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Continuous Contact Model of Metamorphic Groove Pin Pair with Clearance

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Abstract

Metamorphic mechanism has the advantages of variable topology and variable degrees of freedom, which can realize the requirements of multi-conditions and multi-tasks, and has a good application prospect. The configuration transformation is prominent feature of the metamorphic mechanism. The number of constraints or properties of the kinematic pairs provided by the metamorphic kinematic pairs will change under certain conditions, its dynamic performance is much more complex than that of traditional kinematic pairs with immutable constraints. However, the clearance model about traditional kinematic pairs with immutable constraints established by long-term research is difficult to be directly applied to the metachromatic kinematic pairs. Referring to the experience of the traditional kinematic pairs with immutable constraints, the continuous contact model of Metamorphic Groove pin pair with clearance is established. According to the traditional continuous contact model of the kinematic pairs with immutable constraints, the forces between the elements of kinematic pair of the mechanism with clearance and the ideal mechanism without clearance are regarded as the same, and the inertia force and inertia moment of the components are also calculated according to the acceleration of the ideal mechanism. The clearance is regarded as a massless virtual bar with length r. For the rotating pair part, the massless virtual bar length r is the difference between the radius of the shaft and the hole, and for the sliding pair part, the massless virtual bar length r is half of the difference between the height of the slider and the guide groove. According to the new mechanism without gap after adding the imaginary bar, kinetic energy and potential energy of the system are calculated for the two configurations of mechanism with metamorphic Groove pin pair with clearance. The kinetic energy and potential energy of the system are calculated according to the new mechanism without clearance after adding the massless virtual bar. The kinetic energy, potential energy and generalized force are substituted into the Lagrangian equation to obtain the motion equation of the metamorphic mechanism, which lays the foundation for the dynamic performance study of the mechanism with metamorphic groove pin pair with clearance.

Keywords

Metamorphic Mechanism, Metamorphic Pair with Clearance, Continuous Contact Model, Metamorphic Groove Pin Pair

1. Introduction

A kinematic pair represents a movable connection between components, and naturally, there exists clearance between the elements. This clearance not only causes the actual motion of the mechanism to deviate from the intended motion, but may also lead to impacts between kinematic pair elements, generate vibrations and noise, and increase the dynamic stress of the mechanism. Therefore, mechanism researchers have conducted comprehensive studies on kinematic pair clearance for a long time. In terms of research on the dynamic performance of kinematic pair clearance, the most representative works include the three-state model, the two-state model, and the continuous contact model. Among them, although the continuous contact model cannot study the collision and impact performance between kinematic pair elements like the three-state and two-state models, it can still reflect the influence of kinematic pair clearance on the dynamic behavior of the mechanism. Corresponding dynamic curves undergo abrupt changes, indicating mutations in the contact state of the kinematic pair elements, transitioning from contact on one side to the other. Due to its convenience in applying to the dynamic performance analysis of mechanisms, it is favored by mechanism researchers and engineers and has been widely adopted. The continuous contact model [1]-[3] assumes that the elements of the kinematic pair are always in continuous contact, neglecting the separated state. In modeling, the clearance is treated as a rigid bar (clearance bar) without mass and elasticity while the elastic deformation and friction of kinematic pair elements are not considered. The original mechanism with clearance kinematic pairs is transformed into a multi-bar, multi-degree-of-freedom mechanism without clearance, and then Lagrange's equations are used to derive the motion equations of the mechanism. Not considering the stiffness, damping, friction of kinematic pair elements, it cannot reflect the impact situation between them, but it is very convenient to analyze the state of the mechanism. Feng Zhiyou [4] applied this model to the dynamic analysis of elastic mechanisms with clearance, incorporating the kinematic pair clearance as a massless rigid bar element into the finite element model of KED. Furuhashi [5], Erkaya [6] [7] have conducted numerous studies on the massless bar method like using neural networks to optimize the rotation angle of mechanisms. Yang Lixin [8] make a combination of a massless bar and a massless slider based on the massless bar of the continuous contact model, which is closer to actual conditions. Ganwei Cai [9] proposed a finite element unit with clearance kinematic pairs, which take the center of the pin and the center of the hole as unit nodes. The stiffness matrix of the unit reflects the contact stiffness of kinematic pair elements, and the mass matrix of the unit reflects the mass of the pin and hub. This finite element unit facilitates the dynamic modeling of elastic linkage mechanisms with clearance kinematic pairs. Experiments show that the calculated results of vibration response are close to the experimental results when the clearance of the kinematic pair is small. Zou Huijun [10] conducted in-depth research and achieved widely influential results on how clearance in various cams, such as spring-locked and geometrically locked types, affects the dynamic performance of mechanisms. The numerous achievements made by mechanism researchers in the dynamics of mechanisms with traditional fixed-constraint kinematic pair clearance have provided valuable references for the study of the dynamics of mechanisms with metamorphic kinematic pairs with clearance over the decades.

With the advantages of variable topological structure and variable degrees of freedom, the metamorphic mechanism can meet the requirements of multiple operating conditions and tasks, which holds promising application prospects in various fields, including aerospace, robotics, medical equipment, manufacturing systems, and special loaders for unique operating conditions [11]-[14]. Extensive research conducted by scholars at domestic and international has primarily focused on metamorphic methods, configuration description, structural analysis, structural synthesis, and kinematic analysis of metamorphic mechanisms. However, research on the dynamics of metamorphic mechanisms is relatively scarce. The metamorphic kinematic pairs of a metamorphic mechanism provide that the constraint number or the properties of the kinematic pairs will change under certain conditions, and its dynamic performance is much more complicated than that of ordinary kinematic pairs with fixed constraints, and its influence on the mechanism is much greater. The existing theories and models established through long-term and extensive research for clearance states in kinematic pairs with fixed constraints cannot be directly applied to metamorphic kinematic pairs. Therefore, there is an urgent need to establish a clearance model theory of metamorphic kinematic pairs, both for theoretical research and engineering applications of metamorphic mechanisms.

Taking the groove-pin metamorphic kinematic pair [15] as the research object and using the continuous contact model of clearance in ordinary kinematic pairs with fixed constraints for reference, the clearance of the metamorphic kinematic pair is regarded as a clearance bar without mass and elasticity. The metamorphic mechanism is then transformed into a multi-degree-of-freedom mechanism without clearance. The motion equations of the multi-degree-of-freedom mechanism of each configuration of the metamorphic mechanism are established by using Lagrange equations respectively, and the motion equations of the whole configuration of the metamorphic mechanism are composed.

2. The Groove Pin Pair with Clearance and Its Metamorphic Mechanism

Figure 1 illustrates a schematic diagram of the metamorphic mechanism of the groove pin pair with clearance [15], while **Figure 2** presents a photograph of the dynamic experimental rig for the metamorphic groove pin pair mechanism developed by the authors of this paper. The working principle of this metamorphic mechanism is as follows: components 1 and 2 are connected by the metamorphic groove pin pair, component 1 is connected to the frame by a revolute pair, components 2 and 3 are connected by another revolute pair, while component 3 is connected to the frame by a prismatic pair, which is constrained by an auxiliary spring force. The metamorphic groove pin pair represents a variable-constraint pair. When the cylindrical pin is positioned at either end A or B of the groove, it functions equivalently to a revolute pair, which can provide two constraints. When the cylindrical pin is located at any position between the two ends of the groove, it acts as a line-contact prismatic pair, which just offers one constraint. This metamorphic mechanism possesses two configurations. In Configuration 1, as the cylindrical pin slides between ends A and B of the groove, slider 3 remains stationary under the constraint of the spring force, while components 1 and 2

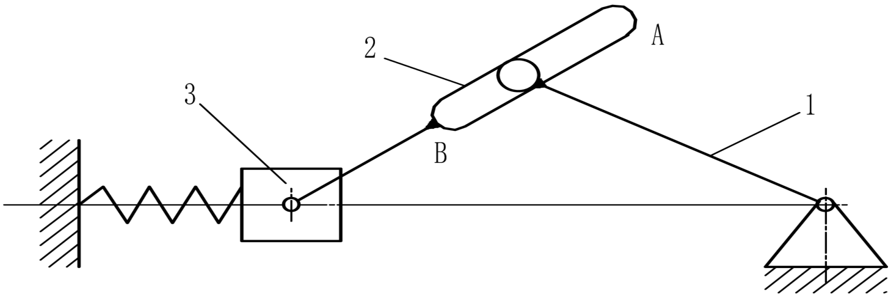

Figure 1. Schematic diagram of a metamorphic mechanism composed of a groove pin metamorphic pair and a prismatic pair with additional spring force constraint.

Figure 2. Experimental platform for dynamic performance of metamorphic mechanism for the groove pin pair with clearance.

rotate around a fixed point. When the cylindrical pin reaches either end A or B of the groove, it ceases sliding relative to the groove and instead rotates at end A or B, driving component 2 to perform planar motion. Under the force exerted by component 2, component 3 overcomes the spring force constraint and moves. The magnitude of the elastic force is set according to the force gradient requirements of the metamorphic mechanism [16] [17], ensuring that component 3 (the slider) remains stationary under the spring force to ensure when the cylindrical pin on component 1 slides within the groove of component 2, component 3 is able to move by overcoming the spring force when the cylindrical pin on component 1 reaches the end of the groove on component 2 and cease sliding, transitioning to rotation instead. When the pin once again moves within the groove, component 3 returns to a stationary state under the action of the spring force.

The motion cycle of the metamorphic mechanism featuring a groove-pin pair with clearance mentioned in this study, can be divided into two configurations and four stages. As depicted in **Figure 3**, the driving lever 1 rotates counterclockwise. The transition from θ_{14} to θ_{11} marks the first stage, where the cylindrical pin on lever 1 drives lever 2 to perform planar motion. Slider 3 moves left and right under the combined action of the X-direction component force from lever 2 and the spring constraint force. The transition from θ_{11} to θ_{12} constitutes the second stage, during which the cylindrical pin on lever 1 slides within the groove of lever 2, while both levers rotate around fixed points. Slider 3 remains stationary under the spring constraint force. The interval from θ_{12} to θ_{13} is the third stage, where the cylindrical pin on lever 1 once again promotes lever 2 into planar motion. Slider 3 moves left and right under the influence of the X-direction component force and the spring constraint force from lever 2 on slider 3. The period from θ_{13} to θ_{14} represents the fourth stage, during which the cylindrical pin on lever 1 slides within the groove of lever 2, and both levers rotate around fixed points, with slider 3 remaining stationary under the spring constraint force. The first and third stages constitute configuration one, while the second and fourth stages make up

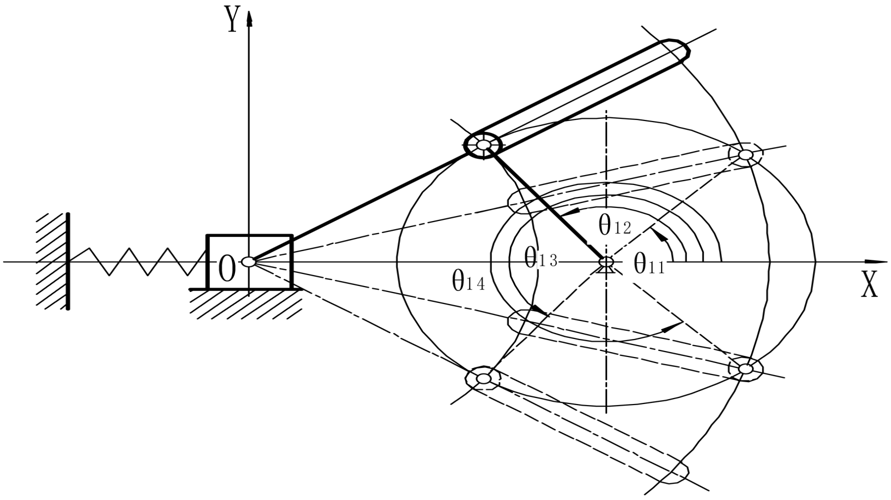

Figure 3. Working cycle of the metamorphic mechanism of the groove pin kinematic pair.

configuration two. In this study, only the clearance in the groove pin pair is considered, while the other fixed constraint kinematic pairs assumed to be ideal with no clearance. Additionally, it is assumed that the clearance remains constant at r both when the cylindrical pin slides within the groove and when it rotates at the ends of the groove.

3. Configuration 1 for the Continuous Contact Model of the Metamorphic Groove Pin Pair with Clearance

Focusing on the first phase where the driving link 1 moves from θ_{14} to θ_{11}, this study investigates the continuous contact model for Configuration 1, noting that the third phase from θ_{12} to θ_{13} follows a similar pattern. **Figure 4** illustrates the new gap-free mechanism after adding a hypothetical link. It is assumed that the forces acting between the kinematic pair elements of the mechanism with clearance are identical to those in an ideal, gap-free mechanism with reference to the traditional continuous contact model for kinematic pairs with fixed constraint clearances. Furthermore, the inertial forces and moments of inertia of the components are calculated based on the accelerations of the ideal mechanism. The clearance is regarded as a massless hypothetical link with a length of r, for the revolute pair in **Figure 4**, corresponds to the difference between the radii of the shaft and the hole. By incorporating this massless hypothetical link, the metamorphic mechanism gains an additional degree of freedom, becoming a two-degree-freedom mechanism. The angle α between the hypothetical link and the horizontal axis is considered as a generalized coordinate, while the angle θ_1 between link 1 and the horizontal axis serves as the other. The kinetic and potential energies of the system are computed by using the gap-free new mechanism with the hypothetical link as depicted in **Figure 4**. By substituting the kinetic energy, potential energy, and generalized forces into the Lagrange equations, the equations of motion for the metamorphic kinematic pair with clearance can be derived. Employing numerical methods such as the Runge-Kutta method to solve these equations, it can produce various quantities, such as α, θ, X_M, $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{X}_M$, $\ddot{\theta}_1$, $\ddot{\theta}_2$, $\ddot{X}_M$ and so on.

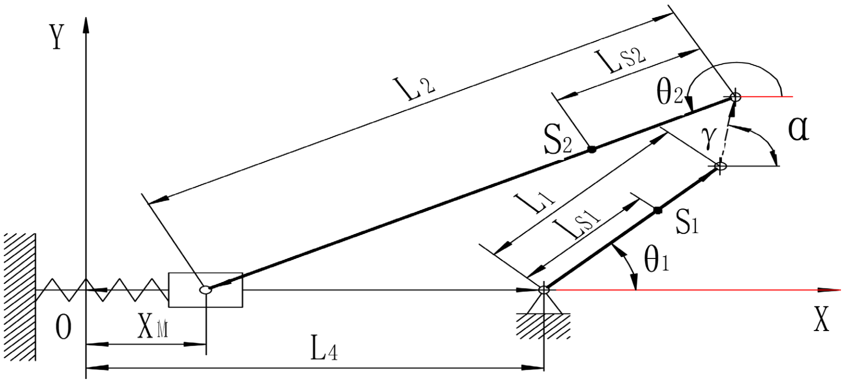

Figure 4. Gap-free new mechanism design: optimizing configuration 1 through the addition of a hypothetical link.

3.1. Positional Relations of Configuration 1

Consider each component and the gap "*r*" as vectors, the directions of which are as illustrated in **Figure 4**. Both θ_i and α are measured counterclockwise from the positive direction of the X-axis, and the corresponding vectors are depicted in **Figure 4**. Among them, the directional angles for the slider M and the fourth link (the frame) are noted as follows: $\theta_x = \pi$, $\theta_4 = 0$.

According to the kinematic analysis method of linkage mechanisms, the five vectors in **Figure 4** can be considered as a closed loop, leading to the vector equation as follows:

$$\boldsymbol{l}_1 + \boldsymbol{r} + \boldsymbol{l}_2 + \boldsymbol{x}_M + \boldsymbol{l}_4 = \mathbf{0} \tag{1}$$

As $\theta_x = -\pi$, $\theta_4 = 0$, projected in the X and Y directions

$$l_1 \cos\theta_1 + r\cos\alpha + l_2\cos\theta_2 - x_M + l_4 = 0 \tag{2}$$

$$l_1 \sin\theta_1 + r\sin\alpha + l_2\sin\theta_2 = 0 \tag{3}$$

It is obtained from Equation (2):

$$\cos\theta_2 = \frac{1}{l_2}\left(x_M - l_4 - l_1\cos\theta_1 - r\cos\alpha\right) \tag{4}$$

It is obtained from Equation (3):

$$\sin\theta_2 = -\frac{1}{l_2}\left(l_1\sin\theta_1 + r\sin\alpha\right) \tag{5}$$

From **Figure 4**, we can derive the geometric relationship as follows:

$$x_M = l_4 + l_1\cos\theta_1 + r\cos\alpha - \sqrt{l_2^2 - \left(l_1\sin\theta_1 + r\sin\alpha\right)^2} \tag{6}$$

Substituting Equation (6) into Equation (4), the following equations can be derived:

$$\cos\theta_2 = -\frac{1}{l_2}\sqrt{l_2^2 - \left(l_1\sin\theta_1 + r\sin\alpha\right)^2} \tag{7}$$

3.2. The Position, Speed and Acceleration of the Centroid of Each Component

$$\begin{aligned} x_{s1} &= l_{s1}\cos\theta_1 + l_4 \\ y_{s1} &= l_{s1}\sin\theta_1 \end{aligned} \tag{8}$$

$$\begin{aligned} x_{s2} &= l_1\cos\theta_1 + r\cos\alpha + l_{s2}\cos\theta_2 + l_4 \\ y_{s2} &= l_1\sin\theta_1 + r\sin\alpha + l_{s2}\sin\theta_2 \end{aligned} \tag{9}$$

Angular velocity and linear velocity of the centroid.

The following equations can be derived from Equation (3) to get the partial derivative of θ_1:

$$l_1\cos\theta_1 + l_2\cos\theta_2\frac{\partial\theta_2}{\partial\theta_1} = 0 \tag{10}$$

$$\frac{\partial\theta_2}{\partial\theta_1} = -\frac{l_1}{l_2}\frac{\cos\theta_1}{\cos\theta_2} \tag{11}$$

The following equations can be derived from Equation (3) to get the partial derivative of α:

$$r\cos\alpha + l_2\cos\theta_2\frac{\partial\theta_2}{\partial\alpha} = 0 \tag{12}$$

$$\frac{\partial\theta_2}{\partial\alpha} = -\frac{r}{l_2}\frac{\cos\alpha}{\cos\theta_2} \tag{13}$$

The following equations can be derived from Equation (2) to get the partial derivative of θ_1:

$$-l_1\sin\theta_1 - l_2\sin\theta_2\frac{\partial\theta_2}{\partial\theta_1} - \frac{\partial x_M}{\partial\theta_1} = 0 \tag{14}$$

Substituting Equation (11) into Equation (14), the following equations can be derived:

$$\frac{\partial x_M}{\partial\theta_1} = \frac{l_1\sin\theta_2\cos\theta_1}{\cos\theta_2} - l_1\sin\theta_1 \tag{15}$$

The following equations can be derived from Equation (2) to get the partial derivative of α:

$$-\mathrm{r}\sin\alpha - l_2\sin\theta_2\frac{\partial\theta_2}{\partial\alpha} - \frac{\partial x_M}{\partial\alpha} = 0 \tag{16}$$

Substituting Equation (13) into Equation (16), the following equations can be derived:

$$\frac{\partial x_M}{\partial\alpha} = \frac{r\sin\theta_2\cos\alpha}{\cos\theta_2} - r\sin\alpha \tag{17}$$

The following equations can be derived from Equation (11) into Equation (13):

$$\dot{\theta}_2 = \frac{\mathrm{d}\theta_2}{\mathrm{d}t} = \frac{\partial\theta_2}{\partial\theta_1}\dot{\theta}_1 + \frac{\partial\theta_2}{\partial\alpha}\dot{\alpha} = -\frac{l_1}{l_2}\frac{\cos\theta_1}{\cos\theta_2}\dot{\theta}_1 - \frac{r}{l_2}\frac{\cos\alpha}{\cos\theta_2}\dot{\alpha} \tag{18}$$

The following equations can be derived from Equation (15) into Equation (17):

$$\begin{aligned}\dot{x}_M &= \frac{\mathrm{d}x_M}{\mathrm{d}t} = \frac{\partial x_M}{\partial\theta_1}\dot{\theta}_1 + \frac{\partial x_M}{\partial\alpha}\dot{\alpha}\\ &= \left(\frac{l_1\sin\theta_2\cos\theta_1}{\cos\theta_2} - l_1\sin\theta_1\right)\dot{\theta}_1 + \left(\frac{r\sin\theta_2\cos\alpha}{\cos\theta_2} - r\sin\alpha\right)\dot{\alpha}\end{aligned} \tag{19}$$

In Equation (18) and Equation (19), $\sin\theta_2$ and $\cos\theta_2$ are derived from Equation (5) and Equation (7). The following equations can be derived from Equation (8):

$$\frac{\partial x_{s1}}{\partial\theta_1} = -l_{s1}\sin\theta_1,\quad \frac{\partial x_{s1}}{\partial\alpha} = 0$$

$$\frac{\partial y_{s1}}{\partial\theta_1} = l_{s1}\cos\theta_1,\quad \frac{\partial y_{s1}}{\partial\alpha} = 0$$

$$\dot{x}_{s1} = \frac{\mathrm{d}x_{s1}}{\mathrm{d}t} = \frac{\partial x_{s1}}{\partial\theta_1}\dot{\theta}_1 + \frac{\partial x_{s1}}{\partial\alpha}\dot{\alpha} = -l_{s1}\sin\theta_1\dot{\theta}_1$$

$$\dot{y}_{s1}=\frac{\mathrm{d}y_{s1}}{\mathrm{d}t}=\frac{\partial y_{s1}}{\partial\theta_1}\dot{\theta}_1+\frac{\partial y_{s1}}{\partial\alpha}\dot{\alpha}=l_{s1}\cos\theta_1\,\dot{\theta}_1 \tag{20}$$

The following equations can be derived from Equation (9):

$$\begin{aligned}
&\frac{\partial x_{s2}}{\partial\theta_1}=-l_1\sin\theta_1-l_{s2}\sin\theta_2\frac{\partial\theta_2}{\partial\theta_1},\quad \frac{\partial y_{s2}}{\partial\theta_1}=l_1\cos\theta_1+l_{s2}\cos\theta_2\frac{\partial\theta_2}{\partial\theta_1}\\
&\frac{\partial x_{s2}}{\partial\alpha}=-r\sin\alpha-l_{s2}\sin\theta_2\frac{\partial\theta_2}{\partial\alpha},\quad \frac{\partial y_{s2}}{\partial\alpha}=r\cos\alpha+l_{s2}\cos\theta_2\frac{\partial\theta_2}{\partial\alpha}\\
&\dot{x}_{s2}=\frac{\mathrm{d}x_{s2}}{\mathrm{d}t}=\frac{\partial x_{s2}}{\partial\theta_1}\dot{\theta}_1+\frac{\partial x_{s2}}{\partial\alpha}\dot{\alpha}\\
&\quad=-l_1\sin\theta_1\,\dot{\theta}_1-l_{s2}\sin\theta_2\frac{\partial\theta_2}{\partial\theta_1}\dot{\theta}_1-r\sin\alpha\,\dot{\alpha}-l_{s2}\cos\theta_2\frac{\partial\theta_2}{\partial\alpha}\dot{\alpha}\\
&\dot{y}_{s2}=\frac{\mathrm{d}y_{s2}}{\mathrm{d}t}=\frac{\partial y_{s2}}{\partial\theta_1}\dot{\theta}_1+\frac{\partial y_{s2}}{\partial\alpha}\dot{\alpha}\\
&\quad=l_1\cos\theta_1\,\dot{\theta}_1+l_{s2}\cos\theta_2\frac{\partial\theta_2}{\partial\theta_1}\dot{\theta}_1+r\cos\alpha\,\dot{\alpha}+l_{s2}\cos\theta_2\frac{\partial\theta_2}{\partial\alpha}\dot{\alpha}
\end{aligned} \tag{21}$$

Among which, $\sin\theta_2,\cos\theta_2,\frac{\partial\theta_2}{\partial\theta_1},\frac{\partial\theta_2}{\partial\alpha}$ are respectively derived from Equations (9)-(11) and (13).

3.3. Angular Acceleration and Centroidal Acceleration

The following equations can be derived from Equation (10) to get the partial derivative of θ_1:

$$-l_1\sin\theta_1-l_2\sin\theta_2\left(\frac{\partial\theta_2}{\partial\theta_1}\right)^2+l_2\cos\theta_2\frac{\partial^2\theta_2}{\partial\theta_1^2}=0 \tag{22}$$

Substituting Equation (11) into Equation (22), the following equations can be derived:

$$\frac{\partial^2\theta_2}{\partial\theta_1^2}=\frac{l_1\sin\theta_1}{l_2\cos\theta_2}+\frac{l_1^2\sin\theta_2\cos^2\theta_1}{l_2^2\cos^3\theta_2} \tag{23}$$

Among which, $\sin\theta_2,\cos\theta_2$ are derived from Equation (5) into Equation (7).

The following equations can be derived from Equation (12) to get the partial derivative of θ_1:

$$-l_2\sin\theta_2\frac{\partial\theta_2}{\partial\theta_1}\frac{\partial\theta_2}{\partial\alpha}+l_2\cos\theta_2\frac{\partial^2\theta_2}{\partial\alpha\partial\theta_1}=0 \tag{24}$$

Substituting Equation (11) and (13) into Equation (24), the following equations can be derived:

$$\frac{\partial^2\theta_2}{\partial\alpha\partial\theta_1}=\frac{rl_1}{l_2^2}\frac{\sin\theta_2\cos\theta_1\cos\alpha}{\cos^3\theta_2} \tag{25}$$

Among which, $\sin\theta_2,\cos\theta_2$ are derived from Equation (5) into Equation (7).

Due to the continuity of mixed partial derivatives, not considering the order of differentiation, the following equations can be derived:

$$\frac{\partial^2\theta_2}{\partial\theta_1\partial\alpha}=\frac{\partial^2\theta_2}{\partial\alpha\partial\theta_1}=\frac{rl_1}{l_2^2}\frac{\sin\theta_2\cos\theta_1\cos\alpha}{\cos^3\theta_2}$$

The following equations can be derived from Equation (12) to get the derivative of α:

$$-r\sin\alpha-l_2\sin\theta_2\left(\frac{\partial\theta_2}{\partial\alpha}\right)^2+l_2\cos\theta_2\frac{\partial^2\theta_2}{\partial\alpha^2}=0 \tag{26}$$

Substituting Equation (13) into Equation (26), the following equations can be derived:

$$\frac{\partial^2\theta_2}{\partial\alpha^2}=\frac{r\sin\alpha}{l_2\cos\theta_2}+\frac{r^2}{l_2^2}\frac{\sin\theta_2\cos^2\alpha}{\cos^3\theta_2} \tag{27}$$

Among which, $\sin\theta_2,\cos\theta_2$ are derived from Equation (5) into Equation (7).

Based on Equation (18), the following equations can be derived:

$$\begin{aligned}\ddot{\theta}_2&=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\theta_2}{\partial\theta_1}\right)\dot{\theta}_1+\frac{\partial\theta_2}{\partial\theta_1}\ddot{\theta}_1+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\theta_2}{\partial\alpha}\right)\dot{\alpha}+\frac{\partial\theta_2}{\partial\alpha}\ddot{\alpha}\\&=\left(\frac{\partial^2\theta_2}{\partial\theta_1^2}\dot{\theta}_1+\frac{\partial^2\theta_2}{\partial\theta_1\partial\alpha}\dot{\alpha}\right)\dot{\theta}_1+\frac{\partial\theta_2}{\partial\theta_1}\ddot{\theta}_1+\left(\frac{\partial^2\theta_2}{\partial\alpha\partial\theta_1}\dot{\theta}_1+\frac{\partial^2\theta_2}{\partial\alpha^2}\dot{\alpha}\right)\dot{\alpha}+\frac{\partial\theta_2}{\partial\alpha}\ddot{\alpha}\\&=\frac{\partial\theta_2}{\partial\theta_1}\ddot{\theta}_1+\frac{\partial\theta_2}{\partial\alpha}\ddot{\alpha}+\frac{\partial^2\theta_2}{\partial\theta_1^2}\dot{\theta}_1^2+\frac{\partial^2\theta_2}{\partial\alpha^2}\dot{\alpha}^2+2\frac{\partial^2\theta_2}{\partial\alpha\partial\theta_1}\dot{\theta}_1\dot{\alpha}\end{aligned} \tag{28}$$

Among which, $\frac{\partial\theta_2}{\partial\theta_1},\frac{\partial\theta_2}{\partial\alpha},\frac{\partial^2\theta_2}{\partial\theta_1^2},\frac{\partial^2\theta_2}{\partial^2\alpha},\frac{\partial^2\theta_2}{\partial\alpha\partial\theta_1}$ are derived from Equations (11), (13), (23), (27), (25).

The following equations can be derived from Equation (14) to get the derivative of θ_1:

$$-l_1\cos\theta_1-l_2\cos\theta_2\left(\frac{\partial\theta_2}{\partial\theta_1}\right)^2-l_2\sin\theta_2\frac{\partial^2\theta_2}{\partial\theta_1^2}-\frac{\partial^2x_M}{\partial\theta_1^2}=0$$

$$\frac{\partial^2x_M}{\partial\theta_1^2}=-l_1\cos\theta_1-l_2\cos\theta_2\left(\frac{\partial\theta_2}{\partial\theta_1}\right)^2-l_2\sin\theta_2\frac{\partial^2\theta_2}{\partial\theta_1^2}$$

$$\frac{\partial^2x_M}{\partial\theta_1\partial\alpha}=-l_2\cos\theta_2\frac{\partial\theta_2}{\partial\alpha}\frac{\partial\theta_2}{\partial\theta_1}-l_2\sin\theta_2\frac{\partial^2\theta_2}{\partial\theta_1\partial\alpha} \tag{29}$$

Among which, $\sin\theta_2,\cos\theta_2,\frac{\partial\theta_2}{\partial\theta_1},\frac{\partial^2\theta_2}{\partial\theta_1^2}$ are derived from Equations (5), (7), (11), (23).

The following equations can be derived from Equation (14) to get the derivative of α:

$$-l_2\cos\theta_2\frac{\partial\theta_2}{\partial\alpha}\frac{\partial\theta_2}{\partial\theta_1}-l_2\sin\theta_2\frac{\partial^2\theta_2}{\partial\theta_1\partial\alpha}-\frac{\partial^2x_M}{\partial\theta_1\partial\alpha}=0 \tag{30}$$

Among which, $\sin\theta_2,\cos\theta_2,\frac{\partial\theta_2}{\partial\theta_1},\frac{\partial\theta_2}{\partial\alpha},\frac{\partial^2\theta_2}{\partial\theta_1\partial\alpha}$ are derived from Equations (5), (7), (11), (13), (25).

The following equations can be derived from Equation (16) to get the derivative of α:

$$\frac{\partial^2 x_M}{\partial \alpha^2} = -r\cos\alpha - l_2\cos\theta_2\left(\frac{\partial \theta_2}{\partial \alpha}\right)^2 - l_2\sin\theta_2\frac{\partial^2\theta_2}{\partial\alpha^2} \tag{31}$$

Among which, $\sin\theta_2, \cos\theta_2, \frac{\partial\theta_2}{\partial\alpha}, \frac{\partial^2\theta_2}{\partial\alpha^2}$ are derived from Equations (5), (7), (13), (27).

Based on Equation (19)

$$\begin{aligned}\ddot{x}_M &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_M}{\partial\theta_1}\right)\dot{\theta}_1 + \frac{\partial x_M}{\partial\theta_1}\ddot{\theta}_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_M}{\partial\alpha}\right)\dot{\alpha} + \frac{\partial x_M}{\partial\alpha}\ddot{\alpha}\\ &= \left(\frac{\partial^2 x_M}{\partial\theta_1}\dot{\theta}_1 + \frac{\partial^2 x_M}{\partial\theta_1\partial\alpha}\dot{\alpha}\right)\dot{\theta}_1 + \frac{\partial x_M}{\partial\theta_1}\ddot{\theta}_1 + \left(\frac{\partial^2 x_M}{\partial\alpha\partial\theta_1}\dot{\theta}_1 + \frac{\partial^2 x_M}{\partial^2\alpha}\dot{\alpha}\right)\dot{\alpha} + \frac{\partial x_M}{\partial\alpha}\ddot{\alpha} \\ &= \frac{\partial x_M}{\partial\theta_1}\ddot{\theta}_1 + \frac{\partial x_M}{\partial\alpha}\ddot{\alpha} + \frac{\partial^2 x_M}{\partial\theta_1^2}\dot{\theta}_1^2 + \frac{\partial^2 x_M}{\partial^2\alpha}\dot{\alpha}^2 + 2\frac{\partial^2 x_M}{\partial\alpha\partial\theta_1}\dot{\theta}_1\dot{\alpha}\end{aligned} \tag{32}$$

Among which, $\frac{\partial x_M}{\partial\theta_1}, \frac{\partial x_M}{\partial\alpha}, \frac{\partial^2 x_M}{\partial\theta_1^2}, \frac{\partial^2 x_M}{\partial^2\alpha}, \frac{\partial^2 x_M}{\partial\alpha\partial\theta_1}$ are derived from Equations (15), (17), (29)-(31).

Based on Equation (20)

$$\begin{aligned}&\frac{\partial^2 x_{s1}}{\partial\theta_1^2} = -l_{s1}\cos\theta_1, \frac{\partial^2 x_{s1}}{\partial\alpha^2} = 0\\ &\frac{\partial^2 x_{s1}}{\partial\theta_1\partial\alpha} = 0, \frac{\partial^2 y_{s1}}{\partial\theta_1^2} = -l_{S1}\sin\theta_1\\ &\frac{\partial^2 y_{s1}}{\partial\alpha^2} = 0, \frac{\partial^2 y_{s1}}{\partial\theta_1\partial\alpha} = 0\end{aligned} \tag{33}$$

The following equations can be derived from Equations (20) and (33):

$$\begin{aligned}\ddot{x}_{s1} &= \frac{\mathrm{d}\dot{x}_{s1}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_{s1}}{\partial\theta_1}\right)\dot{\theta}_1 + \frac{\partial x_{s1}}{\partial\alpha}\ddot{\theta}_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_{s1}}{\partial\alpha}\right)\ddot{\alpha}\\ &= \frac{\partial^2 x_{s1}}{\partial\theta_1^2}\dot{\theta}_1^2 + \frac{\partial^2 x_{s1}}{\partial\theta_1\partial\alpha}\dot{\theta}_1\dot{\alpha} + \frac{\partial x_{s1}}{\partial\theta_1}\ddot{\theta}_1 + \frac{\partial^2 x_{s1}}{\partial\theta_1\partial\alpha}\dot{\theta}_1\dot{\alpha} + \frac{\partial^2 x_{s1}}{\partial\alpha^2}\dot{\alpha}^2 + \frac{\partial x_{s1}}{\partial\alpha}\ddot{\alpha}\\ &= -l_{s1}\cos\theta_1\dot{\theta}_1^2 + 0 - l_{s1}\sin\theta_1\ddot{\theta}_1 + 0 + 0\\ &= -l_{s1}\cos\theta_1\dot{\theta}_1^2 - l_{s1}\sin\theta_1\ddot{\theta}_1\end{aligned} \tag{34}$$

$$\begin{aligned}\ddot{y}_{s1} &= \frac{\mathrm{d}\dot{y}_{s1}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial y_{s1}}{\partial\theta_1}\right)\dot{\theta}_1 + \frac{\partial y_{s1}}{\partial\theta_1}\ddot{\theta}_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial y_{s1}}{\partial\alpha}\right)\dot{\alpha} + \frac{\partial y_{s1}}{\partial\alpha}\ddot{\alpha}\\ &= \frac{\partial^2 y_{s1}}{\partial\theta_1^2}\dot{\theta}_1^2 + \frac{\partial^2 y_{s1}}{\partial\theta_1\partial\alpha}\dot{\theta}_1\dot{\alpha} + \frac{\partial y_{s1}}{\partial\theta_1}\ddot{\theta}_1 + \frac{\partial^2 y_{s1}}{\partial\alpha\partial\theta_1}\dot{\theta}_1\dot{\alpha} + \frac{\partial^2 y_{s1}}{\partial\alpha^2}\dot{\alpha}^2 + \frac{\partial y_{s1}}{\partial\alpha}\ddot{\alpha}\\ &= -l_{s1}\sin\theta_1\dot{\theta}_1^2 + 0 + l_{s1}\cos\theta_1\ddot{\theta}_1 + 0 + 0\\ &= l_{s1}\cos\theta_1\ddot{\theta}_1 - l_{s1}\sin\theta_1\dot{\theta}_1^2\end{aligned} \tag{35}$$

Based on Equation (21)

$$\begin{aligned}
&\frac{\partial^2 x_{s2}}{\partial \theta_1^2} = -l_{s1}\cos\theta_1 - l_{s2}\cos\theta_2\left(\frac{\partial\theta_2}{\partial\theta_1}\right)^2 - l_{s2}\sin\theta_2\frac{\partial^2\theta_2}{\partial\theta_1^2}\\
&\frac{\partial^2 x_{s2}}{\partial \alpha^2} = -r\cos\alpha - l_{s2}\cos\theta_2\left(\frac{\partial\theta_2}{\partial\alpha}\right)^2 - l_{s2}\sin\theta_2\frac{\partial^2\theta_2}{\partial\alpha^2}\\
&\frac{\partial^2 x_{s2}}{\partial \theta_1\partial\alpha} = -l_{s2}\cos\theta_2\frac{\partial\theta_2}{\partial\alpha}\frac{\partial\theta_2}{\partial\theta_1} - l_{s2}\sin\theta_2\frac{\partial^2\theta_1}{\partial\theta_1\partial\alpha}\\
&\frac{\partial^2 y_{s2}}{\partial \theta_1^2} = -l_1\sin\theta_1 - l_{s2}\sin\theta_2\left(\frac{\partial\theta_2}{\partial\theta_1}\right)^2 + l_{s2}\cos\theta_2\frac{\partial^2\theta_2}{\partial\theta_1^2}\\
&\frac{\partial^2 y_{s2}}{\partial \alpha^2} = -r\sin\alpha - l_{s2}\sin\theta_2\left(\frac{\partial\theta_2}{\partial\alpha}\right)^2 + l_{s2}\cos\theta_2\frac{\partial^2\theta_2}{\partial\alpha^2}\\
&\frac{\partial^2 y_{s2}}{\partial \theta_1\partial\alpha} = -l_{s2}\sin\theta_2\frac{\partial\theta_2}{\partial\alpha}\frac{\partial\theta_2}{\partial\theta_1} + l_{s2}\cos\theta_2\frac{\partial^2\theta_2}{\partial\theta_1\partial\alpha}
\end{aligned}\tag{36}$$

Among which, $\sin\theta_2, \cos\theta_2, \frac{\partial\theta_2}{\partial\theta_1}, \frac{\partial\theta_2}{\partial\alpha}\frac{\partial\theta_2^2}{\partial\theta_1^2}, \frac{\partial\theta_2^2}{\partial\alpha^2}, \frac{\partial\theta_2^2}{\partial\theta_1\partial\alpha}$ are derived from Equations (5), (7), (11), (13), (27) and (25).

Based on Equation (21)

$$\begin{aligned}
\ddot{x}_{s2} &= \frac{\mathrm{d}\dot{x}_2}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_{s2}}{\partial\theta_1}\right)\dot{\theta}_1 + \frac{\partial x_{s2}}{\partial\theta_1}\ddot{\theta}_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_{s2}}{\partial\alpha}\right)\dot{\alpha} + \frac{\partial x_{s2}}{\partial\alpha}\ddot{\alpha}\\
&= \frac{\partial^2 x_{s2}}{\partial\theta_1^2}\dot{\theta}_1^2 + \frac{\partial^2 x_{s2}}{\partial\theta_1\partial\alpha}\dot{\theta}_1\dot{\alpha} + \frac{\partial x_{s2}}{\partial\theta_1}\ddot{\theta}_1 + \frac{\partial^2 x_{s2}}{\partial\theta_1\partial\alpha}\dot{\theta}_1\dot{\alpha} + \frac{\partial^2 x_{s2}}{\partial\alpha^2}\dot{\alpha}^2 + \frac{\partial x_{s2}}{\partial\alpha}\ddot{\alpha}\\
\ddot{y}_{s2} &= \frac{\mathrm{d}\dot{y}_{s2}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial y_{s2}}{\partial\theta_1}\right)\dot{\theta}_1 + \frac{\partial y_{s2}}{\partial\theta_1}\ddot{\theta}_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial y_{s2}}{\partial\alpha}\right)\dot{\alpha} + \frac{\partial y_{s2}}{\partial\alpha}\ddot{\alpha}\\
&= \frac{\partial^2 y_{s2}}{\partial\theta_1^2}\dot{\theta}_1^2 + \frac{\partial^2 y_{s2}}{\partial\theta_1\partial\alpha}\dot{\theta}_1\dot{\alpha} + \frac{\partial y_{s2}}{\partial\theta_1}\ddot{\theta}_1 + \frac{\partial^2 y_{s2}}{\partial\theta_1\partial\alpha}\dot{\theta}_1\dot{\alpha} + \frac{\partial^2 y_{s2}}{\partial\alpha^2}\dot{\alpha}^2 + \frac{\partial y_{s2}}{\partial\alpha}\ddot{\alpha}
\end{aligned}\tag{37}$$

Among which, $\frac{\partial x_{s2}}{\partial\theta_1}, \frac{\partial x_{s2}}{\partial\alpha}, \frac{\partial y_{s2}}{\partial\theta_1}, \frac{\partial y_{s2}}{\partial\alpha}$ are derived from Equation (21), $\frac{\partial x_{s2}^2}{\partial\theta_1^2}, \frac{\partial x_{s2}^2}{\partial\alpha^2}, \frac{\partial x_{s2}}{\partial\theta_1\partial\alpha}, \frac{\partial y_{s2}^2}{\partial\theta_1^2}, \frac{\partial y_{s2}^2}{\partial\alpha^2}, \frac{\partial y_{s2}^2}{\partial\theta_1\partial\alpha}$ are derived from Equation (36).

3.4. Equation of Motion

The following equations can be derived from Equation (18) to get the derivative of θ_1:

$$\begin{aligned}
&\frac{\partial\dot{\theta}_2}{\partial\dot{\theta}_1} = \frac{\partial\theta_2}{\partial\theta_1}\\
&\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{\theta}_2}{\partial\dot{\theta}_1}\right) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\theta_2}{\partial\theta_1}\right) = \frac{\partial\dot{\theta}_2}{\partial\theta_1}
\end{aligned}\tag{38}$$

The following equations can be derived from Equation (18) to get the derivative of $\dot{\alpha}$:

$$\frac{\partial\dot{\theta}_2}{\partial\dot{\alpha}} = \frac{\partial\theta_2}{\partial\alpha}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{\theta}_2}{\partial\dot{\alpha}}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\theta_2}{\partial\alpha}\right)=\frac{\partial\dot{\theta}_2}{\partial\alpha} \tag{39}$$

The following equations can be derived from Equation (19) to get the derivative of $\dot{\theta}_1$:

$$\frac{\partial\dot{x}_M}{\partial\dot{\theta}_1}=\frac{\partial x_M}{\partial\theta_1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{x}_M}{\partial\dot{\theta}_1}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_M}{\partial\theta_1}\right)=\frac{\partial\dot{x}_M}{\partial\theta_1} \tag{40}$$

The following equations can be derived from Equation (19) to get the derivative of $\dot{\alpha}$:

$$\frac{\partial\dot{x}_M}{\partial\dot{\alpha}}=\frac{\partial x_M}{\partial\alpha}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{x}_M}{\partial\dot{\alpha}}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_M}{\partial\alpha}\right)=\frac{\partial\dot{x}_M}{\partial\alpha} \tag{41}$$

The motion equation for configuration 1 of the mechanism by using Lagrange's equations can be set up as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial\dot{q}_j}\right)-\frac{\partial E}{\partial q_j}+\frac{\partial U}{\partial q_j}=F_j \qquad j=1,2 \tag{42}$$

where the generalized coordinates of q_j are θ_1,α.

The kinetic energy *E* of the system is the sum of the kinetic energies of bar 1, bar 2, and the slider *M*.

$$E=\frac{1}{2}m_M\dot{x}_M^2+\sum_{i=1}^{2}\frac{1}{2}m_i\left(\dot{x}_{si}^2+\dot{y}_{si}^2\right)+\sum_{i=1}^{2}\frac{1}{2}J_i\dot{\theta}_i^2 \tag{43}$$

where *M* is the mass of the slider, and m_i, j_i are the masses and moments of inertia of bar 1 and bar 2, respectively.

The potential energy of the system is the sum of the gravitational potential energy of bars 1 and 2, and the elastic potential energy of spring *k*.

$$U=m_i g y_{si}+\frac{1}{2}kx_M^2 \qquad i=1,2 \tag{44}$$

"*g*" represents the gravitational acceleration, and the natural position of spring *K* is taken as the zero point of potential energy.

The external force F_j is the driving torque *T* acting on bar 1, $F_1=T$, $F_2=0$.

Substituting Equation (43) and Equation (44) into Equation (42), $j=1$, $q_1=\theta_1$.

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{1}{2}m_M\frac{\partial x_M^2}{\partial\dot{\theta}_1}+\frac{1}{2}m_1\left(\frac{\partial x_{s1}^2}{\partial\dot{\theta}_1}+\frac{\partial y_{s1}^2}{\partial\dot{\theta}_1}\right)+\frac{1}{2}J_1\frac{\partial\dot{\theta}_1^2}{\partial\dot{\theta}_1}+\frac{1}{2}m_2\left(\frac{\partial x_{s2}^2}{\partial\dot{\theta}_1}+\frac{\partial y_{s2}^2}{\partial\dot{\theta}_1}\right)\right.\\&\left.+\frac{1}{2}J_2\frac{\partial\dot{\theta}_2}{\partial\dot{\theta}_1}\right\}-\left\{\frac{1}{2}m_M\frac{\partial x_M^2}{\partial\dot{\theta}_1}+\frac{1}{2}m_1\left(\frac{\partial x_{s1}^2}{\partial\dot{\theta}_1}+\frac{\partial y_{s1}^2}{\partial\dot{\theta}_1}\right)+\frac{1}{2}J_1\frac{\partial\dot{\theta}_1^2}{\partial\dot{\theta}_1}\right.\\&\left.+\frac{1}{2}m_2\left(\frac{\partial x_{s2}^2}{\partial\dot{\theta}_1}+\frac{\partial y_{s2}^2}{\partial\dot{\theta}_1}\right)+\frac{1}{2}J_2\frac{\partial\dot{\theta}_2^2}{\partial\dot{\theta}_1}\right\}+m_1g\frac{\partial y_{s1}}{\partial\theta}+m_2g\frac{\partial y_{s2}}{\partial\theta}+\frac{1}{2}k\frac{\partial x_M^2}{\partial\dot{\theta}_1}=T\end{aligned} \tag{45}$$

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\left\{m_M\dot{x}_M\frac{\partial\dot{x}_M}{\partial\dot{\theta}_1}+m_1\left(\partial\dot{x}_{s1}\frac{\partial\dot{x}_{s1}}{\partial\dot{\theta}_1}+\partial\dot{y}_{s1}\frac{\partial\dot{y}_{s1}}{\partial\dot{\theta}_1}\right)+J_1\dot{\theta}_1\frac{\partial\dot{\theta}_1}{\partial\dot{\theta}_1}+m_2\left(\dot{x}_{s2}\frac{\partial\dot{x}_{s2}}{\partial\dot{\theta}_1}+\dot{y}_{s2}\frac{\partial\dot{y}_{s2}}{\partial\dot{\theta}_1}\right)\right.\\
&\left.+J_2\dot{\theta}_2\frac{\partial\dot{\theta}_2}{\partial\dot{\theta}_1}\right\}-\left\{m_M\dot{x}_M\frac{\partial\dot{x}_M}{\partial\theta_1}+m_1\left(\dot{x}_{s1}\frac{\partial\dot{x}_{s1}}{\partial\theta_1}+\dot{y}_{s1}\frac{\partial\dot{y}_{s1}}{\partial\theta_1}\right)+J_1\dot{\theta}_1\frac{\partial\dot{\theta}_1}{\partial\theta_1}\right.\\
&\left.+m_2\left(\dot{x}_{s2}\frac{\partial\dot{x}_{s2}}{\partial\theta_1}+\dot{y}_{s2}\frac{\partial\dot{y}_{s2}}{\partial\theta_1}\right)+J_2\dot{\theta}_2\frac{\partial\dot{\theta}_2}{\partial\theta_1}\right\}+m_1g\frac{\partial y_{s1}}{\partial\theta_1}+m_2g\frac{\partial y_{s2}}{\partial\theta_1}+kx_M\frac{\partial x_M}{\partial\theta_1}=T
\end{aligned}
\tag{46}
$$

The following equations can be derived by expanding the above equation:

$$
\begin{aligned}
&m_M\ddot{x}_M\frac{\partial\dot{x}_M}{\partial\dot{\theta}_1}+m_M\dot{x}_M\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{x}_M}{\partial\dot{\theta}_1}\right)+m_1\ddot{x}_{s1}\frac{\partial\dot{x}_{s1}}{\partial\dot{\theta}_1}+m_1\dot{x}_{s1}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{x}_{s1}}{\partial\dot{\theta}_1}\right)+m_1\ddot{y}_{s1}\frac{\partial\dot{y}_{s1}}{\partial\dot{\theta}_1}\\
&+m_1\dot{y}_{s1}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{y}_{s1}}{\partial\dot{\theta}_1}\right)+J_1\ddot{\theta}_1\frac{\partial\dot{\theta}_1}{\partial\dot{\theta}_1}+J_1\dot{\theta}_1\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{\theta}_1}{\partial\dot{\theta}_1}\right)+m_2\ddot{x}_{s2}\frac{\partial\dot{x}_{s2}}{\partial\dot{\theta}_1}+m_2\dot{x}_{s2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{x}_{s2}}{\partial\dot{\theta}_1}\right)\\
&+m_2\ddot{y}_{s2}\frac{\partial\dot{y}_{s2}}{\partial\dot{\theta}_1}+m_2\dot{y}_{s2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{y}_{s2}}{\partial\dot{\theta}_1}\right)+J_2\ddot{\theta}_2\frac{\partial\dot{\theta}_2}{\partial\dot{\theta}_1}+J_2\dot{\theta}_2\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{\theta}_2}{\partial\dot{\theta}_1}\right)-m_M\dot{x}_M\frac{\partial\dot{x}_M}{\partial\theta_1}\\
&-m_1\dot{x}_{s1}\frac{\partial\dot{x}_{s1}}{\partial\theta_1}-m_1\dot{y}_{s1}\frac{\partial\dot{y}_{s1}}{\partial\theta_1}-J_1\dot{\theta}_1\frac{\partial\dot{\theta}_1}{\partial\theta_1}-m_2\dot{x}_{s2}\frac{\partial\dot{x}_{s2}}{\partial\theta_1}-m_2\dot{y}_{s2}\frac{\partial\dot{y}_{s2}}{\partial\theta_1}-J_2\dot{\theta}_2\frac{\partial\dot{\theta}_2}{\partial\theta_1}\\
&+m_1g\frac{\partial y_{s1}}{\partial\theta_1}+m_2g\frac{\partial y_{s2}}{\partial\theta_1}+kx_M\frac{\partial x_M}{\partial\theta_1}=T
\end{aligned}
\tag{47}
$$

Substituting Equations (38)-(41) into Equation (47), the following equations can be derived:

$$
\begin{aligned}
&m_M\ddot{x}_M\frac{\partial x_M}{\partial\theta_1}+m_M\dot{x}_M\frac{\partial\dot{x}_M}{\partial\theta_1}+m_1\ddot{x}_{s1}\frac{\partial x_{s1}}{\partial\theta_1}+m_1\dot{x}_{s1}\frac{\partial\dot{x}_{s1}}{\partial\theta_1}+m_1\ddot{y}_{s1}\frac{\partial y_{s1}}{\partial\theta_1}\\
&+m_1\dot{y}_{s1}\frac{\partial\dot{y}_{s1}}{\partial\theta_1}+J_1\ddot{\theta}_1+J_1\dot{\theta}_1\frac{\partial\dot{\theta}_1}{\partial\theta_1}+m_2\ddot{x}_{s2}\frac{\partial x_{s2}}{\partial\theta_1}+m_2\dot{x}_{s2}\frac{\partial\dot{x}_{s2}}{\partial\theta_1}+m_2\ddot{y}_{s2}\frac{\partial y_{s2}}{\partial\theta_1}\\
&+m_2\dot{y}_{s2}\frac{\partial\dot{y}_{s2}}{\partial\theta_1}+J_2\ddot{\theta}_2\frac{\partial\dot{\theta}_2}{\partial\theta_1}+J_2\dot{\theta}_2\frac{\partial\dot{\theta}_2}{\partial\theta_1}-m_M\dot{x}_M\frac{\partial\dot{x}_M}{\partial\theta_1}-m_1\dot{x}_{s1}\frac{\partial\dot{x}_{s1}}{\partial\theta_1}\\
&-m_1\dot{y}_{s1}\frac{\partial\dot{y}_{s1}}{\partial\theta_1}-J_1\dot{\theta}_1\frac{\partial\dot{\theta}_1}{\partial\theta_1}-m_2\dot{x}_{s2}\frac{\partial\dot{x}_{s2}}{\partial\theta_1}-m_2\dot{y}_{s2}\frac{\partial\dot{y}_{s2}}{\partial\theta_1}-J_2\dot{\theta}_2\frac{\partial\dot{\theta}_2}{\partial\theta_1}\\
&+m_1g\frac{\partial y_{s1}}{\partial\theta_1}+m_2g\frac{\partial y_{s2}}{\partial\theta_1}+kx_M\frac{\partial x_M}{\partial\theta_1}=T
\end{aligned}
\tag{48}
$$

$$
\begin{aligned}
&m_M\ddot{x}_M\frac{\partial x_M}{\partial\theta_1}+m_1\ddot{x}_{s1}\frac{\partial x_{s1}}{\partial\theta_1}+m_1\ddot{y}_{s1}\frac{\partial y_{s1}}{\partial\theta_1}+J_1\ddot{\theta}_1+m_2\ddot{x}_{s2}\frac{\partial x_{s2}}{\partial\theta_1}+m_2\ddot{y}_{s2}\frac{\partial y_{s2}}{\partial\theta_1}\\
&+J_2\ddot{\theta}_2\frac{\partial\dot{\theta}_2}{\partial\theta_1}+m_1g\frac{\partial y_{s1}}{\partial\theta_1}+m_2g\frac{\partial y_{s2}}{\partial\theta_1}+kx_M\frac{\partial x_M}{\partial\theta_1}=T
\end{aligned}
\tag{49}
$$

Substituting $\ddot{x}_M,\ddot{x}_{s1},\ddot{y}_{s1},\ddot{x}_{s2},\ddot{y}_{s2},\ddot{\theta}_2$ in Equations (28), (32), (34), (35), (37) into Equation (49), the following equations can be derived:

$$
\begin{aligned}
&m_M\frac{\partial x_M}{\partial\theta_1}\left(\ddot{\theta}_1\frac{\partial x_M}{\partial\theta_1}+\ddot{\alpha}\frac{\partial x_M}{\partial\alpha}+\dot{\theta}_1^2\frac{\partial^2x_M}{\partial\theta_1^2}+\dot{\alpha}^2\frac{\partial^2x_M}{\partial\alpha^2}+2\dot{\theta}_1\dot{\alpha}\frac{\partial^2x_M}{\partial\theta_1\partial\alpha}\right)\\
&+m_1\frac{\partial x_{s1}}{\partial\theta_1}\left(\ddot{\theta}_1\frac{\partial x_{s1}}{\partial\theta_1}+\ddot{\alpha}\frac{\partial x_{s1}}{\partial\alpha}+\dot{\theta}_1^2\frac{\partial^2x_{s1}}{\partial\theta_1^2}+\dot{\alpha}^2\frac{\partial^2x_{s1}}{\partial\alpha^2}+2\dot{\theta}_1\dot{\alpha}\frac{\partial^2x_{s1}}{\partial\theta_1\partial\alpha}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+m_1\frac{\partial y_{s1}}{\partial\theta_1}\left(\ddot{\theta}_1\frac{\partial y_{s1}}{\partial\theta_1}+\ddot{\alpha}\frac{\partial y_{s1}}{\partial\alpha}+\dot{\theta}_1^2\frac{\partial^2 y_{s1}}{\partial\theta_1^2}+\dot{\alpha}^2\frac{\partial^2 y_{s1}}{\partial\alpha^2}+2\dot{\theta}_1\dot{\alpha}\frac{\partial^2 y_{s1}}{\partial\theta_1\partial\alpha}\right)\\
&+J_1\ddot{\theta}_1+m_2\frac{\partial x_{s2}}{\partial\theta_1}\left(\ddot{\theta}_1\frac{\partial x_{s2}}{\partial\theta_1}+\ddot{\alpha}\frac{\partial x_{s2}}{\partial\alpha}+\dot{\theta}_1^2\frac{\partial^2 x_{s2}}{\partial\theta_1^2}+\dot{\alpha}^2\frac{\partial^2 x_{s2}}{\partial\alpha^2}+2\dot{\theta}_1\dot{\alpha}\frac{\partial^2 x_{s2}}{\partial\theta_1\partial\alpha}\right)\\
&+m_2\frac{\partial y_{s2}}{\partial\theta_1}\left(\ddot{\theta}_1\frac{\partial y_{s2}}{\partial\theta_1}+\ddot{\alpha}\frac{\partial y_{s2}}{\partial\alpha}+\dot{\theta}_1^2\frac{\partial^2 y_{s2}}{\partial\theta_1^2}+\dot{\alpha}^2\frac{\partial^2 y_{s2}}{\partial\alpha^2}+2\dot{\theta}_1\dot{\alpha}\frac{\partial^2 y_{s2}}{\partial\theta_1\partial\alpha}\right)\\
&+J_2\frac{\partial\theta_2}{\partial\theta_1}\left(\ddot{\theta}_1\frac{\partial\theta_2}{\partial\theta_1}+\ddot{\alpha}\frac{\partial\theta_2}{\partial\alpha}+\dot{\theta}_1^2\frac{\partial^2\theta_2}{\partial\theta_1^2}+\dot{\alpha}^2\frac{\partial^2\theta_2}{\partial\alpha^2}+2\dot{\theta}_1\dot{\alpha}\frac{\partial^2\theta_2}{\partial\theta_1\partial\alpha}\right)\\
&+m_1 g\frac{\partial y_{s1}}{\partial\theta_1}+m_2 g\frac{\partial y_{s2}}{\partial\theta_2}+kx_M\frac{\partial x_M}{\partial\theta_1}=T
\end{aligned}
\tag{50}
$$

The following equations are derived from Equation (50):

$$
\begin{aligned}
&\left\{J_1+J_2\left(\frac{\partial\theta_2}{\partial\theta_1}\right)^2+m_M\left(\frac{\partial x_M}{\partial\theta_1}\right)^2+m_1\left(\frac{\partial x_{s1}}{\partial\theta_1}\right)^2+m_1\left(\frac{\partial y_{s1}}{\partial\theta_1}\right)^2+m_2\left(\frac{\partial x_{s2}}{\partial\theta_1}\right)^2\right.\\
&\left.+m_2\left(\frac{\partial y_{s2}}{\partial\theta_1}\right)^2\right\}\ddot{\theta}_1+\left\{J_2\frac{\partial\theta_2}{\partial\theta_1}\frac{\partial\theta_2}{\partial\alpha}+m_M\frac{\partial x_M}{\partial\theta_1}\frac{\partial x_M}{\partial\alpha}+m_1\frac{\partial x_{s1}}{\partial\theta_1}\frac{\partial x_{s1}}{\partial\alpha}\right.\\
&\left.+m_1\frac{\partial y_{s1}}{\partial\theta_1}\frac{\partial y_{s1}}{\partial\alpha}+m_2\frac{\partial x_{s2}}{\partial\theta_1}\frac{\partial x_{s2}}{\partial\alpha}+m_2\frac{\partial y_{s2}}{\partial\theta_1}\frac{\partial y_{s2}}{\partial\alpha}\right\}\ddot{\alpha}\\
&=T-\left\{J_2\frac{\partial\theta_2}{\partial\theta_1}\frac{\partial^2\theta_2}{\partial\theta_1^2}+m_M\frac{\partial x_M}{\partial\theta_1}\frac{\partial^2 x_M}{\partial\theta_1^2}+m_1\frac{\partial x_{s1}}{\partial\theta_1}\frac{\partial^2 x_{s1}}{\partial\theta_1^2}+m_1\frac{\partial x_{s1}}{\partial\theta_1}\frac{\partial^2 y_{s1}}{\partial\theta_1^2}\right.\\
&\left.+m_2\frac{\partial x_{s1}}{\partial\theta_1}\frac{\partial^2 x_{s1}}{\partial\theta_1^2}+m_2\frac{\partial y_{s1}}{\partial\theta_1}\frac{\partial^2 y_{s1}}{\partial\theta_1^2}\right\}\dot{\theta}_1^2-\left\{J_2\frac{\partial\theta_2}{\partial\theta_1}\frac{\partial^2\theta_2}{\partial\alpha^2}+m_M\frac{\partial x_M}{\partial\theta_1}\frac{\partial^2 x_M}{\partial\alpha^2}\right.\\
&\left.+m_1\frac{\partial x_{s1}}{\partial\theta_1}\frac{\partial^2 x_{s1}}{\partial\alpha^2}+m_1\frac{\partial y_{s1}}{\partial\theta_1}\frac{\partial^2 y_{s1}}{\partial\alpha^2}+m_2\frac{\partial x_{s2}}{\partial\theta_1}\frac{\partial^2 x_{s2}}{\partial\alpha^2}+m_2\frac{\partial y_{s2}}{\partial\theta_1}\frac{\partial^2 y_{s2}}{\partial\alpha^2}\right\}\dot{\alpha}^2\\
&-\left\{2J_2\frac{\partial\theta_2}{\partial\theta_1}\frac{\partial^2\theta_2}{\partial\theta_1\partial\alpha}+2m_M\frac{\partial x_M}{\partial\theta_1}\frac{\partial^2 x_M}{\partial\theta_1\partial\alpha}+2m_1\frac{\partial x_{s1}}{\partial\theta_1}\frac{\partial^2 x_{s1}}{\partial\theta_1\partial\alpha}+2m_1\frac{\partial y_{s1}}{\partial\theta_1}\frac{\partial^2 y_{s1}}{\partial\theta_1\partial\alpha}\right.\\
&\left.+2m_2\frac{\partial x_{s2}}{\partial\theta_1}\frac{\partial^2 x_{s2}}{\partial\theta_1\partial\alpha}+2m_2\frac{\partial y_{s2}}{\partial\theta_1}\frac{\partial^2 y_{s2}}{\partial\theta_1\partial\alpha}\right\}\dot{\theta}_1\dot{\alpha}-m_1 g\frac{\partial y_{s1}}{\partial\theta_1}-m_2 g\frac{\partial y_{s2}}{\partial\theta_1}-kx_M\frac{\partial x_M}{\partial\theta_1}
\end{aligned}
\tag{51}
$$

Equation (51) can be written in abbreviated form as follows:

$$A_1\ddot{\theta}_1+A_2\ddot{\alpha}=B_1 \tag{52}$$

Among which, A_1, A_2 are the functions of θ_1, α, B_1 is the function of $\theta_1, \alpha, \dot{\theta}_1, \dot{\alpha}$.

When $j=2, q_2=\alpha, F_2=0$, substituting Equations (43), (44) into Equation (42), the following equations are derived:

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{1}{2}m_M\frac{\partial\dot{x}_M^2}{\partial\dot{\alpha}}+\frac{1}{2}m_1\left(\frac{\partial\dot{x}_{s1}^2}{\partial\dot{\alpha}}+^2\frac{\partial\dot{y}_{s1}}{\partial\dot{\alpha}}\right)+\frac{1}{2}J_1\frac{\partial\dot{\theta}_1^2}{\partial\dot{\alpha}}+\frac{1}{2}m_2\left(\frac{\partial\dot{x}_{s2}^2}{\partial\dot{\alpha}}+\frac{\partial\dot{y}_{s2}^2}{\partial\dot{\alpha}}\right)\right.\\
&\left.+\frac{1}{2}J_2\frac{\partial\dot{\theta}_2}{\partial\dot{\alpha}}\right\}-\left\{\frac{1}{2}m_M\frac{\alpha\dot{x}_M}{\partial\alpha}+\frac{1}{2}m_1\left(\frac{\partial\dot{x}_{s1}^2}{\partial\alpha}+\frac{\partial\dot{y}_{s1}^2}{\partial\alpha}\right)+\frac{1}{2}J_1\frac{\partial\dot{\theta}_1^2}{\partial\alpha}\right.\\
&\left.+\frac{1}{2}m_2\left(\frac{\partial\dot{x}_{s2}^2}{\partial\alpha}+\frac{\partial\dot{y}_{s2}^2}{\partial\alpha}\right)+\frac{1}{2}J_2\frac{\partial\dot{\theta}_2^2}{\partial\alpha}\right\}+m_1 g\frac{\partial y_{s1}}{\partial\alpha}+m_2 g\frac{\partial y_{s2}}{\partial\alpha}+\frac{1}{2}k\frac{\partial x_M^2}{\partial\alpha}=0
\end{aligned}
\tag{53}
$$

As Equation (53) is identical in form to Equation (45), Equation (53) can be obtained through the same derivation process as follows:

$$\begin{aligned}
&\left\{J_2\frac{\partial\theta_2}{\partial\alpha}\frac{\partial\theta_2}{\partial\theta_1}+m_M\frac{\partial x_M}{\partial\alpha}\frac{\partial x_M}{\partial\theta_1}+m_1\frac{\partial x_{s1}}{\partial\alpha}\frac{\partial x_{s1}}{\partial\theta_1}+m_1\frac{\partial y_{s1}}{\partial\alpha}\frac{\partial y_{s1}}{\partial\theta_1}+m_2\frac{\partial x_{s2}}{\partial\alpha}\frac{\partial x_{s2}}{\partial\theta_1}\right.\\
&\left.+m_2\frac{\partial y_{s2}}{\partial\alpha}\frac{\partial y_{s2}}{\partial\theta_1}\right\}\ddot{\theta}_1+\left\{J_2\left(\frac{\partial\theta_2}{\partial\alpha}\right)^2+m_M\left(\frac{\partial x_M}{\partial\alpha}\right)^2+m_1\left(\frac{\partial x_{s1}}{\partial\alpha}\right)^2+m_1\left(\frac{\partial y_{s1}}{\partial\alpha}\right)^2\right.\\
&\left.+m_2\left(\frac{\partial x_{s2}}{\partial\alpha}\right)^2+m_2\left(\frac{\partial y_{s2}}{\partial\alpha}\right)^2\right\}\ddot{\alpha}\\
&=-\left\{J_2\frac{\partial\theta_2}{\partial\alpha}\frac{\partial^2\theta_2}{\partial\theta_1^2}+m_M\frac{\partial x_M}{\partial\alpha}\frac{\partial^2 x_M}{\partial\theta_1^2}+m_1\frac{\partial x_{s1}}{\partial\alpha}\frac{\partial^2 x_{s1}}{\partial\theta_1^2}+m_1\frac{\partial y_{s1}}{\partial\alpha}\frac{\partial^2 y_{s1}}{\partial\theta_1^2}\right.\\
&\left.+m_2\frac{\partial x_{s2}}{\partial\alpha}\frac{\partial^2 x_{s2}}{\partial\theta_1^2}+m_2\frac{\partial y_{s2}}{\partial\alpha}\frac{\partial^2 y_{s2}}{\partial\theta_1^2}\right\}\dot{\theta}_1^2-\left\{J_2\frac{\partial\theta_2}{\partial\alpha}\frac{\partial^2\theta_2}{\partial\alpha^2}+m_M\frac{\partial x_M}{\partial\alpha}\frac{\partial^2 x_M}{\partial\alpha^2}\right.\\
&\left.+m_1\frac{\partial x_{s1}}{\partial\alpha}\frac{\partial^2 x_{s1}}{\partial\alpha^2}+m_1\frac{\partial y_{s1}}{\partial\alpha}\frac{\partial^2 y_{s1}}{\partial\alpha^2}+m_2\frac{\partial x_{s2}}{\partial\alpha}\frac{\partial^2 x_{s2}}{\partial\alpha^2}+m_2\frac{\partial y_{s2}}{\partial\alpha}\frac{\partial^2 y_{s2}}{\partial\alpha^2}\right\}\dot{\alpha}^2\\
&-\left\{2J_2\frac{\partial\theta_2}{\partial\alpha}\frac{\partial^2\theta_2}{\partial\theta_1\partial\alpha}+2m_M\frac{\partial x_M}{\partial\alpha}\frac{\partial^2 x_M}{\partial\theta_1\partial\alpha}+2m_1\frac{\partial x_{s1}}{\partial\alpha}\frac{\partial^2 x_{s1}}{\partial\theta_1\partial\alpha}+2m_1\frac{\partial y_{s1}}{\partial\alpha}\frac{\partial^2 y_{s1}}{\partial\theta_1\partial\alpha}\right.\\
&\left.+2m_2\frac{\partial x_{s2}}{\partial\alpha}\frac{\partial^2 x_{s2}}{\partial\theta_1\partial\alpha}+2m_2\frac{\partial y_{s2}}{\partial\alpha}\frac{\partial^2 y_{s2}}{\partial\theta_1\partial\alpha}\right\}\dot{\theta}_1\dot{\alpha}-m_1g\frac{\partial y_{s1}}{\partial\alpha}-m_2g\frac{\partial y_{s2}}{\partial\alpha}-kx_M\frac{\partial x_M}{\partial\alpha}
\end{aligned}\tag{54}$$

Equation (54) can be written in abbreviated form as follows:

$$A_3\ddot{\theta}_1+A_4\ddot{\alpha}=B_2 \tag{55}$$

Among which, A_3, A_4 are the functions of θ_1,α. B_2 is the function of $\theta_1,\alpha,\dot{\theta}_1,\dot{\alpha}$.

By solving the simultaneous equations derived from Equation (52) and Equation (55), we can obtain:

$$\ddot{\theta}_1=f_1\left(\theta_1,\alpha,\dot{\theta}_1,\dot{\alpha}\right),\quad \ddot{\alpha}=f_2\left(\theta_1,\alpha,\dot{\theta}_1,\dot{\alpha}\right).$$

This second-order nonlinear equation represents the differential equation of motion, and the solution is a function of time. If the driving link, Crank 1 rotates uniformly, the solution becomes simpler, where ω is constant. In this case, $\ddot{\theta}_1=0$, $\theta_1=\omega t$ becomes a known function in Equation (55). Thus, from Equation (55), we can derive $\ddot{\alpha}=f_2\left(t,\alpha,\dot{\alpha}\right)$. Substituting the obtained $\alpha=\alpha\left(t\right)$ into Equation (52), it can be achieved $T=T\left(t\right)$, which represents the torque function required to ensure the uniform rotation of the driving link, Crank 1.

4. Configuration 2 for the Continuous Contact Model of the Metamorphic Groove Pin Pair with Clearance

4.1. Positional Relations of Configuration 2

In Configuration 2, the virtual slider moves along link 2. Imagine the distance from the slider to point O be denoted as lS, the following results can be obtained according to geometric relationships:

The metamorphic mechanism has a degree of freedom of 2, with θ_1 and α selected as the generalized coordinates.

$$l_s = \sqrt{\left(l_3 + l_1 \cos\theta_1 + r\cos\alpha\right)^2 + \left(l_1 \sin\theta_1 + r\sin\alpha\right)^2} \tag{56}$$

$$\sin\theta_{2A} = \frac{l_1 \sin\theta_1 + r\sin\alpha}{l_S} \tag{57}$$

$$\begin{gathered} \theta_2' = \theta_2 - \pi, \theta_2 = \theta_2' + \pi \\ \dot{\theta}_2' = \dot{\theta}_2, \ddot{\theta}_2' = \ddot{\theta}_2 \\ \frac{\partial\theta_2'}{\partial\theta_1} = \frac{\partial\theta_2}{\partial\theta_1}, \frac{\partial\theta_2'}{\partial\alpha} = \frac{\partial\theta_2}{\partial\alpha} \end{gathered} \tag{58}$$

The vector equations are as follows:

$$l_1 + r + l_s + l_3 = 0 \tag{59}$$

Projected onto the X and Y directions, the following equations are derived:

$$\begin{gathered} l_1\cos\theta_1 + r\cos\alpha + l_s\cos\theta_2 + l_3\cos\theta_3 = 0 \\ l_1\sin\theta_1 + r\sin\alpha + l_s\sin\theta_2 + 0 = 0 \end{gathered} \tag{60}$$

As $\theta_3 = 0, \theta_2 = \theta_{2A} + \pi$, the following equations are derived:

$$l_1\cos\theta_1 + r\cos\alpha - l_s\cos\theta_{2A} + l_3 = 0 \tag{61}$$

$$\cos\theta_{2A} = \frac{1}{l_s}\left(l_1\cos\theta_1 + r\cos\alpha + l_3\right) \tag{62}$$

$$l_1\sin\theta_1 + r\sin\alpha - l_s\sin\theta_{2A} = 0 \tag{63}$$

$$\sin\theta_{2A} = \frac{1}{l_s}\left(l_1\sin\theta_1 + r\sin\alpha\right) \tag{64}$$

The following equations can be derived from Equations (61) and (63) to get the derivative of θ_1:

$$-l_1\sin\theta_1 - \frac{\partial l_s}{\partial\theta_1}\cos\theta_{2A} + l_s\sin\theta_{2A}\frac{\partial\theta_{2A}}{\partial\theta_1} = 0 \tag{65}$$

$$l_1\cos\theta_1 - \frac{\partial l_s}{\partial\theta_1}\sin\theta_{2A} - l_s\cos\theta_{2A}\frac{\partial\theta_{2A}}{\partial\theta_1} = 0 \tag{66}$$

The following equations are derived from Equations (65) and (66).

$$\frac{\partial\theta_{2A}}{\partial\theta_1} = \frac{l_1}{l_s}\cos\left(\theta_{2A} - \theta_1\right) \tag{67}$$

Substituting Equation (67) into Equation (65), the following equations can be derived:

$$\frac{\partial l_s}{\partial\theta_1} = l_1\frac{\sin\theta_{2A}}{\cos\theta_{2A}}\cos\left(\theta_{2A} - \theta_1\right) - l_1\frac{\sin\theta_1}{\cos\theta_{2A}} \tag{68}$$

The following equations are derived from Equations (61) and (63) to get the derivative of α:

$$-r\sin\alpha - \frac{\partial l_s}{\partial\alpha}\cos\theta_{2A} + l_s\sin\theta_{2A}\frac{\partial\theta_{2A}}{\partial\alpha} = 0 \tag{69}$$

$$r\cos\alpha - \frac{\partial l_s}{\partial \alpha}\sin\theta_{2A} - l_s\cos\theta_{2A}\frac{\partial \theta_{2A}}{\partial \alpha} = 0 \tag{70}$$

The following equations are derived from Equations (69) and (70).

$$\frac{\partial \theta_{2A}}{\partial \alpha} = \frac{r}{l_s}\cos\left(\theta_{2A} - \alpha\right) \tag{71}$$

Substituting Equation (71) into Equation (69), the following equations can be derived:

$$\frac{\partial l_s}{\partial \alpha} = r\frac{\sin\theta_{2A}}{\cos\theta_{2A}}\cos\left(\theta_{2A} - \alpha\right) - r\frac{\sin\alpha}{\cos\theta_{2A}} \tag{72}$$

Among which, $\sin\theta_{2A}, \cos\theta_{2A}$ are derived from Equations (64) and (62).

From **Figure 5**, the coordinates of the component centroids are as follows:

$$\begin{aligned} x_{s1} &= l_3 + l_{s1}\cos\theta_1 \\ y_{s1} &= l_{s1}\sin\theta_1 \end{aligned} \tag{73}$$

In **Figure 5**, the mass of the imaginary slider is equivalent to the mass of the actual cylindrical pin, which is actually included in the mass of Component 1 and affects the centroid position of Component 1. So the mass of the imaginary slider is no longer considered in the analysis. From **Figure 5**, the centroid coordinates of component 2 are as follows:

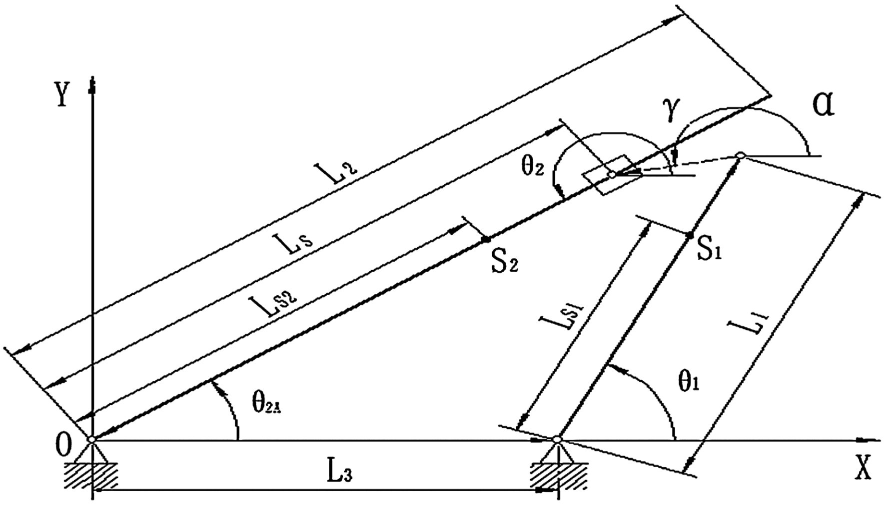

Figure 5. The new mechanism without clearance by adding imaginary links in Configuration 2.

$$\begin{aligned} x_{s2} &= l_{s2}\cos\theta_{2A} \\ y_{s2} &= l_{s2}\sin\theta_{2A} \end{aligned} \tag{74}$$

Among which, l_{s2} is a constant that doesn't change.

4.2. Angular Velocity and Linear Velocity of the Component's Centroid

The angular velocity θ_{2A} and the linear velocity of the component's centroid are functions of the independent variables θ_1, α, as well as their velocities.

The following equations are derived from Equation (74) to get the derivative of: θ_1, α

$$\begin{aligned} &\frac{\partial x_{s1}}{\partial \theta_1} = -l_{s1}\sin\theta_1 \quad \frac{\alpha x_{s1}}{\partial \alpha} = 0 \\ &\frac{\partial y_{s1}}{\partial \theta_1} = l_{s1}\cos\theta_1 \quad \frac{\alpha y_{s1}}{\partial \alpha} = 0 \end{aligned} \tag{75}$$

The following equations are derived from Equation (74) to get the derivative of: θ_1, α

$$\begin{aligned} \frac{\partial x_{s2}}{\partial \theta_1} &= -l_{s2}\sin\theta_{2A}\frac{\partial \theta_{2A}}{\partial \theta_1} \\ \frac{\partial y_{s2}}{\partial \theta_1} &= l_{s2}\cos\theta_{2A}\frac{\partial \theta_{2A}}{\partial \theta_1} \\ \frac{\partial x_{s2}}{\partial \alpha} &= -l_{s2}\cos\theta_{2A}\frac{\partial \theta_{2A}}{\partial \alpha} \\ \frac{\partial y_{s2}}{\partial \alpha} &= l_{s2}\cos\theta_{2A}\frac{\partial \theta_{2A}}{\partial \alpha} \end{aligned} \tag{76}$$

Among which, $\frac{\partial \theta_{2A}}{\partial \theta_1}, \frac{\partial \theta_{2A}}{\partial \alpha}$ is derived from Equations (67) and (71), so the following equations are derived as follows:

$$\dot{\theta}_2 = \dot{\theta}_{2A} = \frac{\partial \theta_{2A}}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial \theta_{2A}}{\partial \alpha}\dot{\alpha} \tag{77}$$

$$\dot{x}_{s1} = \frac{\partial x_{s1}}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial x_{s1}}{\partial \alpha}\dot{\alpha} \tag{78}$$

$$\dot{y}_{s1} = \frac{\partial y_{s1}}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial y_{s1}}{\partial \alpha}\dot{\alpha} \tag{79}$$

$$\dot{x}_{s2} = \frac{\partial x_{s2}}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial x_{s2}}{\partial \alpha}\dot{\alpha} \tag{80}$$

$$\dot{y}_{s2} = \frac{\partial y_{s2}}{\partial \theta_1}\dot{\theta}_1 + \frac{\partial y_{s2}}{\partial \alpha}\dot{\alpha} \tag{81}$$

Among which, $\frac{\partial \theta_{2A}}{\partial \theta_1}, \frac{\partial \theta_{2A}}{\partial \alpha}, \frac{\partial x_{s1}}{\partial \theta_1}, \frac{\partial y_{s1}}{\partial \theta_1}, \frac{\partial x_{s2}}{\partial \theta_1}, \frac{\partial y_{s2}}{\partial \theta_1}, \frac{\partial x_{s2}}{\partial \alpha}, \frac{\partial y_{s2}}{\partial \alpha}$ are derived from Equations (67), (71), (75) and (76).

4.3. Angular Acceleration and Linear Acceleration of the Centroid

The following equations are derived from Equation (67):

$$\begin{aligned} \frac{\partial^2 \theta_{2A}}{\partial \theta_1^2} &= \frac{-l_s l_1 \sin\left(\theta_{2A} - \theta_1\right)\left(\frac{\partial \theta_{2A}}{\partial \theta_1} - 1\right) - l_1 \cos\left(\theta_{2A} - \theta_1\right)\frac{\partial l_s}{\partial \theta_1}}{l_s^2} \\ &= \frac{l_1}{l_s}\sin\left(\theta_{2A} - \theta_1\right)\left(\frac{\partial \theta_{2A}}{\partial \theta_1} - 1\right) - \frac{l_1}{l_s^2}\cos\left(\theta_{2A} - \theta_1\right)\frac{\partial l_s}{\partial \theta_1} \end{aligned} \tag{82}$$

The following equations are derived from Equation (71):

$$\begin{aligned}\frac{\partial \theta_{2A}^2}{\partial \alpha^2} &= \frac{-l_s r \sin\left(\theta_{2A}-\alpha\right)\left(\frac{\partial \theta_{2A}}{\partial \alpha}-1\right)-r\cos\left(\theta_{2A}-\alpha\right)\frac{\partial l_s}{\partial \alpha}}{l_s^2} \\ &= -\frac{r}{l_s}\sin\left(\theta_{2A}-\alpha\right)\left(\frac{\partial \theta_{2A}}{\partial \alpha}-1\right)-\frac{r}{l_s^2}\cos\left(\theta_{2A}-\alpha\right)\frac{\partial l_s}{\partial \alpha}\end{aligned} \tag{83}$$

$$\begin{aligned}\frac{\partial \theta_{2A}^2}{\partial \alpha \partial \theta_1} &= \frac{-l_s r \sin\left(\theta_{2A}-\alpha\right)\frac{\partial \theta_{2A}}{\partial \theta_1}-r\cos\left(\theta_{2A}-\alpha\right)\frac{\partial l_s}{\partial \theta_1}}{l_s^2} \\ &= -\frac{r}{l_s}\sin\left(\theta_{2A}-\alpha\right)\frac{\partial \theta_{2A}}{\partial \theta_1}-\frac{r}{l_s^2}\cos\left(\theta_{2A}-\alpha\right)\frac{\partial l_s}{\partial \theta_1}\end{aligned} \tag{84}$$

In Equations (82)-(84), $\frac{\partial \theta_{2A}}{\partial \theta_1}, \frac{\partial l_s}{\partial \theta_1}, \frac{\partial \theta_{2A}}{\partial \alpha}, \frac{\partial l_s}{\partial \alpha}$ are respectively derived from Equations (67), (68), (71) and (72).

The following equations are derived from Equation (75):

$$\begin{aligned}&\frac{\partial^2 x_{s1}}{\partial \theta_1^2} = -l_{s1}\cos\theta_1 \quad \frac{\partial^2 x_{s1}}{\partial \alpha^2} = 0 \\ &\frac{\partial^2 y_{s1}}{\partial \theta_1^2} = -l_{s1}\sin\theta_1 \quad \frac{\partial^2 y_{s1}}{\partial \alpha^2} = 0\end{aligned} \tag{85}$$

The following equations are derived from Equation (76):

$$\begin{aligned}\frac{\partial^2 x_{s2}}{\partial \theta_1^2} &= -l_{s2}\cos\theta_{2A}\left(\frac{\partial \theta_{2A}}{\partial \theta_1}\right)^2 - l_{s2}\sin\theta_{2A}\frac{\partial^2 \theta_{2A}}{\partial \theta_1^2} \\ \frac{\partial^2 y_{s2}}{\partial \theta_1^2} &= -l_{s2}\sin\theta_{2A}\left(\frac{\partial \theta_{2A}}{\partial \theta_1}\right)^2 + l_{s2}\cos\theta_{2A}\frac{\partial^2 \theta_{2A}}{\partial \theta_1^2} \\ \frac{\partial^2 x_{s2}}{\partial \alpha^2} &= l_{s2}\sin\theta_{2A}\left(\frac{\partial \theta_{2A}}{\partial \alpha}\right)^2 - l_{s2}\cos\theta_{2A}\frac{\partial^2 \theta_{2A}}{\partial \alpha^2} \\ \frac{\partial^2 y_{s2}}{\partial \alpha^2} &= -l_{s2}\sin\theta_{2A}\left(\frac{\partial \theta_{2A}}{\partial \alpha}\right)^2 - l_{s2}\cos\theta_{2A}\frac{\partial^2 \theta_{2A}}{\partial \alpha^2} \\ \frac{\partial^2 x_{s2}}{\partial \theta_1 \partial \alpha} &= -l_{s2}\cos\theta_{2A}\frac{\partial \theta_{2A}}{\partial \alpha}\frac{\partial \theta_{2A}}{\partial \theta_1} - l_{s2}\sin\theta_{2A}\frac{\partial^2 \theta_{2A}}{\partial \theta_1 \partial \alpha} \\ \frac{\partial^2 y_{S2}}{\partial \theta_1 \partial \alpha} &= -l_{S2}\sin\theta_{2A}\frac{\partial \theta_{2A}}{\partial \alpha}\frac{\partial \theta_{2A}}{\partial \theta_1} + l_{S2}\cos\theta_{2A}\frac{\partial \theta_{2A}}{\partial \theta_1 \partial \alpha}\end{aligned} \tag{86}$$

In the equations, $\frac{\partial \theta_{2A}}{\partial \theta_1}, \frac{\partial \theta_{2A}}{\partial \alpha}, \frac{\partial^2 \theta_{2A}}{\partial \theta_1^2}, \frac{\partial \theta_{2A}}{\partial \alpha^2}, \frac{\partial^2 \theta_{2A}}{\partial \theta_1 \partial \alpha}$ are respectively derived from Equations (67), (71), (82)-(84).

And thus, the equations are derived from Equations (77)-(81).

$$\begin{aligned}\ddot{\theta}_2=\ddot{\theta}_{2A}&=\left(\frac{\partial^2\theta_{2A}}{\partial\theta_1^2}\dot{\theta}_1+\frac{\partial^2\theta_{2A}}{\partial\theta_1\partial\alpha}\dot{\alpha}\right)\dot{\theta}_1+\frac{\partial\theta_{2A}}{\partial\theta_1}\ddot{\theta}_1+\left(\frac{\partial^2\theta_{2A}}{\partial\alpha\partial\theta_1}\dot{\theta}_1+\frac{\partial^2\theta_{2A}}{\partial\alpha^2}\dot{\alpha}\right)\dot{\alpha}+\frac{\partial\theta_{2A}}{\partial\alpha}\ddot{\alpha}\\&=\frac{\partial\theta_{2A}}{\partial\theta_1}\ddot{\theta}_1+\frac{\partial^2\theta_{2A}}{\partial\theta_1^2}\dot{\theta}_1^2+2\frac{\partial^2\theta_{2A}}{\partial\theta_1\partial\alpha}\dot{\alpha}\dot{\theta}_1+\frac{\partial\theta_{2A}}{\partial\alpha}\ddot{\alpha}+\frac{\partial^2\theta_{2A}}{\partial\alpha^2}\dot{\alpha}^2\end{aligned}\tag{87}$$

$$\begin{aligned}\ddot{x}_{s1}&=\left(\frac{\alpha^2x_{s1}}{\partial\theta_1^2}\dot{\theta}_1+\frac{\alpha^2x_{s1}}{\partial\theta_1\partial\alpha}\dot{\alpha}\right)\dot{\theta}_1+\frac{\partial x_{s1}}{\partial\theta_1}\ddot{\theta}_1+\left(\frac{\alpha^2x_{s1}}{\partial\theta_1\partial\alpha}\dot{\theta}_1+\frac{\partial^2x_{s1}}{\partial\alpha^2}\dot{\alpha}\right)\dot{\alpha}+\frac{\partial x_{s1}}{\partial\alpha}\ddot{\alpha}\\&=\frac{\alpha^2x_{s1}}{\partial\theta_1^2}\dot{\theta}_1^2+2\frac{\alpha^2x_{s1}}{\partial\theta_1\partial\alpha}\dot{\alpha}\dot{\theta}_1+\frac{\partial x_{s1}}{\partial\theta_1}\ddot{\theta}_1+\frac{\partial^2x_{s1}}{\partial\alpha^2}\dot{\alpha}^2+\frac{\partial x_{s1}}{\partial\alpha}\ddot{\alpha}\end{aligned}\tag{88}$$

$$\begin{aligned}\ddot{x}_{s2}&=\left(\frac{\alpha^2x_{s2}}{\partial\theta_1^2}\dot{\theta}_1+\frac{\alpha^2x_{s2}}{\partial\theta_1\partial\alpha}\dot{\alpha}\right)\dot{\theta}_1+\frac{\partial x_{s2}}{\partial\theta_1}\ddot{\theta}_1+\left(\frac{\alpha^2x_{s2}}{\partial\theta_1\partial\alpha}\dot{\theta}_1+\frac{\partial^2x_{s2}}{\partial\alpha^2}\dot{\alpha}\right)\dot{\alpha}+\frac{\partial x_{s2}}{\partial\alpha}\ddot{\alpha}\\&=\frac{\alpha^2x_{s2}}{\partial\theta_1^2}\dot{\theta}_1^2+2\frac{\alpha^2x_{s2}}{\partial\theta_1\partial\alpha}\dot{\alpha}\dot{\theta}_1+\frac{\partial x_{s2}}{\partial\theta_1}\ddot{\theta}_1+\frac{\partial^2x_{s2}}{\partial\alpha^2}\dot{\alpha}^2+\frac{\partial x_{s2}}{\partial\alpha}\ddot{\alpha}\end{aligned}\tag{89}$$

$$\begin{aligned}\ddot{y}_{s1}&=\left(\frac{\alpha^2y_{s1}}{\partial\theta_1^2}\dot{\theta}_1+\frac{\alpha^2y_{s1}}{\partial\theta_1\partial\alpha}\dot{\alpha}\right)\dot{\theta}_1+\frac{\partial y_{s1}}{\partial\theta_1}\ddot{\theta}_1+\left(\frac{\alpha^2y_{s1}}{\partial\theta_1\partial\alpha}\dot{\theta}_1+\frac{\partial^2y_{s1}}{\partial\alpha^2}\dot{\alpha}\right)\dot{\alpha}+\frac{\partial y_{s1}}{\partial\alpha}\ddot{\alpha}\\&=\frac{\alpha^2y_{s1}}{\partial\theta_1^2}\dot{\theta}_1^2+2\frac{\alpha^2y_{s1}}{\partial\theta_1\partial\alpha}\dot{\alpha}\dot{\theta}_1+\frac{\partial y_{s1}}{\partial\theta_1}\ddot{\theta}_1+\frac{\partial^2y_{s1}}{\partial\alpha^2}\dot{\alpha}^2+\frac{\partial y_{s1}}{\partial\alpha}\ddot{\alpha}\end{aligned}\tag{90}$$

$$\begin{aligned}\ddot{y}_{s2}&=\left(\frac{\alpha^2y_{s2}}{\partial\theta_1^2}\dot{\theta}_1+\frac{\alpha^2y_{s2}}{\partial\theta_1\partial\alpha}\dot{\alpha}\right)\dot{\theta}_1+\frac{\partial y_{s2}}{\partial\theta_1}\ddot{\theta}_1+\left(\frac{\alpha^2y_{s2}}{\partial\theta_1\partial\alpha}\dot{\theta}_1+\frac{\partial^2y_{s2}}{\partial\alpha^2}\dot{\alpha}\right)\dot{\alpha}+\frac{\partial y_{s2}}{\partial\alpha}\ddot{\alpha}\\&=\frac{\alpha^2y_{s2}}{\partial\theta_1^2}\dot{\theta}_1^2+2\frac{\alpha^2y_{s2}}{\partial\theta_1\partial\alpha}\dot{\alpha}\dot{\theta}_1+\frac{\partial y_{s2}}{\partial\theta_1}\ddot{\theta}_1+\frac{\partial^2y_{s2}}{\partial\alpha^2}\dot{\alpha}^2+\frac{\partial y_{s2}}{\partial\alpha}\ddot{\alpha}\end{aligned}\tag{91}$$

4.4. Equation of Motion in Configuration 2

The kinetic energy of the metamorphic mechanism in Configuration 2 is as follows:

$$E=\frac{1}{2}m_1\left(\dot{x}_{s1}^2+\dot{y}_{s1}^2\right)+\frac{1}{2}J_1\dot{\theta}_1^2+\frac{1}{2}m_2\left(\dot{x}_{s2}^2+\dot{y}_{s2}^2\right)+\frac{1}{2}J_2\dot{\theta}_2^2\tag{92}$$

The potential energy of the metamorphic mechanism in Configuration 2 is as follows:

$$U=m_1gy_{s1}+m_2gy_{s2}\tag{93}$$

The new mechanism without clearance by adding a hypothetical link in Configuration 2 has two degrees of freedom. Taking " $q_1=\theta_1$, $q_2=a$ " as the generalized coordinate, the generalized force is the driving torque acting on link 1.

$$F_1=T,F_2=0\tag{94}$$

The following equations are derived from Equation (77) to get the derivative of $\dot{\theta}_1,\dot{\alpha}$:

$$\begin{aligned}\frac{\partial\dot{\theta}_{2A}}{\partial\dot{\theta}_1}&=\frac{\partial\theta_{2A}}{\partial\theta_1}\\\frac{\partial\dot{\theta}_{2A}}{\partial\dot{\alpha}}&=\frac{\partial\theta_{2A}}{\partial\alpha}\end{aligned}\tag{95}$$

So

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{\theta}_{2A}}{\partial\dot{\theta}_1}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\theta_{2A}}{\partial\theta_1}\right)=\frac{\partial\dot{\theta}_{2A}}{\partial\theta_1}\\ \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{\theta}_{2A}}{\partial\dot{\alpha}}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\theta_{2A}}{\partial\alpha}\right)=\frac{\partial\dot{\theta}_{2A}}{\partial\alpha}\end{aligned}\tag{96}$$

The following equations are derived from Equation (78) to get the derivative of $\dot{\theta}_1,\dot{\alpha}$:

$$\begin{aligned}\frac{\partial\dot{x}_{s1}}{\partial\dot{\theta}_1}=\frac{\partial x_{s1}}{\partial\theta_1}\\ \frac{\partial\dot{x}_{s1}}{\partial\dot{\alpha}}=\frac{\partial x_{s1}}{\partial\alpha}\end{aligned}\tag{97}$$

So

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{x}_{s1}}{\partial\dot{\theta}_1}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_{s1}}{\partial\theta_1}\right)=\frac{\partial\dot{x}_{s1}}{\partial\theta_1}\\ \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{x}_{s1}}{\partial\dot{\alpha}}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_{s1}}{\partial\alpha}\right)=\frac{\partial\dot{x}_{s1}}{\partial\alpha}\end{aligned}\tag{98}$$

The following equations are derived from Equation (79) to get the derivative of $\dot{\theta}_1,\dot{\alpha}$:

$$\begin{aligned}\frac{\partial\dot{y}_{s1}}{\partial\dot{\theta}_1}=\frac{\partial y_{s1}}{\partial\theta_1}\\ \frac{\partial\dot{y}_{s1}}{\partial\dot{\alpha}}=\frac{\partial y_{s1}}{\partial\alpha}\end{aligned}\tag{99}$$

So

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{y}_{s1}}{\partial\dot{\theta}_1}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial y_{s1}}{\partial\theta_1}\right)=\frac{\partial\dot{y}_{s1}}{\partial\theta_1}\\ \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{y}_{s1}}{\partial\dot{\alpha}}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial y_{s1}}{\partial\alpha}\right)=\frac{\partial\dot{y}_{s1}}{\partial\alpha}\end{aligned}\tag{100}$$

The following equations are derived from Equation (80) to get the derivative of $\dot{\theta}_1,\dot{\alpha}$:

$$\begin{aligned}\frac{\partial\dot{x}_{s2}}{\partial\dot{\theta}_1}=\frac{\partial x_{s2}}{\partial\theta_1}\\ \frac{\partial\dot{x}_{s2}}{\partial\dot{\alpha}}=\frac{\partial x_{s2}}{\partial\alpha}\end{aligned}\tag{101}$$

So

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{x}_{s2}}{\partial\dot{\theta}_1}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_{s2}}{\partial\theta_1}\right)=\frac{\partial\dot{x}_{s2}}{\partial\theta_1}\\ \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\dot{x}_{s2}}{\partial\dot{\alpha}}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial x_{s2}}{\partial\alpha}\right)=\frac{\partial\dot{x}_{s2}}{\partial\alpha}\end{aligned}\tag{102}$$

The following equations are derived from Equation (81) to get the derivative of

$\dot{\theta}_1, \dot{\alpha}$:

$$\begin{aligned} \frac{\partial \dot{y}_{s2}}{\partial \dot{\theta}_1} &= \frac{\partial y_{s2}}{\partial \theta_1} \\ \frac{\partial \dot{y}_{s2}}{\partial \dot{\alpha}} &= \frac{\partial y_{s2}}{\partial \alpha} \end{aligned} \tag{103}$$

So

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \dot{y}_{s2}}{\partial \dot{\theta}_1}\right) &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial y_{s2}}{\partial \theta_1}\right) = \frac{\partial \dot{y}_{s2}}{\partial \theta_1} \\ \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \dot{y}_{s2}}{\partial \dot{\alpha}}\right) &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial y_{s2}}{\partial \alpha}\right) = \frac{\partial \dot{y}_{s2}}{\partial \alpha} \end{aligned} \tag{104}$$

Therefore, the following equations are derived by substituting Equations (92)-(94) into the Lagrange equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial \dot{q}_j}\right) - \frac{\partial E}{\partial q_j} + \frac{\partial U}{\partial q_j} = F_j \qquad j = 1, 2$$

$$j = 1,\ q_1 = \theta_1,\ F_1 = T$$

So, the following equation is derived:

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{1}{2}m_1\left(\frac{\partial \dot{x}_{s1}^2}{\partial \dot{\theta}_1} + \frac{\partial \dot{y}_{s1}^2}{\partial \dot{\theta}_1}\right) + \frac{1}{2}J_1\frac{\partial \dot{\theta}_1^2}{\partial \dot{\theta}_1} + \frac{1}{2}m_2\left(\frac{\partial \dot{x}_{s2}^2}{\partial \dot{\theta}_1} + \frac{\partial \dot{y}_{s2}^2}{\partial \dot{\theta}_1}\right) + \frac{1}{2}J_2\frac{\partial \dot{\theta}_2^2}{\partial \dot{\theta}_1}\right\} \\ &-\left\{\frac{1}{2}m_1\left(\frac{\partial \dot{x}_{s1}^2}{\partial \theta_1} + \frac{\partial \dot{y}_{s1}^2}{\partial \theta_1}\right) + \frac{1}{2}J_1\frac{\partial \dot{\theta}_1^2}{\partial \theta_1} + \frac{1}{2}m_2\left(\frac{\partial \dot{x}_{s2}^2}{\partial \theta_1} + \frac{\partial \dot{y}_{s2}^2}{\partial \theta_1}\right) + \frac{1}{2}J_2\frac{\partial \dot{\theta}_2^2}{\partial \theta_1}\right\} \\ &+ m_1 g\frac{\partial y_{s1}}{\partial \theta_1} + m_2 g\frac{y_{s2}}{\partial \theta_1} = T \end{aligned} \tag{105}$$

$$\begin{aligned} &m_1\left\{\ddot{x}_{s1}\frac{\partial \dot{x}_{s1}}{\partial \dot{\theta}_1} + \dot{x}_{s1}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \dot{x}_{s1}}{\partial \dot{\theta}_1}\right) + \ddot{y}_{s1}\frac{\partial \dot{y}_{s1}}{\partial \dot{\theta}_1} + \dot{y}_{s1}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \dot{y}_{s1}}{\partial \dot{\theta}_1}\right)\right\} + J_1\ddot{\theta}_1\frac{\partial \dot{\theta}_1}{\partial \dot{\theta}_1} \\ &+ J_1\dot{\theta}_1\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \dot{\theta}_1}{\partial \dot{\theta}_1}\right) + m_2\left\{\ddot{x}_{s2}\frac{\partial \dot{x}_{s2}}{\partial \dot{\theta}_1} + \dot{x}_{s2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \dot{x}_{s2}}{\partial \dot{\theta}_1}\right) + \ddot{y}_{s2}\frac{\partial \dot{y}_{s2}}{\partial \dot{\theta}_1} + \dot{y}_{s2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \dot{y}_{s2}}{\partial \dot{\theta}_1}\right)\right\} \\ &+ J_2\ddot{\theta}_2\frac{\partial \dot{\theta}_2}{\partial \dot{\theta}_1} + J_2\dot{\theta}_2\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \dot{\theta}_2}{\partial \dot{\theta}_1}\right) - \left\{m_1\left(\dot{x}_{s1}\frac{\partial \dot{x}_{s1}}{\partial \theta_1} + \dot{y}_{s1}\frac{\partial \dot{y}_{s1}}{\partial \theta_1}\right) + J_1\dot{\theta}_1\frac{\partial \dot{\theta}_1}{\partial \theta_1}\right. \\ &\left. + m_2\left(\dot{x}_{s2}\frac{\partial \dot{x}_{s2}}{\partial \theta_1} + \dot{y}_{s2}\frac{\partial \dot{y}_{s2}}{\partial \theta_1}\right) + J_2\dot{\theta}_2\frac{\partial \dot{\theta}_2}{\partial \theta_1}\right\} + m_1 g\frac{\partial y_{s1}}{\partial \theta_1} + m_2 g\frac{y_{s2}}{\partial \theta_1} = T \end{aligned} \tag{106}$$

Substituting the Equations (from (95) to (104)) into Equation (69), the following equations are derived:

$$\begin{aligned} &m_1\frac{\partial x_{s1}}{\partial \theta_1}\ddot{x}_{s1} + m_1\frac{\partial y_{s1}}{\partial \theta_1}\ddot{y}_{s1} + J_1\ddot{\theta}_1 + m_2\frac{\partial x_{s2}}{\partial \theta_1}\ddot{x}_{s2} + m_2\frac{\partial y_{s2}}{\partial \theta_1}\ddot{y}_{s2} \\ &+ J_2\frac{\partial \theta_2}{\partial \theta_1}\ddot{\theta}_2 + m_1 g\frac{\partial y_{s1}}{\partial \theta_1} + m_2 g\frac{y_{s2}}{\partial \theta_1} = T \end{aligned} \tag{107}$$

Substituting $\ddot{\theta}_2, \ddot{x}_{s1}, \ddot{y}_{s1}, \ddot{x}_{s2}, \ddot{y}_{s2}$ in the Equations (from (87) to (91)) into Equation (107) and considering Equation (58), the following equations are derived:

$$\begin{aligned}
&J_1\ddot{\theta}_1+J_2\frac{\partial\theta_{2A}}{\partial\theta_1}\left(\frac{\partial\theta_{2A}}{\partial\theta_1}\ddot{\theta}_1+\frac{\partial\theta_{2A}}{\partial\alpha}\ddot{\alpha}+\frac{\partial^2\theta_{2A}}{\partial\theta_1^2}\dot{\theta}_1^2+\frac{\partial^2\theta_{2A}}{\partial\alpha^2}\dot{\alpha}^2+2\frac{\partial^2\theta_{2A}}{\partial\theta_1\partial\alpha}\dot{\alpha}\dot{\theta}_1\right)\\
&+m_1\frac{\partial x_{s1}}{\partial\theta_1}\left(\frac{\partial x_{s1}}{\partial\theta_1}\ddot{\theta}_1+\frac{\partial x_{s1}}{\partial\alpha}\ddot{\alpha}+\frac{\partial^2 x_{s1}}{\partial\theta_1^2}\dot{\theta}_1^2+\frac{\partial^2 x_{s1}}{\partial\alpha^2}\dot{\alpha}^2+2\frac{\partial^2 x_{s1}}{\partial\theta_1\partial\alpha}\dot{\theta}_1\dot{\alpha}\right)\\
&+m_1\frac{\partial y_{s1}}{\partial\theta_1}\left(\frac{\partial y_{s1}}{\partial\theta_1}\ddot{\theta}_1+\frac{\partial y_{s1}}{\partial\alpha}\ddot{\alpha}+\frac{\partial^2 y_{s1}}{\partial\theta_1^2}\dot{\theta}_1^2+\frac{\partial^2 y_{s1}}{\partial\alpha^2}\dot{\alpha}^2+2\frac{\partial^2 y_{s1}}{\partial\theta_1\partial\alpha}\dot{\theta}_1\dot{\alpha}\right)\\
&+m_2\frac{\partial x_{s2}}{\partial\theta_1}\left(\frac{\partial x_{s2}}{\partial\theta_1}\ddot{\theta}_1+\frac{\partial x_{s2}}{\partial\alpha}\ddot{\alpha}+\frac{\partial^2 x_{s2}}{\partial\theta_1^2}\dot{\theta}_1^2+\frac{\partial^2 x_{s2}}{\partial\alpha^2}\dot{\alpha}^2+2\frac{\partial^2 x_{s2}}{\partial\theta_1\partial\alpha}\dot{\theta}_1\dot{\alpha}\right)\\
&+m_2\frac{\partial y_{s2}}{\partial\theta_1}\left(\frac{\partial y_{s2}}{\partial\theta_1}\ddot{\theta}_1+\frac{\partial y_{s2}}{\partial\alpha}\ddot{\alpha}+\frac{\partial^2 y_{s2}}{\partial\theta_1^2}\dot{\theta}_1^2+\frac{\partial^2 y_{s2}}{\partial\alpha^2}\dot{\alpha}^2+2\frac{\partial^2 y_{s2}}{\partial\theta_1\partial\alpha}\dot{\theta}_1\dot{\alpha}\right)\\
&+m_1gy_{s1}+m_2gy_{s2}=T
\end{aligned}\tag{108}$$

From Equation (108), the following equations are derived:

$$\begin{aligned}
&\left\{J_1+J_2\left(\frac{\partial\theta_{2A}}{\partial\theta_1}\right)^2+m_1\left(\frac{\partial x_{s1}}{\partial\theta_1}\right)^2+m_1\left(\frac{\partial y_{s1}}{\partial\theta_1}\right)^2+m_2\left(\frac{\partial x_{s2}}{\partial\theta_1}\right)^2+m_2\left(\frac{\partial y_{s2}}{\partial\theta_1}\right)^2\right\}\ddot{\theta}_1\\
&+\left\{J_2\frac{\partial\theta_{2A}}{\partial\theta_1}\frac{\partial\theta_{2A}}{\partial\alpha}+m_1\frac{\partial x_{s1}}{\partial\theta_1}\frac{\partial x_{s1}}{\partial\alpha}+m_1\frac{\partial y_{s1}}{\partial\theta_1}\frac{\partial y_{s1}}{\partial\alpha}+m_2\frac{\partial x_{s2}}{\partial\theta_1}\frac{\partial x_{s2}}{\partial\alpha}+m_2\frac{\partial y_{s2}}{\partial\theta_1}\frac{\partial y_{s2}}{\partial\alpha}\right\}\ddot{\alpha}=T\\
&-\left\{J_2\frac{\partial\theta_{2A}}{\partial\theta_1}\frac{\partial^2\theta_{2A}}{\partial\theta_1^2}+m_1\frac{\partial x_{s1}}{\partial\theta_1}\frac{\partial^2 x_{s1}}{\partial\theta_1^2}+m_1\frac{\partial y_{s1}}{\partial\theta_1}\frac{\partial^2 y_{s1}}{\partial\theta_1^2}+m_2\frac{\partial x_{s2}}{\partial\theta_1}\frac{\partial^2 x_{s2}}{\partial\theta_1^2}+m_2\frac{\partial y_{s2}}{\partial\theta_1}\frac{\partial y_{s2}}{\partial\alpha}\right\}\dot{\theta}_1^2\\
&-\left\{J_2\frac{\partial\theta_{2A}}{\partial\theta_1}\frac{\partial^2\theta_{2A}}{\partial\alpha^2}+m_1\frac{\partial x_{s1}}{\partial\theta_1}\frac{\partial^2 x_{s1}}{\partial\alpha^2}+m_1\frac{\partial y_{s1}}{\partial\theta_1}\frac{\partial^2 y_{s1}}{\partial\alpha^2}+m_2\frac{\partial x_{s2}}{\partial\theta_1}\frac{\partial^2 x_{s2}}{\partial\alpha^2}+m_2\frac{\partial y_{s2}}{\partial\theta_1}\frac{\partial^2 y_{s2}}{\partial\alpha^2}\right\}\dot{\alpha}^2\\
&-2\left\{J_2\frac{\partial\theta_{2A}}{\partial\theta_1}\frac{\partial^2\theta_{2A}}{\partial\alpha^2}+m_1\frac{\partial x_{s1}}{\partial\theta_1}\frac{\partial^2 x_{s1}}{\partial\theta_1\partial\alpha}+m_1\frac{\partial y_{s1}}{\partial\theta_1}\frac{\partial^2 y_{s1}}{\partial\theta_1\partial\alpha}+m_2\frac{\partial x_{s2}}{\partial\theta_1}\frac{\partial^2 x_{s2}}{\partial\theta_1\partial\alpha}\theta_1\right.\\
&\left.+m_2\frac{\partial y_{s2}}{\partial\theta_1}\frac{\partial^2 y_{s2}}{\partial\theta_1\partial\alpha}\right\}\dot{\theta}_1\dot{\alpha}-m_1g\frac{\partial y_{s1}}{\partial\alpha}-m_2g\frac{\partial y_{s2}}{\partial\alpha}
\end{aligned}\tag{109}$$

Equation (109) is written in a simplified form as follows:

$$A_5\ddot{\theta}_1+A_6\ddot{\alpha}=B_3\tag{110}$$

Among which, A_5, A_6 is the functions of θ_1, α. B_3 is the function of $\theta_1, \alpha, \dot{\theta}_1, \dot{\alpha}$.

When $j=2, q_2=\alpha, F_2=0$, the equations are derived from the Lagrange equation.

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{1}{2}m_1\left(2x_{s1}\frac{\partial\dot{x}_{s1}}{\partial\dot{\alpha}}+2y_{s1}\frac{\partial\dot{y}_{s1}}{\partial\dot{\alpha}}\right)+\frac{1}{2}J_1 2\dot{\theta}_1\frac{\partial\dot{\theta}_1}{\partial\dot{\alpha}}+\frac{1}{2}m_1\left(x_{s1}\frac{\partial\dot{x}_{s2}}{\partial\dot{\alpha}}+2y_{s1}\frac{\partial\dot{y}_{s2}}{\partial\dot{\alpha}}\right)\right.\\
&\left.\frac{1}{2}J_2 2\dot{\theta}_2\frac{\partial\dot{\theta}_2}{\partial\dot{\alpha}}\right\}-\left\{\frac{1}{2}m_1\left(2x_{s1}\frac{\partial\dot{x}_{s1}}{\partial\alpha}+2y_{s1}\frac{\partial\dot{y}_{s1}}{\partial\alpha}\right)+J_1\dot{\theta}_1\frac{\partial\dot{\theta}_1}{\partial\alpha}+\frac{1}{2}m_2\left(2x_{s1}\frac{\partial\dot{x}_{s1}}{\partial\alpha}\right.\right.\\
&\left.\left.+2y_{s1}\frac{\partial\dot{y}_{s1}}{\partial\alpha}+J_2 2\dot{\theta}_2\frac{\partial\dot{\theta}_2}{\partial\alpha}\right)\right\}+m_1g\frac{\partial y_{s1}}{\partial\alpha}+m_2g\frac{\partial y_{s2}}{\partial\alpha}=0
\end{aligned}\tag{111}$$

Since Equation (111) shares the same structure as Equation (105), the equations are derived through the same process as follows:

$$\begin{aligned}
&\left\{J_2\frac{\partial\theta_{2A}}{\partial\alpha}\frac{\partial\theta_{2A}}{\partial\theta_1}+m_1\frac{\partial x_{s1}}{\partial\alpha}\frac{\partial x_{s1}}{\partial\theta_1}+m_1\frac{\partial y_{s1}}{\partial\alpha}\frac{\partial y_{s1}}{\partial\theta_1}+m_2\frac{\partial x_{s2}}{\partial\alpha}\frac{\partial x_{s2}}{\partial\theta_1}+m_2\frac{\partial y_{s2}}{\partial\alpha}\frac{\partial y_{s2}}{\partial\theta_1}\right\}\ddot{\theta}_1\\
&+\left\{J_2\left(\frac{\partial\theta_{2A}}{\partial\alpha}\right)^2+m_1\left(\frac{\partial x_{s1}}{\partial\alpha}\right)^2+m_1\left(\frac{\partial y_{s1}}{\partial\alpha}\right)^2+m_2\left(\frac{\partial x_{s2}}{\partial\alpha}\right)^2+m_2\left(\frac{\partial y_{s2}}{\partial\alpha}\right)^2\right\}\ddot{\alpha}\\
&=-\left\{J_2\frac{\partial\theta_{2A}}{\partial\alpha}\frac{\partial^2\theta_2}{\partial\theta_1^2}+m_1\frac{\partial x_{s1}}{\partial\alpha}\frac{\partial^2 x_{s1}}{\partial\theta_1^2}+m_1\frac{\partial y_{s1}}{\partial\alpha}\frac{\partial^2 y_{s1}}{\partial\theta_1^2}+m_2\frac{\partial x_{s2}}{\partial\alpha}\frac{\partial^2 x_{s2}}{\partial\theta_1^2}\right.\\
&\left.+m_2\frac{\partial y_{s2}}{\partial\alpha}\frac{\partial^2 y_{s2}}{\partial\theta_1^2}\right\}\dot{\theta}_1^2-\left\{J_2\frac{\partial\theta_{2A}}{\partial\alpha}\frac{\partial^2\theta_{2A}}{\partial\theta_1^2}+m_1\frac{\partial x_{s1}}{\partial\alpha}\frac{\partial^2 x_{s1}}{\partial\alpha_1^2}+m_1\frac{\partial y_{s1}}{\partial\alpha}\frac{\partial^2 y_{s1}}{\partial\alpha^2}\right.\\
&\left.+m_2\frac{\partial x_{s2}}{\partial\alpha}\frac{\partial^2 x_{s2}}{\partial\alpha^2}+m_2\frac{\partial y_{s2}}{\partial\alpha}\frac{\partial^2 y_{s2}}{\partial\alpha^2}\right\}\dot{\alpha}^2-2\left\{J_2\frac{\theta_{2A}}{\partial\alpha}\frac{\partial^2\theta_{2A}}{\partial\theta_1\partial\alpha}+m_1\frac{\partial x_{s1}}{\partial\alpha}\frac{\partial^2 x_{s1}}{\partial\theta_1\partial\alpha}\right.\\
&\left.+m_1\frac{\partial y_{s1}}{\partial\alpha}\frac{\partial^2 y_{s1}}{\partial\theta_1\partial\alpha}+m_2\frac{\partial x_{s2}}{\partial\alpha}\frac{\partial^2 x_{s2}}{\partial\theta_1\partial\alpha}+m_2\frac{\partial y_{s2}}{\partial\alpha}\frac{\partial^2 y_{s2}}{\partial\theta_1\partial\alpha}\right\}\dot{\theta}_1\dot{\alpha}-m_1g\frac{\partial y_{s1}}{\partial\alpha}-m_2g\frac{\partial y_{s2}}{\partial\alpha}
\end{aligned}\tag{112}$$

Equation (112) is written in a simplified form as follows:

$$A_1\ddot{\theta}_1+A_8\ddot{\alpha}=B_4 \tag{113}$$

Among which, A_1, A_8 are the functions of θ_1,α. B_4 is the function of $\theta_1,\alpha,\dot{\theta}_1,\dot{\alpha}$.

By solving the simultaneous equations derived from Equation (110) and Equation (113), we can obtain:

$$\ddot{\theta}_1 = f_1\left(\theta_1,\alpha,\dot{\theta}_1,\dot{\alpha}\right),\ \ \ddot{\alpha} = f_2\left(\theta_1,\alpha,\dot{\theta}_1,\dot{\alpha}\right).$$

This second-order nonlinear equation represents the differential equation of motion, and the solution is a function of time. If the driving link, Crank 1 rotates uniformly, the solution becomes simpler, where ω is constant. In this case, $\ddot{\theta}_1=0$, $\theta_1=\omega t$ becomes a known function in Equation (113). Thus, from Equation (113), we can derive $\ddot{\alpha}=f_2\left(t,\alpha,\dot{\alpha}\right)$. Substituting the obtained $\alpha=\alpha\left(t\right)$ into Equation (110), it can be achieved $T=T\left(t\right)$, which represents the torque function required to ensure the uniform rotation of the driving link, Crank 1.

5. Conclusions

Research on traditional fixed-constraint joint clearance models has shown that although continuous contact models are unable to investigate phenomena such as collisions and impacts between pair elements, they can reflect the influence of kinematic pair with Clearance on the dynamics of mechanisms. Due to their ease of application in dynamic performance analysis of mechanisms, they are favored by mechanism researchers as well as engineers, and have been widely adopted. Compared to traditional fixed-constraint transmission pairs with clearance, variable-constraint metamorphic pairs with clearance share common aspects but also exhibit unique characteristics. Generally, as the configuration of the mechanism changes, the constraints and properties of metamorphic pairs may alter.

The clearance in metamorphic pairs has a significant impact on the dynamic performance of mechanisms, yet there has been scant research in this area. Taking the groove-pin metamorphic kinematic pair as the subject, continuous contact models are established separately for its two configurations within one cycle, which reveals the internal relationships between driving torque and the displacement, velocity, acceleration, clearance state, and other attributes of each component, providing a convenient analytical model for further research on the influence of clearance in metamorphic pairs on the dynamic performance of mechanisms.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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