the mistake of saying algebra and giving the wrong answer $-6y + 3xz$.

For the last item in this subtopic, $-r(2q + r)$, the error made by the student is the type of process skill. 2 (3.0%) of the students made this type of error due to failure to handle algebraic pronunciation with negative sign and error while calculating. The wrong final answer is given $-2rq + r^2$. An analysis of the types of errors for each item in this subtopic is shown in **Table 4** as follows.

Overall, the most common types of errors made in this section are the types of negligence and process skills. Students were careless in developing algebraic expressions, especially those involving fractions. For errors in process type skills, most students know how to perform operations and can use correct operations and methods, but fail to perform proper calculations. This type of negligence and process skills process is most common when involving algebraic fractures.

Table 4. The type of error in determining the expansion of a product from an expression.

Item	Frequency of Errors According to the				Total**
	Transformation	Process Skills	Encoding	Negligence	
1		4		1	5
2	1	9	1	19	30
3		1		4	5
4		2		1	3
5		2			2
Total**	1	18	1	25	45

Note: (**) The sum indicates the number of times an error was detected by item (row) and by error (column).

However, there are no reading and comprehension errors made by students in this subtopic.

2) Type of error in determining the development of the product of two expressions

For the subtopic of determining the development of the product of two expressions, the most common type of student error is the encoding and process skills. The solution to the first item of this section, $(x + 2)(x + 2)$ indicates that 3 (4.5%) of the students made some encoding error. Students know how to use the proper operations and methods and can perform the right calculations, but failed to identify and write the final answer correctly. So, the final answer was only up to $x^2 + 2x + 2x + 4$. There were 2 (3.0%) of the students who made the wrong type of process skills due to incorrect calculation work and gave wrong final answers such as $x^2 + 2x$ and $x + 4x + 4$.

Process skills are the most common type of error for the next item, namely $(x - 2)(x - 3)$. A total of 7 (10.4%) students made an error in the process skills for failing to make the correct calculations for similar pronunciations and involving negative marks. For example, the most common type of error in process processing was $(x - 2)(x - 3) = x^2 - x + 2x + 6$. The student also did the type of error in coding for this item. 2 (3.0%) of the students who made this type of error gave $x^2 - 3x - 2x + 6$ answers and failed to write the final answer correctly.

The same type of encoding error was repeated by the same student for the item $(2x + 7)(2x - 7)$. A total of 8 (11.9%) students made an error in the coding and failed to identify and write the final correct answer. Most of the final answers given were $4x^2 - 14x + 14x - 49$. They did not continue the settlement process after the development. This type of error in process skills was also a major issue when students failed to develop items that involve negative markings and errors in computation. A total of 9 (13.4%) students made this mistake. Among the wrong answers given were $4x^2 + 49$ and $4x - 14$.

The same type of encoding error was repeated by the same student for the next item $(a + b)(a + b)$. A total of 13 (19.4%) students made this type of error and all of them gave the final answer as $a^2 + ab + ab + b^2$. There were also 11

(16.4%) students who made a mistake in the process skills because they could not perform the correct operation when adding two identical algebraic expressions, namely ab . Examples of final answers given were $a^2 + a^2b^2 + b^2$, $a^2 + ab^2 + b^2$ and $a^2 + 2a + 2b + b^2$. In addition, 5 (7.5%) of the students made a mistake by solving $(a + b)(a + b) = a^2 + ab + b^2$ for being careless when adding two similar ab terms. However, they were able to respond correctly after being asked to try out for the second time. Only one (1.5%) student made the type of transformation error.

For the last item in this subtopic, which is $(2p - q)^2$, the most common error is the type of process skills. A total of 12 (17.9%) students made this type of error for failing to develop the expression properly and gave the final answer like $4p^2 - q^2$. In addition to the process type error, there were 3 (4.5%) students who made the error due to negligence, 3 (4.5%) students made the error type of encoding, and only one (1.5%) student made the error type of transformation. Analysis of the types of errors in determining the development of the product of two expressions is detailed in **Table 5**.

On the whole, the most common error made by the students in subtopics of determining the development of the product of two expressions is the type of process skills. Errors of the process skills are often encountered, especially for items with negative terms and expressions that do not involve any figures or numbers such as expressions $(a + b)(a + b)$. Another common type of error is encoding. There are no reading and comprehension errors made by students in this subtopic.

3.4. Types of Errors in Algebraic Expressions Factorization

The types of errors analysis in the subtopics related to Algebraic Expressions Factorisation are described in detail as follows:

1) Type of error in converting algebraic expressions that contain two terms to the product of one term with one expression

One type of error that students often make in converting algebraic expressions that contain two terms to the product of one term with one expression is the

Table 5. Type of error in determining the development of the product of two expressions.

Frequency of Errors Based on Newman's Error Hierarchy Model					
Item	Transformation	Process Skills	Encoding	Negligence	Total**
6		3	2		5
7		7	2		9
8		9	8		17
9	1	11	13	5	30
10	1	12	3	3	19
Total**	2	42	28	8	80

Note: (**) The sum indicates the number of times an error was detected by item (row) and by error (column).

process skills. The findings show that for $8pq - 12q$ items, 6 (9.0%) of the students failed to get the correct answer due to the error type of process skills. Students know how to use the correct operations and methods for factoring in these expressions but failed to compute the calculations properly. Examples of solutions shown were $2q(4p - 6)$ and $2(4pq - 6q)$. There were also 2 (3.0%) students who made an error due to negligence by giving answers such as $4q(2 - 3)$. This error may have been caused by the student's inability to answer the question. Only one (1.5%) of the students made the error type of transformation.

For the next item, factoring in the expression $3mn^2 + 21m$, the most common error made is the process skills. However, the number of students who make this type of error is small, only 3 (4.5%). Errors were also caused by the failure to find common factors for both expressions. Some of the wrong answers given were $3mn(n + 7)$, $3m(2mn + 7)$ and $7mn(n + 3)$. In addition, only one (1.5%) student made an error due to negligence and one (1.5%) student made the error type of transformation. An example of wrong answers given for negligence was $3m(n + 7)$.

The type of process skills error increases when it comes to expression, where one of the terms does not have any coefficients such as the $3kp - k^2p$ item. A total of 18 (26.9%) students made this type of error. Of these, 9 (13.4%) of them gave answers $k(3p - kp)$ and the rest gave answers such as $3kp(1 - k)$, $3k(p - kp)$, $3p(k - k^2)$ and others. There were 2 (3.0%) of the students who made this type of transformation error incorrectly because they failed to describe the question into a form that allowed them to use the appropriate operation for the algebraic expressions factorization given.

The most common type of error in the next item, $21ab^2c + 14bcd$ is also the process skills. A total of 16 (23.9%) of the students made the error type of process skills for this item. This is also due to the students' weakness in finding common factors for both terms in a given expression, and this further increases their difficulty in finding solutions when involving more variables in a term. In addition, 5 (7.5%) of the students made the error type of negligence by providing solutions such as $7bc(3a + 2d)$ and $7b(3ab + 2d)$. They can give the right answer when they try again. Another type of error was the transformation, where 3 (4.5%) of the students made this error. Students understand the requirements of the question, but cannot find and formulate a method for the algebraic expressions factorization.

Students also make the same type of error on items $48m^2n + 12mn^2$, which is process skills. A total of 21 (31.3%) students made this type of error. Most of them give answers such as $4mn(12m + 3n)$, $6mn(8m + 2n)$ and $12(4m^2n + mn^2)$. This error also occurs due to students' inability to find the largest common factor for each term in a given expression. On the other hand, the type of errors made was negligence and transformation. Among the answers given for negligence was $12mn(4 + n)$. Students will be able to provide the correct answer when asked to try again. Analysis of the type of errors for this subtopic is summarized in **Table 6** as follows:

Table 6. A type of error in converting algebraic expressions that contain two terms to a product of one term with one expression.

Frequency of Errors Based on Newman's Error Hierarchy Model					
Item	Transformation	Process Skills	Encoding	Negligence	Total**
11	1	6		2	9
12	1	3		1	5
13	2	18			20
14	3	16		5	24
15	2	21		4	27
Total**	9	64		25	85

Note: (**) The sum indicates the number of times an error was detected by item (row) and by error (column).

The findings show that the type of errors in process skills is often made when involving expressions or terms that have a variable with the highest power square. Hal This causes students to become confused in the process of finding common factors that may cause them to make mistakes inadvertently. However, there was no error in the reading, comprehension, and encoding of the students for this subtopic.

2) Type of error in converting algebraic expressions that contain two terms to the product of two expressions

Items in the subtopic: converting algebraic expressions that contain two terms to the product of two expressions: item 16 and item 17. Items in this subtopic require students to use their identities in algebraic expressions, namely $a^2 - b^2 = (a + b)(a - b)$. The most common type of errors is comprehension and process skills.

For item 16, namely $x^2 - 64$, there were 3 (4.5%) students who made the type of error in process skills. Most of them may not even know the identity of the algebraic expression. So, students do not know how to perform operations and only make random guesses. Examples of random response answers given were $(x + 16)(x - 16)$. In addition, 2 (3.0%) of the students made an error in comprehension and 2 (3.0%) of the students made an error in transformation. The type of error in comprehension occurs because students do not understand the question and they failed to understand the term "the product of two expressions". So they don't know how to implement a solution and just make a random guess. The type of error in transformation occurs because students use algebraic expressions r incorrectly, by solving $x^2 - 64 = (x - 8)^2$. Only one (1.5%) student made an error in encoding. For example, students do not proceed with the solution after obtaining $x^2 - 64 = x^2 - 82$.

The students also made the same type of error for the next item, $4y^2 - 36$, namely process skills. A total of 22 (32.8%) of the students made this type of error also because they did not know how to use the algebraic expressions and gave incorrect answers such as $(2y^2 + 6)(2y^2 - 6)$. There were also 7 (10.4%) stu-

dents who made an error in comprehension because they failed to understand the term “the product of two expressions”. Analysis of the type of errors for this subtopic is summarized in **Table 7** as follows:

On the whole, the most common type of error in converting algebraic expressions that contain two terms to the product of two expressions is process skills. This error may occur because students were unclear about the use of identities in algebraic expressions. Another type of error made was comprehension, transformation, and encoding.

3) Type of error in converting algebraic expressions that contain three terms to the product of two expressions

The type of errors made by students in this section consists of process skills and comprehension. Process skills were the type of error made by 13 (19.4%) students for item $x^2 - 7x + 12$ and 12 (17.9%) students for item $x^2 + 2x - 15$. Most students made an error in finding the right pair of factors and wrongly placing positive and negative marks on the selected factor pairs such as $x^2 - 7x + 12 = (x + 3)(x + 4)$ and $x^2 + 2x - 15 = (x - 5)(x + 3)$. For item $3x^2 + 9x + 6$, 40 (59.7%) of the students also made the same type of process skills due to the same factors.

For all three items, only one (1.5%) student made the same type of comprehension error, and the other student made the same error. Students gave random responses that do not mean any solution to the three items. Examples of wrong answers shown were factorisation $x^2 - 7x + 12$ as $x(-7x + 12)$, $x^2 + 2x - 15$ as $x(2x - 15)$ and $3x^2 + 9x + 6$ as $3x^2(9x + 6)$. Analysis of the type of errors for this subtopic is summarized in **Table 8** as follows:

Overall, the most significant type of error in this subtopic is process skills. This type of error occurs because the student fails to perform the cross-sectional process to find the appropriate factor pairs for the given expression. In addition, some students made a comprehension error. However, no student has made an error in reading, transformation, encoding and negligence in this subtopic.

A summary of the types of errors for the entire topic is given in **Table 9**.

The Relationship Between Achievements In The Topics of Algebraic Expressions With Lower Secondary Assessment (PMR) Achievements.

Table 7. A type of error in converting algebraic expressions that contain two terms to the product of two expressions.

Frequency of Errors Based on Newman’s Error Hierarchy Model					
Item	Comprehension	Transformation	Process Skills	Encoding	Total**
16	2	2	3	1	8
17	7		22		29
Total**	9	2	25	1	37

Note: (**) The sum indicates the number of times an error was detected by item (row) and by error (column).

Table 8. Type of error in converting algebraic expressions that contain three terms to the product of two expressions.

Frequency of Errors Based on Newman's Error Hierarchy Model					
Item	Comprehension	Process Skills	Encoding	Negligence	Total**
18	1	13			14
19	1	12			13
20	1	40			41
Total**	3	65			68

Note: (**) The sum indicates the number of times an error was detected by item (row) and by error (column).

Table 9. The whole type of errors in the topic of algebraic expressions.

Item	Reading	Comprehension	Trans-formation	Process Skills	Encoding	Negligence	Total*
1				4		1	5
2			1	9	1	19	30
3				1		4	5
4				2		1	3
5				2			2
6				3	2		5
7				7	2		9
8				9	8		17
9			1	11	13	5	30
10			1	12	3	3	19
11			1	6		2	9
12			1	3		1	5
13			2	18			20
14			3	16		5	24
15			2	21		4	27
16		2	2	3	1		8
17		7		22			29
18		1		13			14
19		1		12			13
20		1		40			41
Total*		12	14	214	30	45	315

The relationship between respondents' achievement in algebraic expressions and the mathematical achievement of respondents in the Lower Secondary Assessment (PMR) examination was determined by Pearson correlation analysis, $r = 0.618$. Thus, respondents' achievement in the topic of algebraic expression has a strong correlation with the mathematics achievement of the respondents in the

PMR examination.

4. Conclusion

The findings showed that the most common type of error made by students is process skills. Some students made other types of errors such as comprehension, transformation, encoding, and negligence. However, the number of students who made this type of error is small. There are no errors in reading shown by the students on this topic. The most significant errors made by the students are algebraic expressions factorization, especially for an expression with three terms. In addition, the most significant errors can also be seen when it comes to items that contain algebraic fractions. The findings also showed that when students fail to master a certain level of learning in algebraic expressions, students will have difficulty mastering the next level of learning or skills in the topic. The quality of education that teachers provide to students is dependent upon what teachers do in their classrooms (Zakaria & Iksan, 2007). Execution of duties in cooperative learning can develop self-confidence in pupils. A study by Zakaria, Chin, & Daud (2010) found that cooperative learning improves students' achievement in mathematics. Further, cooperative learning is an effective approach that mathematics teachers need to incorporate into their teaching. Lately, one of the initiatives proposed by the government is the Massive Online Open Course (MOOC) is web-based learning that can be accessed anywhere and anytime. Integrating the technology into the learning process can help improve understanding of the subject matter such as mathematics (Abdul Wahab et al., 2018; Nordin et al., 2016).

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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