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CO_2 Forcing of Changes in Lower Tropospheric Temperatures: A Time Series Analysis

Peter R. Hartley

Economics Department, and Rice Scholar in the Center for Energy Studies, James A. Baker III Institute for Public Policy, Rice University, Houston, USA
Email: hartley@rice.edu

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Abstract

A simple forcing-feedback model of the effect of CO_2 on temperatures implies that the natural logarithm of the concentration of CO_2 in the atmosphere, $\ln C$, and satellite-measured lower tropospheric temperatures, T, should have the same order of integration. Multiple tests provide strong evidence that $\ln C$ has a unit root (it is integrated of order 1). However, the hypothesis that T has a unit root is strongly rejected both for the globe as a whole and for a set of non-overlapping geographic subsets. A possible explanation is that CO_2 accumulation is associated with some other trending process that moderates its effects on atmospheric temperatures in such a way as to prevent T from also having a unit root. The analysis also reveals interactions between temperature anomalies in the non-overlapping geographic subsets that further limit the range of models consistent with the evidence.

Keywords

Satellite-Measured Temperature Anomalies, Forcing-Feedback Climate Model, Stationarity, Autoregressive Integrated Moving-Average (ARIMA), Autoregressive Fractionally Integrated Moving-Average (ARFIMA), Vector Autoregression (VAR)

1. Introduction

Romps et al. (2022) observe that it is "well known" and a "basic fact" of climate science that "the radiative forcing from carbon dioxide is approximately logarithmic in its concentration".[1] Lightfoot & Mamer (2014) noted that Arrhenius postulated in 1896 that the relationship was logarithmic. They credit the Third

[1] Romps et al. (2022) observe that the literature offers two different explanations for this result. One is based on the CO_2 absorption spectrum, while the other claims that it "stems from the troposphere's lapse rate". They favor the former explanation.

Assessment Report of the IPCC with developing the simplified logarithmic equation $\Delta RF = 5.35\ln\left(C/C_0\right)$ as a reasonable approximation.

An initial increase in temperatures produced by increased forcing induces further changes that enhance the initial warming. These include increases in atmospheric water vapor, which is itself a powerful greenhouse gas. Despite the complexity of climate models, many papers have shown that they predict an approximately linear relationship between radiative forcing and induced temperature. For example, Webb et al. (2013) examine the predicted temperature sensitivity to increased CO_2 forcing in 27 climate models. They find it to be approximately linear in all of them, although the calculated sensitivity varies across the models and across latitude bands.

These two considerations can be represented in a simple forcing-feedback model of the relationship between CO_2 accumulation in the atmosphere C_t and lower tropospheric temperatures T_t as follows:

$$T_t = \alpha + \frac{R}{1-f}\ln C_t + u_t \tag{1}$$

where α, R, and the feedback parameter $f < 1$ are constants, and the subscript t represents time, which will be months in our data. The random error term u_t represents effects on T_t apart from CO_2 accumulation and measurement errors for the variables in the model. Under the hypothesis that greenhouse gas emissions by humans are the only destabilizing influences on T_t in the long run, u_t will be stationary.[2] Equation (1) then implies that T_t and $\ln C_t$ should have the same *order of integration*.[3] Section 3 tests this prediction using satellite observations of global monthly temperature anomalies for the lower troposphere produced by the University of Alabama at Huntsville (UAH). The evidence rejects the hypothesis by finding T_t to be stationary while $\ln C_t$ is integrated of order 1. Section 4 extends the analysis by examining a complete set of non-overlapping geographic subsets of the temperature series. It supplies several additional lines of evidence that reinforce the result that lower troposphere temperatures are stationary.

2. CO_2 and Global Average Temperature Anomalies

To completely characterize the time series relationship between T and $\ln C$, feedback from T to $\ln C$ must be taken into account. Models of CO_2 accumulation in the atmosphere, such as *Carbon Tracker* developed by the National

[2]A time series X_t is stationary (strictly speaking "weakly covariance stationary") if the mean $\mathbb{E}X_t \equiv \mu_t$, variance $\mathbb{E}\left(X_t - \mu_t\right)^2 \equiv \sigma_t$, and k-th lag autocovariances $\operatorname{cov}\left(X_t, X_{t-k}\right) \equiv \gamma_{t,k}$ do not depend on t.

[3] X_t is said to follow a random walk with drift g, to be integrated of order 1, or to have a unit root if it can be written as $X_t - X_{t-1} \equiv (1-L)X_t = g + u_t$ where L is the lag operator, g is constant, and u_t is a mean-zero random variable with a constant distribution. If the *rate of growth* of X_t, approximated by the change in natural logarithm, follows a random walk with drift g, then $(1-L)\ln X_t = g + u_t$. If the growth rate of X_t changes at a random acceleration $a + u_t$, the process is integrated of order 2 and can be written $\left(\ln X_t - \ln X_{t-1}\right) - \left(\ln X_{t-1} - \ln X_{t-2}\right) \equiv (1-L)^2 \ln X_t = a + u_t$. More generally, if it takes a minimum of d first differences to make $(1-L)^d X_t$ stationary, X_t is integrated of order d.

Oceanic and Atmospheric Administration Global Monitoring Laboratory, imply that, absent anthropogenic emissions, C would converge to a natural level as a result of equilibrating flows between the atmosphere and other reservoirs known as sources or sinks. Dominant among these are biomass (which absorbs CO_2 via photosynthesis and releases it via respiration and decay of organic matter) and the oceans. The ability of the oceans to absorb CO_2, and biochemical processes such as photosynthesis, respiration, and organic decay depend on T.

Dengler (2024) shows that, in the absence of new anthropogenic emissions, atmospheric CO_2 concentration converges in an exponential fashion to a natural level. A simple algebraic representation has a constant fraction $0 < \beta < 1$ of the current period gap between atmospheric CO_2 concentration and the natural level closed in the next period. However, Dengler (2024) also shows that allowing net sequestration in sinks to decrease with T, and hence CO_2 in the atmosphere to increase with T, improves the model.

To keep the model simple, the same endogenous variables as in Equation (1) are used to represent this equilibration process. Specifically, let $\ln \bar{C} + \delta T_t$ denote the long-run equilibrium value that $\ln C_t$ would converge to *if* temperature were to remain at T_t and anthropogenic emissions at zero. The current gap driving net absorption into sinks is then $\ln C_t - \left(\ln \bar{C} + \delta T_t\right)$. Letting $\ln Z_t$ be the direct effect on $\ln C_t$ from current combustion of fossil fuels, the equilibrating process can be written:

$$\ln C_t - \ln \bar{C} - \delta T_t = \beta\left(\ln C_{t-1} - \ln \bar{C} - \delta T_{t-1}\right) + \ln Z_t + v_t \quad (2)$$

where v_t represents measurement errors for the variables in the model and unmeasured influences on CO_2 accumulation in the atmosphere.

Substituting (1) into (2) gives a relationship between Z_t and *net* accumulation C_t:

$$(1 - \beta L)\left[\psi \ln C_t - \ln \bar{C} - \delta\alpha\right] = \ln Z_t + v_t + \delta u_t - \beta\delta u_{t-1} \quad (3)$$

where $\psi \equiv 1 - \delta R/(1 - f)$ is required to be positive for C_t to be positively related to emissions from fossil fuel combustion, as evidence implies. For $f < 1$, this in turn requires $1 > f + \delta R$. Observe also that $f < 1$ implies $\psi < 1$.

Since the industrial revolution, growing economic output has produced growing Z_t. The reason is that energy, which enables a force to do work, is an essential input into economic activity, and fossil fuels still supply more than 80% of the world's primary energy. Assume, therefore, that Z_t is given by:

$$(1 - L)\ln Z_t = \gamma + z_t \quad (4)$$

where $\gamma > 0$ is the *mean* growth rate of emissions from fossil fuel combustion and z_t is another random variable. For this discussion, z_t is assumed to be white noise (its distribution is unchanging). However, the algebra below can be easily modified to accommodate z_t following any stationary ARMA (p, q) process $\phi(L) z_t = a + \theta(L) v_t$, where the autoregressive (AR) $\phi(L) = \sum_{k=1}^{p} \phi_k L^k$ and moving average (MA) $\theta(L) = \sum_{k=1}^{q} \theta_k L^k$ polynomials in L are of order

p and q respectively. For stationarity, the inverse roots of $\phi(L)$ all need to be less than 1 in modulus. If the inverse roots of $\theta(L)$ are also less than 1 in modulus, $\theta(L)$ is invertible into an infinite autoregressive process, $\theta^{-1}(L)\phi(L)z_t = a\theta^{-1}(1) + v_t$, where $\theta^{-1}(1)$, with the number 1 replacing L, is just another constant. If $d \geq 1$ is the minimum integer such that $\phi(L)(1-L)^d z_t = a + \theta(L)v_t$ is a stationary ARMA (p,q), then z_t is integrated of order d and is designated ARIMA (p,d,q).

Take the first difference of (3) by multiplying both sides by $1-L$. Then from (4), the *reduced form* equation characterizing the time series process followed by C_t can be written:

$$(1-\beta L)(1-L)\ln C_t = \frac{\gamma}{\psi} + \frac{1}{\psi}\left[z_t + \delta u_t - \delta(1+\beta)u_{t-1} + \delta\beta u_{t-2} + v_t - v_{t-1}\right] \quad (5)$$

If z_t, u_t and v_t are white noise, $z_t + \delta u_t - \delta(1+\beta)u_{t-1} + \delta\beta u_{t-2} + v_t - v_{t-1}$ can be written in terms of a *fundamental* white noise process, denoted w_t, as $w_t - \theta_1 w_{t-1} + \theta_2 w_{t-2}$ and Equation (5) would imply that $\ln C_t$ follows an ARIMA (1, 1, 2) process.[4] In particular, continued growth in CO_2 emissions imparts a unit root into atmospheric CO_2 concentration.

Multiplying Equation (1) by $(1-\beta L)(1-L)$ and using Equation (5), and after assuming that z_t, u_t and v_t are contemporaneously uncorrelated white noise processes, temperature T_t should also follow an ARIMA (1, 1, 2) process, albeit one with a different constant and different moving average coefficients:

$$\begin{aligned}(1-\beta L)(1-L)T_t = &\frac{\gamma R}{1-f-\delta R} + \frac{R}{1-f-\delta R}\left(1-\theta_1 L + \theta_2 L^2\right)w_t \\ &+ \left[1-(1+\beta)L + \beta L^2\right]u_t\end{aligned} \quad (6)$$

Although Equation (5) and Equation (6) imply that $\ln C$ and T should both have unit roots, Equation (1) and the hypothesis that u_t is stationary imply that an ordinary least squares regression of T_t on $\ln C_t$ should yield residuals that are stationary. In other words, the processes $\ln C$ and T ought to be cointegrated.[5]

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[4]Assuming z_t, u_t and v_t are contemporaneously uncorrelated σ_w^2, θ_1 and θ_2 would satisfy the equations:

$$\begin{aligned}&\left(1+\theta_1^2+\theta_2^2\right)\sigma_w^2 = \sigma_z^2 + 2\delta^2\left(1+\beta+\beta^2\right)\sigma_u^2 + 2\sigma_v^2 \\ &\theta_1\sigma_w^2 = \delta^2(1+\beta)^2\sigma_u^2 + \sigma_v^2 \\ &\theta_2\sigma_w^2 = \delta^2\beta\sigma_u^2\end{aligned}$$

These three equations would include covariances if z_t, u_t and v_t are contemporaneously correlated. Also, the moving average in (5) could be of higher order if z_t, u_t or v_t were themselves autocorrelated.

[5]Two time series that are integrated of the same order (≥1) are cointegrated if a linear combination of them results in a time series that has a lower order of integration. For example, two time series X_t and Y_t that are both integrated of order 1 are cointegrated if there exists a constant α such that $u_t \equiv Y_t - \alpha X_t$ is integrated of order 0, that is, u_t is stationary. The vector $[1-\alpha]$ is called the cointegrating vector.

The hypothesis that $\ln C$ and T are cointegrated has been tested in many papers, starting with Stern & Kaufmann (2000). They examined annual data from 1856 for 11 different time series—northern hemisphere, southern hemisphere, and global temperatures, atmospheric concentrations of CO_2, CH_4, CFC, N_2O, solar activity, stratospheric sulfates, and SO_x emissions—but the results for CO_2 and temperature are most relevant for this paper. Prior to the availability of CO_2 measurements from Mauna Loa in 1958, they used ice core data from Law Dome with missing values interpolated by cubic splines. The CO_2 is then converted to a "forcing equivalent" by taking the logarithm and multiplying by a constant. The temperature data was the HadCRUT series from the Hadley Center of the UK Met Office. As a preliminary to testing for cointegration, Stern & Kaufmann (2000) test for the degree of integration of each time series. They found CO_2 forcing to be integrated of order 2 while the temperature series was integrated of order 1.

Liu & Rodríguez (2005) examined cointegration of global temperature anomalies, CO_2, CH_4, and N_2O data from the Goddard Institute for Space Studies (GISS). They also converted the greenhouse gas concentrations to radiative forcing measures using the logarithmic transformation. The variables were again measured at the annual frequency and covered the period 1856-2001. They also found ln (CO_2) to be integrated of order 2 and temperature integrated of order 1.

Balcombe et al. (2019) use a linear sum of the greenhouse gases, sulphur dioxides, and solar components as an aggregate forcing measure. They found both it and the HadCRUT temperature series to be integrated of order 1. In contrast to previous authors, they used a Bayesian analysis in addition to a classical approach to examine whether the two series are cointegrated. They found that two different models were equally consistent with the evidence, with only one of them implying cointegration between the two series.

In a paper closer to this one, Gil-Alana & Monge (2020) considered fractional integration. By using a binomial expansion in L, $(1-L)^d$ for fractional $-1<d<1$ can be written as an infinite AR. A process X_t such that $(1-L)^d X_t$ is an ARMA (p,q) for fractional d is said to be *fractionally integrated* of order d and is designated ARFIMA (p,d,q). It will be stationary if $-0.5<d<0.5$. Generalizing, if $(1-L)^{d_i}(1-L)^{d-d_i}X_t$ is an ARMA(p,q) for $d_i \ge 1$ an integer and $|d-d_i|<0.5$, X_t is written as a *non-stationary* ARFIMA (p,d,q). While the autocorrelations $\operatorname{cov}(X_t, X_{t-k})$ of an ARMA (p,q) decay exponentially with lag length k, those in a stationary ARFIMA (p,d,q) decay at a hyperbolic rate. The process is said the have a long memory.

Gil-Alana & Monge (2020) analyzed annual data from 1880-2015 on global CO_2 emissions from fossil fuel burning, the logarithm of CO_2 emissions, and the HadCRUT land and combined land and ocean temperatures. In the latter case, they also examined temperatures in the northern and southern hemispheres separately, yielding a total of four temperature series. In their base specification, they found the estimated values of the degree d of fractional integration to be

1.30 and 0.97, respectively, for the unlogged and logged CO_2 emissions. The hypothesis that the log of emissions is integrated of order 1 could not be rejected. By contrast, the estimated values of d for the global temperatures ranged from 0.48 to 0.64 and were statistically significantly different from 1.

Dagsvik et al. (2020) provide a result that justifies examining fractional integration as an alternative hypothesis to the prediction from Equation (6) that T should be integrated of order 1. They consider an observation T_t of a temperature series measured at some sampling frequency indexed by t (for example, months) to be an average of m observations of an underlying stochastic process recorded at a finer "basic" timescale (for example, daily). They cite a result from Giraitis et al. (2012) that shows that if the underlying process is stationary and satisfies some regularity conditions, allowing $m \to \infty$ will produce a continuous time fractional Brownian motion process with Hurst index $0 < H < 1$. Standard Brownian motion corresponds to $H = 0.5$, while the process has long memory or persistent behavior when $H > 0.5$, and anti-persistent behavior when $H < 0.5$. The innovation in a Brownian motion when $H > 0.5$ has autocorrelations that decay at the same hyperbolic rate in lag length k as those in a stationary ARFIMA $(0, d, 0)$ with $d = H - 0.5$.

Dagsvik et al. (2020) examined 96 time series of monthly mean temperature observations from weather stations in 32 countries. They seasonally adjusted the monthly series by subtracting monthly means and dividing by monthly standard deviations. They rejected stationarity at the 5% level for 14 to 18 series. However, they rejected stationarity at the 5% level for only 1 of the annual averages of the unadjusted monthly means, perhaps suggesting that their control for seasonality was inadequate.[6] Using several estimation methods, they found the Hurst index H for the stationary series ranged from 0.63 to 0.95.

3. Tests Based on Lower Troposphere Anomalies

In contrast to the existing literature examining the relationship between CO_2 and surface temperatures, this paper analyzes satellite-based observations of temperature anomalies for the lower troposphere produced by the University of Alabama at Huntsville (UAH). In addition to the global anomaly examined in this section, the next section examines temperature anomalies for separate land versus ocean regions in five different non-overlapping latitude bands (tropics, 20˚S - 20˚N, mid-latitude regions, 20˚N - 60˚N and 20˚S - 60˚S, and the polar regions above 60˚N and below 60˚S).[7] Since the temperature series are measured as anomalies relative to a 30-year average for the same time of year, the monthly mean de-seasonalized CO_2 measurements made by the Scripps Institution of

[6]Dagsvik & Moen (2023) examined 75 time series from 32 countries, most of which updated the series used in Dagsvik et al. (2020). They rejected stationarity at the 5% level for 10 monthly and 3 annual series.

[7]The data available at http://vortex.nsstc.uah.edu/data/msu/v6.0/tlt/uahncdc_lt_6.0.txt does not separate anomalies into non-overlapping regions that fully cover the globe. I thank John Christy for modifying the algorithm to produce these.

Oceanography at Mauna Loa[8] are used for C_t.

Several tests are used to assess the order of integration of $\ln C_t$ and T_t. The Phillips-Perron unit root test (Phillips & Perron, 1988) is like a Dickey-Fuller (DF) test (Dickey & Fuller, 1979) made robust to serial correlation using the Newey & West (1987) heteroskedasticity and autocorrelation consistent covariance matrix estimator. It assumes, as a null hypothesis, that the time series has a unit root (is integrated of order 1). Elliott et al. (1996) proposed a modified unit root test (DF-GLS) that first transforms the time series via generalized least squares (GLS) regression before performing the DF test. They and subsequent authors have presented Monte Carlo evidence that DF-GLS has significantly greater power than DF. In other words, it is more likely to correctly reject the null hypothesis that the series has a unit root when it is false. The generalized Kwiatkowski, Phillips, Schmidt, and Shin ((Kwiatkowski et al., 1992), generalized by Hobijn et al. (2004) to use a quadratic spectral kernel to weight the serial dependence and automatic bandwidth selection to determine the lag truncation parameter) assumes, as a null hypothesis, that the series is stationary around a mean.

Table 1. Tests for unit root stationarity.

	Phillips-Perron tests		
variable	Z_ρ	Z_τ	KPSS test
$\ln C$	−2.923	−1.114	2.317
T	−125.230	−8.439	0.091
	Critical values		
1%	−29.500	−3.960	0.218
5%	−21.800	−3.410	0.148
10%	−18.300	−3.120	0.119
$(1-L)\ln C$	−657.532	−36.222	0.517
	Critical values		
1%	−20.700	−3.430	0.744
5%	−14.100	−2.860	0.460
10%	−11.300	−2.570	0.347

Table 1 presents the Phillips-Perron and KPSS results along with critical values at the 1%, 5% and 10% levels. **Table 2** presents the test DF-GLS results and critical values at the 1%, 5% and 10% levels with the maximum number of lags chosen via the Schwert criterion. Since $(1-L)(a+bt)=b$ and Equation (5) and Equation (6) have positive constant terms, the tests allow the time series for $\ln C$ and T to have a deterministic trend. On the other hand, the tests for $(1-L)\ln C_t$

[8]The data are available from NOAA at https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_mm_mlo.txt.

assume that it has a non-zero mean but no trend.

Table 2. DF-GLS tests for unit root stationarity.

	$\ln C$	T	Critical values		
lag [s]	DF-GLS τ	DF-GLS τ	1%	5%	10%
18	−0.516	−4.652	−3.48	−2.821	−2.538
17	−0.480	−4.790	−3.48	−2.824	−2.541
16	−0.473	−4.770	−3.48	−2.828	−2.544
15	−0.468	−5.177	−3.48	−2.832	−2.548
14	−0.420	−5.052	−3.48	−2.835	−2.551
13	−0.366	−5.250	−3.48	−2.839	−2.554
12	−0.585	−5.063	−3.48	−2.842	−2.557
11	−0.652	−5.065	−3.48	−2.845	−2.560
10	−0.584	−5.053	−3.48	−2.849	−2.563
9	−0.538	−5.332	−3.48	−2.852	−2.566
8	−0.503	−5.525	−3.48	−2.855	−2.569
7	−0.389	−5.404	−3.48	−2.858	−2.572
6	−0.345	−5.309	−3.48	−2.861	−2.574
5	−0.327	−5.289	−3.48	−2.864	−2.577
4	−0.385	−5.338	−3.48	−2.867	−2.579
3	−0.429	−5.741	−3.48	−2.869	−2.582
2	−0.526	−5.819	−3.48	−2.872	−2.584
1	−0.757	−6.177	−3.48	−2.875	−2.587

The hypothesis that $\ln C$ has a unit root is not rejected at even the 10% level (Phillips-Perron and DF-GLS tests), while the hypothesis that it is stationary around a mean is rejected at even the 1% level (KPSS test). The Phillips-Perron tests reject the null hypothesis of a unit root in $(1-L)\ln C$ at an extremely low level. The KPSS test rejects the null hypothesis that $(1-L)\ln C$ is stationary around a mean at the 5% level but not at the 1% level. It follows that $(1-L)\ln C$ is likely stationary. This conclusion is reinforced by the fact that the (unreported) autocorrelations in $(1-L)\ln C$ decline quickly with lag.

The Phillips-Perron and DF-GLS tests reject the null hypothesis that T has a unit root at a much lower than 1% level, while the KPSS test does not reject the null hypothesis that T is stationary around a mean at even the 10% level. Hence, T definitely does not have a unit root.

In summary, the results in **Table 1** and **Table 2** strongly reject the implication from Equation (5) and Equation (6) that both series should be integrated of order 1. These conclusions can be further tested by estimating the time series processes followed by $\ln C$ and T.

In an ARFIMA ($0,d,0$) model for $\ln C$, the fractional difference parameter

d has an estimated value 0.77 with an estimated Huber-White[9] robust standard error of 0.023. It is thus statistically different from both 0.5 and 1.0, implying $\ln C$ is a non-stationary fractionally integrated process. However, the estimated residuals remain strongly autocorrelated at several lags and especially at lag 1. After allowing for a moving average of order 1, the estimated value of d changes to 1.077 with an estimated robust standard error of 0.095, implying it is not statistically different from 1. Portmanteau tests that the autocorrelations in the estimated residuals from an ARIMA (0, 1, 1) are all zero produced *p*-values of 0.9934 for the first 6 lags, 0.8904 for the first 12 lags, and 0.3108 for the first 24 lags. The estimated parameter values, with corresponding estimated robust standard errors in parentheses, were as follows:

$$(1-L)\ln C_t = \underset{(0.000019)}{0.00042} + \varepsilon_t - \underset{(0.0419)}{0.4237}\,\varepsilon_{t-1} \tag{7}$$

Equation (7) implies an average seasonally adjusted growth rate of CO_2 in the atmosphere of 0.042% per month (statistically significantly different from zero at a better than 0.1% level). A deviation from that growth rate in any month is offset by more than half in the immediately following month. Growth of CO_2 then

Table 3. Test statistics for ARFIMA models of T.

Statistic	Model (8)	Model (9)
log likelihood	391.7937	391.5951
AIC	−775.5873	−773.1901
BIC	−758.4958	−751.8163
	p-values	
H_0: $d=0$		0.015
H_0: $d=0.5$	0.037	0.146
H_0: $d=1$	0.000	
Portmanteau tests:		
lag 1	0.8368	0.5867
lag 2	0.9783	0.5850
lag 3	0.7365	0.7693
lag 4	0.8546	0.6266
lag 5	0.8050	0.7034
lag 6	0.8683	0.8058
lag 7	0.9026	0.8661
lag 8	0.9460	0.9208
lag 9	0.8185	0.8256
lag 10	0.7922	0.8163
lag 11	0.8539	0.8562
lag 12	0.8959	0.9030

[9]Developed independently by Huber (1967) and White (1980, 1982) and extended by many later authors.

returns to trend absent another shock. This rapid adjustment contradicts the autoregressive adjustment process assumed in Equation (2).

Two different ARFIMA models, namely

$$(1-L)^{\overset{0.5946}{(0.0454)}} T_t = \underset{(0.0006)}{0.0011} + \xi_t + \underset{(0.0491)}{0.1227}\, \xi_{t-2} \tag{8}$$

and

$$\Big(1-\underset{(0.0642)}{0.8394}\,L\Big)(1-L)^{\overset{0.3128}{(0.1287)}} T_t = -\underset{(0.1319)}{0.0752} + \xi_t - \underset{(0.0882)}{0.5299}\, \xi_{t-1} \tag{9}$$

appear to fit the T series quite well. Again, the estimated robust standard errors are in parentheses. In each case, Portmanteau tests applied to the estimated residuals revealed remaining autocorrelations were not statistically significantly different from zero.

Table 3 presents various test statistics relating to these two models. The tests mostly suggest that the non-stationary model (8) is superior to the stationary model (9). Nevertheless, either (8) or (9) confirm the conclusion from the tests in Table 1 that $\ln C$ and T have different levels of integration. This is also a strong rejection of the assumption that T and $\ln C$ are cointegrated as in Equation (1).

4. Temperatures for Geographic Subdivisions

The forcing-feedback model implies that regional temperatures also should have the same order of integration as $\ln C$. This can be tested using geographic subsets of tropospheric temperature anomalies over land versus ocean in five non-overlapping latitude bands. Table 4 gives the Phillips-Perron unit root and generalized KPSS test statistics in each region. The null is again that the series follows a random walk with or without drift for the Phillips-Perron tests, and is stationary around a trend for the KPSS test. The critical values are the same as in the top half of Table 1.

Table 4. Stationarity tests for regional temperatures.

	Phillips-Perron tests		
variable	Z_ρ	Z_τ	KPSS test
Tropics Land	−119.676	−8.192	0.047
Tropics Ocean	−76.818	−6.387	0.057
N. Mid-Lat Land	−363.801	−15.851	0.091
N. Mid-Lat Ocean	−354.861	−15.374	0.220
S. Mid-Lat Land	−414.669	−17.343	0.088
S. Mid-Lat Ocean	−361.935	−15.634	0.103
N. Polar Land	−469.804	−19.055	0.113
N. Polar Ocean	−433.233	−18.255	0.150
S. Polar Land	−376.669	−16.692	0.030
S. Polar Ocean	−404.758	−17.350	0.075

Comparing the Phillips-Perron results in **Table 4** with those for the global temperature anomaly in **Table 1**, the rejection of the null hypothesis of a unit root is even stronger for the regional temperatures than for the global series. The KPSS test for stationarity around a trend is rejected for N. Mid-Lat Ocean at the 1% level and N. Polar Ocean at the 5% level. For the remaining regions, the Phillips-Perron and KPSS results together imply that the series are unambiguously stationary. Perhaps temperatures over northern hemisphere oceans are influenced by long-term cycles in ocean currents that appear non-stationary over the time span of the data. However, another interpretation is that these series are fractionally integrated with a value of d close to 0.5, leading to the KPSS test rejections.

In all regions except the tropics, attempting to estimate an ARFIMA ($0, d, 0$) model with $d > 0.5$ failed as the estimating algorithm produced a value for d that tended toward 0.5 without the convergence criteria ever being satisfied. Allowing $d < 0.5$ then yielded a maximizing solution in each case. Once a suitable ARFIMA ($0, d, 0$) model was found, the residuals were examined to choose a suitable ARMA short-run adjustment process and the ARFIMA (p, d, q) was then estimated. The procedure was repeated until the residuals passed Portmanteau tests for an absence of autocorrelation at all lags out to 40 months. In the two south polar regions, the hypothesis that $d = 0$ could not be rejected once the ARMA dynamics were estimated. Setting $d = 0$ then produced ARMA models with both AIC and BIC lower than in the best ARFIMA model.

For land areas in the tropics, a model with $d > 0.5$ could be estimated, but an MA (2) was required to whiten the residuals. The estimated value of d was then 0.5331 with a robust standard error of 0.0894. A test of $d = 0.5$ gave a *p*-value of 0.711. Allowing $d < 0.5$ required an ARMA (1, 1) to whiten the residuals. The estimated value of d was then 0.2379 with a robust standard error of 0.1088. The hypothesis that $d = 0.5$ was rejected with a *p*-value of 0.016. Both the AIC and BIC values were lower for the stationary model. The Portmanteau tests for non-autocorrelated residuals also implied that the autocorrelations are more likely to be zero for the stationary model, especially at lags in excess of 24 months.

For ocean areas in the tropics, a model with $d = 0.7610$ and a robust standard error of 0.0527 could be estimated, but a moving average with some large lags was required to whiten the residuals.[10] This suggests that allowing $d > 0.5$ resulted in over-differencing. Yet allowing $d < 0.5$ produced an estimated value of $d = 0.4985$ with a standard error of 0.0002, which also suggests the series could be non-stationary. The residuals from this model were highly autocorrelated, however, and after allowing for an ARMA (1, 2) model of the error term, the estimated value for $d = 0.0622$ with a robust standard error of 0.1253, which is not significantly different from zero at even the 50% level. Setting $d = 0$, the estimated residuals could still be whitened with an ARMA (1, 2) model. Both the AIC and BIC were lower for the ARIMA (1, 0, 2) model than for the non-

[10]Coefficients statistically significantly different from zero were found at lag 2 (0.186, s.e. 0.065), lag 24 (−0.104, s.e. 0.043), lag 25 (−0.108, s.e. 0.039) and lag 34 (0.132, s.e. 0.044).

stationary ARFIMA model. The Portmanteau tests for non-autocorrelated residuals also implied that the autocorrelations are much more likely to be zero for the stationary model at all lags up to 24 months and most lags up to 36 months.

Table 5 reports the preferred model for each region (with estimated robust standard errors in parentheses). Consistent with the KPSS test results in Table 4, the evidence that the two tropical and two south polar series are stationary is especially strong. The estimated d is statistically identical in the northern mid-latitude land and ocean areas, the southern mid-latitude land area, and the north polar ocean. A value of approximately 0.44 is also close to 0.5, implying significant temperature anomalies in these regions can persist for many months. The estimated value of d is slightly lower in the southern mid-latitude ocean area, and noticeably lower in the north polar land and tropics land areas. Although the temperature anomalies in the tropics ocean region are not fractionally integrated, they have the largest autoregressive coefficient, which will also lead to relatively long adjustment lags. The relatively rapid adjustment of south polar temperature anomalies indicates they are more independent of temperatures in the other regions.

Table 5. ARFIMA/ARIMA models for regional temperatures.

Series	constant	d	AR (1)	MA (1)	MA (2)
Tropics Land	−0.0998 (0.1202)	0.2379 (0.1088)	0.7900 (0.0692)	−0.3233 (0.0799)	
Tropics Ocean	−0.0861 (0.0561)		0.8711 (0.0271)	−0.0893 (0.0495)	0.1980 (0.0589)
N. Mid-Lat Land	−0.0582 (0.2954)	0.4482 (0.0532)	0.5868 (0.1488)	−0.6827 (0.1395)	
N. Mid-Lat Ocean	−0.0381 (0.2554)	0.4433 (0.0472)		−0.1093 (0.0675)	
S. Mid-Lat Land	−0.0995 (0.2878)	0.4316 (0.0698)	0.4823 (0.1527)	−0.6387 (0.1585)	
S. Mid-Lat Ocean	−0.0364 (0.1214)	0.3737 (0.0313)			
N. Polar Land	−0.1074 (0.1400)	0.2569 (0.0300)			
N. Polar Ocean	−0.1104 (0.3734)	0.4440 (0.0515)	0.5730 (0.1068)	−0.7689 (0.0922)	
S. Polar Land	−0.0481 (0.0644)		0.3239 (0.0408)		
S. Polar Ocean	−0.0082 (0.0315)		0.5581 (0.1167)	−0.2944 (0.1401)	

Inter-regional interactions between temperature anomalies can be investigated by stacking the series into a 10 × 1 vector T_t and estimating a vector autoregression (VAR). In a k-th order VAR, a vector of variables depends on k lags of every variable in the vector:

$$T_t = \mu + \sum_{i=1}^{k} A_i T_{t-i} + u_t \tag{10}$$

The 10 × 1 vector μ and the 10 × 10 matrices A_i are parameters to be estimated. The 10-dimensional multivariate process u_t is not autocorrelated, but can have non-zero contemporaneous covariances between the different components of u_t.

Information criteria statistics suggested that the optimal k is either 1 (BIC) or 2 (AIC). An optimal VAR lag of only two months appears to contradict the univariate ARFIMA analysis, which implied that all temperature anomalies, except those in the tropical ocean and two southern polar regions, are fractionally integrated. In the VAR system, however, feedback operating via the matrices A_i transmits anomalies from one region across the remaining regions and then back to the originating region. These interactions can significantly prolong the persistence of any anomaly. Formally, Equation (10) with $k = 2$ can be written in terms of a matrix polynomial operator as

$$\mathcal{A}(L)T_t \equiv \left(I - A_1 L - A_2 L^2\right)T_t = \mu + u_t \tag{11}$$

Inverting the matrix $\mathcal{A}(L)$, where $\mathcal{A}(L)^{-1} \equiv \det\left(\mathcal{A}(L)\right)^{-1} \operatorname{adj}\left(\mathcal{A}(L)\right)$ yields

$$\det\left(\mathcal{A}(L)\right)T_t = \operatorname{adj}\left(\mathcal{A}(L)\right)\mu + \operatorname{adj}\left(\mathcal{A}(L)\right)u_t \tag{12}$$

where the determinant $\det\left(\mathcal{A}(L)\right)$ will be a polynomial in L of degree 20.

Table 6 gives summary statistics for the 10 equations in the VAR with $k = 2$ listed in order of the R^2. The latter indicates the proportion of variability in each temperature series that can be explained by lagged values of all the series. This again suggests that the south polar temperatures are more independent of temperatures in the other regions.

Table 6. VAR summary statistics.

Dependent variable (variable name)	R^2	H0: $A_1 \equiv 0$ χ^2_{10} (p-value)	H0: $A_2 \equiv 0$ χ^2_{10} (p-value)
Tropics Ocean (TO)	0.8013	284.72 (0.000)	35.62 (0.000)
Tropics Land (TL)	0.7614	227.99 (0.000)	14.97 (0.133)
N. Mid-Lat Ocean (NMO)	0.5404	66.46 (0.000)	45.14 (0.000)
N. Mid-Lat Land (NML)	0.4907	93.94 (0.000)	15.59 (0.112)
S. Mid-Lat Ocean (SMO)	0.4824	87.01 (0.000)	21.45 (0.018)
S. Mid-Lat Land (SML)	0.4093	61.15 (0.000)	6.31 (0.789)
N. Polar Ocean (NPO)	0.3308	51.10 (0.000)	22.90 (0.011)
N. Polar Land (NPL)	0.3025	42.50 (0.000)	32.36 (0.000)
S. Polar Land (SPL)	0.1353	55.60 (0.000)	8.12 (0.617)
S. Polar Ocean (SPO)	0.1219	44.40 (0.000)	10.45 (0.402)

Table 7 gives the estimated VAR coefficients (with robust standard errors in parentheses below each coefficient). The largest coefficients in Table 7 tend to be on the main diagonal. The implication is that anomalies in a given geographic region tend to respond more to lagged values of anomalies in the same region than to anomalies in other regions. Anomalies over land or ocean in a given latitude band tend to respond strongly to anomalies over ocean or land (respectively) in the same latitude band.

Table 7. VAR estimated coefficients.

Dependent variable:	*TO*	*TL*	*NMO*	*NML*	*SMO*	*SML*	*NPO*	*NPL*	*SPL*	*SPO*
TO_{t-1}	0.7723** (0.0712)	0.4032** (0.0797)	0.0243 (0.0949)	0.0835 (0.1593)	−0.0892 (0.0854)	−0.1919 (0.1515)	0.2832 (0.3038)	0.8943** (0.2784)	0.3294 (0.4654)	0.2831 (0.1906)
TO_{t-2}	0.3581** (0.0713)	0.0637 (0.0778)	−0.0254 (0.0982)	−0.1146 (0.1459)	−0.0142 (0.0908)	0.2161 (0.1638)	−0.3936 (0.2515)	−0.5416* (0.2347)	−0.2163 (0.5103)	0.0600 (0.1972)
TL_{t-1}	−0.0134 (0.0612)	0.3042** (0.0678)	0.0291 (0.0766)	0.0537 (0.1230)	0.1322 (0.0759)	0.4305** (0.1358)	−0.0568 (0.2640)	−0.5416* (0.2349)	−0.0934 (0.4444)	−0.1725 (0.1850)
TL_{t-2}	−0.2463** (0.0617)	0.0841 (0.0692)	0.1260 (0.0830)	0.0536 (0.1259)	0.1438 (0.0797)	−0.0448 (0.1453)	0.3858 (0.2312)	0.2564 (0.2214)	0.0224 (0.4366)	−0.0612 (0.1717)
NMO_{t-1}	−0.0312 (0.0336)	0.0307 (0.0382)	0.2363** (0.0485)	0.2400** ** (0.0841)	0.0523 (0.0463)	0.0714 (0.0705)	−0.0370 (0.1506)	0.2864* (0.1317)	0.0055 (0.2379)	−0.0381 (0.1085)
NMO_{t-2}	0.0293 (0.0339)	−0.0430 (0.0413)	0.1246* (0.0505)	−0.0053 (0.0744)	−0.0830 (0.0445)	0.0119 (0.0771)	−0.0111 (0.1381)	0.0511 (0.1243)	0.2435 (0.2395)	0.0624 (0.1103)
NML_{t-1}	−0.0011 (0.0239)	−0.0063 (0.0269)	0.0717 (0.0376)	0.2593** (0.0570)	−0.0274 (0.0319)	0.0143 (0.0541)	0.1559 (0.1099)	0.1005 (0.0987)	0.1312 (0.1673)	−0.0127 (0.0792)
NML_{t-2}	0.0231 (0.0236)	0.0356 (0.0279)	0.0355 (0.0367)	0.1120* (0.0533)	0.0609 (0.0327)	0.0406 (0.0525)	0.1921 (0.0999)	0.3072** (0.0943)	−0.1640 (0.1612)	−0.0483 (0.0766)
SMO_{t-1}	−0.0543 (0.0408)	0.0070 (0.0461)	−0.0157 (0.0541)	0.1497 (0.0825)	0.2658** (0.0485)	−0.0806 (0.0880)	0.2034 (0.1440)	0.0539 (0.1391)	0.3461 (0.2690)	0.2201 (0.1281)
SMO_{t-2}	−0.0267 (0.0368)	−0.0379 (0.0448)	0.1924** (0.0500)	0.0854 (0.0797)	0.0866 (0.0495)	−0.0381 (0.0878)	0.0899 (0.1616)	−0.0144 (0.1509)	−0.0827 (0.2439)	−0.0163 (0.1152)
SML_{t-1}	0.0072 (0.0214)	0.0283 (0.0248)	0.0699* (0.0310)	−0.0377 (0.0494)	0.0459 (0.0298)	0.2212** (0.0493)	−0.0037 (0.0918)	−0.0514 (0.0906)	−0.1023 (0.1530)	−0.1250 (0.0655)
SML_{t-2}	0.0184 (0.0214)	0.0105 (0.0249)	−0.0310 (0.0323)	0.0461 (0.0494)	0.0231 (0.0305)	0.0246 (0.0514)	0.0445 (0.0897)	0.1002 (0.0815)	0.0955 (0.1499)	0.0460 (0.0648)
NPO_{t-1}	0.0103 (0.0140)	0.0048 (0.0157)	0.0281 (0.0204)	0.0834** (0.0296)	0.0150 (0.0179)	0.0207 (0.0296)	0.1875** (0.0638)	0.0566 (0.0545)	−0.0569 (0.0963)	−0.0215 (0.0436)
NPO_{t-2}	0.0186 (0.0139)	0.0188 (0.0169)	−0.0038 (0.0196)	0.0451 (0.0313)	0.0017 (0.0179)	0.0230 (0.0302)	−0.0048 (0.0582)	−0.0394 (0.0568)	−0.0827 (0.0920)	−0.0407 (0.0394)
NPL_{t-1}	0.0042 (0.0144)	0.0307 (0.0162)	−0.0065 (0.0204)	0.0040 (0.0313)	0.0283 (0.0203)	0.0700* (0.0324)	0.0546 (0.0623	0.1211* (0.0577)	0.0053 (0.0909)	−0.0206 (0.0415)
NPL_{t-2}	−0.0167 (0.0140)	−0.0031 (0.0167)	0.0166 (0.0220)	−0.0039 (0.0344)	0.0130 (0.0168)	−0.0194 (0.0302)	0.1431* (0.0580)	0.1274* (0.0557)	0.2199* (0.0958)	0.0963* (0.0415)
SPL_{t-1}	0.0058 (0.0082)	0.0077 (0.0101)	−0.0033 (0.0115)	0.0112 (0.0178)	0.0295** (0.0112)	0.0490* (0.0216)	0.0426 (0.0346)	0.0054 (0.0309)	0.2820** (0.0592)	0.0499 (0.0267)
SPL_{t-2}	0.0180* (0.0086)	0.0128 (0.0099)	−0.0057 (0.0115)	−0.0164 (0.0178)	−0.0005 (0.0112)	0.0077 (0.0194)	−0.0031 (0.0345)	0.0119 (0.0328)	0.0260 (0.0595)	−0.0333 (0.0285)
SPO_{t-1}	−0.0028 (0.0184)	−0.0163 (0.0209)	−0.0125 (0.0258)	−0.0776 (0.0407)	−0.0854** (0.0237)	−0.0894* (0.0443)	0.0940 (0.0777)	−0.0037 (0.0720)	0.1032 (0.1295)	0.1938** (0.0618)
SPO_{t-2}	−0.0399* (0.0195)	−0.0291 (0.0216)	0.0187 (0.0279)	0.0682 (0.0425)	−0.0035 (0.0245)	−0.0188 (0.0418)	−0.0157 (0.0760)	0.0095 (0.0724)	0.0143 (0.1339)	0.1133 (0.0590)
constant	−0.0086 (0.0068)	−0.0065 (0.0077)	0.0046 (0.0092)	0.0018 (0.0140)	0.0053 (0.0080)	−0.0171 (0.0132)	−0.0233 (0.0264)	−0.0190 (0.0245)	−0.0017 (0.0442)	0.0004 (0.0207)

**Statistically significantly different from zero at the 1% level; *Statistically significantly different from zero at the 5% level.

The largest off-diagonal effects tend to occur in the mid-latitude and north polar regions. Ocean regions tend to interact more across latitude bands. Temperature departures in tropical oceans exhibit the strongest autocorrelation. These dynamics could reflect a prominent role for ocean currents in slowly transmitting temperature anomalies across regions (atmospheric pressure systems would transmit anomalies in less than a month).

The critical question for the present analysis is whether the estimated model implies the dynamic response of the regional temperatures to shocks is stationary. In the case of the VAR, a stronger condition,[11] known as stability, requires that all eigenvalues of the companion matrix have a modulus less than 1. The companion matrix is obtained by rewriting Equation (10) as a first-order vector autoregression after defining a new vector X_t and new coefficient matrices M and A to yield:

$$X_t \equiv \begin{bmatrix} T_t \\ T_{t-1} \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \end{bmatrix} + \begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix} \begin{bmatrix} T_{t-1} \\ T_{t-2} \end{bmatrix} + \begin{bmatrix} u_t \\ 0 \end{bmatrix} \equiv M + AX_{t-1} + U_t \tag{13}$$

where A is the companion matrix with $A = P\Lambda P^{-1}$ for Λ a diagonal matrix with the eigenvalues of A on the leading diagonal. Then $A^j = P\Lambda^j P^{-1} \to 0$ when the eigenvalues are all smaller than 1 in modulus. For the parameter estimates in Table 7, the eigenvalues of the companion matrix are given in Table 8. The results imply that the temperature series are jointly stationary.

Table 8. VAR eigenvalues.

Eigenvalue	Modulus
$0.8979 \pm 0.0567i$	0.8997
$0.4980 \pm 0.1195i$	0.5121
$0.5031 \pm 0.0578i$	0.5064
$0.3380 \pm 0.2373i$	0.4130
$-0.3190 \pm 0.1914i$	0.3720
$0.2889 \pm 0.0225i$	0.2897
$-0.0904 \pm 0.2147i$	0.2329
$-0.2067 \pm 0.0426i$	0.2110
$-0.1376 \pm 0.0827i$	0.1605
−0.4080	0.4080
−0.2932	0.2932

5. Discussion of the Results

The literature review at the end of Section 2 noted that previous papers have tended to find global surface temperature anomalies to be integrated of order 1.

[11]For example, Proposition 2.1 in Lütkepohl (1991) asserts that a stable VAR is stationary, but a stationary VAR need not be stable.

Most of these papers considered only integer values of d. Gil-Alana & Monge (2020) allowed for fractional integration and estimated values of d ranging from 0.48 to 0.60, which exceed the values found in this paper and are non-stationary for $d > 0.5$. Dagsvik et al. (2020) and Dagsvik & Moen (2023) found that the vast majority of monthly temperatures from many individual city weather stations they examined were stationary, but fractionally integrated.

Studies using surface temperatures cover a much longer period, for example, starting in 1856 in the case of Stern & Kaufmann (2000) and Stern & Kaufmann (2000), and 1880 in the case of Gil-Alana & Monge (2020). On the one hand, a longer time period is better for detecting long-term trends in time series. On the other hand, these long-term temperature series are compiled from records using different measuring instruments and techniques. It is by no means obvious that it is reasonable to treat them as a single time series. Theorem 1 in Dagsvik et al. (2020) shows that a weighted sum of two independent fractional Brownian motion processes W_1 and W_2 with Hurst indexes H_1 and H_2 respectively, with $H_1 \geq H_2$, will converge weakly toward a fractional Brownian motion process with Hurst index H_1. If measured temperature series are a combination of a "true" underlying temperature series and a measurement error that has a higher Hurst index, the measured series will appear closer to non-stationary than the underlying true series.

Another potential problem is that the land portion of the surface temperature data sets has been compiled from temperature records dominated by urban areas. Increases in population and economic growth could then directly increase measured temperatures, imparting a component of the integrated variable Z_t in Equation (2) into the error term u_t in Equation (1). The data processing techniques used to derive the main surface temperature data sets are aimed at eliminating such non-climatic influences on temperatures, but several lines of evidence suggest they may not be completely successful.

McKitrick & Michaels (2004) estimated deterministic time trends in monthly surface temperatures from 1979-2000 recorded at 218 sites in 93 countries spread across 7 continents used in compiling the GISS surface temperature series. They also estimated deterministic trends in the 5 × 5 gridded HadCRUT surface temperature anomalies encompassing the same 218 sites. They then regressed these time trends on variables reflecting climatic factors, the population of the city, town, or rural area where the thermometer is located, national economic growth, coal use (as a proxy for local sulfate aerosol pollution), and literacy, among other factors. They concluded that the surface temperature measures they examined are significantly affected by non-climatic influences. The statistically significant influences differ across high and low income countries (defined by per capita income), but some non-climatic influences are found significant in each group. The effects persisted after allowing for separate cold-season and warm-season trends, the removal of outliers, and performing other robustness checks. In a follow-up paper, McKitrick & Michaels (2007) examined deterministic time trends in the monthly temperature anomalies in 440 land-based 5 × 5 grid cells from the HadCRUT data set over the period 1979:1-2002:12. The time period selected allowed them to include as a regressor the time trend of the UAH temperatures in the lower troposphere in the same grid cell. They strongly rejected the hypothesis that the HadCRUT temperature trends are independent of socioeconomic variables. The conclusion again survived a number of robustness checks.

Christy & McNider (2017) compared changes in the vertical profile of temperature anomalies predicted by 25 climate models with observations from satellites, radiosondes, and surface measurements. Compared to the model results, the trend in surface temperatures was too high relative to the trends at higher altitudes.

Finally, the UAH temperature anomalies for the contiguous 48 United States can be compared with surface temperature anomalies for the same region compiled by Berkeley Earth Surface Temperatures (BEST) and the USCRN data from the National Centers for Environmental Information at NOAA. The BEST data, like the GISS and HadCRUT data, are compiled from daily maximum and minimum temperatures at mostly urban weather stations. The USCRN data is derived from a set of weather stations placed in long-term sites protected from land-use changes. The USCRN stations use high-quality, calibrated, and well-maintained instruments to provide a reliable series of continuously measured observations. The correlation between the UAH and USCRN monthly temperature measurements from the start of the USCRN data in 2005 to the end of our UAH data set was 0.8816. By comparison, the correlation between the UAH and BEST measurements over the same period was 0.8272. Regressing the UAH data on the other two series yields

$$\text{UAH}_t = \underset{(0.057)}{0.069} + \underset{(0.071)}{0.636}\,\text{USCRN}_t - \underset{(0.068)}{0.060}\,\text{BEST}_t \tag{14}$$

The insignificant and negative coefficient on BEST once USCRN is included in the regression implies USCRN is much more closely related to UAH. In addition, although the time period may be too short to yield robust estimates, separate ARFIMA models were estimated for these three series. After whitening the residuals with ARMA models (not reported), the degree of fractional integration in the UAH data, namely $d = 0.2068$ with a robust standard error 0.0476, did not differ significantly from the estimate $d = 0.2146$ with a robust standard error 0.0533 found for the USCRN data. By contrast, the degree of fractional integration in the BEST data was estimated to be $d = 0.3145$ with a robust standard error of 0.0930.

In summary, non-climatic influences likely affect the widely used globally averaged ground temperature series, which many have found to be non-stationary. The result that lower tropospheric temperatures are stationary therefore likely provides a better guide to how temperatures in the bulk of the atmosphere have evolved over the last 45 years.

6. Concluding Remarks

Time series analysis of atmospheric CO_2 concentration and lower troposphere temperature data reveals that a simple forcing-feedback model with constant impact and feedback coefficients cannot explain the UAH tropospheric temperature anomalies T. The logarithm of the stock of CO_2 in the atmosphere, $\ln C$, has a unit root, and its first difference has a positive mean. Thus, it is unambiguously trending over time. By contrast, the hypothesis that T has a unit root is soundly rejected by a wide range of tests. Evidently, changes in $\ln C$ must be associated with another trending process X, such as concomitant emissions of sulfate aerosols or induced changes in cloud cover, that modifies the warming effects of CO_2 in such a way as to prevent T from also having a unit root. Cointegration of $\ln C$ and X would then allow T to depend on a linear combination of $\ln C$ and X.

Characterizing the processes followed by two time series is a weak test of models of the interaction between them in so far as many different structural models can yield the same reduced form. A corresponding strength of time series analysis, however, is that it can reveal that a wide class of structural models is inconsistent with the evidence. In this case, the evidence of different orders of integration of atmospheric CO_2 accumulation and temperature anomalies contradicts proportionality between the logarithm of CO_2 accumulation in the atmosphere and lower tropospheric temperature anomalies.

Conflicts of Interest

I declare no potential conflict of interest and declare that no external funding was received to produce this research. I thank an anonymous referee for very valuable comments.

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